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Robust H_∞ Filtering for Continuous Time-Varying Uncertain Systems with Adaptive Mechanism

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***Abstract*— This paper studies the problem of designing robust H_∞ filters for linear uncertain systems. The uncertainty parameters are assumed to be time-varying, unknown, but bounded, which appear affinely in the matrices of system models. An adaptive mechanism is introduced to construct novel filters with variable gains, which can reduce the conservativeness of traditional robust H_∞ filters. The proposed adaptive filter design conditions are given in terms of linear matrix inequalities (LMIs) to guarantee the asymptotically stability and optimize H_∞ performances of the resulting closed-loop systems. A numerical example is presented to illustrate the effectiveness of the proposed strategy.**

Key words: Robust filtering; H_∞ filtering; time-varying uncertainty; linear matrix inequalities (LMIs); indirect adaptive method.

I. INTRODUCTION

It is well-known that the Kalman filter is no longer applicable when a priori information on the external noise is not precise known. In such cases, H_∞ filtering was introduced in [3], which is to make the worse case H_∞ norm from the process noise to the estimation error minimized. Comparing with H_2 filtering, the advantages of H_∞ filtering approach are twofold. First, the assumption of boundness of the noise variance is loosen. Second, the H_∞ filter tends to be more robust when there exist additional uncertainties in systems, such as quantization errors, delays and unmodeled dynamics [14]. Since modelling error and system uncertainties often exist in the plant model, much works has been done to the design of robust H_∞ filters [4], [6], [9], [11]-[13], [15]-[20].

Norm bounded uncertainty is a particular uncertainty representation where the mathematical model of the uncertain system explicitly exhibits a nominal model located at the center of the hyper ellipsoid of uncertainty in the parameter space. Some of the above-mentioned results deal with the so-called norm-bounded uncertainty by means of the Riccati-like approaches [4], [11], [13], [17] and [18]. The norm-bounded uncertainty is converted into some scaling parameters, which significantly simplify the robust H_∞ filtering problems and make it possible to use the standard H_∞ filtering results. However, the introduction of the scaling parameters make the resulting conditions difficult to solve. Further, the norm-bounded uncertainty assumptions is somewhat conservative in many applications [7] and [10].

Another uncertain representation is convex polytopic uncertainty [7], which represents an uncertain domain more precisely than the norm-bounded uncertainty and causes no conservatism for a particular structure. In the past few years, robust H_∞ filtering problem for systems with polytopic-type parameter uncertainty has been treated based on parameter-independent Lyapunov function [9], [20] or parameter-dependent Lyapunov function [6], [12], [19] using LMI methodologies, which are computationally simple and numerically reliable for solving convex optimization problems [1] and [2]. In fact, parameter-dependent Lyapunov method can reduce conservativeness compared with parameter-independent one when the uncertain parameters are time-invariant. Also parameter-dependent Lyapunov method can include the traditional quadratic stability approach as a special case if the time-varying parameters and their rate of variation are assumed to belong to a given convex-bounded polyhedral domain. However, while the uncertain parameters is time-varying and the bound of its derivative is unknown, only the parameter-independent Lyapunov function method can be applied.

This paper develops a new robust H_∞ filter design method for linear uncertain continuous-time systems. The uncertainty parameters are assumed to be time-varying, unknown, but bounded, which appear affinely in the matrices of system models. Apart from using traditional filters with fixed filter parameters, the proposed filter parameters are adjustable based on the introduced indirect adaptive mechanism [8]. The derived filter design conditions are given in terms of LMIs, which can reduce conservativeness compared with the corresponding conditions of traditional robust H_∞ filters. The potential of the method is demonstrated by a numerical example that illustrates the H_∞ performance improvement.

This paper is organized as follows. Section 2 introduces the problem and some preliminaries. It is followed by the adaptive robust H_∞ filter design method in Section 3. An illustrative example is given in Section 4 to demonstrate the proposed method. Finally, Section 5 concludes the paper.

