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Robust High-Order Repetitive Control

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Abstract— **High-order repetitive control has previously been introduced to either improve the robustness for period-time uncertainty or reduce the sensitivity to non-periodic inputs of standard repetitive control schemes. This paper presents a systematic, semidefinite programming based approach to compute high-order repetitive controllers that yield an optimal trade-off between these two performance criteria. Additionally, the capability of the proposed design approach to reproduce and outperform existing high-order repetitive control approaches is illustrated.**

I. Introduction

Repetitive controllers [1], [2], [3] are feedback controllers specifically designed for periodic inputs. Their design is based on the internal model principle [4], which states that, if a disturbance/reference signal can be regarded as the output of an autonomous system, including this system in a stable feedback loop guarantees asymptotically perfect rejection/tracking. Figure 1 shows the most general generator of signals with period T_0 [s]. Including this system in the controller results in an infinite loop gain, and hence, a zero closed-loop sensitivity, at all multiples of $\omega_0 = 2\pi/T_0$ [rad/s]. Consequently, the closed-loop system yields perfect *nominal periodic performance*: any periodic input is perfectly rejected/tracked asymptotically, provided that its period is exactly T_0.

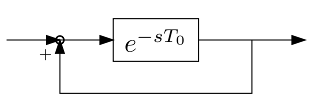

Fig. 1. Generator of periodic signals with period T_0.

Most repetitive controller designs in literature suffer from two disadvantages. First, the periodic performance of the closed-loop system is very sensitive to uncertainty on the period of the input. Hence, while yielding perfect nominal periodic performance the *robust periodic performance* of the closed-loop system is in general not satisfactory. Second, the repetitive controller negatively affects the closed-loop performance for non-periodic inputs, i.e., the *non-periodic performance*: due to the Bode Sensitivity Integral [5], pushing the sensitivity down to zero at the multiples of ω_0 is paid for by an increase of the sensitivity at the intermediate frequencies.

To overcome these disadvantages, so-called high-order repetitive control has been proposed. Inoue [6] and Chang et al. [7] design high-order repetitive controllers to improve the non-periodic performance under the constraint of perfect nominal periodic performance, while the design of Steinbuch [8] improves the robust periodic performance under the same constraint. A unified framework that is able to reproduce the results of both [7] and [8] is proposed in [9].

While the approaches [7], [8], [9] focus either on the non-periodic performance or on the robust periodic performance, this paper proposes a novel high-order repetitive controller design approach that realizes an optimal trade-off between these two performance criteria. Perfect nominal periodic performance is no longer the starting point of the design, but can be added as an additional constraint. The optimal repetitive controller design is reformulated as a semidefinite program (SDP), guaranteeing an efficient and reliable computation of the global optimum. As illustrated in Sec. V, the proposed design approach is able to reproduce and outperform the results of [7], [8].

The paper is organized as follows: Section II details the control setup used in this paper and analyzes the closed-loop system without and with repetitive controller. High-order repetitive control is introduced in Sec. III while Sec. IV develops the optimization framework. Section V numerically illustrates the potential of the proposed design approach and Sec. VI summarizes the conclusions of this paper.

The notation used in this paper is standard. $\mathcal{S}_n$ indicates the set of symmetric $n \times n$ matrices. I_n corresponds to the $n \times n$ identity matrix, while $0_{n,m}$ indicates the zero-matrix of dimension $n \times m$. To alleviate notation, the frequency response function (FRF) of a discrete-time system $H(z)$ is denoted by $H(\omega)$ instead of $H\left(e^{j\omega T_s}\right)$.

II. Add-on Repetitive Control System

The repetitive controller is considered as an add-on device, that is, it is added to the loop gain of an existing feedback control system. This is the more common repetitive control scheme in literature [3]. Section II-A details the corresponding control setup, while Sec. II-B analyzes the closed-loop system without and with repetitive controller. Section II-C introduces two performance indices to quantify the effect of the repetitive controller on the closed-loop non-periodic and robust periodic performance.

