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Nonlinear Sampled-data Stabilization Control for Ships

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Abstract— **In this paper we consider the nonlinear sampled-data stabilization control for ships. Using the nonlinear sampled-data control theory developed by Nesic et al and the integrator backstepping technique for the Euler approximate model, we design both semiglobally practically asymptotically (SPA) stabilizing state feedback laws and SPA stabilizing output feedback controllers. We give a numerical example to illustrate the design methods.**

I. INTRODUCTION

From the beginining of 20th century, the design of control systems for ships has been actively considered. PID contropl, linear quadratic optimal control, Kalman filtering, H_∞ control, sliding mode control, feedback linearization and etc have been applied to design control systems for ships (for details see [2], [12] and references therein). Recently nonlinear backstepping design techniques ([2], [13]), the control problems of underactuated dynamic positioning [14] and the nonlinear observer design technique [6] for ships have been also discussed. The analysis and synthesis of the control problems for ships have been considered based on linear or nonlinear continuous-time model of ships and the design methods of continuous-time controllers have been mainly discussed.

Practical and modern control systems usually use digital computers as discrete-time controllers with samplers (A/D converters) and zero-order holds (D/A converters) to control continuous-time systems [1]. Such a control system involves both continuous-time and discrete-time signals in a continuous-time framework and is called a sampled-data system. For linear systems, the sampled-data control theory has been widely studied. Specially, using so called lifting thechnique, the design methods of sampled-data H_2 controllers and sampled-data H_∞ controllers have been developed [1]. On the other hand, due to the difficulty to find equivalent and useful discrete-time models of sampled-data nonlinear systems, the sampled-data control theory for nonlinear systems has not been developed compared to linear systems. But recently, the framework for design of nonlinear sampled-data systems by discrete-time approximate models of the nonlinear sampled-data systems is proposed ([5], [10] and [11]). Several design methods such as the emulation, the controller redesign of continuous-time controllers [7], integrator backstepping for discrete-time nonlinear systems [9] and the controller design by receding horizon methods [8] have been proposed to guarantee the stability of nonlinear sampled-data systems. But as mentioned in [5], case studies and practical implementations of the controllers designed in the framework of the nonlinear sampled-data systems have not been discussed in the literatures.

In this paper we consider the design of stabilization controllers for ships in this framework. We first summarize the framework for the design of nonlinear sampled-data systems by its approximate model proposed in [5], [10] and [11]. We then extend the integrator backstepping method for nonlinear single-input discrete-time systems [9] to the case of nonlinear multi-input discrete-time systems. We apply the extended result to the stabilization control for ships. We first apply the extended result directly to design semiglobally practically asymptotically (SPA) stabilizing state feedback laws. Following the approach [2] and [3] we then design global asymptotical convergent nonlinear observers for the Euler approximate model and SPA stabilizing outpu feedback controllers. We also give a numerical example to illustrate the design methods.

Notations: Let $\mathbf{N}$, $\mathbf{R}$ and $\mathbf{R}_{\geq 0}$ be the sets of natural numbers, real numbers and nonnegative real numbers, respectively. Let $\mathbf{C}^- = \{\lambda = \alpha + i\beta | \alpha < 0\}$ and $\mathbf{D} = \{\lambda = \alpha + i\beta | \sqrt{\alpha^2 + \beta^2} < 1\}$. Let $\sigma(M)$ be the set of all eigenvalues of a square matrix M. Let $\| x \|$ be the norm of a vector x given by $\| x \| = \sqrt{x^T x}$. A function α is of class K if it is continuous, zero at zero and strictly increasing. It is of class K_∞ if it is of class K and unbounded. A function β: $\mathbf{R}_{\geq 0} \times \mathbf{R}_{\geq 0} \to \mathbf{R}_{\geq 0}$ is of class KL if for any fixed $t \geq 0$, the function $\beta(\cdot, t)$ is of class K and for each fixed $s \geq 0$ the function $\beta(s, \cdot)$ is deceasing to zero as its argument tends to infinity [4].

II. DESIGN OF NONLINEAR SAMPLED-DATA SYSTEMS BY EULER APPROXIMATE MODEL

A. The Framework for the Design of Nonlinear Sampled-data Systems

Here we summarize the framework for the design of nonlinear sampled-data systems by Euler approximate model. For details see [5], [10] and [11].

Consider the nonlinear continuous-time system

$$\dot{x} = f(x, u), \ x(0) = x_0 \tag{1}$$

where $x \in \mathbf{R}^n$ is the state, $u \in \mathbf{R}^m$ is the control input realized through a zero-order hold, i.e., $u(t) = u(k)$, $^\forall t \in [kT, (k+1)T)$ and $T > 0$ is a sampling period. Here we assume that for each initial condition and each constant control, there exists a unique solution of (1) defined on some bounded interval of the form $[0, \tau)$. We also assume that the sampling period is a design parameter and can be assigned arbitrarily.