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II. Problem Statement and Preliminaries

A. Problem Statement

Consider a linear uncertain model described by

$$\begin{aligned}\dot{x}(t) &= A(\delta(t))x(t) + B_\omega \omega(t)\\ z(t) &= C_1 x(t)\\ y(t) &= C_2 x(t) + D_{21}\omega(t)\end{aligned} \tag{1}$$

where $x(t) \in R^n$ is the state, $y(t) \in R^p$ is the measured output and $z(t) \in R^q$ is the output signal vector to be estimated, respectively. $\omega(t) \in R^v$ is the exogenous disturbance in $L_2[0,\infty)$. And

$$A(\delta(t)) = A_0 + \sum_{i=1}^{N_0} \delta_i(t) A_i,$$

$A_0, A_i, B_\omega, C_1, C_2$ and D_{21} are known constant matrices of appropriate dimensions. $\delta_i(t)(i = 1\cdots N_0)$ are unknown time-varying uncertainty, which satisfy $\underline{\delta}_i \leq \delta_i(t) \leq \bar{\delta}_i$. Here $\underline{\delta}_i$ and $\bar{\delta}_i$ are known lower and upper bounds of $\delta_i(t)$, respectively. Since $C_2 \in R^{p\times n}$ and $\mathrm{rank}(C_2) = p_1 \leq p$, then there exists a matrix $T_c \in R^{p_1 \times p}$ such that $\mathrm{rank}(T_c C_2) = p_1$. Furthermore, there exists a matrix C_{cn} such that rank $\begin{bmatrix} T_c C_2 \\ C_{cn}\end{bmatrix} = n$. Denote $T_{cn} = \begin{bmatrix} T_c C_2 \\ C_{cn}\end{bmatrix}^{-1}$.
Throughout this paper, we make the following assumption.
Assumption 1: System (1) is asymptotically stable.
For traditional robust filtering, the following form filter is usually used.

$$\begin{aligned}\dot{\xi}_1(t) &= A_{Ff}\xi_1(t) + B_{Ff} y(t)\\ z_{F1}(t) &= C_{Ff}\xi_1(t)\end{aligned} \tag{2}$$

where $A_{Ff} \in R^{n\times n}$, $B_{Ff} \in R^{n\times p}$ and $C_{Ff} \in R^{q\times n}$ are the filter parameter matrices to be designed. Here, we assume that the filter is of the same order as the system model.
Then combing (2) with (1), it follows

$$\begin{aligned}\dot{x}_{ef}(t) &= A_{ef}x_{ef}(t) + B_{ef}\omega(t)\\ z_{ef}(t) &= C_{ef}x_{ef}(t)\end{aligned} \tag{3}$$

where $x_{ef}(t) = [x^T(t)\ \xi_1^T(t)]^T$ is the state estimation error, $z_{ef}(t) = z(t) - z_{F1}(t)$ is the estimated error and

$$A_{ef} = \begin{bmatrix} A(\delta) & 0 \\ B_{Ff}C_2 & A_{Ff}\end{bmatrix},\quad B_e = \begin{bmatrix} B_\omega \\ B_{Ff}D_{21}\end{bmatrix},$$
$$C_{ef} = [C_1 \quad -C_{Ff}].$$

In this paper, the following adaptive robust filter with variable gains is considered.

$$\begin{aligned}\dot{\xi}(t) &= A_F(\hat{\delta}(t))\xi(t) + B_F(\hat{\delta}(t))y(t)\\ z_F(t) &= C_F(\hat{\delta}(t))\xi(t)\end{aligned} \tag{4}$$

where $\hat{\delta}_i(t)(i = 1\cdots N_0)$ are the estimations of $\delta_i(t)$, which are obtained according to the introduced adaptive mechanism. $A_F(\hat{\delta}) \in R^{n\times n}$, $B_F(\hat{\delta}) \in R^{n\times p}$ and $C_F(\hat{\delta}) \in R^{m\times n}$ have the following forms, that is

$$A_F(\hat{\delta}) = A_{F0} + \sum_{i=1}^{N_0}\hat{\delta}_i A_{Fi},\quad B_F(\hat{\delta}) = B_{F0} + \sum_{i=1}^{N_0}\hat{\delta}_i B_{Fi},$$

