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Formation Control of Multiple Mobile Robots with Uncertainty

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***Abstract*— This paper considers the formation control problem of multiple mobile robots with parameter uncertainty. Decentralized adaptive control laws are proposed with the aid of the passivity property of the system dynamics and the results of graph theory.**

I. INTRODUCTION

Decentralized control of multiple mobile robots is challenging. One of reasons is that each robot's behavior is affected by its neighbor's actions and the required performance for a group of robots is the team performance instead of each individual performances.

Though study of a group of systems traces back many years, there was a surge of new results in cooperative control of multiple systems a couple of years ago [1, 2]. In [3], a distributed smooth time-varying feedback control law was proposed with analysis based on averaging theory for coordinating the motion of multiple nonholonomic mobile robots to capture/enclose a target. In [4], formation control of several mobile robots was considered with the aid of the dynamic feedback linearization technique, resulting in cooperative control laws based on multiple double integrator systems. In [5], the authors used decentralized control theory to propose and analyze controllers for multiple cooperating robotic vehicles. In [6, 7], the stability of multiple mobile robots in cyclic pursuit was studied. In [8] and [9], steering control laws were proposed for mobile robots to achieve both rectilinear and circular formations. In [10], hybrid control laws were proposed for formation control of robots. In addition, several general control methods were proposed for multiple robots. There are behavior-based control [4, 11–13], virtual structure [14–16], and leader-follower [17–20] methods.

In the control of multiple vehicles, results of graph theory have been applied in cooperative control of multiple linear systems by various authors [2, 21–25]. In these papers, the structure of the communication network between vehicles was described by Laplacian matrices. Each vehicle was treated as a vertex and the communication links between vehicles were treated as edges. The stability of the whole system was guaranteed by the stability of each modified individual linear system, where the modification to the linear system accounts for the structure of the communication network. Article [26] considered the stability of multiple agents with nonlinear models in discrete time and time-dependent communication links. Necessary and/or sufficient conditions for the convergence of the state of each individual agent to a consensus vector were presented with the aid of graph theory and convexity. These results of [26] for discrete-time systems were extended to continuous time systems in [27]. In [28–30], cooperative control of multiple nonholonomic agents was considered. Cooperative control laws were proposed for fixed and switching communication digraphs.

In this paper, we consider the cooperative control of multiple mobile robots with parameter uncertainty such that they come into a desired stationary geometric pattern. Since there is parameter uncertainty in the dynamics of each robot, the cooperative control problem is challenging. To solve this problem, adaptive decentralized controllers are proposed with the aid of results of graph theory and the passivity property of each robot's dynamics. It is shown that our proposed results can make the group of mobile robots converges to a desired stationary geometric pattern. The stability of closed-loop systems with communication delay is also studied. It is shown that the proposed control laws make the closed-loop system stable in the presence of constant communication delay. The contribution of this paper is that cooperative control of multiple wheeled mobile robots with uncertainty in dynamics is first solved by decentralized control laws which are robust to constant time delays.

II. PROBLEM STATEMENT

Consider a group of m wheeled mobile robots, indexed by j for $1 \leq j \leq m$. For the j-th robot, assume its motion is defined in the following form

$$\begin{aligned} M_j(q_j)\ddot{q}_j + C_j(q_j, \dot{q}_j)\dot{q}_j + G_j(q_j) = \\ B_j(q_j)\tau_j + [\sin\theta_j, -\cos\theta_j, 0]^\top \lambda_j, \quad (1) \\ \dot{x}_j \sin\theta_j - \dot{y}\cos\theta_j = 0 \quad (2) \end{aligned}$$

where $q_j = [x_j, y_j, \theta_j]^\top$ is the state of the j-th system, $M_j(q_j)$ is an 3×3 bounded positive-definite symmetric matrix, $C_j(q_j, \dot{q}_j)\dot{q}_j$ presents centripetal and Coriolis generalized forces, $G_j(q^j)$ is gravitational force, $B_j(q_j)$ is an $3{\times}2$ input transformation matrix, τ_j is an 2-vector of control input, λ_j is the force that ensures that the nonholonomic constraint of eqn. (2) is satisfied, and the superscript $\top$ denotes the transpose.

Eqn. (1) has the following two properties for each $1 \leq j \leq m$ [31].

Property 1: Matrix $\dot{M}_j - 2C_j$ is skew-symmetric for a proper definition of C_j.

Property 2: For any differentiable vector $\xi \in R^3$, the left side of eqn. (1) can be written as

$$M_j(q_j)\dot{\xi} + C_j(q_j, \dot{q}_j)\xi + G_j(q_j) = Y_j(q_j, \dot{q}_j, \xi, \dot{\xi})a_j$$

where a_j is an inertia parameter vector, Y_j is a regressor matrix.

