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P·quasi-I·D Control for MIMO Systems
—Stabilization Based on High Gain Output Feedback—

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***Abstract*— This paper is concerned with regulator problem for MIMO systems with P·quasi-I·D control. The P·quasi-I·D control is constructed by modifying the integral operation of the PID control. The purpose is to design a P · quasi-I · D controller for aysmptotic stabilization and to adjust PID parameter matrices for improving convergence speed of responces under guaranteeing the stability. In our method, we consider a certain hypothetical system derived from the closed loop system with P·quasi-I·D control in order to apply high gain output feedback. Then the PID parameter matrices are adjusted by making zero dynamics of the hypothetical system asymptotically stable and performing the high gain output feedback. The proposed method is fundamentally based on the high gain output feedback theorem. The effectiveness of the method is confirmed by simulation results for unstable MIMO plants.**

I. Introduction

PID control is widely used as a classical dynamic controller for SISO system. So there exist lots of PID parameter tuning methods for SISO system (e.g. Ziegler-Nichols method [8], C-H-R method [8] and Kitamori's method [6]). But it is often difficult to apply them to MIMO system. Although there are several researches of PID control for MIMO system, they are usually restricted to stable and/or minimum phase system. Thus, there is enough room to study in case of general MIMO system being non-minimum phase and/or unstable.

As tuning methods of PID control for MIMO system there exist several researches [1], [5], [13] based on classical control theory. Recently, several researches [3], [4], [11], [10] adopted approaches from modern control theory which is effective for analysis of MIMO system. Refs. [3], [4] try to determine PID parameter matrices by solving LMI after one formulates PID control as static output feedback for the extended system. As a method based on the static output feedback, a method by eigenvalue assignment has been proposed in [11]. Ref.[10] proposes the extended PID control of velocity type and its adjustment method by applying the high gain output feedback.

In this paper, we investigate on regulator problem of MIMO systems by a P · quasi-I · D control. It is constructed by modifying the integral operation of the usual PID control. Compared with standard PID control having 3 parameters k_P, k_I, k_D, the P · quasi-I · D control possesses 4 parameters K_P, K_I, K_D plus a new matrix D included in the quasi-I dynamics. These parameters can be designed systematically by adjusting a high gain $\mathcal{L}$ based on the high gain feedback theorem[9]. The P·quasi-I·D control has a structure different from the one proposed in [10].

In our method, we consider a certain hypothetical system related to the closed loop system with P · quasi-I · D control. Then after making zero dynamics of the hypothetical system asymptotically stable, we apply the the high gain output feedback to design a P · quasi-I · D controller aysmptotically stabilizing the closed loop system and to adjust PID parameter matrices in order to improve the convergence speed of responces under guaranteeing the stability.

More concretely, we transform the hypothetical system into the normal form and caluculate its zero-dynamics. Then we propose two methods for determining the PD parameter matrices K_P, K_D and intermediate parameter matrix H_I which stabilize the zero-dynamics. One is the simpler method using the state feedback under the order condition $2m > n$. The other is more complicated method applying an eigenvalue assignment method by the static output feedback. However, this method requires the order condition $2m+r > n$ that is weeker than the condition of the first one. Then, the I-parameter matrix K_I can be determined by multiplying the intermediate parameter matrix H_I by the high gain coefficient $\mathcal{L}$. D is also determined by multiplying $\mathcal{L}$. It is noted that the response speed can be also improved by adjusting the high gain coefficient under the assurance of stabilily of the closed-loop system.

Finally, the effectiveness of the proposed method is confirmed with various simulation results.

II. Formulation of P·quasi-I·D Control

Consider the following MIMO system:

$$\dot{\boldsymbol{x}}(t) = A\boldsymbol{x}(t) + B\boldsymbol{u}(t) \tag{1}$$

$$\boldsymbol{y}(t) = C\boldsymbol{x}(t) \tag{2}$$

where $\boldsymbol{x}(t) \in \mathbb{R}^n,\ \boldsymbol{u}(t) \in \mathbb{R}^r,\ \boldsymbol{y}(t) \in \mathbb{R}^m$ are the state vector, the input vector and the output vector, respectively.

PID control is usually given as

$$\boldsymbol{u}(t) = K_P\boldsymbol{e}(t) + K_I \int_0^t \boldsymbol{e}(\tau)d\tau + K_D\dot{\boldsymbol{e}}(t) \tag{3}$$

where $K_P, K_I, K_D \in \mathbb{R}^{r\times m}$ are PID parameter matrices. And $\boldsymbol{e}(t) = \boldsymbol{r}(t) - \boldsymbol{y}(t)$ denotes the error of output from the desired value $\boldsymbol{r}(t)$.

Since we consider here a regulator problem with $\boldsymbol{r}(t) = \boldsymbol{0}$, (3) becomes

$$\boldsymbol{u}(t) = -K_P\boldsymbol{y}(t) - K_I \int_0^t \boldsymbol{y}(\tau)d\tau - K_D\dot{\boldsymbol{y}}(t) \tag{4}$$

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In this paper, we propose the following P·quasi-I·D control

$$\boldsymbol{u}(t) = -K_P\boldsymbol{y}(t) - K_I\boldsymbol{z}(t) - K_D\dot{\boldsymbol{y}}(t) \tag{5}$$

$$\boldsymbol{z}(t) = \int_0^t (\boldsymbol{y}(\tau) - D\boldsymbol{z}(\tau))d\tau,\ \boldsymbol{z}(0) = \boldsymbol{0} \tag{6}$$

in which a feedback path with gain $D \in \mathbb{R}^{m\times m}$ is added to the integral part of (4) . Note that for convenience we call K_I the I-parameter matrix, although (6) differs from exact I-operation.

