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Land Vehicle Control Using a Command Filtered Backstepping Approach

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Abstract— **This article describes the derivation, design, and simulation implementation of a nonlinear controller for a nonholonomic vehicles in 2D space. The controller is designed using a command filtered, vector backstepping approach. This article focuses on the trajectory tracking capability. The trajectory tracking controller generates yaw and velocity commands. A yaw controller transforms the yaw commands to yaw rate commands. The velocity and yaw rate controller generates the two actuator force and torque commands to achieve the yaw rate and velocity commands. Each controller is nonlinear to address the kinematics and vehicle dynamics. The approach guarantees exponential stability of a compensated tracking error in the sense of Lyapunov. Both the stability analysis and simulation results are included.**

I. Introduction

Different approaches for motion control of autonomous vehicles (land, air, surface, and underwater robots) have been analyzed in recent past [14]. The literature, generally, distinguishes among three different motion control problems:

1) point stabilization - where the goal is to stabilize the robot at a given target point, with a desired orientation,
2) path following - where the robot is required to converge to and follow a path where only spatial convergence is necessary without any temporal requirement, and
3) trajectory tracking - where the robot is required to track a time parameterized reference with temporal requirement.

In this paper, we focus on the trajectory tracking problem for nonholonomic vehicles. This problem is solved for the fully actuated systems and solutions can be found in the nonlinear control textbooks, such as in [12], pages 540-544. The trajectory tracking control for nonholonomic systems is an active area of research interest. Example of such vehicles are certain land vehicles.

In other prior work researchers solved the problem of stabilization of a nonholonomic system by linearization around desired trajectory [15]. Since this approach gives an explicit control law which locally exponentially stabilizes the system to the desired trajectory it will only work close to the desired trajectory. Other approaches included a feedback linearization method [10]. Lyapunov based control laws, including backstepping technique (this technique is well described in [13], pages 25-37), have been proposed in [9], [3], and [11]. Fierro and Lewis propose the dynamical extension which provides a rigorous method of taking into account the specific vehicle dynamics to convert a steering system (kinematic model) command into control inputs for the actual vehicle [9]. Encarnacao and Pascoal combine the trajectory tracking and path following problems and develop a control scheme that can yield good tracking performance while keeping some of the desired properties of path following [3]. Godhavn considers a full nonlinear model of both the dynamics and the kinematics, and allows large variation in both forward and lateral velocities. He used splines in order to make smooth trajectories and backstepping to stabilize them [11].

It is well known, by Brockett's necessary conditions for stability [1], that nonholonomic systems cannot be stabilized to a point using smooth-static state feedback. We are interested in trajectory tracking problems where the objective is to force the system position output $y(t) \in \Re^m$ to track a desired ideal output $y_d(t) \in \Re^m$ where $\|\dot{y}_d\| \geq \epsilon > 0$; therefore, we are stabilizing such a system about a trajectory, which is feasible as proven in [15] and [11].

In this paper, we focus on trajectory tracking which forces the system to follow a given point as it moves along a user-defined trajectory. Based on the vehicle kinematics and dynamics, we use a command filtered, vector backstepping approach to design a control law to track the position in 2D. We prove that the tracking error is exponentially decreasing to zero. The simulation confirms arbitrarily small tracking error. The nonlinear controller accounts for nonlinear transformations.

Several places in this paper refer to filtering of a signal x_c^o to produce a signal x_c and its derivative $\dot{x}_c$. This is referred to as command filtering. The motivation of command filtering is to determine the signals $x_c(t)$ and $\dot{x}_c(t)$ with $|x_c^o(t) - x_c(t)|$ being small as needed for the next iteration of the backstepping procedure [13], without having to analytically or numerically differentiate x_c^o. The effects of command filtering on the backstepping stability analysis are analyzed in [6], [7], [8] and will not be repeated herein. The summary of that analysis is that for a properly designed command filter (unity DC gain to the first output and the second output being the derivative of the first output) the closed-loop command filtered implementation of the backstepping controller will be stable and the tracking error will be $\mathcal{O}\left(\frac{1}{\omega_n}\right)$ where ω_n is the bandwidth of the command filter. Therefore, the effect of command filtering on tracking error can be made arbitrarily small by increasing the parameter ω_n.