A. Control Setup

Figure 2 shows the control setup of an add-on repetitive control system. The controller is designed in discrete time,

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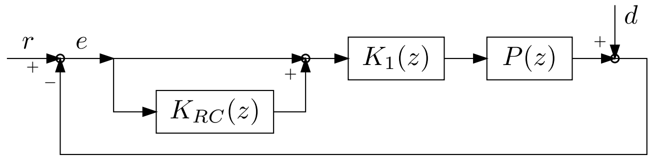

Fig. 2. Control setup of an add-on repetitive control system.

where the sampling period is denoted by T_s [s] and the index k refers to the discrete time instants kT_s. The plant is a single-input single-output linear time-invariant system with transfer function $P(z)$. The signals $r(k)$, $d(k)$ and $e(k)$ correspond to the reference input, the disturbance input and the tracking error, respectively. In closed loop, $e(k)$ is given by

$$e(k) = S(z)\,\underbrace{(r(k) - d(k))}_{w(k)} , \tag{1}$$

where $S(z)$ denotes the closed-loop sensitivity function. The signal $w(k)$ is referred to as the *input* of the closed-loop system and features both a periodic and a non-periodic part. The periodic part has a nominal period T_0 where the corresponding fundamental frequency is denoted by ω_0. The sample frequency is chosen such that T_0 corresponds to an integer number N of samples: $T_0 = NT_s$. Uncertainty on T_0 is modeled as a relative uncertainty on ω_0, bounded by Δ:

$$\omega_{0,\delta} = \omega_0\,(1+\delta) \qquad |\delta| \leq \Delta , \tag{2a}$$

$$T_{0,\delta} = \frac{T_0}{1+\delta} \qquad |\delta| \leq \Delta . \tag{2b}$$

The index l indicates the harmonics of the periodic part of the input:

$$l \in \{1, 2, \cdots, L\} , \tag{3}$$

where L corresponds to the highest harmonic to be suppressed.

B. Closed-Loop System Without and With $K_{RC}(z)$

The repetitive controller $K_{RC}(z)$ is considered as an add-on device, that is, the feedback controller $K_1(z)$ is assumed to be properly designed first, while $K_{RC}(z)$ is added afterwards to improve the closed-loop periodic performance.

In the absence of $K_{RC}(z)$, the closed-loop sensitivity function $S_1(z)$ and complementary sensitivity function $T_1(z)$ are given by

$$S_1(z) = \frac{1}{1 + K_1(z)P(z)} , \tag{4}$$

$$T_1(z) = \frac{K_1(z)P(z)}{1 + K_1(z)P(z)} . \tag{5}$$

A proper design of $K_1(z)$ implies (i) a stable closed-loop system; (ii) high-gain feedback at low frequencies; (iii) sufficient roll-off of $|T_1(\omega)|$ at high frequencies; and (iv) a small modulus margin, defined as $\|S_1(z)\|_\infty$. See for instance [10] for more details.

Property (ii) is referred to as *good non-periodic performance*, since it yields small values of $|S_1(\omega)|$ at low frequencies, resulting in a small tracking error $e(k)$ for *any* low-frequency input $w(k)$. The bandwidth of the original closed-loop system, ω_{BW}, is defined as the first frequency at which $|S_1(\omega)|$ crosses -3dB from below. Hence, $K_1(z)$ yields good non-periodic performance up to ω_{BW}.

When $K_{RC}(z)$ is added to the loop, the closed-loop sensitivity changes from $S_1(z)$ to $S(z)$:

$$S(z) = S_1(z)\, M_S(z) , \tag{6}$$

where $M_S(z)$ is given by

$$M_S(z) = \frac{1}{1 + K_{RC}(z)T_1(z)} . \tag{7}$$

$M_S(z)$ is called the *modifying sensitivity function* [9] and gathers the effect of $K_{RC}(z)$ on the closed-loop sensitivity.

$K_{RC}(z)$ is designed such that the closed-loop periodic performance is improved, while keeping the inevitable non-periodic performance degradation within acceptable limits. Additionally, $K_{RC}(z)$ must not compromise the modulus margin and high-frequency roll-off of the original feedback system. The effect of $K_{RC}(z)$ on the closed-loop performance is quantified by two performance indices, defined in the following section.

C. Performance Indices of $K_{RC}(z)$

The influence of $K_{RC}(z)$ on the closed-loop nominal periodic performance is quantified by the *nominal periodic performance index* γ_p, defined as the smallest relative reduction of $|S(l\omega_0)|$ over all harmonics l:

$$\gamma_p \equiv \max_{l \leq L} \left\{ \frac{|S(l\omega_0)|}{|S_1(l\omega_0)|} \right\} , \tag{8a}$$

$$= \max_{l \leq L} \{|M_S(l\omega_0)|\} . \tag{8b}$$

In the presence of uncertainty on the period of the input, the worst-case value of (8b) is taken over all potential values $\omega_{0,\delta}$. Hence, the *robust periodic performance index* $\gamma_{p,\Delta}$ is given by

$$\gamma_{p,\Delta} \equiv \max_{|\delta| \leq \Delta} \left\{ \max_{l \leq L} \{|M_S(l\,\omega_{0,\delta})|\} \right\} , \tag{9a}$$