Property 1 is called the passivity property of the system (1). In this paper, for each system we assume that $Y_j(q_j, \dot{q}_j, \xi, \dot{\xi})$ are known functions and parameter vector a_j are unknown.

The communication between the robots can be described by the edges $\mathcal{E}$ of the digraph $\mathcal{G} = \{\mathcal{V}, \mathcal{E}\}$ where the m mobile robots are represented by the m nodes in $\mathcal{V}$ [32, 33]. The existence of an edge $(l, j) \in \mathcal{E}$ means that the state q_{*l} of robot l is available to robot j for control (i.e., unidirectional communication). Bidirectional communication, if it exists, would be represented by the edge (j, l) also being in the digraph $\mathcal{G}$. The symbol $\mathcal{N}_j$ denotes the neighbors of node j and is the set of indices of agents whose state is available to robot j. The information available to robot j for the controller design is the $j-$th robot's own state and the state of each robot l such that $l \in \mathcal{N}_j$. Due to sensor range limitations and bounded communication bandwidth between robots, $\mathcal{N}_j$ may change with time, which means that the edge set $\mathcal{E}$ may be time-varying and consequently the Laplacian matrix L corresponding to $\mathcal{G}$ may be time-varying. In this paper, we make the following assumption.

Assumption 1: The communication between robots are bidirectional and the communication graph $\mathcal{G}$ is strongly connected.

Graph theoretic concepts such as spanning trees are discussed in, for example, [33]. An important implication of Assumption 1 is that the Laplacian matrix of the graph has only one eigenvalue which is zero.

Given a desired geometric pattern $\mathcal{P}$ defined by constant vectors $[p_{jx}, p_{jy}]^\top (1 \leq j \leq m)$, the control problem discussed in this article is defined as follows.

Formation Control Problem: Design a control law τ_j for system j with unknown inertia parameter vector a_j using $(q_j, \dot{q}_j)$, the relative state information between robot j and robot l for $l \in \mathcal{N}_j$ such that

$$\lim_{t\to\infty} \begin{bmatrix} x_i - x_j \\ y_i - y_j \end{bmatrix} = \begin{bmatrix} p_{ix} - p_{jx} \\ p_{iy} - p_{jy} \end{bmatrix}, 1 \leq i \neq j \leq m \quad (3)$$

and

$$\lim_{t\to\infty} \begin{bmatrix} \sum_{j=1}^{m} \frac{x_j}{m} \\ \sum_{j=1}^{m} \frac{y_j}{m} \end{bmatrix} = \begin{bmatrix} \sum_{j=1}^{m} \frac{p_{jx}}{m} \\ \sum_{j=1}^{m} \frac{p_{jy}}{m} \end{bmatrix}. \quad (4)$$

Remark 1: In this problem formulation, the orientation θ_j is not explicitly specified. The proposed control law will result in $\theta_j(t) \to w_1(t)$ where w_1 is a designer specified signal (see the next section). In the formation control problem, the control law for robot j is designed based on the state of robot j and the relative information between robot j and robot l for $l \in \mathcal{N}_j$. Noting the definition of the problem, eqn. (3) means that the group of robots converges to a desired geometric pattern. Eqn. (4) means that the desired geometric pattern is stationary and its geometric center is $(\frac{1}{m}\sum_{j=1}^{m} p_{jx}, \frac{1}{m}\sum_{j=1}^{m} p_{jy})$.

To solve the formation control problem, we convert eqns. (1)-(2) into a more suitable form. Following the method in [34], by eqn. (2) we have

$$\dot{q}_j = \begin{bmatrix} \cos\theta_j & 0 \\ \sin\theta_j & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} v_{1j} \\ v_{2j} \end{bmatrix} =: g_j v_j \quad (5)$$

where $v_j = [v_{1j}, v_{2j}]^\top$ are suitable vectors and are determined from eqn. (1). Differentiating both sides of eqn. (5) and substituting this expression into eqn. (1) and multiplying both sides of eqn. (1) by $g^T(q^j)$, we obtain

$$\overline{M}_j(q_j)\dot{v}_j + \overline{C}_j(q_j, \dot{q}_j)v_j + \overline{G}_j(q_j) = \overline{B}_j(q_j)\tau_j \quad (6)$$

where $\overline{M}_j(q_j) = g_j^\top M_j(q_j)g_j$, $\overline{C}_j(q_j, \dot{q}_j) = g_j^\top M_j(q_j)\dot{g}_j + g_j^\top C_j(q_j, \dot{q}_j)g_j$, $\overline{G}_j(q_j) = g_j^\top G_j(q_j)$, and $\overline{B}_j(q_j) = g_j^\top B_j(q_j)$.