$$C_F(\hat{\delta}) = C_{F0} + \sum_{i=1}^{N_0}\hat{\delta}_i C_{Fi}$$

where A_{F0}, A_{Fi}, B_{F0}, B_{Fi}, C_{F0}, C_{Fi} are fixed parameter matrices to be designed. Here, the designed filter is of the same order as the system model.
Applying the robust filter (4) to the system (1), it follows

$$\begin{aligned}\dot{x}_e(t) &= A_e x_e(t) + B_e\omega(t)\\ z_e(t) &= C_e x_e(t)\end{aligned} \tag{5}$$

where $x_e(t) = [x^T(t)\ \xi^T(t)]^T$ is the state estimation error, $z_e(t) = z(t) - z_F(t)$ is the estimated output error. and

$$A_e = \begin{bmatrix} A(\delta) & 0 \\ B_F(\hat{\delta})C_2 & A_F(\hat{\delta})\end{bmatrix},\quad B_e = \begin{bmatrix} B_\omega \\ B_F(\hat{\delta})D_{21}\end{bmatrix},$$
$$C_e = [C_1 \quad -C_F(\hat{\delta})]$$

The adaptive robust H_∞ filtering problem associated with the system (1) is as follows: Given $\gamma > 0$, find a filter of the form (4) such the corresponding error dynamics (5) is asymptotically stable and satisfies

$$\|T_{z_e\omega}\|_\infty < \gamma,\quad x_e(0) = 0. \tag{6}$$

B. Preliminaries

The following lemma presents a condition for the system (3) to have robust H_∞ performance bound.
Lemma 1: Consider the system described by (3), and let $\gamma > 0$ be given constant. Then the following two statements are equivalent:
(i) there exist a symmetric matrix $X > 0$ and a robust filter described by (2) such that

$$A_{ef}^T X + XA_{ef} + \frac{1}{\gamma^2}XB_{ef}B_{ef}^T X + C_{ef}^T C_{ef} < 0 \tag{7}$$

holds for $\delta_i \in [\underline{\delta}_i, \bar{\delta}_i]$
(ii) there exist symmetric matrices $0 < N < Y$, and a robust filter described by (2) with $A_{Ff} = A_{Fe1}$, $B_{Ff} = B_{Fe1}$ and $C_{Ff} = C_{Fe1}$ such that

$$V_a = \begin{bmatrix} V_{11} & V_{12} & YB_\omega - NB_{Fe1}D_{21} & C_1^T \\ * & V_{22} & -NB_\omega + NB_{Fe1}D_{21} & -C_{Fe1}^T \\ * & * & -\gamma^2 I & 0 \\ * & * & * & -I\end{bmatrix} < 0 \tag{8}$$

holds for $\delta_i \in [\underline{\delta}_i, \bar{\delta}_i]$ where

$$\begin{aligned} V_{11} &= YA(\delta) - NB_{Fe1}C_2 + (YA(\delta) - NB_{Fe1}C_2)^T\\ V_{12} &= -NA_{Fe1} - A^T(\delta)N + C_2^T B_{Fe1}^T N^T\\ V_{22} &= NA_{Fe1} + (NA_{Fe1})^T\end{aligned}$$

Proof:(7) $\Longleftrightarrow$ (8). (7) holds for $X > 0$ is equivalent to that there exists

$$X = \begin{bmatrix} X_{11} & X_{12}^T \\ X_{12} & X_{22}\end{bmatrix} \tag{9}$$

with $X_{11} \in R^{n\times n}$ and X_{12} nonsingular such that

$$\begin{bmatrix} A_{ef}^T X + XA_{ef} & XB_{ef} & C_{ef}^T \\ * & -\gamma^2 I & 0 \\ * & * & -I\end{bmatrix} < 0 \tag{10}$$

holds. Let $A_{Fe1} = (X_{12}^{-1})^T X_{22} A_{Ff} X_{22}^{-1} X_{12}^T$, $B_{Fe1} = -(X_{12}^{-1})^T X_{22} B_{Ff}$, $C_{Fe1} = -C_{Ff} X_{22}^{-1} X_{12}^T$, $X_{11} = Y$ and $N = X_{12} X_{22}^{-1} X_{12}^T$. Then

$$X_a := \begin{bmatrix} I & 0 \\ 0 & -X_{12}X_{22}^{-1} \end{bmatrix} X \begin{bmatrix} I & 0 \\ 0 & -X_{12}X_{22}^{-1} \end{bmatrix}^T = \begin{bmatrix} Y & -N \\ -N & N \end{bmatrix} \tag{11}$$