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The difference equations corresponding to the exact discrete-time model and the Euler approximate model of (1) are denoted by

$$x(k+1) = F_T^e(x(k), u(k)), \quad (2)$$
$$x(k+1) = F_T^{Euler}(x(k), u(k)) \quad (3)$$

respectively, where $x(k) := x(kT)$ to avoid the complexity of notations and F_T^e, F_T^{Euler} are given by

$$F_T^e(x(k), u(k)) = x(k) + \int_{kT}^{(k+1)T} f(x(s), u(k))ds,$$

and $F_T^{Euler}(x(k), u(k)) = x(k) + Tf(x(k), u(k))$, respectively. To define semiglobal practical asymptotic (SPA) stability and SPA stability Lyapunov functions, we consider the following discrete-time system

$$x(k+1) = F_T(x(k), u_T(x(k))). \quad (4)$$

Definition 2.1: The system (4) is semiglobal practical asymptotic (SPA) stable if there exists $\beta \in \text{class}KL$ such that for any strictly positive real numbers (D, d), there exists $T^* > 0$ such that for all $T \in (0, T^*)$ and all initial state $x(0)$ with $\| x(0) \| \leq D$, the solution of (4) satisfies $\| x(k) \| \leq \beta(\| x(0) \|, kT) + d$.

Definition 2.2: Let $\hat{T} > 0$ be given and for each $T \in (0, \hat{T})$ let function $V_T : \mathbf{R}^n \to \mathbf{R}_{\geq 0}$ and $u_T : \mathbf{R}^n \to \mathbf{R}^m$ be defined. Then (u_T, V_T) is called a SPA stabilizing pair for F_T if there exist α_1, α_2, $\alpha_3 \in \text{class}K_\infty$ such that for any strictly positive real numbers (Δ, δ) there exist strictly positive real numbers (T^*, L, M) with $T^* < \hat{T}$ such that for all x, $z \in \mathbf{R}^n$ with $\max\{\| x \|, \| z \|\} \leq \Delta$ and $T \in (0, T^*)$

$$\alpha_1(\| x \|) \leq V_T(x) \leq \alpha_2(\| x \|), \quad (5)$$
$$V_T(F_T(x, u_T(x))) - V_T(x) \leq -T\alpha_3(\| x \|) + T\delta, \quad (6)$$
$$|V_T(x) - V_T(z)| \leq L \| x - z \|, \quad (7)$$
$$\| u_T \| \leq M. \quad (8)$$

Theorem 2.1: If (u_T, V_T) is a SPA stabilizing pair for F_T^{Euler}, then u_T SPA stabilizes F_T^e.

Remark 2.1: 1) If the sampling period is sufficiently small and F_T^e is locally Lipschitz, then u_T, which SPA stabilizes F_T^e, SPA stabilizes (1), i.e, the system $\dot{x} = f(x, u_T(x(k)))$, $t \in [kT, (k+1)T)$ is SPA stable ([5], [11]).
2) If u_T SPA (or globally asymptotically (GA)) stabilizes F_T^{Euler}, then u_T SPA stablizes (1) under the conditions that the sampling period $T > 0$ is sufficiently small and F_T^e is locally Lipschitz.
3) If the original nonlinear system has a strictly feedback form, then its Euler approximate model also has a strict feedback form.

B. Integrator Backstepping

Consider the nonlinear system of a strict feedback form

$$\dot{x}_1 = f(x_1) + g(x_1)x_2, \quad \dot{x}_2 = u \quad (9)$$

where $x_1 \in \mathbf{R}^n$, $x_2 \in \mathbf{R}^m$, $f(0) = 0$, f, g are differentiable sufficiently many times and the control input $u(t)$ is realized through a zero-order hold. Then the Euler approximate model of (9) is given by

$$x_1(k+1) = r_T(x_1(k), x_2(k)), \quad (10)$$
$$x_2(k+1) = x_2(k) + Tu(k) \quad (11)$$

where $r_T(x_1, x_2) = x_1 + T[f(x_1) + g(x_1)x_2]$. Next theorem is an extension of the result in [9] to the multi-input nonlinear discrete-time system (10) and (11). The proof of this theorem is given in Appendix.

Theorem 2.2: Assume that there exist $\hat{T} > 0$ and (ϕ_T, W_T) that is defined for each $T \in (0, \hat{T})$ and that is a SPA stabilizing pair for the subsystem (10) with a vertial control $x_2 \in \mathbf{R}^m$. Suppose
1) ϕ_T and W_T are continuously differentiable for any $T \in (0, \hat{T})$.
2) there exists $\tilde{\varphi} \in \text{class}K_\infty$ such that $\| \phi_T(x_1) \| \leq \tilde{\varphi}(\| x_1 \|)$ for all $x_1 \in \mathbf{R}^n$ and $T \in (0, \hat{T})$.
3) for any $\tilde{\Delta} > 0$ there exist strictly positive numbers $(\tilde{T}, \tilde{M})$ such that for each $T \in (0, \tilde{T})$ and $\| x_1 \| \leq \tilde{\Delta}$ we have

$$\max\left\{\| \frac{\partial W_T}{\partial x_1} \|, \| \frac{\partial \phi_T}{\partial x_1} \|\right\} \leq \tilde{M}.$$

Then there exists a SPA stabilizing pair (u_T, V_T) for (10) and (11). In particular we can take

$$u_T(x) = -c[x_2 - \phi_T(x_1)] - \frac{\Delta\tilde{W}_T(x)}{T} + \frac{\Delta\phi_T(x)}{T}, \quad (12)$$
$$V_T(x) = W_T(x_1) + \frac{1}{2} \| x_2 - \phi_T(x_1) \|^2 \quad (13)$$

where $c > 0$ is arbitrary, $x = [\, x_1^T \quad x_2^T \,]^T$ and

$$\Delta\phi_T(x) = \phi_T(r_T) - \phi_T(x_1),$$
$$\Delta\tilde{W}_T(x) = \begin{cases} \frac{\Delta\bar{W}_T(x)[x_2 - \phi_T(x_1)]}{\|x_2 - \phi_T(x_1)\|^2}, & x_2 \neq \phi_T(x_1), \\ Tg^T(x_1)\left(\frac{\partial W_T}{\partial x_1}\right)^T(r_T), & x_2 = \phi_T(x_1), \end{cases}$$
$$\Delta\bar{W}_T(x) = W_T(r_T) - W_T(r_T^\phi),$$
$$r_T^\phi = x_1 + T[f(x_1) + g(x_1)\phi_T(x_1)].$$