Now we define the integral parameter matrix K_I as

$$K_I \overset{\triangle}{=} H_I\mathcal{L} \tag{7}$$

where $H_I \in \mathbb{R}^{r\times m}$ and $\mathcal{L} \in \mathbb{R}^{m\times m}$ $(\det\mathcal{L} \neq 0)$ are called the intermediate parameter matrix and the adjustable parameter matrix, respectively. So P·quasi-I·D control (5) can be represented as

$$\boldsymbol{u}(t) = -K_P\boldsymbol{y}(t) + H_I\boldsymbol{z}'(t) - K_D\dot{\boldsymbol{y}}(t) \tag{8}$$

$$\text{where}\quad \boldsymbol{z}'(t) \overset{\triangle}{=} -\mathcal{L}\boldsymbol{z}(t) \tag{9}$$

Since the following relations

$$\begin{aligned} \boldsymbol{y}(t) &= C\boldsymbol{x}(t) && (10a)\\ \dot{\boldsymbol{y}}(t) &= CA\boldsymbol{x}(t) + CB\boldsymbol{u}(t) && (10b) \end{aligned}$$

are obtaind from (2), substitute them into (8) to get

$$\boldsymbol{u}(t) = H_I\boldsymbol{z}'(t) - K_PC\boldsymbol{x}(t) - K_D(CA\boldsymbol{x} + CB\boldsymbol{u}(t))$$

Furthermore, arranging this equation, we obtain

$$\begin{aligned} \boldsymbol{u}(t) &= -(I_r + K_DCB)^{-1}(K_PC + K_DCA)\boldsymbol{x}(t)\\ &\quad +(I_r + K_DCB)^{-1}H_I\boldsymbol{z}'(t)\\ &= -K_X\boldsymbol{x}(t) + K_Z\boldsymbol{z}'(t) \end{aligned} \tag{11}$$

$$\begin{aligned} \text{where } K_X &\overset{\triangle}{=} (I_r + K_DCB)^{-1}(K_PC + K_DCA)\\ K_Z &\overset{\triangle}{=} (I_r + K_DCB)^{-1}H_I \end{aligned}$$

And from (9) and (6) the time derivative of $\boldsymbol{z}'(t)$ becomes

$$\begin{aligned} \dot{\boldsymbol{z}}'(t) &= -\mathcal{L}\Big(\boldsymbol{y}(t) - D\boldsymbol{z}(t)\Big)\\ &= -\mathcal{L}\Big(C\boldsymbol{x}(t) + D\mathcal{L}^{-1}\boldsymbol{z}'(t)\Big)\\ &= -\mathcal{L}C\boldsymbol{x}(t) - \mathcal{L}D'\boldsymbol{z}'(t) \end{aligned} \tag{12}$$

$$\text{where}\quad D' \overset{\triangle}{=} D\mathcal{L}^{-1} \tag{13}$$

Accordingly, by substituting (11) into (1) and combining (12), the closed-loop system with the P·quasi-I·D control becomes

$$\begin{bmatrix} \dot{\boldsymbol{x}}(t)\\ \dot{\boldsymbol{z}}'(t) \end{bmatrix} = \begin{bmatrix} A - BK_X & BK_Z\\ -\mathcal{L}C & -\mathcal{L}D' \end{bmatrix} \begin{bmatrix} \boldsymbol{x}(t)\\ \boldsymbol{z}'(t) \end{bmatrix} \tag{14}$$

Now let us consider the following hypothetical system based on the closed-loop sytem (14) with P·quasi-I·D control:

$$\begin{aligned} \begin{bmatrix} \dot{\boldsymbol{x}}(t)\\ \dot{\boldsymbol{z}}'(t) \end{bmatrix} &= \begin{bmatrix} A - BK_X & BK_Z\\ O & O \end{bmatrix} \begin{bmatrix} \boldsymbol{x}(t)\\ \boldsymbol{z}'(t) \end{bmatrix}\\ &\quad + \begin{bmatrix} O\\ I_m \end{bmatrix}\boldsymbol{v}(t) := \widetilde{A}\widetilde{\boldsymbol{x}}(t) + \widetilde{B}\boldsymbol{v}(t) && (15)\\ \widetilde{\boldsymbol{y}}(t) &= \begin{bmatrix} C & D' \end{bmatrix}\begin{bmatrix} \boldsymbol{x}(t)\\ \boldsymbol{z}'(t) \end{bmatrix} := \widetilde{C}\widetilde{\boldsymbol{x}}(t) && (16) \end{aligned}$$

where $\boldsymbol{v}(t) \in \mathbb{R}^m, \widetilde{\boldsymbol{y}} \in \mathbb{R}^m$ are the input and the output of the hypothetical system $\{\widetilde{A}, \widetilde{B}, \widetilde{C}\}$. Here applying Proposition 1 in the next section, the input $\boldsymbol{v}(t)$ is given as

$$\boldsymbol{v}(t) = -\mathcal{L}\widetilde{\boldsymbol{y}}(t) = -\mathcal{L}\begin{bmatrix} C & D' \end{bmatrix}\begin{bmatrix} \boldsymbol{x}(t)\\ \boldsymbol{z}'(t) \end{bmatrix} \tag{17}$$

where $\mathcal{L}$ is the output feedback gain. At this time, the closed-loop sytem of the hypothetical system becomes

$$\begin{bmatrix} \dot{\boldsymbol{x}}(t)\\ \dot{\boldsymbol{z}}'(t) \end{bmatrix} = \begin{bmatrix} A - BK_X & BK_Z\\ -\mathcal{L}C & -\mathcal{L}D' \end{bmatrix} \begin{bmatrix} \boldsymbol{x}(t)\\ \boldsymbol{z}'(t) \end{bmatrix} \tag{18}$$

It is clear that (18) equals to the closed-loop system (14) with P·quasi-I·D control by the output feedback gain $\mathcal{L}$ being equal to the adjustable parameter matrix of (7). Therefore, (18) is the closed-loop system with the P·quasi-I·D control

$$\boldsymbol{u}(t) = -K_P\boldsymbol{y}(t) - K_I\boldsymbol{z}(t) - K_D\dot{\boldsymbol{y}}(t) \tag{19}$$

$$\dot{\boldsymbol{z}}(t) = \boldsymbol{y}(t) - D\boldsymbol{z}(t),\ \boldsymbol{z}(0) = \boldsymbol{0} \tag{20}$$

$$\Big(\text{namely,}\quad \boldsymbol{z}(t) = \int_0^t \boldsymbol{y}(\tau) - D\boldsymbol{z}(\tau)d\tau,\ \boldsymbol{z}(0) = \boldsymbol{0}\Big)$$

where K_I and D are chosen as

$$K_I = H_I\mathcal{L},\quad D = D'\mathcal{L} \tag{21}$$

III. Design of stabilizing P · quasi-I · D controller

In our method, we use the high gain output feedback theorem [9] in order to design the P·quasi-I·D controller. So at the beginning we prepare some terminology.