The paper is organized as follows. Section II introduces the problem of trajectory tracking control for a land robot. Section II defines robot dynamics. Section III outlines the control law signals. Sections IV, V, and VI present a detailed derivation of trajectory tracking control laws to deal with vehicle dynamics. A Lyapunov stability analysis is performed in Section VII. The performance of the control system proposed is illustrated in simulation in Section VIII. Finally, Section IX contains the conclusions and describes some problems that warrant further research.

II. Problem Statement

The kinematic and dynamic equations for a land vehicle are described as

$$\dot{x} = u\cos(\psi) \quad (1)$$
$$\dot{y} = u\sin(\psi) \quad (2)$$
$$\dot{\psi} = r \quad (3)$$
$$\dot{u} = g(u,r) + F \quad (4)$$
$$\dot{r} = f(u,r) + \tau \quad (5)$$

where x and y are the earth relative position, ψ is the yaw angle, u is the velocity in body frame, r is the yaw rate in body frame, F is the body-frame control force, τ is the body-frame control moment, $g(u,r)$ and $f(u,r)$ are friction and other forces acting on the robot. In this formulation, we have assumed that the position and velocity are measured at the center of the rear horizontal axle; therefore, the lateral velocity v is zero. For this article, we assume unit values for the mass and inertia. Accounting for non-unit values is straightforward.

We are interested in the trajectory tracking problem where the objective is to force the system output $z(t) = [x(t), y(t)]^\top \in \Re^2$ to track a desired ideal output $z_d(t) = [x_c(t), y_c(t)]^\top \in \Re^2$. We use the command filtering backstepping approach [4], [5], [6], [7], [8] assuming that z_d and $\dot{z}_d$ are available as inputs to the control law.

III. Control Signal Implementation

This section summarizes the control law and the stability properties of the closed loop system. The following equations describe the control signals

$$u_c^o = \gamma \| \; [-K_{xy}\tilde{x} + \dot{x}_c, -K_{xy}\tilde{y} + \dot{y}_c] \; \| \quad (6)$$
$$\psi_c^o = atan2\left[\gamma(-K_{xy}\tilde{y} + \dot{y}_c), \gamma(-K_{xy}\tilde{x} + \dot{x}_c)\right] \quad (7)$$
$$r_c^o = -K_\psi\tilde{\psi} + \dot{\psi}_c - \psi_{bs} \quad (8)$$
$$F = -g(u,r) - K_u\tilde{u} + \dot{u}_c - u_{bs} \quad (9)$$
$$\tau = -f(u,r) - K_r\tilde{r} + \dot{r}_c - r_{bs}, \quad (10)$$

where ψ_{bs}, u_{bs}, and r_{bs} are defined in eqns. (34), (35), and (36), respectively. The symbols K_ψ, K_u, and K_r represent positive design parameters. The symbol K_{xy} is a positive, possibly time-varying, design parameter. The parameter $\gamma = \pm 1$ is a constant selected by the user. The sign of γ will determine the sign of the commanded speed in body frame; therefore, for $\gamma = 1$ the vehicle drives forward while for $\gamma = -1$ the vehicle drives backward. In addition, the control law implements the signals ζ_x, ζ_y, and ζ_ψ using eqns. (22) and (26). Including the three command filters defined below, the controller has nine states: ζ_x, ζ_y, ζ_ψ, ψ_c, $\dot{\psi}_c$, u_c, $\dot{u}_c$, r_c and $\dot{r}_c$. The control law is derived in sections IV, V, and VI.

We will use a certain subscript and superscript notation throughout the article. For example, in addition to the variable u, we introduce the variables u_c^o and u_c. The symbol u_c^o represents the ideal desired value for u. The symbol u_c represents a filtered version of u_c^o. The filter, with bandwidth determined by a parameter ω_n, is defined in Appendix I. This notation will also be used similarly to define ψ_c^o, ψ_c, r_c^o, and r_c. Given this notation, the tracking error variables are defined as

$$\begin{array}{ll} \tilde{x} = x - x_c & \tilde{u} = u - u_c \\ \tilde{y} = y - y_c & \tilde{r} = r - r_c \\ \tilde{\psi} = \psi - \psi_c & \end{array}$$

If $\psi_c = \psi_c^o$, $u_c = u_c^o$, and $r_c = r_c^o$, then eqns. (6 – 10) would implement a conventional backstepping control law. However, the conventional backstepping approach would require analytic expressions for $\dot{\psi}_c^o$, $\dot{u}_c^o$, and $\dot{r}_c^o$, which can be quite complicated, especially when the designer chooses K_{xy} as a function of the state as in Appendix III. The command filtered backstepping approach avoids the analytic derivation of these expressions by the use of filters. The approach is designed to maintain the exponential stability properties of the backstepping approach for a set of compensated tracking errors denoted by v_x, v_y, v_ψ, v_u, and v_r, and to ensure that the control signals ψ_c, u_c, and r_c (see (21), (25), (30), and (31)) are the same as those of the conventional backstepping approach to within an error proportional to $\frac{1}{\omega_n}$. The closed loop system has the stability properties stated in Theorems. 1 and 2.