III. Nonlinear Sampled-data Control for Ships

We first introduce the following notations to describe the equation of motion of a ship. Let n, e and ψ be the North and the East positions of a ship and the yaw angle (orientation) of a ship, respectively in the Earth-fixed coordinate system and let μ, v and r be linear velocities in surge, sway and the angular velocity in yaw, i.e., $r = \dot{\psi}$, respectively, decomposed in the body-fixed coordinate system (Figure 1). Let $\eta = [\, n \quad e \quad \psi \,]^T$ and $\nu = [\, \mu \quad v \quad r \,]^T$.

In the dynamic positioning (DP) problems, the speed of a ship is quite small ($\mu \simeq 0$, $v \simeq 0$, $r \simeq 0$) and we can assume that the damping forces are linear [2]. Hence the equation of motion of a ship can be written as

$$\dot{\eta} = R(\psi(t))\nu, \quad (14)$$
$$\dot{\nu} = A\nu + Bu \quad (15)$$

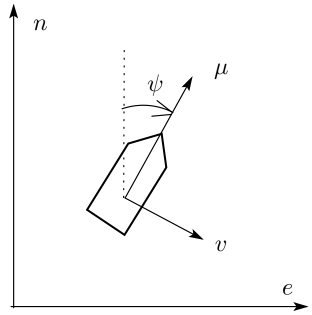

Fig. 1. Coordinate systems

where $A = -M^{-1}D$, $B = M^{-1}$,

$$R(\psi) = \begin{bmatrix} \cos\psi & -\sin\psi & 0 \\ \sin\psi & \cos\psi & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

is the rotation matrix in yaw, M is the inertia matrix including hydrodynamic added inertia, D is the damping matrix and the control forces and moment $u = [\, u_1 \quad u_2 \quad u_3 \,]^T$ are provided by thrusters. Note that $R^{-1}(\psi) = R^T(\psi)$ and $R(\psi)$ is bounded for any ψ.

If we set $u(t) = u(k)$, $t \in [kT, (k+1)T)$, then the Euler approximate model of (14) and (15) is given by

$$\begin{aligned} \eta(k+1) &= \eta(k) + TR(\psi(k))\nu(k), && (16) \\ \nu(k+1) &= \nu(k) + T[A\nu(k) + Bu(k)] && (17) \end{aligned}$$

where $\eta(k) := \eta(kT)$, $\nu(k) := \nu(kT)$ and $\psi(k) := \psi(k)$.

A. Design of SPA Stabilizing State Feedback Laws

We first assume that both η and ν are available to the controller. Using the input transformation $u = B^{-1}(u_a - A\nu)$, (16) and (17) can be rewritten as

$$\begin{aligned} \eta(k+1) &= \eta(k) + TR(\psi(k))\nu(k), && (18) \\ \nu(k+1) &= \nu(k) + Tu_a(k). && (19) \end{aligned}$$

Then a state feedback law $\phi_T(\eta)$ which globally asymptotically stabilizes the subsystem (18) with ν viewed as a vertial input and the corresponding Lyapunov function $W_T(\eta)$ are given by $\phi_T(\eta) = R^T(\psi)L\eta$ and $W_T(\eta) = \frac{1}{2}\eta^T\eta$, respectively, where L can be chosen such that $L = \mathrm{diag}\{l_1, l_2, l_3\}$ with $|1 + Tl_i| < 1$. Since $R(\psi)$ is bounded for any ψ, it is obvious that $\phi_T(\eta)$ and $W_T(\eta)$ satisfy all conditions in Thereom 2.2 and we have $r_T^\phi = (I + TL)\eta$ and $\partial W_T/\partial\eta = \eta^T$. The feedback law which SPA stabilizes (18) and (19) is given by

$$u_{aT}(x) = -c[\nu - \phi_T(\eta)] - \frac{\Delta\tilde{W}_T(x)}{T} + \frac{\Delta\phi_T(x)}{T} \qquad (20)$$

where $c > 0$ is arbitrary, $x = [\, \eta^T \quad \nu^T \,]^T$ and

$$\begin{aligned} \Delta\phi_T(x) &= \phi_T(\eta + TR(\psi)\nu) - \phi_T(\eta), \\ \Delta\tilde{W}_T(x) &= \begin{cases} \frac{\Delta\bar{W}_T(x)[\nu - \phi_T(\eta)]}{\|\nu - \phi_T(\eta)\|^2}, & \nu \neq \phi_T(\eta), \\ TR^T(\psi)[\eta + TR(\psi)\nu], & \nu = \phi_T(\eta), \end{cases} \\ \Delta\bar{W}_T(x) &= W_T(\eta + TR(\psi)\nu) - W_T((I + TL)\eta). \end{aligned}$$