Consider the following general MIMO system

$$\dot{\widetilde{\boldsymbol{x}}}(t) = \widetilde{A}\widetilde{\boldsymbol{x}}(t) + \widetilde{B}\boldsymbol{v}(t) \tag{22}$$

$$\widetilde{\boldsymbol{y}}(t) = \widetilde{C}\widetilde{\boldsymbol{x}}(t) \tag{23}$$

where $\widetilde{\boldsymbol{x}}(t) \in \mathbb{R}^N, \boldsymbol{v}(t) \in \mathbb{R}^m, \widetilde{\boldsymbol{y}}(t) \in \mathbb{R}^m$.

[Definition 1] (relative degree) System (22), (23) is said to have relative degree $\{q_1, q_2, \cdots, q_r\}$, when the following relations concerning $\widetilde{y}_i^{(k)}$ (k times derivative of $\widetilde{y}_i$) hold.

1) In the neighborhood of $\widetilde{\boldsymbol{x}} = \widetilde{\boldsymbol{x}}_e$, for all $k < q_i$

$$\frac{\partial\widetilde{y}_i^{(k)}}{\partial v_j} = 0,\quad \text{for all } 1 \le j \le m$$

2) In the neighborhood of $\widetilde{\boldsymbol{x}} = \widetilde{\boldsymbol{x}}_e$, $m \times m$ matrix

$$\left[\frac{\partial\widetilde{y}_i^{(q_i)}}{\partial v_j}\right]_{1\le i,j\le m}$$

is nonsingular.

If system (22), (23) has the relative degree $\{1, 1, \cdots, 1\}$ such that $\widetilde{C}\widetilde{B}$ is nonsingular, then the system can be transformed into the normal form[2], [7]. That is, by coordinate transformation

$$\begin{bmatrix} \boldsymbol{\xi}\\ \boldsymbol{\eta} \end{bmatrix} = \begin{bmatrix} \widetilde{C}\\ \widetilde{T} \end{bmatrix}\widetilde{\boldsymbol{x}},\quad \boldsymbol{\xi} \in \mathbb{R}^m,\ \boldsymbol{\eta} \in \mathbb{R}^{(N-m)} \tag{24a}$$

$$\widetilde{T}\widetilde{B} = O \tag{24b}$$

we can transform system (22), (23) into the normal form

$$\dot{\boldsymbol{\xi}}(t) = Q_{11}\boldsymbol{\xi}(t) + Q_{12}\boldsymbol{\eta}(t) + \widetilde{C}\widetilde{B}\boldsymbol{v}(t) \quad (25a)$$

$$\dot{\boldsymbol{\eta}}(t) = Q_{21}\boldsymbol{\xi}(t) + Q_{22}\boldsymbol{\eta}(t) \quad (25b)$$

$$\widetilde{\boldsymbol{y}}(t) = \boldsymbol{\xi}(t) \quad (26)$$

where $Q_{11} \in \mathbb{R}^{m\times m}$, $Q_{12} \in \mathbb{R}^{m\times(N-m)}, Q_{21} \in \mathbb{R}^{(N-m)\times m}$ $, Q_{22} \in \mathbb{R}^{(N-m)\times(N-m)}$ are coefficient matreces after the coordinate transformation.

[Definition 2] (zero dynamics) In (25b),

$$\dot{\boldsymbol{\eta}}(t) = Q_{22}\boldsymbol{\eta}(t) \quad (27)$$

is called the zero-dynamics which contributes to stability of the system with zero output. When the zero dynamics (27) is asymptotically stable, the system (22), (23) is said to be minimum phase.

Using the properties defined above, we have[9]:

[**Propositon 1**] (**high gain output feedback**)

Suppose that system (22), (23) has relative degree $\{1, 1, \cdots, 1\}$ at an equillibrium $\widetilde{\boldsymbol{x}}_e = \mathbf{0}$ (i.e. $\widetilde{C}\widetilde{B}$ is nonsingular) and suppose that the system is minimum-phase (i.e. the zero dynamics are asymptotically stable). Consider an output feedback control

$$\boldsymbol{v}(t) = -\mathcal{L}\widetilde{\boldsymbol{y}}(t) \quad (28)$$

with gain matrix $\mathcal{L} \in R^{m\times m}$. Then there exist constants $\overline{\alpha}_{i0}$, γ_{i0} such that the closed-loop system (22), (23), (28) is asymptotically stable, provided that $\mathcal{L}$ is chosen as (i) or (ii) below.

(i) Choose $\mathcal{L} = (\widetilde{C}\widetilde{B})^{-1}\overline{\mathcal{L}}$, where $\overline{\mathcal{L}}$ is a sufficiently large positive definite diagonal matrix $\overline{\mathcal{L}} = \mathrm{diag}(\overline{\alpha}_1, \overline{\alpha}_2, \cdots, \overline{\alpha}_r)$, $\overline{\alpha}_i \geq \overline{\alpha}_{i0} > 0$.

(ii) Choose $\mathcal{L} = (\widetilde{C}\widetilde{B})^{-1}(Q_{11} + \Gamma)$ with $\Gamma = \mathrm{diag}(\gamma_1, \gamma_2, \cdots, \gamma_r)$, $\gamma_i \geq \gamma_{i0} > 0$, where Q_{11} is the matrix of (25a).

[**proof**] given in [9].

Now let us consider to apply Proposition 1 to the hypothetical system (15), (16) in order to design the P$\cdot$quasi-I$\cdot$D controller.