$X > 0$ is equivalent to $0 < N < Y$, and (10) is equivalent to

$$V_{a0} := \begin{bmatrix} A_{ea}^T X_a + X_a A_{ea} & X_a B_{ea} & C_{ea}^T \\ * & -\gamma^2 I & 0 \\ * & * & -I \end{bmatrix} < 0 \tag{12}$$

where

$$A_{ea} = \begin{bmatrix} A(\delta) & 0 \\ B_{Fe1}C_2 & A_{Fe1} \end{bmatrix}, \quad B_{ea} = \begin{bmatrix} B_\omega \\ B_{Fe1}D_{21} \end{bmatrix}$$

$$C_{ea} = \begin{bmatrix} C_1 & -C_{Fe1} \end{bmatrix}$$

Furthermore, after some algebraic computation, $V_{a0} < 0$ can be written as $V_a < 0$ in (8). Thus, the proof is complete. □

Remark 1: It should be noted that when NA_{Ff} and NB_{Ff} are defined as new variables, conditions (8) become LMIs.

Remark 2: In fact ,the affine time-varying uncertainty considered in this paper is a special form of polytopic type time-varying uncertainty. From Lemma 1, it is easy to see the condition (8) for traditional robust filter design is equivalent to Theorem 2 in [9], where robust H_∞ filtering for polytopic time-varying uncertain system is investigated based on parameter-independent Lypunov function.

The following algorithm is given to optimize the robust H_∞ performance of the closed-loop systems (3).

Algorithm 1: Let γ denotes the robust H_∞ performance bound of the closed-loop system (3). Let $NA_{Ff} = \bar{A}_{Ff}$ and $NB_{Ff} = \bar{B}_{Ff}$.

$$\min \eta \text{ s.t. } \; 0 < N < Y \quad (8)$$

where $\eta = \gamma^2$. Then the resultant gains of robust filter (2) are $A_{Ff} = \bar{A}_{Ff} N^{-1}$, $B_{Ff} = \bar{B}_{Ff} N^{-1}$ and C_{Ff}.

III. Adaptive Robust H_∞ Filter Design

In this section, the problem of designing an adaptive robust H_∞ filter for system (1) is studied. An adaptive mechanism is introduced to reduce the conservativeness compared with traditional robust filters.

Theorem 1: The closed-loop system (5) is stable and H_∞ disturbance attenuation is no large than γ, if there exist matrices $0 < N < Y, A_{F0}, A_{Fi}, B_{F0}, B_{Fi}, C_{F0}, C_{Fi}, i = 1 \cdots N_0$ such that for $\delta_i(t), \hat{\delta}_i(t) \in [\underline{\delta}_i, \bar{\delta}_i]$ the following matrix inequalities hold:

$$\begin{bmatrix} T_1 + T_1^T & T_2 & T_3 & C_1^T \\ * & T_4 & T_5 & -C_F^T(\hat{\delta}) \\ * & * & -\gamma^2 I + T_6 & 0 \\ * & * & * & -I \end{bmatrix} < 0 \tag{13}$$

with

$$T_1 = YA(\delta) - NB_F(\delta)C_2 - \sum_{i=1}^{N_0}(\hat{\delta}_i - \delta_i) N_3^T N B_{Fi} C_2$$

$$T_2 = -NA_F(\delta) - A^T(\delta)N + C_2^T B_F^T(\delta) N - \sum_{i=1}^{N_0}(\hat{\delta}_i - \delta_i) N_3^T N A_{Fi}$$