Note that

$$\begin{aligned} &\phi_T(\eta(k) + TR(\psi(k))\nu(k)) \\ = \;& R^T(\psi(k+1))L[\eta(k) + TR(\psi(k)\nu(k)] \end{aligned}$$

and $\psi(k+1)$ is given by $\psi(k+1) = \psi(k) + Tr(k)$. Hence the state feedback controller

$$u(x(k)) = B^{-1}[u_{aT}(x(k)) - A\nu(k)] \qquad (21)$$

SPA stabilizes (16) and (17), i.e., the closed-loop system $z(k+1) = \Phi(x(k))z(k)$ is SPA stable where $z = [\, x^T \quad z_2^T \,]^T$, $z_2 = \nu - \phi_T(\eta)$ and

$$\Phi = \begin{cases} \begin{bmatrix} I + TL & TR(\psi) \\ 0 & (1 - cT - \frac{\Delta\bar{W}_T(x)}{\|z_2\|^2})I \end{bmatrix}, \\ \qquad\qquad \nu \neq \phi_T(\eta), \\ \begin{bmatrix} I + TL & TR(\psi) \\ -TR^T(\psi)(I + TL) & (1 - cT - T^2)I \end{bmatrix}, \\ \qquad\qquad \nu = \phi_T(\eta). \end{cases} \qquad (22)$$

B. Design of Output Feedback SPA Stabilizing Controllers

Since the position η is usually available to the controller in the control problems of a ship, we introduce the sampled observation $y(k) = \eta(k)$ for (14) and (15). We assume that A is stable, i.e., $\sigma(A) \subset \mathbf{C}^-$. This is a technical assumption, but some small size or middle size ships satisfy this assumption [2]. Then the Euler approximate model of (14), (15) with $y(k) = \eta(k)$ is given by

$$\begin{aligned} \eta(k+1) &= \eta(k) + TR(\psi(k))\nu(k), && (23) \\ \nu(k+1) &= A_d\nu(k) + B_du(k), && (24) \\ y(k) &= \eta(k) && (25) \end{aligned}$$

where $A_d = I + TA$ and $B_d = TB$. Since $\sigma(A) \subset \mathbf{C}^-$, we can make $\sigma(A_d) \subset \mathbf{D} \setminus \{0\}$ for sufficiently small $T > 0$. Following the approach [2] and [3] we shall design global asymptotical convergent observers and output feedback SPA stabilizing controllers.

For (23)-(25) we consider the observer of the form

$$\begin{aligned} \begin{bmatrix} \hat{\eta} \\ \hat{\nu} \end{bmatrix}(k+1) &= \begin{bmatrix} I & TR(\psi(k)) \\ 0 & A_d \end{bmatrix}\begin{bmatrix} \hat{\eta} \\ \hat{\nu} \end{bmatrix}(k) + \begin{bmatrix} 0 \\ B_d \end{bmatrix}u(k) \\ &\quad + \begin{bmatrix} K_1 \\ K_2 \end{bmatrix}(k)\{y(k) - \hat{\eta}(k)\}. \qquad (26) \end{aligned}$$

Let $\tilde{\eta} = \eta - \hat{\eta}$ and $\tilde{\nu} = \nu - \hat{\nu}$. Then we have

$$\begin{bmatrix} \tilde{\eta} \\ \tilde{\nu} \end{bmatrix}(k+1) = \begin{bmatrix} I - K_1(k) & TR(\psi(k)) \\ -K_2(k) & A_d \end{bmatrix}\begin{bmatrix} \tilde{\eta} \\ \tilde{\nu} \end{bmatrix}(k). \qquad (27)$$

For (27) we introduce

$$V_o(k) = \frac{1}{2}\begin{bmatrix} \tilde{\eta} \\ \tilde{\nu} \end{bmatrix}^T(k)\begin{bmatrix} P_1 & 0 \\ 0 & P_2 \end{bmatrix}\begin{bmatrix} \tilde{\eta} \\ \tilde{\nu} \end{bmatrix}(k) \qquad (28)$$

where P_1 and P_2 are positive definite matrices. Then we have

$$\begin{aligned} &V_o(k+1) - V_o(k) \\ = \;& \frac{1}{2}\tilde{\eta}^T[(I - K_1)^T P_1(I - K_1) - P_1 + K_2^T P_2 K_2]\tilde{\eta} \\ &+ \frac{1}{2}\tilde{\nu}^T[A_d^T P_2 A_d - P_2 + T^2 R^T(\psi) P_1 R(\psi)]\tilde{\nu} \\ &+ \tilde{\nu}^T[TR^T(\psi)P_1(I - K_1) - A_d^T P_2 K_2]\tilde{\eta}. \end{aligned}$$

Here we want to find P_1, P_2, K_1 and K_2 which satisfy

$$\begin{aligned}(I-K_1)^T P_1 (I-K_1) - P_1 & \\ +K_2^T P_2 K_2 &\leq -Q_1, \quad (29)\\ A_d^T P_2 A_d - P_2 + T^2 R^T(\psi) P_1 R(\psi) &\leq -Q_2, \quad (30)\\ TR^T(\psi)P_1(I-K_1) - A_d^T P_2 K_2 &= 0 \quad (31)\end{aligned}$$

where Q_1 and Q_2 are some positive definite matrices. Since $\sigma(A_d) \subset \mathbf{D} \setminus \{0\}$ for sufficiently small $T > 0$, there exists $P_2 > 0$ such that

$$A_d^T P_2 A_d - P_2 = -(Q_2 + \hat{Q}_2) \quad (32)$$

for any given $Q_2, \hat{Q}_2 > 0$. Since $\sigma(A_d) \subset \mathbf{D} \setminus \{0\}$ and $P_2 > 0$, we can set