First check the relative degree of system (15) and (16). Differentiation of (16) becomes

$$\begin{aligned}\dot{\widetilde{\boldsymbol{y}}}(t) &= \begin{bmatrix} C & D' \end{bmatrix} \begin{bmatrix} \dot{\boldsymbol{x}}(t) \\ \dot{\boldsymbol{z}}'(t) \end{bmatrix} \\ &= \begin{bmatrix} C & D' \end{bmatrix} \left(\begin{bmatrix} A - BK_X & BK_Z \\ O & O \end{bmatrix} \begin{bmatrix} \boldsymbol{x}(t) \\ \boldsymbol{z}'(t) \end{bmatrix} \right. \\ &\quad \left. + \begin{bmatrix} O \\ I_m \end{bmatrix} \boldsymbol{v}(t) \right) \end{aligned} \quad (29)$$

Hence we have

$$\frac{\partial \dot{\widetilde{\boldsymbol{y}}}(t)}{\partial \boldsymbol{v}(t)} = \widetilde{C}\widetilde{B} = \begin{bmatrix} C & D' \end{bmatrix} \begin{bmatrix} O \\ I_m \end{bmatrix} = D'$$

To satisfy that $\{\widetilde{A}, \widetilde{B}, \widetilde{C}\}$ has relative degree $\{1, 1, \cdots, 1\}$, the above matrix has to be nonsingular . Therefore, let us set

$$D' = \widetilde{C}\widetilde{B} \quad (30)$$

as a nonsingular matrix.

Next check the minimum-phase property. Since the relative degree of the hypothetical system (15), (16) is $\{1, 1, \cdots, 1\}$ from the mentioned above, we can transform this system into the normal form to obtain its zero dynamics. Hence let us consider the following transfomation based on (24):

$$\begin{bmatrix} \boldsymbol{\xi} \\ \boldsymbol{\eta} \end{bmatrix} = \begin{bmatrix} \widetilde{C} \\ \widetilde{T} \end{bmatrix} \widetilde{\boldsymbol{x}} = \begin{bmatrix} C & D' \\ I_n & O \end{bmatrix} \begin{bmatrix} \boldsymbol{x} \\ \boldsymbol{z}' \end{bmatrix} \quad (31)$$

where $\widetilde{T} = \begin{bmatrix} I_n & O \end{bmatrix}$, $\widetilde{T}\widetilde{B} = O$

where $\boldsymbol{\xi} \in \mathbb{R}^m$ from $\boldsymbol{\xi} = \widetilde{\boldsymbol{y}}$ and so $\boldsymbol{\eta} \in \mathbb{R}^n$. Note that the inverse matrix of (31) becomes

$$\begin{bmatrix} C & D' \\ I_n & O \end{bmatrix}^{-1} = \begin{bmatrix} O & I_n \\ D'^{-1} & -D'^{-1}C \end{bmatrix}$$

Therefore, from the following calculation

$$\begin{aligned}\begin{bmatrix} \dot{\boldsymbol{\xi}} \\ \dot{\boldsymbol{\eta}} \end{bmatrix} &= \begin{bmatrix} C & D' \\ I_n & O \end{bmatrix} \begin{bmatrix} A - BK_X & BK_Z \\ O & O \end{bmatrix} \\ &\quad \times \begin{bmatrix} O & I_n \\ D'^{-1} & -D'^{-1}C \end{bmatrix} \begin{bmatrix} \boldsymbol{\xi} \\ \boldsymbol{\eta} \end{bmatrix} \\ &\quad + \begin{bmatrix} C & D' \\ I_n & O \end{bmatrix} \begin{bmatrix} O \\ I_m \end{bmatrix} \boldsymbol{v} \end{aligned} \quad (32)$$

, we can obtain the normal form of the hypothetical system (15),(16):

$$\begin{aligned}\dot{\boldsymbol{\xi}} &= CBK_Z D'^{-1}\boldsymbol{\xi} \\ &\quad + C\Big((A - BK_X) - BK_Z D'^{-1}C\Big)\boldsymbol{\eta} + D'\boldsymbol{v} \quad (33a) \\ \dot{\boldsymbol{\eta}} &= BK_Z D'^{-1}\boldsymbol{\xi} \\ &\quad + \Big((A - BK_X) - BK_Z D'^{-1}C\Big)\boldsymbol{\eta} \quad (33b) \\ \widetilde{\boldsymbol{y}} &= \boldsymbol{\xi} \quad (34)\end{aligned}$$

Accordingly, from (33b) the zero dynamics is expressed as

$$\dot{\boldsymbol{\eta}} = \Big(A - B(K_X + K_Z D'^{-1}C)\Big)\boldsymbol{\eta} \quad (35)$$

To satisfy the minimum-phase property , the zero dynamics (35) has to be asymptotically stable. Threfore, let us assume the following:

[Assumption 1] There exists parameter matrices H_I, K_P, K_D such that the zero dynamics $\dot{\boldsymbol{\eta}} = \big(A - B(K_X + K_Z D'^{-1}C)\big)\boldsymbol{\eta}$ is asymptotically stable.

Consequently we can apply Proposition 1, and can obtain $\mathcal{L}$ of the output feedback (17) which asymptotically stabilizes (15) can be obtained. Thus by setting PID parameters as $K_I = H_I\mathcal{L}$, $D = D'\mathcal{L}$, P$\cdot$quasi-I$\cdot$D control

$$\begin{aligned}&\boldsymbol{u}(t) = -K_I\boldsymbol{z}(t) - K_P\boldsymbol{y}(t) - K_D\dot{\boldsymbol{y}}(t) \\ &\dot{\boldsymbol{z}}(t) = \boldsymbol{y}(t) - D\boldsymbol{z}(t),\ \boldsymbol{z}(0) = \mathbf{0} \\ &\left(\text{i.e.,}\ \ \boldsymbol{z}(t) = \int_0^t (\boldsymbol{y}(\tau) - D\boldsymbol{z}(\tau))d\tau,\ \boldsymbol{z}(0) = \mathbf{0}\right)\end{aligned}$$

is obtained and system (1), (2), (5), (6) is asymptotically stable. And from the property of high gain output feedback, we can regulate the converge speed of responce by adjusting $\mathcal{L}$ of $D = D'\mathcal{L}$.

IV. DETERMINATION OF PID PARAMETER MATRICES

The most important task in our method is to satisfy Assumption 1, that is, to determine PD parameter matrices $K_P,\ K_D$ and the intermediate parameter matrix H_I such that the zero dynamics of (35)

$$\begin{aligned}\dot{\boldsymbol{\eta}} &= \Big(A - B(K_X + K_Z D^{'-1}C)\Big)\boldsymbol{\eta} \\ &= \Big(A - B(I_r + K_D CB)^{-1} \\ &\quad \times (K_P C + K_D CA + H_I D^{'-1}C)\Big)\boldsymbol{\eta} \end{aligned} \tag{36}$$

is asymptotically stable.