System (5)-(6) describes the motion of the original j-th system. Therefore, the formation control problem can be considered based on system (5)-(6) instead of system (1)-(2). In order to completely actuate each system, $\overline{B}_j(q)$ should be have full rank. For our robots, $\overline{B}_j$ are indeed full rank matrices.

III. Decentralized Controller Design

To facilitate the control law design, we introduce the following change of states for $1 \leq j \leq m$ [29]

$$\begin{cases} z_{1j} &= \theta_j - \int_0^t w_1(s)ds \\ z_{2j} &= (x_j - p_{jx})\cos\theta_j + (y_j - p_{jy})\sin\theta_j \\ & \quad +\beta w_1 z_{3j} \\ z_{3j} &= (x_j - p_{jx})\sin\theta_j - (y_j - p_{jy})\cos\theta_j \\ u_{1j} &= v_{2j} \\ u_{2j} &= v_{1j} - z_{3j}v_{2j} \end{cases} \quad (7)$$

where $w_1 = \epsilon\sin\omega t$, and the constants β, ϵ, and ω are all positive. Taking derivative of eqn. (7), we have

$$\begin{cases} \dot{z}_{1j} &= u_{1j} - w_1 \\ \dot{z}_{2j} &= u_{2j} + \beta\dot{w}_1 z_{3j} - \beta^2 w_1^2 u_{1j} z_{3j} \\ & \quad +\beta w_1 u_{1j} z_{2j} \\ \dot{z}_{3j} &= -\beta z_{3j} w_1^2 + w_1 z_{2j} \\ & \quad +(u_{1j} - w_1)(z_{2j} - \beta w_1 z_{3j}) \end{cases} \quad (8)$$

$$\tilde{M}_j\dot{u}_j + \tilde{C}_j u_j + \tilde{G}_j = \tilde{B}_j\tau_j \quad (9)$$

where $u_j = [u_{1j}, u_{2j}]^\top$, $\tilde{M}_j = \phi_j^\top \overline{M}_j \phi_j$, $\tilde{C}_j = \phi_j^\top \overline{M}_j \dot{\phi}_j + \phi_j^\top \overline{C}_j \phi_j$, $\tilde{G}_j = \phi_j^\top \overline{G}_j$, $\tilde{B}_j = \phi_j^\top \overline{B}_j$, and

$$\phi_j = \begin{bmatrix} z_{3j} & 1 \\ 1 & 0 \end{bmatrix}.$$

For the dynamics of eqn. (9), we have the following properties.

Property 3: Matrix $\dot{\tilde{M}}_j - 2\tilde{C}_j$ is skew-symmetric.

Property 4: For any differentiable vector $\xi \in R^2$,

$$\tilde{M}_j(q_j)\dot{\xi} + \tilde{C}_j(q_j, \dot{q}_j)\xi + \tilde{G}_j(q_j) = \tilde{Y}_j(q_j, \dot{q}_j, \xi, \dot{\xi})a_j$$

where $\tilde{Y}_j(q_j, \dot{q}_j, \xi, \dot{\xi}) =$

$$(g(q_j)\phi_j)^\top Y_j\left(q_j, \dot{q}_j, g(q_j)\phi_j\xi, \frac{d}{dt}(g(q_j)\phi_j\xi)\right).$$

We design the control laws in two steps by backstepping. In the first step, we consider u_j as virtual control inputs and design cooperative control laws such that eqn. (3) is satisfied. In the second step, we design control laws τ_j such that eqn. (3) is satisfied with the aid of the results in the first step. In the first step, we have the following result.

Lemma 1: For system (8), under Assumption 1, the control laws $u_{1j} = \eta_{1j}$ and $u_{2j} = \eta_{2j}$ for $1 \leq j \leq m$ make (3) hold, where

$$\begin{aligned}\eta_{1j} &= -\sum_{l\in\mathcal{N}_j} b_{jl}[z_{1j} - z_{1l} + z_{3j}(z_{2j} - \beta w_1 z_{3j}) \\ &\quad - z_{3l}(z_{2l} - \beta w_1 z_{3l})] + w_1 \qquad (10)\\ \eta_{2j} &= -\sum_{l\in\mathcal{N}_j} b_{jl}(z_{2j} - z_{2l}) - \beta\dot{w}_1 z_{3j} \\ &\quad + \beta^2 w_1^2 \eta_{1j} z_{3j} - \beta w_1 \eta_{1j} z_{2j} \qquad (11)\end{aligned}$$

the control parameters $b_{jl} = b_{lj}$, β, ϵ, and ω are each positive.

In order to prove Lemma 1 we need the following lemma.