$$K_2(k) = TP_2^{-1} A_d^{-T} R^T(\psi(k)) P_1 (I - K_1). \quad (33)$$

Let

$$K_1 = \mathrm{diag}\{k_1, k_2, k_3\} \quad (34)$$

be fixed for some $0 < k_i < 2$, $i = 1, 2, 3$. Then for any given $Q_1, \hat{Q}_1 > 0$, there exists $P_1 > 0$ such that

$$(I-K_1)^T P_1 (I-K_1) - P_1 = -(Q_1 + \hat{Q}_1). \quad (35)$$

Since we can choose $T > 0$ sufficiently small such that

$$\begin{aligned}K_2^T P_2 K_2 &\leq \hat{Q}_1, \quad (36)\\ T^2 R^T(\psi) P_1 R(\psi) &\leq \hat{Q}_2, \quad (37)\end{aligned}$$

we obtain (29)-(31). Consequently the observer (26) with (33) and (34) achieves the globally asymptotically convergence of $(\hat{\eta}, \hat{\nu})$ to (η, ν) for sufficiently small $T > 0$.

Now we consider the following state feedback law

$$\begin{aligned}u(\hat{x}(k)) &= B^{-1}[\bar{u}_{aT}(\hat{x}(k)) - A\hat{\nu}(k)], \quad (38)\\ \bar{u}_{aT}(\hat{x}) &= -c[\hat{\nu} - \phi_T(\hat{\eta})] - \frac{\Delta \tilde{W}_T(\hat{x})}{T} + \frac{\Delta \bar{\phi}_T(\hat{x})}{T}\end{aligned}$$

where $c > 0$ is arbitrary, $\hat{x} = [\,\hat{\eta}^T \quad \hat{\nu}^T\,]^T$,

$$\begin{aligned}\Delta \tilde{W}_T(\hat{x}) &= \begin{cases} \frac{\Delta \bar{W}_T(\hat{x})[\hat{\nu} - \phi_T(\hat{\eta})]}{\|\hat{\nu} - \phi_T(\hat{\eta})\|^2}, & \hat{\nu} \neq \phi_T(\hat{\eta}),\\ TR^T(\psi)[\hat{\eta} + TR(\psi)\hat{\nu}], & \hat{\nu} = \phi_T(\hat{\eta}),\end{cases}\\ \Delta \bar{W}_T(\hat{x}) &= W_T(\hat{\eta} + TR(\psi)\hat{\nu}) - W_T((I + TL)\hat{\eta}),\\ \Delta \bar{\phi}_T(k) &= \bar{\phi}_T(k+1) - \phi_T(\hat{\eta}(k)),\\ \phi_T(\hat{\eta}(k)) &= R^T(\psi(k)) L \hat{\eta}(k),\\ \bar{\phi}_T(k+1) &= R^T(\hat{\psi}(k+1)) L[\hat{\eta}(k) + TR^T(\psi(k))\hat{\nu}(k)],\\ W_T(\hat{\eta}) &= \frac{1}{2}\hat{\eta}^T \hat{\eta}\end{aligned}$$

and L is chosen such that $L = \mathrm{diag}\{l_1, l_2, l_3\}$ with $|1 + Tl_i| < 1$, $i = 1, 2, 3$. If we can replace $\bar{\phi}_T(k+1)$ by

$$\phi_T(k+1) = R^T(\psi(k+1)) L[\hat{\eta}(k) + TR^T(\psi(k))\hat{\nu}(k)],$$

then, by Theorem 2.2, (38) SPA stabilizes

$$\begin{aligned}\hat{\eta}(k+1) &= \hat{\eta}(k) + TR(\psi(k))\hat{\nu}(k), \quad (39)\\ \hat{\nu}(k+1) &= \hat{\nu}(k) + T[A\hat{\nu}(k) + Bu(k)] \quad (40)\end{aligned}$$

and the corresponding Lyapunov function is given by

$$V_T(\hat{x}) = W_T(\hat{\eta}) + \frac{1}{2} \parallel \hat{\nu} - \phi_T(\hat{\eta}) \parallel^2 .$$

Note that $\bar{u}_{aT}(\hat{x})$ can be rewritten as

$$\bar{u}_{aT}(\hat{x}) = u_{aT}(\hat{x}) + \frac{1}{T}[\bar{\phi}_T(k+1) - \phi_T(k+1)]$$

where $u_{aT}(\hat{x})$ is given by (20) with x replaced by $\hat{x}$.

Now we shall show that the output feedback controller (26), (33), (34) and (38) SPA stabilizes (23)-(25). Let $\hat{z}_2 = \hat{\nu} - \phi_T(\hat{\eta})$, $\hat{z} = [\,\hat{\eta}^T \quad \hat{z}_2^T\,]^T$ and $\tilde{x} = [\,\tilde{\eta}^T \quad \tilde{\nu}^T\,]^T$. Then the closed-loop system (23)-(25) with (26), (33), (34) and (38) can be written as

$$\begin{aligned}\hat{z}(k+1) &= \hat{\Phi}(k)\hat{z}(k) + \Gamma(k)\hat{z}(k) + K(k)\tilde{\eta}(k),\\ \tilde{x}(k+1) &= \begin{bmatrix} I - K_1 & TR(\psi(k))\\ -K_2(k) & A_d\end{bmatrix}\tilde{x}(k) \quad (41)\end{aligned}$$

where $\hat{\Phi}$ is given by (22) with x and z replaced by $\hat{x}$ and $\hat{z}$, respectively and

$$\begin{aligned}\Gamma(k) &= \hat{B}(k+1) L \hat{C}(k),\\ \hat{B}(k+1) &= \begin{bmatrix} 0 \\ R^T(\hat{\psi}(k+1)) - R^T(\psi(k+1))\end{bmatrix},\\ \hat{C}(k) &= [\, I + TL \quad TR(\psi(k))\,],\\ K(k) &= \begin{bmatrix} K_1 \\ K_2(k) - R^T(\psi(k+1)) L K_1\end{bmatrix}.\end{aligned}$$