$$T_3 = YB_\omega - NB_F(\hat{\delta})D_{21} + \sum_{i=1}^{N_0}(\hat{\delta}_i - \delta_i)[C_2^T B_{Fi}^T N N_2 + NB_{Fi}D_{21} - N_3^T N B_{Fi} D_{21}]$$

$$T_4 = NA_F(\delta) + A_F(\delta)^T N$$

$$T_5 = -NB_\omega + NB_F(\hat{\delta})D_{21} + \sum_{i=1}^{N_0}(\hat{\delta}_i - \delta_i)[A_{Fi}^T N N_2 - NB_{Fi}D_{21}]$$

$$T_6 = \sum_{i=1}^{N_0}(\hat{\delta}_i - \delta_i)[N_2^T N B_{Fi} D_{21} + (N_2^T N B_{Fi} D_{21})^T]$$

$$N_1 = T_{cn}\begin{bmatrix} T_c \\ 0 \end{bmatrix}, N_2 = T_{cn}\begin{bmatrix} T_c D_{21} \\ 0 \end{bmatrix}, N_3 = T_{cn}\begin{bmatrix} 0 \\ C_{cn} \end{bmatrix},$$

$$A_F(\delta) = A_{F0} + \sum_{i=1}^{N_0}\delta_i A_{Fi}, \quad B_F(\delta) = B_{F0} + \sum_{i=1}^{N_0}\delta_i B_{Fi}$$

and also $\hat{\delta}_i(t)$ is determined according to the adaptive law

$$\hat{\delta}_i = \begin{cases} \bar{\delta}_i, & \text{if } M_i < 0 \\ \underline{\delta}_i, & \text{if } M_i \geq 0 \end{cases}, \quad i = 1 \cdots N_0 \tag{14}$$

$$M_i = \xi^T N A_{Fi} \xi - y^T N_1^T N A_{Fi} \xi + \xi^T N B_{Fi} y - y^T N_1^T N B_{Fi} y$$

Then the filter gains of the form (4) are given by $A_{F0}, A_{Fi},$, $B_{F0}, B_{Fi}, C_{F0}, C_{Fi}, i = 1 \cdots N_0$.

Proof: Now we choose the following Lyapunov function

$$V(t) = x_e^T(t) P x_e(t).$$

Then A_e can be written as

$$A_e = A_{ea} + A_{eb}$$

where

$$A_{ea} = \begin{bmatrix} A(\delta) & 0 \\ B_F(\delta)C_2 & A_F(\delta) \end{bmatrix},$$

$$A_{eb} = \sum_{i=1}^{N_0}(\hat{\delta}_i - \delta_i)\begin{bmatrix} 0 & 0 \\ B_{Fi}C_2 & A_{Fi} \end{bmatrix}$$

Let P be of the following form,

$$P = \begin{bmatrix} Y & -N \\ -N & N \end{bmatrix}$$

with $0 < N < Y$, which implies $P > 0$. From (1), it follows

$$T_c C_2 x = T_c[y - D_{21}\omega] \tag{15}$$

Thus

$$x = T_{cn} \begin{bmatrix} T_c C_2 x \\ C_{cn} x \end{bmatrix} = N_1 y - N_2 \omega + N_3 x \qquad (16)$$

with $N_1 = T_{cn} \begin{bmatrix} T_c \\ 0 \end{bmatrix}, N_2 = T_{cn} \begin{bmatrix} T_c D_{21} \\ 0 \end{bmatrix}, N_3 = T_{cn} \begin{bmatrix} 0 \\ C_{cn} \end{bmatrix}$.

Furthermore, we have

$$PA_{ea} = \begin{bmatrix} YA(\delta) - NB_F(\delta)C_2 & -NA_F(\delta) \\ -NA(\delta) + NB_F(\delta)C_2 & NA_F(\delta) \end{bmatrix}$$

and

$$PA_{eb} = \sum_{i=1}^{N_0} (\hat{\delta}_i - \delta_i) \begin{bmatrix} -NB_{Fi}C_2 & -NA_{Fi} \\ NB_{Fi}C_2 & NA_{Fi} \end{bmatrix}$$

which follows

$$\begin{aligned} & [x^T \ \xi^T] PA_{eb} [x^T \ \xi^T]^T \\ & = \sum_{i=1}^{N_0} (\hat{\delta}_i - \delta_i) \{ -x^T NB_{Fi} C_2 x - x^T NA_{Fi} \xi \\ & \quad + \xi^T NB_{Fi} C_2 x + \xi^T NA_{Fi} \xi \} \end{aligned}$$