Lemma 2: Let L be the Laplacian matrix of communication graph $\mathcal{G}$ which satisfies Assumption 1 and has weight matrix $\mathcal{B} = [b_{jl}]$ with $b_{jl} = b_{lj} > 0$. For any vector function $\xi(t) \in R^m$, if $\lim_{t\to\infty} \xi^\top(t)L\xi(t) = 0$, then

$$\lim_{t\to\infty}\left(\xi(t) - \sum_{j=1}^{m}\frac{\xi_j(t)}{m}\mathbf{1}\right) = 0 \qquad (12)$$

where $\mathbf{1} = [1, \ldots, 1]^\top$.

Proof: Noting the definition of L, by the Gerschgorin Circle Theorem, each $\lambda_i(L)$ is contained in the union of the m Gerschgorin circles $|z - L_{jj}| \leq L_{jj}$ for $1 \leq j \leq m$. Therefore, either $\lambda_j(L) > 0$ or $\lambda_j(L) = 0$ for $1 \leq j \leq m$. Since $\mathcal{G}$ is strongly connected, there is only one zero eigenvalue [35], i.e., $\lambda_1 = 0$ and $\lambda_m \geq \cdots \geq \lambda_3 \geq \lambda_2 > 0$.

Since L is symmetric and $\lambda_1 = 0$, there exists an orthogonal matrix $Q = [Q_{ij}]$ with its first column being $\mathbf{1}/\sqrt{m}$ such that

$$Q^\top L Q = \mathrm{diag}[0, \lambda_2, \ldots, \lambda_m].$$

So,

$$\lim_{t\to\infty} \xi^\top L\xi = \lim_{t\to\infty}(Q^\top\xi)^\top \mathrm{diag}[0, \lambda_2, \ldots, \lambda_m](Q^\top\xi) = 0.$$

Let $y = [y_1, y_2, \ldots, y_m]^\top = Q^\top\xi$, then $\lim_{t\to\infty} y_i = 0$ for $2 \leq i \leq m$. Noting $y_1 = \frac{1}{\sqrt{m}}\sum_{l=1}^{m}\xi_l$ and

$$\begin{aligned}&\lim_{t\to\infty}\left(\xi - \frac{1}{m}\sum_{l=1}^{m}\xi_l\right) = \lim_{t\to\infty}\left(Qy - \frac{1}{\sqrt{m}}y_1\mathbf{1}\right)\\ &= \lim_{t\to\infty}\left[\sum_{l=2}^{m}Q_{1l}y_l, \cdots, \sum_{l=2}^{m}Q_{ml}y_l\right]^\top = [0, \cdots, 0]^\top \qquad (13)\end{aligned}$$

Therefore, the lemma is proved. ■

Proof of Lemma 1: Applying control laws u_{1j} and u_{2j} to system (8), we have

$$\left\{\begin{aligned}\dot{z}_{1j} &= -\sum_{l\in\mathcal{N}_j} b_{jl}[z_{1j} - z_{1l} + z_{3j}(z_{2j} - \beta w_1 z_{3j}) \\ &\quad - z_{3l}(z_{2l} - \beta w_1 z_{3l})]\\ \dot{z}_{2j} &= -\sum_{i\in\mathcal{N}_j} b_{ji}(z_{2j} - z_{2i})\\ \dot{z}_{3j} &= -\beta z_{3j}w_1^2 + w_1 z_{2j}\\ &\quad + (u_{1j} - w_1)(z_{2j} - \beta w_1 z_{3j})\end{aligned}\right. \qquad (14)$$

where

$$u_{1j} - w_1 = -\sum_{i\in\mathcal{N}_j} b_{ji}(z_{1j} - z_{1i} + \Delta_j - \Delta_i) \qquad (15)$$

$$\Delta_j = z_{3j}(z_{2j} - \beta w_1 z_{3j}). \qquad (16)$$

Define the positive definite Lyapunov function

$$V = \frac{1}{2}\sum_{j=1}^{m}\sum_{i=1}^{3} z_{ij}^2. \qquad (17)$$

Differentiating V along the solutions of eqn. (14), we have

$$\dot{V} = -\sum_{j=1}^{m}\beta w_1^2 z_{3j}^2 - z_{2*}^\top L z_{2*} - (z_{1*} + \Delta)^\top L(z_{1*} + \Delta) \leq 0$$

where $z_{2*} = [z_{21}, \ldots, z_{2m}]^\top$, $z_{1*} = [z_{11}, \ldots, z_{1m}]^\top$, $\Delta = [\Delta_1, \ldots, \Delta_m]^\top$, L is the Laplacian matrix of the graph $\mathcal{G}$ with weight matrix $\mathcal{B} = [b_{jl}]$. Therefore, V is bounded. Hence, z_{ij} are bounded. By Barlalat's Lemma [36], $\lim_{t\to\infty}\dot{V} = 0$. So

$$\begin{aligned}&\lim_{t\to\infty} w_1^2 z_{3j}^2 = 0, \quad 1 \leq j \leq m\\ &\lim_{t\to\infty} z_{2*}^\top L z_{2*} = 0, \lim_{t\to\infty}(z_{1*} + \Delta)^\top L(z_{1*} + \Delta) = 0.\end{aligned}$$