To show that the closed-loop system (41) is SPA stable, we introduce the following Lyapunov function candidate

$$V(k) = V_T(\hat{z}(k)) + V_o(\tilde{x}(k)) \quad (42)$$

where $V_o(\tilde{x}(k))$ is given by (28) and

$$V_T(\hat{z}) = W_T(\hat{\eta}) + \frac{1}{2}\hat{z}_2^T \hat{z}_2 = \frac{1}{2}(\hat{\eta}^T \hat{\eta} + \hat{z}_2^T \hat{z}_2).$$

Then we want to show that the conditions (5)-(8) are satisfied for (26), (33), (34), (38) and (42). It is obvious that the conditions (5), (7) and (8) are satisfied and hence it is enough to show (6). By (29)-(31) we have

$$V_o(\tilde{x}(k+1)) - V_o(\tilde{x}(k)) \leq -\frac{1}{2}\tilde{x}^T(k)\mathrm{diag}\{Q_1, Q_2\}\tilde{x}(k).$$

We also have

$$\begin{aligned}& V_T(\hat{z}(k+1)) - V_T(\hat{z}(k))\\ = & \frac{1}{2}\hat{z}^T(k)[\hat{\Phi}^T(k)\hat{\Phi}(k) - I]\hat{z}(k)\\ & + \frac{1}{2}(\hat{z}^T \Gamma^T \Gamma \hat{z})(k) + \frac{1}{2}(\tilde{\eta}^T K^T K \tilde{\eta})(k)\\ & + (\hat{z}^T \hat{\Phi}^T \Gamma \hat{z})(k) + (\hat{z}^T \Gamma^T K \tilde{\eta})(k) + (\tilde{\eta}^T K^T \hat{\Phi} \hat{z})(k)\\ \leq & \frac{1}{2}\hat{z}^T[\hat{\Phi}^T\hat{\Phi} - I]\hat{z} + \frac{1}{2}\hat{z}^T \Gamma^T \Gamma \hat{z} + \frac{1}{2}\tilde{\eta}^T K^T K \tilde{\eta}\\ & + \frac{a}{2}\hat{z}^T \hat{\Phi}^T \hat{\Phi} \hat{z} + \frac{1}{2a}\hat{z}^T \Gamma^T \Gamma \hat{z} + \frac{b}{2}\hat{z}^T \Gamma^T \Gamma \hat{z}\\ & + \frac{1}{2b}\tilde{\eta}^T K^T K \tilde{\eta} + \frac{c}{2}\tilde{\eta}^T K^T K \tilde{\eta} + \frac{1}{2c}\hat{z}^T \hat{\Phi}^T \hat{\Phi} \hat{z}\\ = & \frac{1}{2}(1 + a + \frac{1}{c})\hat{z}^T[\hat{\Phi}^T\hat{\Phi} - I]\hat{z} + (a + \frac{1}{c}) \parallel \hat{z} \parallel^2\\ & + \frac{1}{2}(1 + \frac{1}{a} + b)\hat{z}^T \Gamma^T \Gamma \hat{z} + \frac{1}{2}(1 + \frac{1}{b} + c)\tilde{\eta}^T K^T K \tilde{\eta}\end{aligned}$$

for any a, b, $c > 0$ where we have used the inequality

$$2x^T N^T My \leq ax^T N^T Nx + \frac{1}{a} y^T M^T My$$

for any $a > 0$ in the first inequality. Since $R(\psi)$ is bounded for any ψ, there exists $m > 0$ such that $\frac{1}{2}(1 + \frac{1}{a} + b)\hat{z}^T \Gamma^T \Gamma \hat{z} \leq m \parallel \hat{z} \parallel^2$ and we have

$$\begin{aligned} & V_T(\hat{z}(k+1)) - V_T(\hat{z}(k)) \\ \leq \ & a_1 \hat{z}^T(k)[\hat{\Phi}^T(k)\hat{\Phi}(k) - I]\hat{z}(k) + a_2 \parallel \hat{z}(k) \parallel^2 \\ & + a_3 \tilde{\eta}^T(k) K^T(k) K(k) \tilde{\eta}(k) \end{aligned}$$

where $a_1 = \frac{1}{2}(1 + a + \frac{1}{c})$, $a_2 = a + \frac{1}{c} + m$ and $a_3 = \frac{1}{2}(1 + \frac{1}{b} + c)$. Hence we have

$$\begin{aligned} & V(k+1) - V(k) \\ \leq \ & a_1 \hat{z}^T(k)[\hat{\Phi}^T(k)\hat{\Phi}(k) - I]\hat{z}(k) + a_2 \parallel \hat{z}(k) \parallel^2 \\ & - \frac{1}{2}\tilde{\eta}^T(k)[Q_1 - 2a_3 K^T(k)K(k)]\tilde{\eta}(k) \\ & - \frac{1}{2}\tilde{\nu}^T(k) Q_2 \tilde{\nu}(k). \end{aligned}$$