Theorem 1: For the system described by eqns. (1–5) with the feedback control law defined in eqns. (6–10), we have the following stability properties:

1) v_x, v_y, v_ψ, v_u, $v_r \to 0$ exponentially,

where v_x, v_y, v_ψ, v_u, v_r are defined in eqns. (21), (25), (30) and (31). △

Theorem 1 shows that the compensated tracking errors of the 2D command filtered backstepping approach have the same properties as the tracking errors of the standard backstepping approach. The proof of Theorem 1 is in Section VII.

Article [4], [5] analyzes the relative performance between the command filtered and conventional backstepping approaches as a function of the bandwidth parameter ω_n of the command filters. Based on the results in [4], [5], the controller also has the properties stated in Theorem 2.

Theorem 2: For the system described by eqns. (1–5) with the feedback control law defined in eqns. (6–10), we have the following stability properties:

1) $\tilde{x}$, $\tilde{y}$, $\tilde{\psi}$, $\tilde{u}$, $\tilde{r} \in L_\infty$;
2) u_c, ψ_c, r_c are $\mathcal{O}\left(\frac{1}{\omega_n}\right)$ with respect to the corresponding signals for the conventional backstepping approach; and,
3) $\tilde{x}$, $\tilde{y}$, $\tilde{\psi}$, $\tilde{u}$, $\tilde{r}$ are $\mathcal{O}\left(\frac{1}{\omega_n}\right)$ with respect to the corresponding signals for the conventional backstepping approach.

△

A signal $y(t,\epsilon)$ is said to by $\mathcal{O}(\epsilon)$ if there exist positive constants k and c such that $|y(t,\epsilon)| \leq k|\epsilon|,\ \forall|\epsilon| < c$ and $t \geq 0$ (see p. 383 in [12]).

IV. Trajectory Following

The inputs to the position control loop are $x_c(t)$, $y_c(t)$, and the derivatives, $\dot{x}_c$, $\dot{y}_c$. We assume that

$$\left\| \begin{array}{c} \dot{x}_c \\ \dot{y}_c \end{array} \right\| \geq \epsilon > 0$$

This section is concerned with the control of $[x,y]$ by specification of desired values for $[u,\psi]$.

A. Notation Definition

For clarity, we rewrite position dynamics as

$$\left[\begin{array}{c} \dot{x} \\ \dot{y} \end{array} \right] = \left[\begin{array}{c} u_x \\ u_y \end{array} \right]$$

where

$$\left. \begin{array}{rcl} u_x &=& u\ \mathrm{c}\psi \\ u_y &=& u\ \mathrm{s}\psi, \end{array} \right\} \tag{11}$$

where the symbols $\mathrm{c}\psi$ and $\mathrm{s}\psi$ represent $cos(\psi)$ and $sin(\psi)$. We also defined this same function for the commanded variables:

$$\left. \begin{array}{rcl} u^o_{x_c} &=& u^o_c\ \mathrm{c}\psi^o_c \\ u^o_{y_c} &=& u^o_c\ \mathrm{s}\psi^o_c, \end{array} \right\} \tag{12}$$

and command filtered variables:

$$\left. \begin{array}{rcl} u_{x_c} &=& u_c\ \mathrm{c}\psi_c \\ u_{y_c} &=& u_c\ \mathrm{s}\psi_c. \end{array} \right\} \tag{13}$$

Eqn. (12) can be inverted to give the desired physical values for u^o_c and ψ^o_c

$$u^o_c = \gamma\sqrt{\left(u^o_{x_c}\right)^2 + \left(u^o_{y_c}\right)^2} \tag{14}$$

$$\psi^o_c = atan2\left(\gamma u^o_{y_c}, \gamma u^o_{x_c}\right), \tag{15}$$

when γ, $u^o_{x_c}$ and $u^o_{x_c}$ are known.