$$= \max_{l \leq L} \left\{ \max_{\omega \in \Omega_l} \{|M_S(\omega)|\} \right\} , \tag{9b}$$

where

$$\Omega_l = [l\omega_0(1-\Delta)\ ,\ l\omega_0(1+\Delta)] . \tag{10}$$

The effect of $K_{RC}(z)$ on the closed-loop non-periodic performance is quantified by the *non-periodic performance index* γ_{np}, defined as the highest relative amplification of $|S(\omega)|$ below ω_{BW}:

$$\gamma_{np} \equiv \max_{\omega \leq \omega_{BW}} \left\{ \frac{|S(\omega)|}{|S_1(\omega)|} \right\} , \tag{11a}$$

$$= \max_{\omega \leq \omega_{BW}} \{|M_S(\omega)|\} , \tag{11b}$$

$$= \|M_S(z)\|_\infty . \tag{11c}$$

The equivalence between (11b) and (11c) holds since $|M_S(\omega)|$ reaches its maximum in the frequency range $0 \leq \omega \leq \omega_0$. This is further discussed in Sec. III-B.

The goal of a repetitive controller is to improve the robust periodic performance: $\gamma_{p,\Delta} < 1$, at the expense of worse non-periodic performance: $\gamma_{np} > 1$.

III. High-Order Repetitive Control

This section introduces high-order repetitive control. Section III-A details the general structure of a high-order repetitive controller. In Sec. III-B some of the design freedom is fixed to preserve the modulus margin and high-frequency roll-off of the original feedback system while Sec. III-C derives the performance indices γ_{np} and $\gamma_{p,\Delta}$ as a function of the remaining design parameters.

A. Structure of a High-Order Repetitive Controller

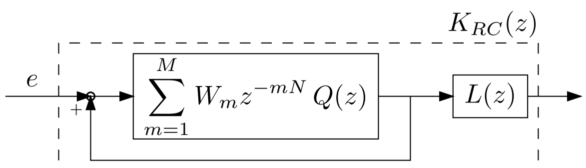

Fig. 3. Structure of a high-order repetitive controller.

Figure 3 shows the structure of a high-order repetitive controller of order M [8]. This structure gives rise to the following expressions for $K_{RC}(z)$ and $M_S(z)$:

$$K_{RC}(z) = \frac{W(z)Q(z)L(z)}{1 - W(z)Q(z)} \ , \tag{12}$$

$$M_S(z) = \frac{1 - W(z)Q(z)}{1 - W(z)Q(z)\left[1 - L(z)T_1(z)\right]} \ , \tag{13}$$

where

$$W(z) = \sum_{m=1}^{M} W_m z^{-mN} \ . \tag{14}$$

Designing the high-order repetitive controller corresponds to designing $W(z)$ and the filters $Q(z)$ and $L(z)$. $Q(z)$ and $L(z)$ are designed first (Sec. III-B) to preserve the modulus margin and high-frequency roll-off of the original feedback system, irrespective of $W(z)$. Subsequently, $W(z)$ is designed such that the corresponding repetitive controller realizes an optimal trade-off between the performance indices $\gamma_{p,\Delta}$ and γ_{np} (Sec. IV).

B. Design of $Q(z)$ and $L(z)$

The filters $Q(z)$ and $L(z)$ are usually designed as follows [1], [6], [8]:

- $Q(z)$ is a low-pass linear-phase FIR filter with unity dc-gain and cut-off frequency ω_{co}.
- $L(z)$ is set equal to $T_1^{-1}(z)$. If $T_1(z)$ is non-minimum phase, the ZPETC inverse [11] is used to obtain a stable $L(z)$. In fact, it suffices that $L(\omega) = T_1^{-1}(\omega)$ in the pass-band of the filter $Q(z)$.

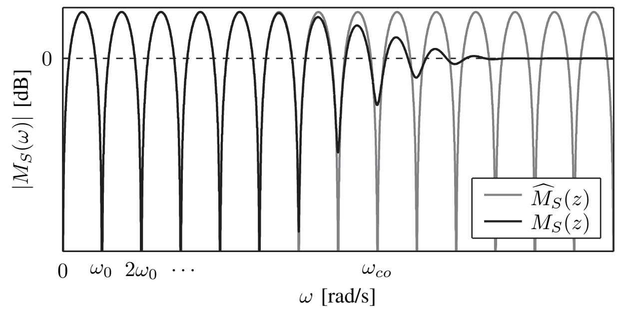

Fig. 4. Comparison between $M_S(\omega)$ and $\widehat{M}_S(\omega)$ for a typical repetitive control system.