In this section, we propose two methods determing $K_P,\ K_D$ and H_I so as to stabilize the matrix of zero dynamics

$$A - B(I_r + K_D CB)^{-1}(K_P C + K_D CA + H_I D^{'-1}C) \tag{37}$$

Method I is simpler than Method II, though an order condition is severe.

<Method I> (a method using state feedback) First, we prepare an asymptotically stable matrix

$$A - BK_\eta \tag{38}$$

To obtain such K_η, one can apply any state feedback stabilizing technique, provided that the system $\{A, B\}$ be stabilizable.

Next in order to let the zero dynamics (37) be asymptotically stable, we choose H_I, K_P, K_D such that (37) accords with the stable matrix (38). Namely, the following matrix equation including the variables H_I, K_P, K_D must be solved.

$$\begin{aligned}&(I_r + K_D CB)^{-1} \\ &\qquad \times (K_P C + K_D CA + H_I D^{'-1}C) = K_\eta\end{aligned} \tag{39}$$

Since this equation can be transformed as

$$(H_I D^{'-1} + K_P)C + K_D(CA - CBK_\eta) = K_\eta, \tag{40}$$

put

$$\overline{K} \triangleq (H_I D^{'-1} + K_P) \quad \in \mathbb{R}^{r\times m} \tag{41}$$

so that (40) can be represented as the following matirx equation:

$$\begin{bmatrix} \overline{K} & K_D \end{bmatrix} \begin{bmatrix} C \\ CA - CBK_\eta \end{bmatrix} = K_\eta \tag{42}$$

By putting

$$X = \begin{bmatrix} \overline{K} & K_D \end{bmatrix}^T \in \mathbb{R}^{2m\times r} \tag{43a}$$

$$L = \begin{bmatrix} C \\ CA - CBK_\eta \end{bmatrix}^T \in \mathbb{R}^{n\times 2m} \tag{43b}$$

(42) can be expressed as the following general form

$$LX = K_\eta^T \tag{44}$$

This equation is generally solvable, if an order condition

$$2m \geq n \tag{45}$$

is satisfied and L has full rank. Here let us suppose such a solution X equal to $\overline{K}, K_D$ is obtained. Since from (41) we have

$$K_P = \overline{K} - H_I D^{'-1} \tag{46}$$

K_P can be obtained by substituting $\overline{K}$ and the adequate intermediate parameter matrix H_I into the above equation.

[Design Procedure I]

Step0 If (45) is satisfied, go to Step1.

Step1 Determine K_η such that $A - BK_\eta$ is asymptotically stable.

Step2 Check whether L has full rank. If so, solve the matrix linear equaion (44) to obtain $\overline{K}, K_D$.

Step3 Give the nonsingular matrix D' and the intermediate parameter matrix H_I adequately, and determine K_P from (46).

Step4 Choose a proper $\mathcal{L}$ of (17) by Proposition 1, and determine the I-parameter matrix $K_I \triangleq H_I\mathcal{L}$ and the additional matrix $D \triangleq D'\mathcal{L}$. Specifically, let us choose $\mathcal{L}$ by the second method (ii), i.e., $\mathcal{L} = (D')^{-1}(CB(I_r + K_D CB)^{-1}H_I D'^{-1} + \Gamma),\ \Gamma = \mathrm{diag}\{\gamma_1, \gamma_2, \cdots, \gamma_m\},\ \gamma_i \geq \gamma_{i0} > 0$.

<Method II> (a method using output feedback)

We first transform the partial matrix $(I_r + K_D CB)^{-1}$ $(K_P C + K_D CA + H_I D^{'-1}C)$ of the zero dynamics (37) into

$$\begin{aligned}&(I_r + K_D CB)^{-1}(K_P C + K_D CA + H_I D^{'-1}C) \\ &= (I_r + K_D CB)^{-1} \\ &\quad \times \begin{bmatrix} K_P + H_I D^{'-1} & K_D \end{bmatrix} \begin{bmatrix} C \\ CA \end{bmatrix}\end{aligned} \tag{47}$$

By defining the following matrix

$$F_{\eta 1} = (I_r + K_D CB)^{-1}(K_P + H_I D^{'-1}) \tag{48}$$

$$F_{\eta 2} = (I_r + K_D CB)^{-1}K_D \tag{49}$$

$$F_\eta = \begin{bmatrix} F_{\eta 1} & F_{\eta 2} \end{bmatrix},\ C_\eta = \begin{bmatrix} C \\ CA \end{bmatrix}, \tag{50}$$

(47) can be represented as

$$(I_r + K_D CB)^{-1}(K_P C + K_D CA + H_I D^{'-1}C) = F_\eta C_\eta$$

Then the matrix of zero dynamics (37) can be expressed as

$$A - BF_\eta C_\eta \tag{51}$$

This can be regarded as the closed-loop system with the output feedback $-F_\eta C_\eta \boldsymbol{\eta}$ for subsystem $\{A, B, C_\eta\}$. Accordingly, in order to get matrices H_I, K_P, K_D asymptotically stabilizing the zero dynamics, we apply static output feedback $-F_\eta C_\eta \boldsymbol{\eta}$ for $\{A, B, C_\eta\}$. After determining the output feedback gain F_η stabilizing (51) , we can obtain H_I, K_P, K_D stabilizing (37) from the relations (48), (49) and (50).

Now, to determine such output feedback gain $F_\eta = \begin{bmatrix} F_{\eta 1} & F_{\eta 2} \end{bmatrix}$, we apply the eigenvalue assignment method with the static output feedback, which we proposed in [11], to the system $\{A, B, C_\eta\}$. That is, we can obtain the output feedback gain F_η assigning the desired eigenvalues Λ_n such that (51) is assymtotically stable, provided that $\{A, B, C_\eta\}$ be controllable and observable and satisfy the order condition

$$2m + r > n \tag{52}$$

And when such $F_\eta = \begin{bmatrix} F_{\eta 1} & F_{\eta 2} \end{bmatrix}$ is obtained,we can determine K_D from the relation (49) as follows.

$$K_D = F_{\eta 2}(I_m - CBF_{\eta 2})^{-1} \tag{53}$$

Further, since

$$K_P = (I_r + K_D CB)F_{\eta 1} - H_I D^{'-1} \tag{54}$$

from (48), K_P can be calculated by substituting K_D of (53) and an adequate H_I into the above equation.