By Lemma 2, we have $\lim_{t\to\infty}(z_{2*}(t) - c_2(t)\mathbf{1}) = 0$ and $\lim_{t\to\infty}(z_{1*}(t) + \Delta(t) - c_1(t)\mathbf{1}) = 0$ where c_1 and c_2 are bounded and are defined as

$$c_2 = \frac{1}{m}\sum_{l=1}^{m} z_{2l}, \quad c_1 = \frac{1}{m}\sum_{l=1}^{m}(z_{1l} + \Delta_l).$$

Therefore, $\lim_{t\to\infty}(z_{2j} - z_{2l}) = 0$ and $\lim_{t\to\infty}(z_{1j} + \Delta_j - z_{1l} - \Delta_l) = 0$ for $1 \leq j \neq l \leq m$. Furthermore, we can prove that $\lim_{t\to\infty}(z_{3j} - z_{3l}) = 0$ for $1 \leq l \neq j \leq n$. Therefore, $\lim_{t\to\infty}(z_{1j} - z_{1l}) = 0$ for $1 \leq j \neq l \leq m$. By the definitions of the variables, eqn. (3) holds. ■

The virtual control laws in Lemma 1 cannot make eqn. (4) hold. In order to make eqns. (3)-(4) hold, we introduce damping terms in the virtual control laws.

Lemma 3: For system (8), under Assumption 1, the control laws $u_{1j} = \eta_{1j}$ and $u_{2j} = \eta_{2j}$ for $1 \leq j \leq m$ make (3)-(4) hold, where

$$\begin{aligned} \eta_{1j} &= -\sum_{l\in\mathcal{N}_j} b_{jl}[z_{1j} - z_{1l} + z_{3j}(z_{2j} - \beta w_1 z_{3j}) \\ &\quad -z_{3l}(z_{2l} - \beta w_1 z_{3l})] - \mu_j[z_{1j} + z_{3j}(z_{2j} \\ &\quad -\beta w_1 z_{3j})] + w_1 \qquad (18) \\ \eta_{2j} &= -\sum_{l\in\mathcal{N}_j} b_{jl}(z_{2j} - z_{2l}) - \mu_j z_{2j} - \beta \dot{w}_1 z_{3j} \\ &\quad +\beta^2 w_1^2 \eta_{1j} z_{3j} - \beta w_1 \eta_{1j} z_{2j} \qquad (19) \end{aligned}$$

the control parameters $b_{jl} = b_{lj}$, β, ϵ, and ω are each positive constant, constants $\mu_j \geq 0$ and $\sum_{j=1}^m \mu_j > 0$.

Proof: Define the positive definite Lyapunov function

$$V = \frac{1}{2}\sum_{j=1}^m \sum_{i=1}^3 z_{ij}^2. \qquad (20)$$

Differentiating V along the solutions of the closed-loop systems, we have

$$\begin{aligned} \dot{V} &= -\sum_{j=1}^m \beta w_1^2 z_{3j}^2 - z_{2*}^\top L z_{2*} - (z_{1*} + \Delta)^\top L(z_{1*} + \Delta) \\ &\quad -\sum_{j=1}^m \mu_j z_{2j}^2 - \sum_{j=1}^m \mu_j (z_{1j} + \Delta_j)^2 \leq 0 \end{aligned}$$

where z_{1*}, z_{2*}, Δ_j, and Δ are defined in the proof of Lemma 1. Therefore, V is bounded. Hence, z_{ij} are bounded. By Barlalat's Lemma [36], $\lim_{t\to\infty} \dot{V} = 0$. So

$$\begin{aligned} &\lim_{t\to\infty} w_1^2 z_{3j}^2 = 0, \quad 1 \leq j \leq m \\ &\lim_{t\to\infty} z_{2*}^\top L z_{2*} = 0, \lim_{t\to\infty} (z_{1*} + \Delta)^\top L(z_{1*} + \Delta) = 0. \\ &\sum_{j=1}^m \mu_j z_{2j}^2 = 0, \quad \sum_{j=1}^m \mu_j (z_{1j} + \Delta_j)^2 = 0. \end{aligned}$$

By Lemma 2, we have $\lim_{t\to\infty}(z_{2*}(t) - c_2(t)\mathbf{1}) = 0$ and $\lim_{t\to\infty}(z_{1*}(t) + \Delta(t) - c_1(t)\mathbf{1}) = 0$ where c_1 and c_2 are bounded and are defined as

$$c_2 = \frac{1}{m}\sum_{l=1}^m z_{2l}, \quad c_1 = \frac{1}{m}\sum_{l=1}^m (z_{1l} + \Delta_l).$$