Substituting $L(z) = T_1^{-1}(z)$ in (7) yields a modifying sensitivity function

$$M_S(z) = 1 - W(z)Q(z) \tag{15}$$

that no longer depends on the original control system, that is, on $S_1(z)$, $T_1(z)$ nor $K_1(z)$.

To illustrate the effect of $Q(z)$ on $M_S(z)$, Fig. 4 compares, for a typical repetitive control system, the FRF of $M_S(z)$ with the FRF of $\widehat{M}_S(z)$, defined as the modifying sensitivity function for $Q(z) \equiv 1$:

$$\widehat{M}_S(z) = 1 - W(z) \ . \tag{16}$$

The low-pass filter $Q(z)$ turns off $K_{RC}(z)$ outside its pass-band, since $Q(\omega) \approx 0$ yields $M_S(\omega) \approx 1$. Hence, if ω_{co} lies well below ω_{BW}, neither the modulus margin nor the high-frequency roll-off are altered by the repetitive controller, irrespective of $W(z)$.

The remainder of the paper relies on two nonrestrictive assumptions. First, all harmonics $l : 1, \ldots, L$ are assumed to lie in the pass-band of $Q(z)$ ([1]). Second, in its pass-band, $Q(z)$ is assumed to equal its dc-gain, as is done in all current high-order repetitive control design approaches [6], [7], [8], [9]. These two assumptions imply that

$$\forall \omega \leq L\omega_0(1 + \Delta) \ : \ M_S(\omega) = \widehat{M}_S(\omega) \ . \tag{17}$$

The implications of these assumptions for the computation of $\gamma_{p,\Delta}$ and γ_{np} are discussed in the following paragraph.

C. Performance Indices as a Function of $\widehat{M}_S(z)$

Assumption (17) allows the replacement of $M_S(\omega)$ by $\widehat{M}_S(\omega)$ in the definition (9) of $\gamma_{p,\Delta}$:

$$\gamma_{p,\Delta} = \max_{l \leq L} \left\{ \max_{\omega \in \Omega_l} \left\{ |\widehat{M}_S(\omega)| \right\} \right\} \ . \tag{18}$$

Due to the periodicity of $\widehat{M}_S(\omega)$, see e.g. Fig. 4, each uncertainty interval Ω_l can be replaced by $\widetilde{\Omega}_l$ around the origin:

$$\widetilde{\Omega}_l = [-l\omega_0\Delta \ , \ l\omega_0\Delta] \ . \tag{19}$$

[1]If this assumption does not hold, L should be redefined as the highest harmonic below ω_{co}, since the repetitive controller simply cannot improve the periodic performance with respect to the harmonics above ω_{co}.

Since for all $l \leq L$, $\widetilde{\Omega}_l \subset \widetilde{\Omega}_L$, it follows that

$$\gamma_{p,\Delta} = \max_{\omega \in \widetilde{\Omega}_L} \left\{ |\widehat{M}_S(\omega)| \right\} . \tag{20}$$

For $\Delta = 0$, this equation reduces to

$$\gamma_p = |\widehat{M}_S(0)| . \tag{21}$$

Since $\|M_S(z)\|_\infty$ is determined by $|M_S(\omega)|$ for $0 \leq \omega \leq \omega_0$, as illustrated in Fig. 4, and by assumption (17), $|M_S(\omega)| = |\widehat{M}_S(\omega)|$ at these frequencies, Eq. (11c) reduces to

$$\gamma_{np} = \|\widehat{M}_S(z)\|_\infty . \tag{22}$$

IV. Optimal Robust Repetitive Control Design

The design variables W_m (14) are computed such that the corresponding repetitive controller realizes an optimal trade-off between the periodic performance improvement and the inevitable non-periodic performance degradation. The repetitive controller design amounts to solving the following optimization problem in $W_1, \ldots, W_M$, γ_{np} and $\gamma_{p,\Delta}$:

$$\text{minimize} \quad \gamma_{p,\Delta} + \alpha \gamma_{np} \tag{23a}$$

$$\text{subject to} \quad |\widehat{M}_S(\omega)| \leq \gamma_{np} , \qquad \forall \omega \in \mathbb{R} \tag{23b}$$

$$|\widehat{M}_S(\omega)| \leq \gamma_{p,\Delta} , \qquad \forall \omega \in \widetilde{\Omega}_L \tag{23c}$$

where the weight α controls the trade-off between $\gamma_{p,\Delta}$ and γ_{np}. This optimization problem is semi-infinite since the constraints require evaluation for infinitely many values of ω. To render this problem numerically tractable, the semi-infinite constraints are transformed into linear matrix inequalities (LMIs) by application of the KYP lemma [12] and the generalized KYP lemma [13], [14]. These transformations are detailed in Sec. IV-A.