[Remark 1] When we apply the eigenvalue assignment method to obtain F_η stabilizing (51), it is important how to choose the desired eigenvalues practically. So as the adequate eigenvalues, we can use the optimal eigenvalues $\Lambda_n = \sigma(A - BK_\eta)$ which can be calculated from the optimal closed-loop matrix $A - BK_\eta$, where the $K_\eta = R^{-1}B^T P$ is obtained by solving the Riccati equation $PA + A^T P + Q - PBR^{-1}B^T P = O, \quad Q > 0, R > 0$.

[Design Procedure II]

Step 0 If $\{A, B, C_\eta\}$ is controllable and observable and satisfy the order condition (52), go to Step1.

Step 1 Set the desired eigenvalues Λ_n for (51) (e.g. using the method in Remark 1).

Step 2 Apply the eigenvalue assignment method [11] to $\{A, B, C_\eta\}$, and determine the output feedback gain $F_\eta = \begin{bmatrix} F_{\eta 1} & F_{\eta 2} \end{bmatrix}$ assigning th desired eigenvalue Λ_n given in Step1 .

Step 3 Give the nonsingular matrix $D^{'} \in \mathbb{R}^{m \times m}$ and the intermediate parameter matrix $H_I \in \mathbb{R}^{r \times m}$, and determine the K_D, K_P from (53),(54).

Step 4 Choose $\mathcal{L}$ in (17) and determine $K_I \triangleq H_I\mathcal{L},\ D \triangleq D'\mathcal{L}$. Here we use the second method (ii) of Proposition 1, that is, $\mathcal{L} = (D')^{-1}(CB(I_r + K_D CB)^{-1}H_I D'^{-1} + \Gamma),\ \Gamma = \text{diag}\{\gamma_1, \gamma_2, \cdots, \gamma_m\},\ \gamma_i \geq \gamma_{i0} > 0$.

V. NUMERICAL EXAMPLE

4 dimensional 2-inputs 2-outputs unstable system: Consider the following MIMO linear system

$$\dot{\boldsymbol{x}}(t) = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 3 & 0 & -3 & 1 \\ -1 & 1 & 4 & -1 \\ 1 & 0 & -1 & 0 \end{bmatrix} \boldsymbol{x}(t) + \begin{bmatrix} 1 & 0 \\ 0 & 1 \\ 0 & 0 \\ 0 & 0 \end{bmatrix} \boldsymbol{u}(t) \tag{55}$$

$$\boldsymbol{y}(t) = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \boldsymbol{x}(t) \tag{56}$$

Eigenvalues of A are $\{\ 3.1883,\ 0.5907 \pm 0.7069i,\ -0.3696\ \}$ which implies that the plant is an unstable system.

Since this example satisfies Step 0 in Design Procedure I, go to Step 1 and determine K_η which asymptotically stabilizes $A - BK_\eta$. So we solve the Ricatti equation

$$PA + A^T P - PBR^{-1}B^T P + Q = O$$
$$\text{where} \quad Q = I_4,\ R = I_2$$

to obtain $K_\eta = R^{-1}B^T P$. K_η becomes

$$K_\eta = \begin{bmatrix} 4.076 & -1.823 & -11.73 & 3.209 \\ -1.823 & 6.611 & 23.02 & -4.572 \end{bmatrix} \tag{57}$$

By Step 2 $\overline{K}, K_D$ are obtained as follows.

$$K_D = \begin{bmatrix} -1.823 & 2.253 \\ 6.611 & 4.788 \end{bmatrix},\ \overline{K} = \begin{bmatrix} -2.186 & 1.385 \\ 1.360 & 2.040 \end{bmatrix}$$

From Step 3, giving adequate matrices

$$D' = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix},\quad H_I = \begin{bmatrix} 4 & 1 \\ 1 & 4 \end{bmatrix}$$

we obtain the P-parameter matrix as follows.

$$K_P = \begin{bmatrix} -6.186 & 0.3854 \\ 0.3601 & -1.960 \end{bmatrix}$$

By Step 4 choosing $\Gamma = \text{diag}\{\gamma_1, \gamma_2\}$ as (a) $\Gamma = \text{diag}\{1, 1\}$, (b) $\Gamma = \text{diag}\{1.5, 1.5\}$, (c) $\Gamma = \text{diag}\{2, 2\}$ and (d) $\Gamma = \text{diag}\{5, 5\}$ and setting $K_I = H_I\mathcal{L},\ D = D'\mathcal{L}$, simulation results was obtained as shown in **Fig.1**. From **Fig.1**, it is observed that the convergence speed can be improved by enlarging Γ. It is also seen that eigenvalues of the closed-loop system move to left in the complex LHP as indicated in **Table 1**.

Table 1 Eigenvalues with changing Γ

Γ	Eigenvalues
(a) diag $\{1, 1\}$	$-2.190 \pm 2.600i,\ -0.4406$ $-0.04494 \pm 0.5178i,\ -3.778$
(b) diag $\{1.5, 1.5\}$	$-2.372 \pm 2.640i,\ -0.6141$ $-0.2014 \pm 0.5868i,\ -3.924$
(c) diag $\{2, 2\}$	$-2.567 \pm 2.658i,\ -0.7636$ $-0.3416 \pm 0.6073i,\ -4.107$
(d) diag $\{5, 5\}$	$-3.888 \pm 2.246i,\ -0.9161$ $-1.036 \pm 0.3388i,\ -5.924$

5 dimensional 2-input 2output unstable system: Consider the second example

$$\dot{\boldsymbol{x}}(t) = \begin{bmatrix} 1 & 0 & 1 & 0 & 1 \\ 0 & 0 & -1 & 0 & 0 \\ -1 & 1 & 0 & 0 & 2 \\ 0 & -1 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 & 0 \end{bmatrix} \boldsymbol{x}(t) + \begin{bmatrix} 0 & 0 \\ 1 & 0 \\ 0 & 1 \\ 1 & 0 \\ 0 & -1 \end{bmatrix} \boldsymbol{u}(t) \tag{58}$$

$$\boldsymbol{y}(t) = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & -1 & 0 \end{bmatrix} \boldsymbol{x}(t) \tag{59}$$

Its eigenvalues are $\{\ 0,\ 0,\ -1,\ 1\pm i\ \}$ which implies that the plant is unstable. Since this example does not satisfy Step 0 of Design Procedure I, we use Design Procedure II.