Since there is at least one integer p such that $\mu_p \neq 0$, $\lim_{t\to\infty} z_{2p} = 0$ and $\lim_{t\to\infty}(z_{1p} + \Delta_p) = 0$. Therefore, $\lim_{t\to\infty} z_{2j} = 0$ and $\lim_{t\to\infty}(z_{1j} + \Delta_j) = 0$ for $1 \leq j \leq m$. Furthermore, we can prove that $\lim_{t\to\infty} z_{3j} = 0$ and $\lim_{t\to\infty} z_{1j} = 0$ for $1 \leq j \leq n$. By the definitions of the variables, eqns. (3)-(4) hold. ■

With the aid of Lemma 1, we can design the cooperative control laws τ_j such that the group of robots come into formation.

Theorem 1: For system (1)-(2), under Assumption 1, the control laws

$$\tau_j = \tilde{B}_j^{-1}\left(-K\tilde{u}_j + \tilde{Y}_j(q_j, \dot{q}_j, \eta_j, \dot{\eta}_j)\hat{a}_j - \Lambda_{*j}\right) \quad (21)$$

and update laws

$$\dot{\hat{a}}_j = -\Gamma_j \tilde{Y}_j^\top(q_j, \dot{q}_j, \eta_j, \dot{\eta}_j)\tilde{u}_j \qquad (22)$$

for $1 \leq j \leq m$ make (3) hold and $\hat{a}_j$ bounded, where symmetric constant matrices $K > 0$ and $\Gamma_j > 0$, $\tilde{u}_j = u_j - \eta_j$, $\eta_j = [\eta_{1j}, \eta_{2j}]^\top$, η_{1j} and η_{2j} are defined in (10)-(11), and

$$\Lambda_{*j} = \begin{bmatrix} z_{1j} + (1 - \beta^2 w_1^2) z_{2j} z_{3j} + \beta w_1 (z_{2j}^2 - z_{3j}^2) \\ z_{2j} \end{bmatrix}.$$

Proof: With the control laws (21) and the update laws (22), we have

$$\left\{ \begin{aligned} \dot{z}_{1j} &= -\textstyle\sum_{l\in\mathcal{N}_j} b_{jl}[z_{1j} - z_{1l} \\ &\quad +z_{3j}(z_{2j} - \beta w_1 z_{3j}) \\ &\quad -z_{3l}(z_{2l} - \beta w_1 z_{3l})] + \tilde{u}_{1j} \\ \dot{z}_{2j} &= -\sum_{i\in\mathcal{N}_j} a_{ji}(z_{2j} - z_{2i}) + \tilde{u}_{2j} \\ &\quad -\beta^2 w_1^2 \tilde{u}_{1j} z_{3j} + \beta w_1 \tilde{u}_{1j} z_{2j} \\ \dot{z}_{3j} &= -\beta z_{3j} w_1^2 + w_1 z_{2j} \\ &\quad +(u_{1j} - w_1)(z_{2j} - \beta w_1 z_{3j}) \end{aligned} \right. \qquad (23)$$

$$\tilde{M}_j \dot{\tilde{u}}_j + \tilde{C}_j \tilde{u}_j = -K\tilde{u}_j + \tilde{Y}_j \tilde{a}_j - \Lambda_{*j} \qquad (24)$$

$$\dot{\tilde{a}}_j = -\Gamma_j \tilde{Y}_j^\top(q_j, \dot{q}_j, \eta_j, \dot{\eta}_j)\tilde{u}_j \qquad (25)$$

where $\tilde{a}_j = \hat{a}_j - a_j$. Define the nonnegative function

$$V = \frac{1}{2}\sum_{j=1}^m (z_{1j}^2 + z_{2j}^2 + z_{3j}^2 + \tilde{u}_j^\top \tilde{M}_j \tilde{u}_j + \tilde{a}_j^\top \Gamma_j^{-1} \tilde{a}_j).$$

Differentiating V along the solutions of eqns. (23)-(25), we have

$$\begin{aligned} \dot{V} &= -(z_{1*} + \Delta)^\top L(z_{1*} + \Delta) - z_{2*}^\top L z_{2*} \\ &\quad -\beta w_1^2 z_{3*}^\top z_{3*} - \tilde{u}_j^\top K \tilde{u}_j \end{aligned}$$

where we use the fact that $(\dot{\tilde{M}}_j - 2\tilde{C}_j)$ is skew symmetric. Therefore, V is non-increasing. Following the proof of Lemma 1, we can prove that $\lim_{t\to\infty} \tilde{u}_j = 0$ and $\lim_{t\to\infty}(z_{ij} - z_{lj}) = 0$ for $1 \leq i \leq 3$ and $1 \leq j \neq l \leq m$. By the definitions of the variables, it can be verified that (3) holds. ■