The error signals

$$\left. \begin{array}{rcl} \tilde{u}_x &=& u_x - u_{x_c} \\ \tilde{u}_y &=& u_y - u_{y_c}, \end{array} \right\} \tag{16}$$

will be important in the subsequent analysis.

B. Control Design and Error Analysis

The dynamic equation for x and y can be written as

$$\left[\begin{array}{c} \dot{x} \\ \dot{y} \end{array} \right] = \left[\begin{array}{c} u^o_{x_c} \\ u^o_{y_c} \end{array} \right] + \left[\begin{array}{c} \tilde{u}_x \\ \tilde{u}_y \end{array} \right] + \left[\begin{array}{c} u_{x_c} - u^o_{x_c} \\ u_{y_c} - u^o_{y_c} \end{array} \right] \tag{17}$$

For the purpose of the control signal design, we select the signals $[u^o_{x_c}, u^o_{y_c}]^\top$ as

$$\left[\begin{array}{c} u^o_{x_c} \\ u^o_{y_c} \end{array} \right] = \left[\begin{array}{c} -K_{xy}\tilde{x} + \dot{x}_c \\ -K_{xy}\tilde{y} + \dot{y}_c \end{array} \right] \tag{18}$$

where $K_{xy} > 0$ can be time varying. The selection of K_{xy} is discussed in Appendix III. With the control signal in eqn. (18) the x and y position error dynamics are

$$\left[\begin{array}{c} \dot{\tilde{x}} \\ \dot{\tilde{y}} \end{array} \right] = \left[\begin{array}{c} -K_{xy}\tilde{x} \\ -K_{xy}\tilde{y} \end{array} \right] + \left[\begin{array}{c} \tilde{u}_x \\ \tilde{u}_y \end{array} \right] + \left[\begin{array}{c} u_{x_c} - u^o_{x_c} \\ u_{y_c} - u^o_{y_c} \end{array} \right]. \tag{19}$$

The $\tilde{u}_x$ and $\tilde{u}_y$ terms can be manipulated by two very similar approaches (see Appendix II). In either case this term can be expressed in the form

$$\left[\begin{array}{c} \tilde{u}_x \\ \tilde{u}_y \end{array} \right] = \left[\begin{array}{cc} A & Bg(\tilde{\psi}) \end{array} \right] \left[\begin{array}{c} \tilde{u} \\ \tilde{\psi} \end{array} \right] \tag{20}$$

Thus, the position error dynamics can be expressed as

$$\begin{array}{rcl} \left[\begin{array}{c} \dot{\tilde{x}} \\ \dot{\tilde{y}} \end{array} \right] &=& \left[\begin{array}{c} -K_{xy}\tilde{x} \\ -K_{xy}\tilde{y} \end{array} \right] + \left[\begin{array}{cc} A & Bg(\tilde{\psi}) \end{array} \right] \left[\begin{array}{c} \tilde{u} \\ \tilde{\psi} \end{array} \right] \\ && + \left[\begin{array}{c} u_{x_c} - u^o_{x_c} \\ u_{y_c} - u^o_{y_c} \end{array} \right] \end{array}$$

which is a form suitable for stability analysis.

Define the compensated tracking error signals v_x and v_y as

$$\left[\begin{array}{c} v_x \\ v_y \end{array} \right] = \left[\begin{array}{c} \tilde{x} - \zeta_x \\ \tilde{y} - \zeta_y \end{array} \right]. \tag{21}$$

where ζ_x and ζ_y are defined as

$$\begin{array}{rcl} \left[\begin{array}{c} \dot{\zeta}_x \\ \dot{\zeta}_y \end{array} \right] &=& \left[\begin{array}{c} -K_{xy}\zeta_x \\ -K_{xy}\zeta_y \end{array} \right] + \left[\begin{array}{cc} A & Bg(\tilde{\psi}) \end{array} \right] \left[\begin{array}{c} \zeta_u \\ \zeta_\psi \end{array} \right] \\ && + \left[\begin{array}{c} u_{x_c} - u^o_{x_c} \\ u_{y_c} - u^o_{y_c} \end{array} \right]. \end{array} \tag{22}$$

with $\zeta_x(0) = 0$ and $\zeta_y(0) = 0$. With these definitions, the dynamics of the compensated tracking errors are

$$\left[\begin{array}{c} \dot{v}_x \\ \dot{v}_y \end{array} \right] = \left[\begin{array}{c} -K_{xy}v_x \\ -K_{xy}v_y \end{array} \right] + \left[\begin{array}{cc} A & Bg(\tilde{\psi}) \end{array} \right] \left[\begin{array}{c} v_u \\ v_\psi \end{array} \right] \tag{23}$$

where v_u and v_ψ are defined in eqns. (25) and (30).