The global optimum of the resulting semidefinite program (SDP) is computed using SDPT3 [15], a dedicated solver for conic programming problems, called via the YALMIP interface [16]. The required CPU time is typically less than one CPU second.

Potentially, a constraint on the nominal periodic performance index γ_p can be added to problem (23). The resulting program is still an SDP and requires a similar amount of computation time.

A. LMI Formulation of the Semi-Infinite Constraints

The *KYP lemma* states that constraint (23b) is equivalent to the matrix inequality

$$\begin{bmatrix} A^T P_{np} A - P_{np} & A^T P_{np} B & C^T \\ B^T P_{np} A & B^T P_{np} B - \gamma_{np} & D^T \\ C & D & -\gamma_{np} \end{bmatrix} \preceq 0 , \tag{24}$$

where $P_{np} \in \mathcal{S}_M$ is a slack matrix variable. The matrices A, B, C and D correspond to a state-space realization of $\widehat{M}_S(z^N)$ ([2]):

$$\widehat{M}_S(z^N) = C(z^N I - A)^{-1} B + D . \tag{25}$$

[2]Due to the periodicity of the FRF of $\widehat{M}_S(z)$ a state-space model of $\widehat{M}_S(z^N)$ instead of $\widehat{M}_S(z)$ is appropriate.

The matrix inequality (24) only corresponds to an LMI if all entries are linear in the optimization variables. For this reason, the control canonical state-space form is used:

$$A = \begin{bmatrix} 0_{M-1,1} & I_{M-1} \\ 0 & 0_{1,M-1} \end{bmatrix} \qquad B = \begin{bmatrix} 0_{M-1,1} \\ 1 \end{bmatrix}$$
$$C = \begin{bmatrix} -W_M & -W_{M-1} & \cdots - W_1 \end{bmatrix} \quad D = \begin{bmatrix} 1 \end{bmatrix} .$$

Since all design variables W_m are grouped into C, the variables in LMI (24) are γ_{np}, C and P_{np}.

The *generalized KYP lemma* states that constraint (23c) is equivalent to the following set of LMI's

$$\begin{bmatrix} A^T P_p A - P_p & A^T P_p B & C^T \\ B^T P_p A & B^T P_p B - \gamma_{p,\Delta} & D^T \\ C & D & -\gamma_{p,\Delta} \end{bmatrix} + \begin{bmatrix} Q_p A + A^T Q_p + \eta Q_p & Q_p B & 0 \\ B^T Q_p & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \preceq 0 , \tag{26a}$$

$$-Q_p \preceq 0 , \tag{26b}$$

where $\eta = -2\cos(2\pi L \Delta)$. The variables in these LMIs are $\gamma_{p,\Delta}$, C and the slack matrix variables $P_p \in \mathcal{S}_M$ and $Q_p \in \mathcal{S}_M$.

V. Numerical Results

This section numerically illustrates the potential of the proposed design approach. Section V-A illustrates the effect of the weight α and the order M on the solution of (23). Section V-B subsequently compares the proposed design approach with the current high-order repetitive control design approaches [7], [8].

A. Effect of α and M

To illustrate the effect of the weight α on the solution of (23), Fig. 5 shows for $M = 3$ and $L\Delta = 10\%$ the optimal $\widehat{M}_S(\omega)$, where α ranges from 0.1 to 1.5 with an increment of 0.1. Large values of α indicate that the control engineer is not really willing to give-up non-periodic performance to improve the robust periodic performance. Hence, the optimal solution approaches $\widehat{M}_S(z) = 1$, which corresponds to $K_{RC}(z) = 0$. If α is reduced, better robust periodic performance is obtained at the expense of an increased γ_{np}. Neither the increase of γ_{np}, nor the decrease of $\gamma_{p,\Delta}$ is proportional with α.

To illustrate the effect of the order M, Fig. 6 shows the optimal $\widehat{M}_S(\omega)$ for $L\Delta = 10\%$ and different orders M. For each order M, the weight α is adjusted to obtain a solution with $\gamma_{np} = 1.7$. This figure shows that increasing the order of the repetitive controller does not always result in a significant decrease of $\gamma_{p,\Delta}$. For instance, little is gained by moving from a first-order repetitive controller ($\gamma_{p,\Delta} = 0.598$) to a second-order controller ($\gamma_{p,\Delta} = 0.593$). However, increasing the order to three does result in significantly better a robust periodic performance: $\gamma_{p,\Delta} = 0.435$, an improvement of 27% compared to $M = 2$.