By (15) and (16), it is easy to see

$$\begin{aligned} -x^T NB_{Fi} C_2 x &= -x^T NB_{Fi} (y - D_{21} \omega) \\ &= -y^T N_1^T NB_{Fi} y + x^T NB_{Fi} D_{21} \omega(t) \\ &\quad + (\omega^T N_2^T - x^T N_3^T) NB_{Fi} (C_2 x + D_{21} \omega) \\ -x^T NA_{Fi} \xi &= -(y^T N_1^T - \omega^T N_2^T + x^T N_3^T) NA_{Fi} \xi \\ \xi^T NB_{Fi} C_2 x &= \xi^T NB_{Fi} (y - D_{21} \omega) \end{aligned}$$

Thus,

$$\begin{aligned} x_e^T PA_{eb} x_e &= x_e^T A_{Pe} x_e + x_e^T B_{Pe} \omega + M \\ &\quad + \sum_{i=1}^{N_0} (\hat{\delta}_i - \delta_i) \omega^T N_2^T NB_{Fi} D_{21} \omega \end{aligned}$$

where

$$\begin{aligned} A_{Pe} &= \sum_{i=1}^{N_0} (\hat{\delta}_i - \delta_i) \begin{bmatrix} -N_3^T NB_{Fi} C_2 & -N_3^T NA_{Fi} \\ 0 & 0 \end{bmatrix} \\ B_{Pe} &= \sum_{i=1}^{N_0} (\hat{\delta}_i - \delta_i) \begin{bmatrix} \Gamma_0 \\ A_{Fi}^T N N_2 - NB_{Fi} D_{21} \end{bmatrix} \\ \Gamma_0 &= C_2^T B_{Fi}^T N N_2 + NB_{Fi} D_{21} - N_3^T NB_{Fi} D_{21} \\ M &= \sum_{i=1}^{N_0} (\hat{\delta}_i - \delta_i) M_i. \end{aligned}$$

where $M_i = \xi^T NA_{Fi} \xi - y^T N_1^T NA_{Fi} \xi + \xi^T NB_{Fi} y - y^T N_1^T NB_{Fi} y$.

Then from the derivative of $V(t)$ along the closed-loop system (5), it follows

$$\begin{aligned} & \dot{V}(t) + z_e^T(t) z_e(t) - \gamma^2 \omega^T(t) \omega(t) \\ = \ & 2x_e^T P(A_e x_e + B_e \omega) + x_e^T C_e^T C_e x_e - \gamma^2(t) \omega^T(t) \omega(t) \\ = \ & 2x_e^T P(A_{ea} x_e + B_e \omega) + x_e^T C_e^T C_e x_e - \gamma^2 \omega^T(t) \omega(t) \\ & +2x_e^T A_{Pe} x_e + 2x_e^T B_{Pe} \omega + 2M \\ & +2 \sum_{i=1}^{N_0} (\hat{\delta}_i - \delta_i) \omega^T(t) N_2^T NB_{Fi} D_{21} \omega(t) \\ \leq \ & \begin{bmatrix} x_e \\ \omega \end{bmatrix}^T W_0 \begin{bmatrix} x_e \\ \omega \end{bmatrix} + 2M \end{aligned}$$

where

$$W_0 = \begin{bmatrix} \Gamma_1 + C_e^T C_e & PB_e + B_{Pe} \\ * & -\gamma^2 I + T_5. \end{bmatrix}$$

where where $\Gamma_1 = PA_{ea} + A_{Pe} + [PA_{ea} + A_{Pe}]^T$ and T_5 is defined below (13).

The design condition $\dot{V}(t) + z_e^T(t) z_e(t) - \gamma^2 \omega^T(t) \omega(t) \leq 0$ is reduced to

$$W_0 < 0 \qquad (17)$$

and

$$M \leq 0. \qquad (18)$$

Since y and ξ are available on line, the adaptive law can be chosen as (14). So (18) can be achieved.