Consider the Lyapunov function $V_{xy} = \frac{1}{2}\left(v_x^2 + v_y^2\right)$. The derivative of V_{xy} is

$$\begin{array}{rcl} \dot{V}_{xy} &=& -K_{xy}\left(v_x^2 + v_y^2\right) + \left(v_x\tilde{u}_x + v_y\tilde{u}_y\right) \\ &=& -K_{xy}\left(v_x^2 + v_y^2\right) \\ && + \left[\begin{array}{cc} v_x & v_y \end{array} \right] \left[\begin{array}{cc} A & Bg(\tilde{\psi}) \end{array} \right] \left[\begin{array}{c} v_u \\ v_\psi \end{array} \right] \\ &=& -K_{xy}\left(v_x^2 + v_y^2\right) + \\ && A^T \left[\begin{array}{c} v_x \\ v_y \end{array} \right] v_u + g(\tilde{\psi})^\top B^T \left[\begin{array}{c} v_x \\ v_y \end{array} \right] v_\psi. \end{array}$$

This equation will be used in the stability analysis of Section VII.

V. Yaw Control

The objective of this section is to design a controller to stabilize the dynamic system of eqns. (1–3). Because the position dynamics were already discussed, this section focuses on selection of r^o_c to stabilize the ψ dynamics. For ψ tracking control, the input is the yaw command $\psi_c(t)$ and its derivative $\dot{\psi}_c(t)$ which are produced by a command filter with input ψ^o_c as discussed in Appendix I.

For yaw control, based on eqn. (3), we define the signal

$$r^o_c = -K_\psi\tilde{\psi} + \dot{\psi}_c - \psi_{bs}$$

where K_ψ is a positive constant. Using this definition, the closed-loop tracking error corresponding to eqn. (3) is

$$\begin{aligned}\dot{\psi} &= r_c^o + (r - r_c) + (r_c - r_c^o) \\ &= -K_\psi \tilde{\psi} + \dot{\psi}_c - \psi_{bs} + \tilde{r} + (r_c - r_c^o) \\ \dot{\tilde{\psi}} &= -K_\psi \tilde{\psi} - \psi_{bs} + \tilde{r} + (r_c - r_c^o). \end{aligned} \tag{24}$$

The compensated tracking error signal for the ψ dynamics is defined as

$$v_\psi = \tilde{\psi} - \zeta_\psi. \tag{25}$$

The signal ζ_ψ is defined as

$$\dot{\zeta}_\psi = -K_\psi \zeta_\psi + (r_c - r_c^o) + \zeta_r \tag{26}$$

with $\zeta_\psi(0) = 0$. With these definitions, the dynamic equation of v_ψ is

$$\begin{aligned}\dot{v}_\psi &= \dot{\tilde{\psi}} - \dot{\zeta}_\psi \\ &= -K_\psi v_\psi - \psi_{bs} + v_r, \end{aligned} \tag{27}$$

where v_r is defined in (31).

Choosing the Lyapunov function as

$$V_\psi = \frac{1}{2} v_\psi^2,$$

its derivative is

$$\dot{V}_\psi = -K_\psi v_\psi^2 + v_\psi v_r - \psi_{bs} v_\psi. \tag{28}$$

This equation will be used in the stability analysis of Section VII.

VI. Velocity and Yaw Rate Control

The objective of this section is to design a controller to stabilize the dynamic system of eqns. (1–5) using backstepping. This section focuses on selection of F and τ to stabilize the u and r tracking error dynamics. For u and r tracking control, the inputs are the horizontal speed and yaw rate commands $u_c(t)$ and $r_c(t)$ and their derivatives $\dot{u}_c(t)$ and $\dot{r}_c(t)$ which are produced by command filters with input u_c^o and r_c^o as discussed in Appendix I.

For tracking control using eqns. (4-5) we select the control force and the control torque as

$$\begin{aligned} F &= -g(u, r) - K_u \tilde{u} + \dot{u}_c - u_{bs} \\ \tau &= -f(u, r) - K_r \tilde{r} + \dot{r}_c - r_{bs}, \end{aligned} \tag{29}$$

where K_u and K_r are positive constants.