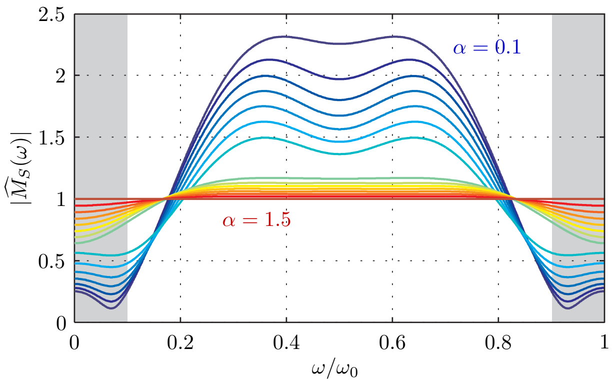

Fig. 5. Optimal $\widehat{M}_S(\omega)$ for $M = 3$ and $L\Delta = 10\%$; α ranges from 0.1 to 1.5 with an increment of 0.1. The grey band indicates the uncertainty interval $\widetilde{\Omega}_L$.

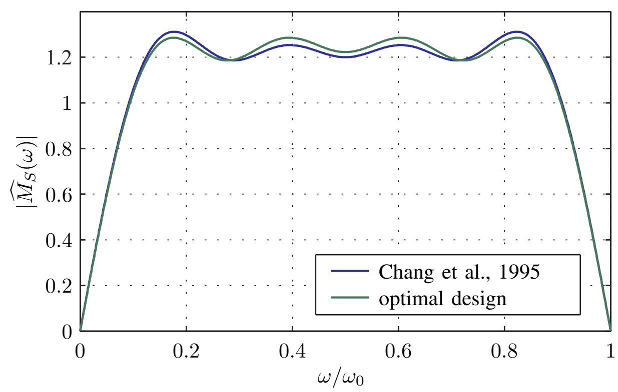

Fig. 7. Comparison of optimal repetitive control design ($\alpha = 10^{-3}$) and the repetitive controller designed by Chang et al. (1995) for $M = 4$.

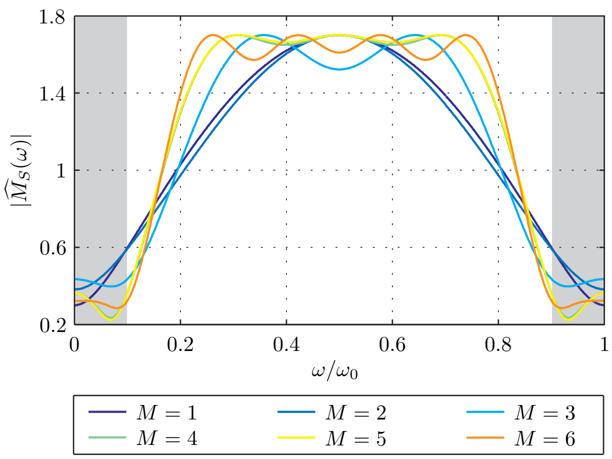

Fig. 6. Optimal $\widehat{M}_S(\omega)$ for $L\Delta = 10\%$; $\gamma_{np} = 1.7$ and different orders M of the repetitive controller. The grey band indicates the uncertainty interval $\widetilde{\Omega}_L$.

B. Comparison With Literature

The high-order repetitive controllers of Chang et al. [7] and Steinbuch [8] can be reproduced by the design framework proposed in this paper.

1) Comparison with Chang et al., 1995: To improve the non-periodic performance index of a repetitive controller, Chang et al. [7] design a high-order controller that minimizes γ_{np} under the constraint $\gamma_p = 0$. The same design is obtained by solving optimization problem (23) with $\Delta = 0$ and a small value of α. To illustrate this, Fig. 7 compares for a fourth order controller the results of [7] with the result of (23) with $\alpha = 10^{-3}$. For this α-value, the optimal result features $\gamma_p = 2.01\,10^{-9}$. The difference observed in Fig. 7 is related to the approximate strategy used by Chang et al. [7] to solve the corresponding semi-infinite optimization problem. Their result yields $\gamma_{np} = 1.31$, while the optimal solution features $\gamma_{np} = 1.29$.