Since Step 0 of Design Procedure II holds , set the desired eigenvalues Λ_n from Step 1. By using the method in Remark 1 with $P = I_7, R = I_2$, the desired eigenvalues are obtained:

$$\Lambda_n = \{-1.104 \pm 1.264i,\ -1.670 \pm 0.5475i,\ -0.8819\}$$

From Step 2, by applying the eigenvalue assignment method [11], F_η which assigns Λ_n is obtained as

$$F_\eta = \begin{bmatrix} 7.245 & -5.815 & 4.587 & -0.6357 \\ -8.815 & -0.7322 & -2.839 & -3.480 \end{bmatrix}$$

By Step3, setting

$$D' = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}, H_I = \begin{bmatrix} 1 & 0.5 \\ 0.5 & 1 \end{bmatrix},$$

we can calculate PD parameter matrices as

$$K_P = \begin{bmatrix} 6.245 & -6.315 \\ -9.314 & -1.732 \end{bmatrix},$$
$$K_D = \begin{bmatrix} 4.587 & -0.6357 \\ -2.839 & -3.480 \end{bmatrix}$$

By step 4, choosing $\Gamma = \mathrm{diag}\{\gamma_1, \gamma_2\}$ as (a) $\Gamma = \mathrm{diag}\{0.5, 0.5\}$, (b) $\Gamma = \mathrm{diag}\{1, 1\}$, (c) $\Gamma = \mathrm{diag}\{5, 5\}$, (d) $\Gamma = \mathrm{diag}\{10, 10\}$ and setting $K_I = H_I\mathcal{L},\ D = D'\mathcal{L}$, we obtained the simulation results as shown in **Fig.2**. From **Fig.2**, it is seen that the response speed can be improved by enlarging Γ. Note that eigenvalues of the closed-loop system move to left in the complex LHP as indicated in **Table 2**.

Table 2 Eigenvalues with changing Γ

Γ	Eigenvalues
(a) $\mathrm{diag}\{0.5, 0.5\}$	$-2.805 \pm 0.4325i, -0.6425 \pm 2.214i$ $-0.0024 \pm 0.2905i, -0.5312$
(b) $\mathrm{diag}\{1, 1\}$	$-0.7327 \pm 2.259i, -2.860 \pm 0.2509i$ $-0.0809 \pm 0.3820i, -1.083$
(c) $\mathrm{diag}\{5, 5\}$	$-2.546, -1.298 \pm 2.112, -4.822$ $-0.4014 \pm 0.5366i, -5.662$
(d) $\mathrm{diag}\{10, 10\}$	$-10.222, -9.929, -1.424 \pm 1.791i$ $-0.5732 \pm 0.5513i, -2.284$

VI. Concluding Remarks

We proposed the P · quasi-I · D control for general MIMO systems. Compared with usual PID control, the P · quasi-I · D control has 4 parameters K_P, K_I, K_D, D. We can determine, however, these parameter matrices systematically by Design Procedure I or II, and determine them by adjusting $\mathcal{L}$. Note that the P · quasi-I · D control is useful not only for stabilization but also for improving the convergence speed by adjusting the gain $\mathcal{L}$.

An advantage of modifying the integrator by (6) is that one can establish systematic adjustment of PID parameteres for any MIMO systems, based on the high gain output feedback.

It is remarked that, however, our high gain output feedback approach is available for the normal PID control with $D = O$ in case that the relative degree of system is less than or equal to 2. (See Ref. [12] in detail.)

Our method can be extended to a setpoint servo problem in which the desired value of output is y^*. Evidently the equilibrium of (1) and (2) must satisfy

$$\mathbf{0} = A\boldsymbol{x}_e + B\overline{\boldsymbol{u}}$$
$$\boldsymbol{y}^* = C\boldsymbol{x}_e$$

Here the desired equilibrium $\boldsymbol{x}^*$ corresponding to $\boldsymbol{y}^*$ and it is obtained as

$$\begin{bmatrix} \boldsymbol{x}^* \\ \overline{\boldsymbol{u}} \end{bmatrix} = \begin{bmatrix} A & B \\ C & D \end{bmatrix}^{-1} \begin{bmatrix} \mathbf{0} \\ \boldsymbol{y}^* \end{bmatrix}$$

Next, let $\boldsymbol{e}_x \triangleq \boldsymbol{x}^* - \boldsymbol{x}$ and $\boldsymbol{e}_y \triangleq \boldsymbol{y}^* - \boldsymbol{y}$ and consider P · quasi-I · D control

$$\dot{\boldsymbol{z}}' = \mathcal{L}\boldsymbol{e}_y - \mathcal{L}D'\boldsymbol{z}'$$
$$\boldsymbol{u} = K_P\boldsymbol{e}_y + H_I\boldsymbol{z}' + K_D\dot{\boldsymbol{e}}_y + \boldsymbol{m}_0$$

where $\boldsymbol{m}_0$ is the manual reset quantity. Then by setting $\boldsymbol{m}_0 = \overline{\boldsymbol{u}}$, we can obtanin the closed-loop error system as follows:

$$\begin{bmatrix} \dot{\boldsymbol{e}}_x \\ \dot{\boldsymbol{z}} \end{bmatrix} = \begin{bmatrix} A - BK_x & -BK_z \\ -\mathcal{L}C & -\mathcal{L}D' \end{bmatrix} \begin{bmatrix} \boldsymbol{e}_x \\ \boldsymbol{z} \end{bmatrix}$$

Therefore, if the above system is asymptotically stable, we have $\boldsymbol{e}_x(t) \to \mathbf{0}$, $\boldsymbol{e}_y(t) \to \mathbf{0}$ as $t \to \infty$, namely $\boldsymbol{y}(t) \to \boldsymbol{y}^*$ is attained. The remainder is the same as the regulation problem stated in Section III and IV.