Remark 2: Cooperative controllers (21) are decentralized and make the group of robots come into the desired geometric pattern. The control law τ_j consists of the relative information between neighbors. The motion of the system is driven by the relative positions and relative velocities among neighbors. The performance of the closed-loop system depends on the connectivity of the communication graph $\mathcal{G}$. The value $\lambda_2(L)$ (λ_2 is the smallest nonzero eigenvalue of L) affects the convergence rate of z_{1*} and z_{2*}. It depends on the topology of the graph $\mathcal{G}$ and the weights b_{jl}. Eigenvalue $\lambda_2(\mathcal{G})$ is known as the algebraic connectivity. Generally, a dense interconnection of $\mathcal{G}$ means a larger value of $\lambda_2(L)$. Therefore, more interconnections facilitate the cooperative performance. However, increasing the number of interconnections does not necessarily imply a larger value of $\lambda_2(L)$. Under the same topology of the communication graph $\mathcal{G}$, different weights b_{jl} may lead to different $\lambda_2(L)$. If L_1 and L_2 are the Laplacian matrices of the graph $\mathcal{G}$ with weight $\mathcal{B}_1 = [b_{ji}]$ and weight $\mathcal{B}_2 = [\rho b_{ji}]$ respectively, then $\lambda_2(L_2) = \rho\lambda_2(L_1)$. Therefore, one can choose the weights b_{jl} to maximize $\lambda_2(L)$ using the methods in [37]. Also, one can simply increase each b_{jl} to be ρb_{jl} ($\rho > 1$) to get a large $Re(\lambda_2)$.

Remark 3: The estimated parameters $\hat{a}_j$ generally do not converge to their actual values. However, they are bounded. To make the adaptive laws robust to disturbances, robust adaptive techniques may be applied [38].

Remark 4: In the theorem, we assume that the communication graph is strongly connected during the control. In practice, this is guaranteed by the proposed control laws because the distances between robots are bounded and converge to their desired values.

With the aid of Lemma 3, we can design the cooperative control laws τ_j such that the group of robots come into a stationary geometric pattern.

Theorem 2: For system (1)-(2), under Assumption 1, control law (21) and update law (22) for $1 \le j \le m$ make (3)-(4) hold and $\hat{a}_j$ bounded, where symmetric constant matrices $K > 0$ and $\Gamma_j > 0$, $\tilde{u}_j = u_j - \eta_j$, $\eta_j = [\eta_{1j}, \eta_{2j}]^\top$, η_{1j} and η_{2j} are defined in (18)-(19), Λ_{*j} are defined in Theorem 1, and the other control parameters are defined in Lemma 3.

Proof: Define the nonnegative function

$$V = \frac{1}{2}\sum_{j=1}^{m}(z_{1j}^2 + z_{2j}^2 + z_{3j}^2 + \tilde{u}_j^\top \tilde{M}_j \tilde{u}_j + \tilde{a}_j^\top \Gamma_j^{-1}\tilde{a}_j).$$

Differentiating V along the solutions of the closed-loop systems, we have

$$\begin{aligned}\dot{V} \;=\; & -(z_{1*}+\Delta)^\top L(z_{1*}+\Delta) - z_{2*}^\top L z_{2*} \\ & -\sum_{j=1}^{m}\mu_j z_{2j}^2 - \beta w_1^2 z_{3*}^\top z_{3*} - \sum_{j=1}^{m}\mu_j(z_{1j}+\Delta_j)^2 \\ & -\tilde{u}_j^\top K \tilde{u}_j\end{aligned}$$

where we use the fact that $(\dot{\tilde{M}}_j - 2\tilde{C}_j)$ is skew symmetric. Therefore, V is non-increasing. Following the proof of Lemma 3 and Theorem 1, we can prove that $\lim_{t\to\infty}\tilde{u}_j = 0$ and $\lim_{t\to\infty} z_{ij} = 0$ for $1 \le i \le 3$ and $1 \le j \le m$. By the definitions of the variables, it can be verified that (3)-(4) hold. ■

Remark 5: Cooperative controllers in Theorem 2 are decentralized and solve the defined problem in this paper. On the relationship between the control parameters and the performance of the closed-loop system, see Remark 2. The estimated parameters $\hat{a}_j$ generally do not converge to their actual values. However, they are bounded.

IV. Closed-Loop System Stability With Communication Delays

In the previous controller design, we did not consider communication delays in the control design and analysis. In practice, there are always delays due to communication and other factors. For simplicity, in this paper we assume that all time delays are constant.

Corresponding to Theorem 2, we have the following delayed version result.