With this choice of the control signal the dynamics of the u and r tracking errors are

$$\begin{aligned}\dot{\tilde{u}} &= -K_u \tilde{u} - u_{bs} \\ \dot{\tilde{r}} &= -K_r \tilde{r} - r_{bs}\end{aligned}$$

The signals ζ_u and ζ_r are identically zero; therefore,

$$v_u = \tilde{u} \tag{30}$$

$$v_r = \tilde{r} \tag{31}$$

and the dynamics of the compensated tracking errors are

$$\left.\begin{aligned}\dot{v}_u &= -K_u v_u - u_{bs} \\ \dot{v}_r &= -K_r v_r - r_{bs}\end{aligned}\right\} \tag{32}$$

Choosing the Lyapunov function as

$$V_i = \frac{1}{2}\left(v_u^2 + v_r^2\right),$$

its derivative is

$$\dot{V}_i = -K_u v_u^2 - u_{bs} v_u - K_r v_r^2 - r_{bs} v_r. \tag{33}$$

This equation will be used in the stability analysis of Section VII.

VII. Stability Proof

Section III presented two theorems that summarize the properties of the 2D command filtered backstepping approach. Theorem 1 is proven in this section. Theorem 2 is proved in [4], [5].

Proof: Define the overall Lyapunov function

$$V = V_{xy} + V_\psi + V_i.$$

The time derivative of V along solutions of the closed-loop system is

$$\begin{aligned}\dot{V} &= \dot{V}_{xy} + \dot{V}_\psi + \dot{V}_i \\ &= -K_{xy}\left(v_x^2 + v_y^2\right) - K_\psi v_\psi^2 - K_u v_u^2 - K_r v_r^2 \\ &\quad + g(\tilde{\psi})^\top B^T \begin{bmatrix} v_x \\ v_y \end{bmatrix} v_\psi + A^T \begin{bmatrix} v_x \\ v_y \end{bmatrix} v_u + v_\psi v_r \\ &\quad - \psi_{bs} v_\psi - u_{bs} v_u - r_{bs} v_r\end{aligned}$$

which is the sum of eqns. (24), (28), (33). To remove the sign indefinite terms on lines two and three of the above result, we define the backstepping terms as

$$\psi_{bs} = g(\tilde{\psi})^\top B^T \begin{bmatrix} v_x \\ v_y \end{bmatrix} \tag{34}$$

$$u_{bs} = A^T \begin{bmatrix} v_x \\ v_y \end{bmatrix} \tag{35}$$

$$r_{bs} = v_\psi. \tag{36}$$

With these definitions, the derivative of $V(t)$ satisfies

$$\dot{V} \leq -K_{xy}\left(v_x^2 + v_y^2\right) - K_\psi v_\psi^2 - K_u v_u^2 - K_r v_r^2. \tag{37}$$

Therefore, by Theorem 4.10 in [12], the equilibrium $v = 0$ of the system described by eqns. (23), (27), and (32) is globally exponentially stable. ■

The above result holds for any ω_n. Selection of a value for ω_n involves a tradeoff between decreasing the effects of measurement noise and increasing trajectory tracking accuracy. As w_n is increased the command filtered variables (e.g., x_c) converge to the ideal desired variables (e.g., x_c^o) more rapidly and track the desired variables more accurately. Also, the signals ζ_x, ζ_y and ζ_ψ are decreasing functions of ω_n. The analysis in [4], [5] proves that by increasing w_n, the solution to the command filtered backstepping closed-loop system can be made arbitrarily close to the backstepping solution that relies on analytic derivatives.

VIII. Simulation Results

Due to space limitation, the simulation results are not included in this paper. They can be downloaded from www.ee.ucr.edu/∼farrell/ACC2008Simulation_vlad_jay_wenjie.pdf.

IX. Conclusion

This article has discussed the design and derivation of a command filtered, vector backstepping approach to design a stable translational controller (i.e., $y = [x(t), y(t)]^\top$) applicable to a nonholonomic land robot. The mission scenario specifies the position and desired speed commands which are command filtered to produce inputs (together with their derivatives) for the translational and yaw controllers. The commands, such as horizontal velocity (u_c) and yaw rate (r_c) are generated by the translational and yaw loop, command filtered, and are inputs (together with their derivatives) to the velocity and yaw rate controllers. The u and r controller determines the appropriates actuator force/torque. The article has presented both a stability proof and simulation results. Although our approach is theoretically applicable to land, air, surface, and underwater vehicles, this article focuses on land robots.