Notice that

$$PB_e = \begin{bmatrix} YB_\omega - NB_F(\hat{\delta}) D_{21} \\ -NB_\omega + NB_F(\hat{\delta}) D_{21} \end{bmatrix}$$

It is easy to see $W_0 < 0$ is equivalent to

$$W_1 = \begin{bmatrix} \Gamma_1 & PB_e + B_{Pe} & C_e^T \\ * & -\gamma^2 I + T_5 & 0 \\ * & * & -I \end{bmatrix} < 0 \qquad (19)$$

If (13) holds, which implies $W_1 < 0$. Thus it follows $W_0 < 0$. Together with adaptive laws (14) , we can get $\dot{V}(t) \leq 0$. Furthermore, we have

$$\dot{V}(t) + z_e^T(t) z_e(t) - \gamma^2 \omega^T(t) \omega(t) \leq 0.$$

Integrate the above-mentioned inequalities from 0 to ∞ on both sides, we obtain

$$V(\infty) - V(0) + \int_0^\infty z_e(t)^T z_e(t) dt \leq \gamma^2 \int_0^\infty \omega(t)^T \omega(t) dt.$$

which implies that the H_∞ disturbance attenuation of the closed-loop system (5) is no larger than γ holds. □

Remark 3: It is easy to see when $NA_{F0}, NA_{Fi}, NB_{F0}, NB_{Fi} (i = 1 \cdots N_0)$ are defined as new variables, conditions (13) become LMIs.

For the comparison between Theorem 1 and Lemma 1, we have the following theorem

Theorem 2: If the condition in Lemma 1 holds for the closed-loop system (3) with traditional robust filter (2), then the condition in Theorem 1 holds for the closed-loop system (5) with adaptive robust filter (4).

Proof: Notice that if $V_a < 0$, then the condition in Theorem

1 is feasible with $A_{F0} = A_{Kf}, B_{F0} = B_{Kf}, C_{F0} = C_{Kf}$ and $A_{Fi} = B_{Fi} = C_{Fi} = 0, i = 1 \cdots N_0$. The proof is complete. □

Remark 4: Theorem 2 shows that the adaptive robust H_∞ filter design method given in Theorem 1 is less conservative than that given in Lemma 1 for traditional robust H_∞ filter design method.

Based on Theorem 1, the following algorithm is proposed to optimize the robust H_∞ performance of the closed-loop systems (5)

Algorithm 2: Let γ denotes the robust H_∞ performance bound of the closed-loop system (5). Let $NA_{F0} = \bar{A}_{F0}$, $NA_{Fi} = \bar{A}_{Fi}$, $NB_{F0} = \bar{B}_{F0}$.and $NB_{Fi} = \bar{B}_{Fi}$.

$$\min \eta \quad \text{s.t.} \quad 0 < N < Y \text{ and } (13),$$

where $\eta = \gamma^2$. Then the resultant gains of adaptive robust filter (4) are $A_{F0} = \bar{A}_{F0}N^{-1}$, $A_{Fi} = \bar{A}_{Fi}N^{-1}$, $B_{F0} = \bar{B}_{F0}N^{-1}$, $B_{Fi} = \bar{B}_{Fi}N^{-1}$, C_{F0} and C_{Fi}, $i = 1 \cdots N_0$.

IV. Numerical Example

Consider the following linear continuous-time system (1) with time-varying uncertainty satisfying

$$A(\delta) = \begin{bmatrix} -5 & 1 \\ 1 & -2 \end{bmatrix} + \delta_1(t) \begin{bmatrix} 1 & 0.1 \\ 0 & -0.6 \end{bmatrix} + \delta_2(t) \begin{bmatrix} 0.5 & -0.2 \\ 0.6 & 0 \end{bmatrix}$$

$$C_1 = \begin{bmatrix} 1 & 0 \\ 5 & 2 \end{bmatrix}, \; B_\omega = \begin{bmatrix} 0 & 1 \\ 0 & 2 \end{bmatrix}, \; C_2 = \begin{bmatrix} 3 & 0 \\ 1 & 0 \end{bmatrix},$$

$$D_{21} = \begin{bmatrix} 2 & 0 \\ 1 & 0 \end{bmatrix}, \; x(0) = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

with $\delta_1(t) = \cos(t)$ and $\delta_2(t) = \sin(t)$.