Since K_1 given by (34) is fixed and $K_2(k)$ is given by (33), we can choose $Q_1 > 0$ such that

$$Q_1 - 2a_3 K^T(k)K(k) > \tilde{Q}_1 \tag{43}$$

for sufficiently small $T > 0$ and for some $\tilde{Q}_1 > 0$. Hence we have

$$\begin{aligned} & V(k+1) - V(k) \\ \leq \ & a_1 \hat{z}^T(k)[\hat{\Phi}^T(k)\hat{\Phi}(k) - I]\hat{z}(k) + a_2 \parallel \hat{z}(k) \parallel^2 \\ & - \frac{1}{2}\tilde{x}^T(k) \begin{bmatrix} \tilde{Q}_1 & 0 \\ 0 & Q_2 \end{bmatrix} \tilde{x}(k). \end{aligned}$$

Since the system $\hat{z}(k+1) = \hat{\Phi}(k)\hat{z}(k)$ is SPA stable and it is guaranteed by the Lyapunov function $V_T(\hat{z})$, there exists $\alpha_3 \in \text{class}K_\infty$ such that for any strictly positive real numbers (Δ, δ) there exists $T^* > 0$ such that for all $\hat{z}$ with $\parallel \hat{z} \parallel < \Delta$ and $T \in (0, T^*)$

$$\hat{z}^T[\hat{\Phi}^T(k)\hat{\Phi}(k) - I]\hat{z} \leq -T\alpha_3(\parallel \hat{z} \parallel) + T\delta.$$

Then we have

$$\begin{aligned} V(k+1) - V(k) \ \leq \ & -Ta_1\alpha_3(\parallel \hat{z}(k) \parallel) + Ta_1\delta + a_2\Delta^2 \\ & -\frac{1}{2}\tilde{x}^T(k)\text{diag}\{\tilde{Q}_1, Q_2\}\tilde{x}(k) \\ = \ & -T\hat{\alpha}_3(\parallel \begin{bmatrix} \hat{z} \\ \tilde{x} \end{bmatrix}(k) \parallel) + T\hat{\delta} \end{aligned}$$

where $\hat{\delta} = a_1\delta + \frac{a_2}{T}\Delta^2$ and

$$\begin{aligned} \hat{\alpha}_3(\parallel \begin{bmatrix} \hat{z} \\ \tilde{x} \end{bmatrix} \parallel) \ = \ & a_1\alpha_3(\parallel \hat{z} \parallel) \\ & + \frac{1}{2T}\lambda_{min}(\text{diag}\{\tilde{Q}_1, Q_2\}) \parallel \tilde{x} \parallel^2 \end{aligned}$$

and $\lambda_{min}(M)$ is the minimum eigenvalue of a matrix M. Since $\hat{\alpha}_3 \in \text{class}K_\infty$, we obtain (6). Consequently the output feedback controller (26), (33), (34) and (38) SPA stabilizes (23)-(25).

C. Numerical Example

We now consider the Bis-scaled system matrices for the supply vessel in Example 11.5 in [2]. The inertia matrix M and the damping matrix D are

$$M = \begin{bmatrix} 1.1274 & 0 & 0 \\ 0 & 1.8902 & -0.0744 \\ 0 & -0.0744 & 0.1278 \end{bmatrix},$$

$$D = \begin{bmatrix} 0.0358 & 0 & 0 \\ 0 & 0.1183 & -0.0124 \\ 0 & -0.0041 & 0.0308 \end{bmatrix}.$$

Then we have $\sigma(A) = \{-0.2428, -0.0627, -0.0318\} \subset \mathbf{C}^-$.

We first design SPA stabilizing state feedback laws. We set the sampling period $T = 10$ [msec], $L = -5I$ and $c = 1$. The we have $I + TL = 0.95I$ and we apply the state feedback law $u(x(k))$ given by (21) to (14) and (15). Let

$$\eta(0) = [\,-4 \quad 3 \quad -\tfrac{\pi}{4}\,]^T, \ \nu(0) = 0_{3\times 1} \tag{44}$$

be the initial condition of the system (14) and (15). Figures 2 and 3 show the time response of the yaw angle $\psi(t)$ and the trajectory of the North-East position of the ship, respectively. As we see Figures 2 and 3, the designed state feedback law stabilizes the ship.

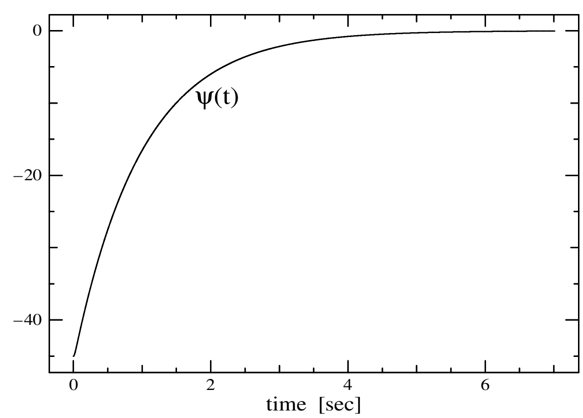

Fig. 2. Time response of yaw angle $\psi(t)$

Next we design SPA stabilizing output feedback controllers. Again we set $T = 10$ [msec]. Then we have $\sigma(A_d) = \{0.9976, 0.9994, 0.9997\} \subset \mathbf{D}$. To design an observer for the Euler approximate model, we set $Q_1 = 40I$, $\hat{Q}_1 = 10I$, $Q_2 = \hat{Q}_2 = 0.1I$ and $K_1 = 0.5I$. Then the solution of (32) and (35) are given by $P_1 = 66.667I$ and

$$P_2 = \begin{bmatrix} 314.9662 & 0 & \\ 0 & 159.5140 & -2.7302 \\ 0 & -2.7302 & 41.2957 \end{bmatrix},$$

respectively and the remaining observer gain K_2 is given by (33). In this case we can easily check that the conditions (36), (37) and (43) with $a = b = c = 1$ are satisfied for any ψ. We set $L = -5$ and $c = 1$ again and we apply the output feedback controller (26) and (38) to the continuous-time system (14) and (15). Ler (44) be the initial condition of