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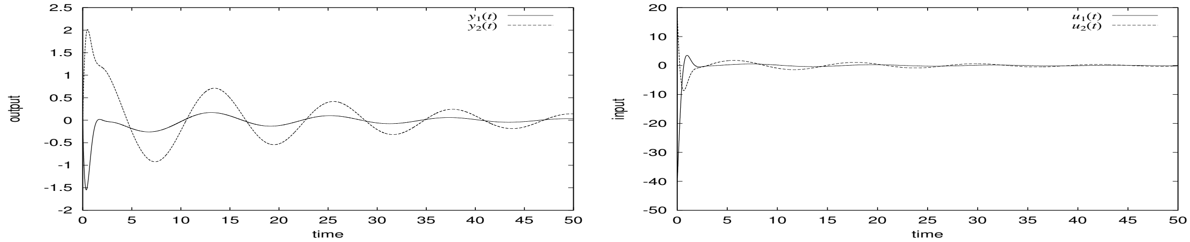
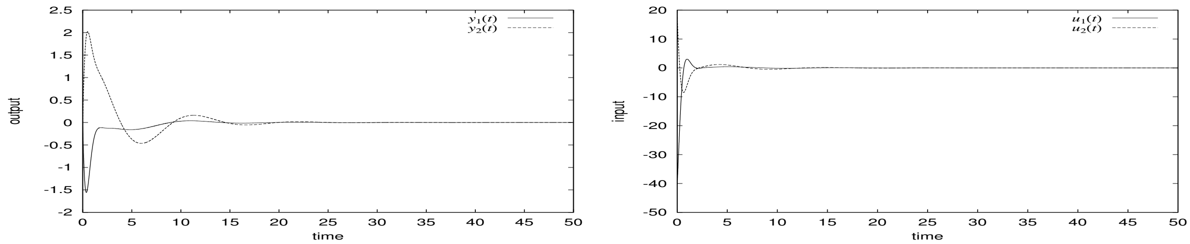
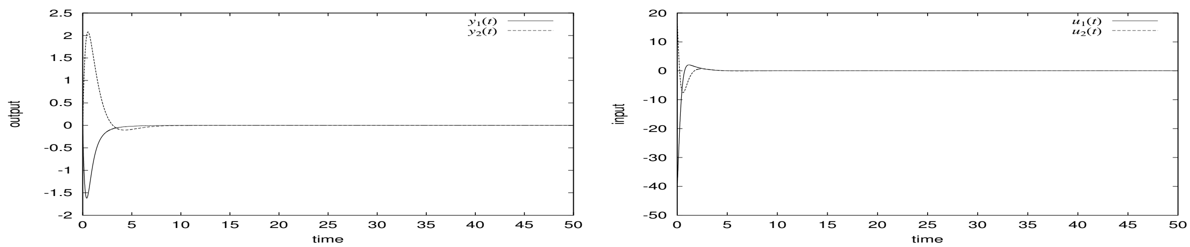


(a) $\Gamma = \mathrm{diag}\{1, 1\}$

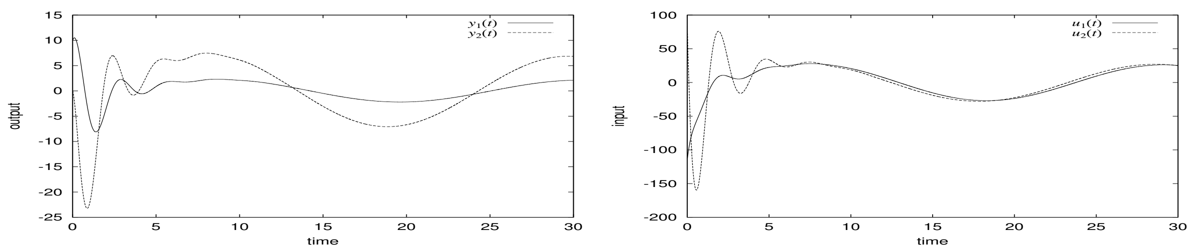


(b) $\Gamma = \mathrm{diag}\{1.5, 1.5\}$

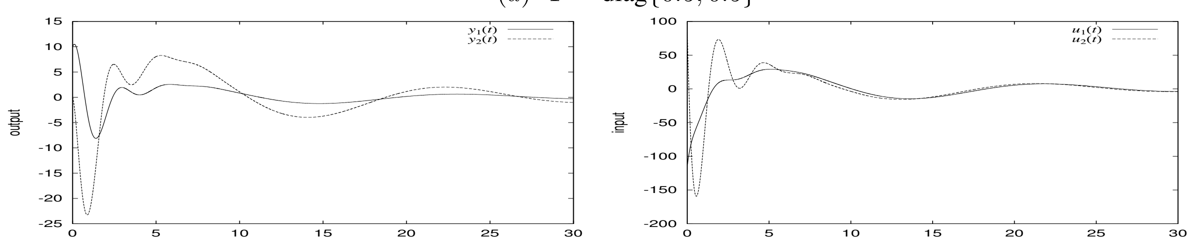


(d) $\Gamma = \mathrm{diag}\{5, 5\}$

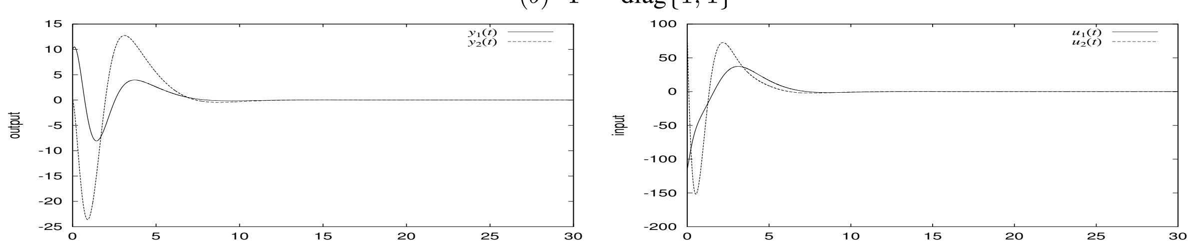
Fig.1 Time histories of input and output by adjusting Γ (4-order, 2-input, 2-output system, Method I)



(a) $\Gamma = \mathrm{diag}\{0.5, 0.5\}$



(b) $\Gamma = \mathrm{diag}\{1, 1\}$



(d) $\Gamma = \mathrm{diag}\{10, 10\}$

Fig.2 Time histories of input and output by adjusting Γ (5-order, 2-input, 2-output system, Method II)