The plan for future is to design a similar controller, to extend it to 3D case and apply it to an underwater vehicle.

Appendix I
Command filter

The purpose of this appendix is to provide an example and discussion of a command filter. Advanced control approaches often assume the availability of a continuous and bounded desired trajectory $y_c(t)$ and its first r derivatives $y_c^{(r)}(t)$. The first time that this assumption is encountered it may seen unreasonable, since a user will often only specify a command signal $y_d(t)$. However, this assumption can always be satisfied by passing the commanded signal $y_d(t)$ through a single-input, multi-output prefilter. This procedure is explained in detail in for example [6].

Throughout, this article refers to filtering of a signal x_c^o to produce a bandwidth limited signal x_c and its derivative $\dot{x}_c$. The state space implementation of such a filter is

$$\begin{aligned} \dot{x}_1 &= x_2 \\ \dot{x}_2 &= -2\zeta w_n x_2 - w_n^2\,(x_1 - x_c^o) \end{aligned}$$

where $x_c = x_1$ and $\dot{x}_c = x_2$. Note that if x_c^o is bounded, then x_c and $\dot{x}_c$ are bounded and continuous. The transfer function from x_c^o to x_c is

$$\frac{X_c(s)}{X_c^o(s)} = H(s) = \frac{w_n^2}{s^2 + 2\zeta w_n s + w_n^2} \qquad (38)$$

which has a unity gain at low frequencies, damping ratio ζ and undamped natural frequency ω_n. The error $|x_c^o(t) - x_c(t)|$ is small if the bandwidth of $x_c^o(s)$ is less than the bandwidth of $H(s)$. If the bandwidth of x_c^o is known and the goal of the filter is to generate x_c and its derivative with $|x_c^o - x_c|$ small, then the designer simply chooses ω_n sufficiently large.

Note that the signal $\dot{x}_c$ is computed by integration, not differentiation. This helps to decrease the effects of measurement noise; nonetheless, noise will impose a tradeoff in how large of a value can be selected for ω_n.

Appendix II
Derivation of Eqn. (20)

The purpose of this appendix is to show that $\tilde{u}_x$ and $\tilde{u}_y$ terms can be expressed in terms of $\tilde{u}$ and $\tilde{\psi}$ as in eqn. (20). From eqns. (11), (13), and (16) we have the relationship

$$\begin{bmatrix} \tilde{u}_x \\ \tilde{u}_y \end{bmatrix} = u \begin{bmatrix} c\psi \\ s\psi \end{bmatrix} - u_c \begin{bmatrix} c\psi_c \\ s\psi_c \end{bmatrix} \qquad (39)$$

Defining the function $g(\tilde{\psi})$ as

$$g(\tilde{\psi}) = \left[\frac{\cos(\tilde{\psi}) - 1}{\tilde{\psi}},\ \frac{\sin(\tilde{\psi})}{\tilde{\psi}} \right]^\top, \qquad (40)$$

two similar approaches can be used.

1) If the vector $[u\ c\psi_c, u\ s\psi_c]^\top$ is added and subtracted from the eqn. (39), then the result after manipulations has the form
$$\begin{bmatrix} \tilde{u}_x \\ \tilde{u}_y \end{bmatrix} = \begin{bmatrix} c\psi_c \\ s\psi_c \end{bmatrix} \tilde{u} + u \begin{bmatrix} c\psi_c & -s\psi_c \\ s\psi_c & c\psi_c \end{bmatrix} g(\tilde{\psi})\tilde{\psi}$$

2) If the vector $[u_c\ c\psi, u_c\ s\psi]^\top$ is added and subtracted from the eqn. (39), then the result after manipulations has the form
$$\begin{bmatrix} \tilde{u}_x \\ \tilde{u}_y \end{bmatrix} = \begin{bmatrix} c\psi \\ s\psi \end{bmatrix} \tilde{u} + u_c \begin{bmatrix} c\psi & -s\psi \\ s\psi & c\psi \end{bmatrix} g(\tilde{\psi})\tilde{\psi}$$

Throughout the body of the article, we will use Approach 1. Therefore, we have

$$A = \begin{bmatrix} c\psi_c \\ s\psi_c \end{bmatrix} \text{ and } B = \begin{bmatrix} c\psi_c & -s\psi_c \\ s\psi_c & c\psi_c \end{bmatrix} \qquad (41)$$

which will be used in Section VII to derive the signals u_{bs} and ψ_{bs}.