Using Matlab LMI tool box [5], Algorithm 1 and Algorithm 2, we get the H_∞ performance index is 2.3934 with the adaptive robust filter while that of traditional robust filter is 2.9859. Just as the theory has proved that the adaptive robust H_∞ filter design method is less conservative than traditional robust filter design method.

In order to see the effectiveness of our method more clearly, some simulation results are also given. Here the disturbance $\omega(t) = \begin{bmatrix} \omega_1(t) & \omega_2(t) \end{bmatrix}^T$ that used is

$$\omega_1(t) = \omega_2(t) = \begin{cases} 1, & 2 \le t \le 3 \text{ (seconds)} \\ 0 & \text{otherwise} \end{cases}$$

Figure 1 and Figure 2 are the responses curves of estimated output errors with adaptive robust H_∞ filter and traditional robust H_∞ filter, respectively. It is easy to see our adaptive robust H_∞ filter has more disturbance attenuation ability than that of traditional robust filter as theory has proved.

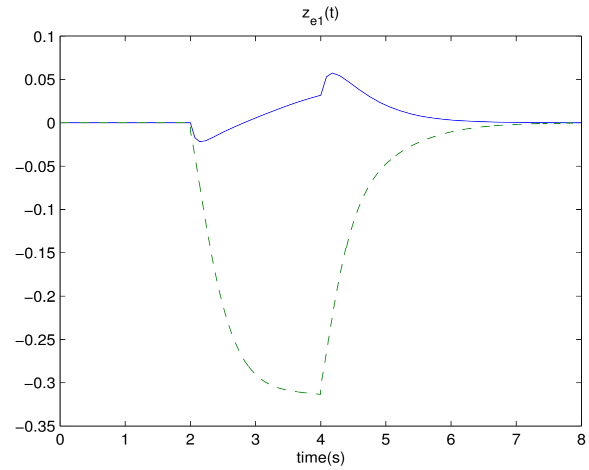

Fig. 1. The first component response of estimated output error $z_e(t)$ with adaptive robust filter (solid) and traditional robust filter (dashed).

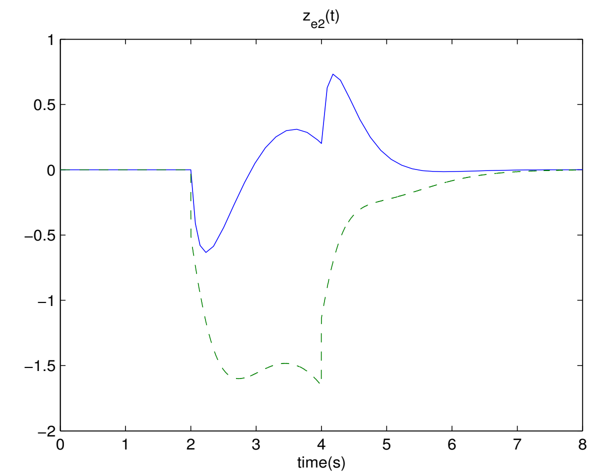

Fig. 2. The second component response of estimated output error $z_e(t)$ with adaptive robust filter (solid) and traditional robust filter (dashed).

V. Conclusions

This paper has investigated the design of robust H_∞ filters for linear uncertain continuous-time systems. The uncertainty parameters are assumed to be time-varying, unknown, but bounded, which appear affinely in the matrices of system models. An adaptive mechanism is introduced to construct robust H_∞ filters with variable gains, which can reduce the conservativeness inherent in the traditional robust H_∞ filter design with fixed gains. New robust H_∞ filter design conditions are derived in the frameworks of LMIs. The proposed method has been applied to a numerical example and exhibited superior performances as compared to the traditional robust H_∞ filter design method.

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