2) Comparison with Steinbuch, 2001: To improve the robust periodic performance under the constraint of perfect nominal periodic performance, Steinbuch [8] enforces the higher-order derivatives of $|\widehat{M}_S(\omega)|$ to equal zero at the multiples of ω_0. Hence, for a repetitive controller of order M, the design variables W_m are designed to satisfy the following set of M equations, linear in W_m:

$$\frac{d^i|\widehat{M}_S(\omega_0)|}{d\,\omega^i} = 0\,, \quad \forall i = 0, \ldots, M-1\,. \tag{27}$$

While this analytical design approach does not involve numerical optimization, the results can be approximated by solving optimization problem (23) with a small uncertainty $L\Delta$ and $\alpha = 0$.

Figure 8 compares, for $M = 3$ the results of [8] with the optimal solution of (23) with $\alpha = 0$. Two sizes for the uncertainty $L\Delta$ are considered: $L\Delta = 2\,\%$ in Fig. 8(a) and $L\Delta = 20\,\%$, Fig. 8(b).

For a small uncertainty, $L\Delta = 2\,\%$, the results of the two design approaches almost coincide. [8] yields $\gamma_{p,\Delta} = 2 \cdot 10^{-3}$ and $\gamma_{np} = 8$, while the optimal design yields $\gamma_{p,\Delta} = 5.84 \cdot 10^{-4}$ and $\gamma_{np} = 7.97$. If an optimal repetitive controller is designed that yields $\gamma_{p,\Delta} = 2 \cdot 10^{-3}$, the same value of [8], a better non-periodic performance index is achieved: $\gamma_{np} = 6.97$ compared to $\gamma_{np} = 8$, a reduction by 13%.

For a large uncertainty, $L\Delta = 20\,\%$, the presented approach differs significantly from the design [8]. The optimal repetitive controller design features both a better robust periodic performance index ($\gamma_{p,\Delta} = 0.37$ compared to $\gamma_{p,\Delta} = 1.62$ for [8]) and a better non-periodic performance index ($\gamma_{np} = 4.83$ compared to $\gamma_{np} = 8$ for [8]). Following the design strategy of [8], the lowest order repetitive controller that yields a robust periodic performance improvement ($\gamma_{p,\Delta} \leq 1$) is of order $M = 18$. This controller features $\gamma_{p,\Delta} = 0.17$ and an unacceptably high $\gamma_{np} = 1360$.

To investigate the effect of adding the constraint of perfect nominal periodic performance, $\gamma_p = 0$, to program (23), Fig. 8(b) also shows the corresponding result for $M = 3$ and $L\Delta = 20\%$. It is observed that for this case adding this constraint has a limited effect on $\gamma_{p,\Delta}$, which increases from 0.37 to 0.39, but a large effect on γ_{np}, which increases form 4.83 to 5.46. Stated otherwise, removing this constraint leaves more design freedom to improve both γ_{np} and $\gamma_{p,\Delta}$. However, even with this additional constraint, the optimal

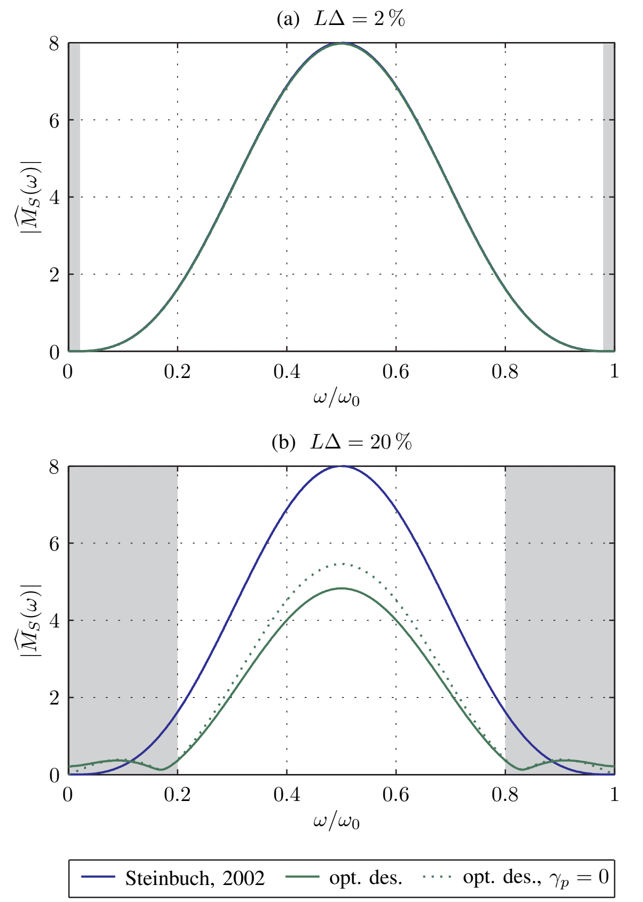

Fig. 8. Comparison of optimal repetitive control design and the repetitive controller designed by Steinbuch (2002) for $M = 3$: (a) $L\Delta = 2\,\%$; (b) $L\Delta = 20\,\%$. In figure (b) the optimal solution with $\gamma_p = 0$ as additional constraint is also shown. The grey band indicates the uncertainty interval $\widetilde{\Omega}_L$.

solution still outperforms the result of [8].