Appendix III
Selection of Control Gain

Eqn. (18) has the form,

$$\begin{bmatrix} u_c^o\ c\psi_c^o \\ u_c^o\ s\psi_c^o \end{bmatrix} = v_d - K_{xy}E$$

where

$$v_d = \begin{bmatrix} \dot{x}_c \\ \dot{y}_c \end{bmatrix} \text{ and } E = \begin{bmatrix} \tilde{x} \\ \tilde{y} \end{bmatrix}$$

The quantity v_d is the velocity vector that causes the vehicle to follow the trajectory given that the vehicle was currently on the trajectory. The quantity $K_{xy}E$ is the feedback term that causes the vehicle to converge toward the trajectory.

The definition of $(u_{x_c}^o, u_{y_c}^o)$ from eqn. (18) defines (u_c^o, ψ_c^o) as in eqn. (15). This subsection discusses technical details related to the selection of K_{xy} in eqn. (18). The gain K_{xy} is changed as a function of the control state information to achieve two objectives. First, the vehicle yaw should never differ from the trajectory heading by more than 90 degrees.

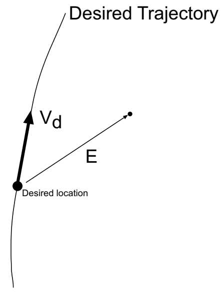

Fig. 1. Trajectory depiction

Second, the norm of the the term $K_{xy}E$ should always be less than a parameter $\bar{u} > 0$. The purpose of the second constraint is to ensure that poor initial conditions do not yield unreasonably large speed commands to the vehicle.

For the stability analysis, the value of K_{xy} must be positive; however, its magnitude can be manipulated to achieve secondary performance objectives such as those stated above. Consider the situation depicted in Figure 1 where the inner product of v_d and E is positive (i.e., the robot is ahead of the current desired trajectory position). If the speed is selected to be positive (i.e., $\gamma = 1$), depending on the value of K_{xy}, the commanded yaw angle could result in the vehicle circling to get to the desired location. In particular, when $K_{xy}\|E\| > \|v_d\|$, the vehicle yaw may be commanded in a direction opposite to the direction of v_d. Typically, this is not desirable. To prevent this we must ensure that the angle between v_d and $v_d - K_{xy}E$ is less than $90\,\text{deg}$:

$$\langle v_d, (v_d - K_{xy}E)\rangle \geq 0 \quad (42)$$

$$\|v_d\|^2 \geq K_{xy}\langle v_d, E\rangle. \quad (43)$$

There are three possible cases:

1) $\langle v_d, E\rangle > 0$: This is the problematic case that could result in the vehicle pointing opposite to the desired velocity if K_{xy} is too big. The value of K_{xy} should be selected such that
$$K_{xy} \leq \frac{\|v_d\|^2}{\langle v_d, E\rangle}.$$
2) $\langle v_d, E\rangle = 0$: In this case, the value of K_{xy} does not matter.
3) $\langle v_d, E\rangle < 0$: In this case, any positive value of K_{xy} satisfies eqn. (43).

Therefore, the designer specifies positive constants $\bar{k}$ and $0 < \alpha < 1$. The parameter $\bar{k}$ is the largest allowable position control feedback gain. At each time instant, the value of K_{xy} is selected as

$$K_{xy}(t) = \begin{cases} \min\left(\bar{k}, \frac{\bar{u}}{\|E\|}\right) & \text{if} \quad \langle v_d, E\rangle \leq 0 \\ \min\left(\frac{\alpha\|v_d\|^2}{\langle v_d, E\rangle}, \bar{k}, \frac{\bar{u}}{\|E\|}\right) & \text{if} \quad \langle v_d, E\rangle > 0. \end{cases}$$

This definition of $K_{xy}(t)$ is a positive, continuous function of time. In situations such as that in Figure 1, this approach results in the vehicle driving towards the trajectory with the tangential component small enough that the trajectory point will ultimately catch up to the vehicle. In the case where the vehicle is on the trajectory directly in front of the desired location, this approach causes the vehicle to drive slower than the desired point is moving, in effect waiting for the desired position to catch up. The same approach works for the $\gamma = -1$ (backward driving) case.

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