Theorem 3: For system (1)-(2), under Assumption 1, if the communication graph is balanced, the control laws

$$\begin{aligned}\tau_j(t) \;=\; & \tilde{B}_j^{-1}(t)\big(-K\tilde{u}_j(t) - \Lambda_{*j}(t) \\ & + \tilde{Y}_j(q_j(t), \dot{q}_j(t), \eta_j(t), \dot{\eta}_j(t))\hat{a}_j(t)\big)\end{aligned} \quad (26)$$

and updated laws

$$\dot{\hat{a}}_j(t) = -\Gamma_j \tilde{Y}_j^\top(q_j(t), \dot{q}_j(t), \eta_j(t), \dot{\eta}_j(t))\tilde{u}_j(t) \quad (27)$$

for $1 \le j \le m$ make (3)-(4) hold and $\hat{a}_j$ bounded, where symmetric constant matrices $K > 0$ and $\Gamma_j > 0$, $\eta_j(t) = [\eta_{1j}(t), \eta_{2j}(t)]^\top$, η_{1j} and η_{2j} are defined as

$$\begin{aligned}\eta_{1j}(t) \;=\; & -\sum_{l\in\mathcal{N}_j} b_{jl}[z_{1j}(t) - z_{1l}(t-d_l) + z_{3j}(t)(z_{2j}(t) \\ & -\beta w_1(t)z_{3j}(t)) - z_{3l}(t-d_l)(z_{2l}(t-d_l) \\ & -\beta w_1(t-d_l)z_{3l}(t-d_l))] + w_1(t) \\ & -\mu_j(z_{1j}(t)+\Delta_j(t))\end{aligned} \quad (28)$$

$$\begin{aligned}\eta_{2j}(t) \;=\; & -\sum_{l\in\mathcal{N}_j} b_{jl}(z_{2j}(t) - z_{2l}(t-d_l)) - \beta\dot{w}_1(t)z_{3j}(t) \\ & +\beta^2 w_1^2(t)\eta_{1j}(t)z_{3j}(t) - \beta w_1(t)\eta_{1j}(t)z_{2j}(t) \\ & -\mu_j z_{2j}(t)\end{aligned} \quad (29)$$

d_i are bounded, $\tilde{u}_j(t) = u_j(t) - \eta_j(t)$, $\Lambda_{*j}(t) = [\Lambda_{1j}(t), \Lambda_{2j}(t)]^\top$, $\Lambda_{2j}(t) = z_{2j}(t)$, and $\Lambda_{1j}(t) = z_{1j}(t) + (1-\beta^2 w_1^2(t))z_{2j}(t)z_{3j}(t) + \beta w_1(t)(z_{2j}^2(t) - z_{3j}^2(t))$.

Proof: Define the nonnegative function

$$\begin{aligned}V = \frac{1}{2}\sum_{j=1}^{m}\Bigg(& z_{3j}^2 + z_{1j}^2 + \sum_{i\in\mathcal{N}_j}\int_{t-d_i}^{t} b_{ji}z_{1i}^2(s)ds + z_{2j}^2 \\ & + \sum_{i\in\mathcal{N}_j}\int_{t-d_i}^{t} b_{ji}z_{2i}^2(s)ds + \tilde{u}_j^\top\tilde{M}_j\tilde{u}_j + \tilde{a}_j^\top\Gamma_j^{-1}\tilde{a}_j\Bigg).\end{aligned}$$

Differentiating it along the closed-loop system, we have

$$\begin{aligned}\dot{V} \;=\; & \sum_{j=1}^{m}\Bigg(-\frac{1}{2}\sum_{i\in\mathcal{N}_j} b_{ji}[z_{2j}(t) - z_{2i}(t-d_i)]^2 - \beta w_1^2 z_{3j}^2 \\ & -\frac{1}{2}\sum_{i\in\mathcal{N}_j} b_{ji}[z_{1j}(t) + \Delta_j(t) - z_{1i}(t-d_i) \\ & -\Delta_i(t-d_i)]^2 - \tilde{u}_j^\top K\tilde{u}_j - \mu_j z_{2j}^2 - \mu_j(z_{1j}+\Delta_j)^2\Bigg)\end{aligned}$$

where

$$\Delta_j(t) = z_{3j}(t)(z_{2j}(t) - \beta w_1(t)z_{3j}(t))$$

we use Property 3 and the fact that $\mathcal{G}$ is balanced. Therefore, z_{ij}, $\tilde{a}_j$, and $\tilde{u}_j$ are bounded. By Barbalat's Lemma, we can prove that $\tilde{u}_j$ and z_{1j} converge to zero for $1 \leq j \leq m$ and $1 \leq i \leq 3$. By the definitions of the variables in (7), eqns. (3)-(4) hold. ■

V. Conclusion

This paper discusses the cooperative control problem of multiple mobile robots with parameter uncertainty. Adaptive cooperative control laws are proposed with the aid of results from graph theory. The obtained results in this paper can be extended to systems with higher state dimension.

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