VI. Conclusions

This paper proposes an optimal repetitive control design approach that accounts for both non-periodic inputs and period-uncertain inputs that enter the control loop. The high-order repetitive controller is designed to yield an optimal trade-off between its non-periodic performance index γ_{np} and its robust periodic performance index $\gamma_{p,\Delta}$. The computation of this optimal repetitive controller is reformulated as an SDP.

The proposed repetitive control design approach is able to reproduce the current design approaches of Chang et al. [7] and Steinbuch [8]. Additionally, the proposed approach outperforms the approach by Steinbuch [8], for the same robust periodic performance index is achieved with significantly less non-periodic performance degradation.

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References

[1] S. Hara, Y. Yamamoto, T. Omata, and M. Nakano, “Repetitive control system: A new type of servo system for periodic exogenous signals,” *IEEE Transactions on Automatic Control*, vol. 33, no. 7, pp. 695–668, 1988.

[2] M. Tomizuka, T.-C. Tsao, and K.-K. Chew, “Analysis and synthesis of discrete-time repetitive controllers,” *Transactions of the ASME: Journal of Dynamic Systems, Measurement, and Control*, vol. 111, pp. 353–358, 1989.

[3] L. Cuiyan, Z. Dongchun, and Z. Xianyi, “A survey of repetitive control,” in *Proceedings of 2004 IEEE/RSJ lnternational Conference on Intelligent Robots and Systems*, Sendai, Japan, September-October 2004, pp. 1160–1166.

[4] B. A. Francis and W. M. Wonham, “The internal model principle of control theory,” *Automatica*, vol. 12, no. 5, pp. 457–465, 1976.

[5] H.-K. Sung and S. Hara, “Properties of sensitivity and complementary sensitivity functions in single-input single-output digital control systems,” *International Journal of Control*, vol. 48, no. 6, pp. 2429–2439, 1988.

[6] T. Inoue, “Practical repetitive control system design,” in *Proc. of the 29th IEEE Conference on Decision and Control*, Honolulu, Hawaii, December 1990, pp. 1673–1678.

[7] W. S. Chang, I. H. Suh, and T. W. Kim, “Analysis and design of two types of digital repetitive control systems,” *Automatica*, vol. 31, no. 5, pp. 741–746, 1995.

[8] M. Steinbuch, “Repetitive control for systems with uncertain period-time,” *Automatica*, vol. 38, no. 12, pp. 2103–2109, 2002.

[9] M. Steinbuch, S. Weiland, and T. Singh, “Design of noise and period-time robust high order repetitive control, with application to optical storage,” 2007, reprint accepted for publication in Automatica.

[10] S. Skogestad and I. Postlethwaite, *Multivariable Feedback Control - Analysis and Design*. Chichester, West Sussex, England: John Wiley & Sons, 2005.

[11] M. Tomizuka, “Zero phase error tracking algorithm for digital control,” *Transactions of the ASME: Journal of Dynamic Systems, Measurement and Control*, vol. 109, no. 1, pp. 65–68, 1987.

[12] C. Scherer and S. Weiland, *Linear Matrix Inequalities in Control*. Notes for a course of the Dutch Institute of Systems and Control, 2004.

[13] T. Iwasaki and S. Hara, “Generalized kyp lemma: unified frequency domain inequalities with design applications,” *IEEE Transactions on Automatic Control*, vol. 50, no. 1, pp. 41–59, 2005.

[14] C. W. Scherer, “Lmi relaxations in robust control,” *European Journal of Control*, vol. 12, no. 1, pp. 3–29, 2006.

[15] K. C. Toh, R. H. Tütüncü, and M. J. Todd, *On the implementation and usage of SDPT3 - a Matlab software package for semidefinite-quadratic-linear programming, version 4.0*, 2006, avalable from `http://www.math.nus.edu.sg/~mattohkc/sdpt3.html`.

[16] J. Löfberg, *YALMIP. Yet another LMI sparser.* University of Linköping, Sweden, 2002.