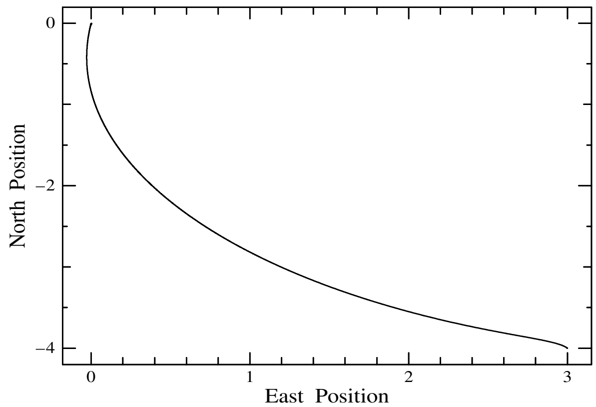

Fig. 3. The trajectory of the North-East position

the system (14) and (15) and let $\hat{\eta}(0) = \hat{\nu}(0) = 0_{3\times 1}$ be the initial condition of the controller. In this case the trajectory of the North-East position of the ship is given in Figure 4 and the time response of the yaw angle $\psi(t)$ is similar to Figure 2. As we see Figure 4, the designed output feedback controller works well.

We also give a brief discussion of the performance between the designed output feedback SPA stabilizing controllers and the Euler approximation of continuous-time output feedback stabilizing controllers designed by a backstepping technique and a nonlinear observers ([2], [4]). Though we do not have an enough space to show simulation results, the time responses of the yaw angle and the trajectories of the ship are not different so much for both controllers. But we can set the sampling period until $T = 600$ [msec] to design the output feedback SPA stabilizing controller (26) and (38). Until $T = 380$ [msec] we can find the Euler approximation of continuous-time output feedback controllers which stabilizes the ship. Hence there is a possibility that the controller design based on the Euler approximate model makes a sampling period longer than the controller design based on the continuous-time model.

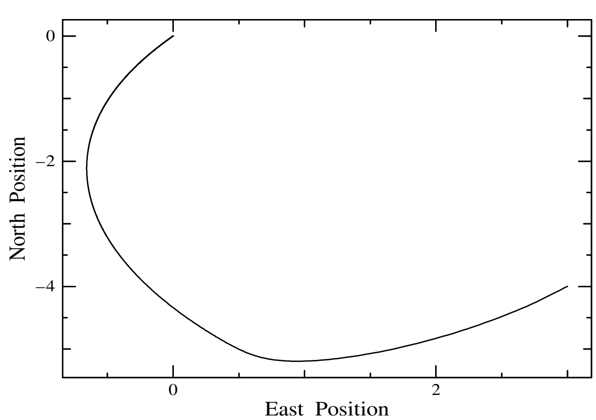

Fig. 4. The trajectory of the North-East position

IV. CONCLUSION

In this paper we have considered the nonlinear sampled-data stabilization control for ships. First we have summarized the framework for the design of nonlinear sampled-data systems proposed in [5], [10] and [11] and we have extended the result in [9] to multi-input nonlinear discrete-time systems. Then we have applied the extended result to the stabilization control for ships. We have designed both SPA stabilizing state feedback laws and SPA stabilizing output feedback controllers for ships. We have given a numerial example to illustrate the design methods.

APPENDIX

Proof of Theorem 2.2: It is enough to show that $(u_T(x), V_T(x))$ given by (12) and (13) satisfy (5)-(8) in Definition 2.2. By Proposition 1 in [9] and the equivalence of p-norms, we can show that $V_T(x)$ satisfies (5). Similar to the proof of Theorem 2 in [9] (7) and (8) are satisfied. Using the relation $\Delta \bar{W}_T = [x_2 - \phi_T(x_1)]^T \Delta \tilde{W}_T$ and the Mean Value Theorem, we can also show (3) similar to the proof of Theorem 2 in [9].

REFERENCES

[1] T. Chen and B. A. Francis, *Optimal Sampled-Data Control Systems*, Springer, 1995.

[2] T. I. Fossen, *Marine Control Systems*, Marine Cybernetics; 2002.

[3] T. I. Fossen and A. Grovlen, Nonlinear output feedback control of dynamically positioned ships using vectorical observer backstepping, *IEEE Trans. Contr. Syst. Technol.*, vol. 6, 1998, pp. 121-128.

[4] H. K. Khalil, *Nonlinear Systems*, Prentice Hall, 2002.

[5] D. S. Laila, D. Nesic and A. Astolfi, Sampled-data control of nonlinear systems, *in Advanced Topics in Control Systems Theory: Lecture Notes from FAP 2005 (A. Loria, F. L-Lagarrigue and E. Panteley (Eds))*, Lecture Notes in Control and Information Sciences, Vol. 328, pp. 91-137, Springer, 2005.

[6] A. Loria, T. I. Fossen and E. Panteley, A separation principle for Dynamic positioning of ships: Theoretical and experimental results, *IEEE Trans. Contr. Syst. Technol.*, vol. 8, 2000, pp. 332-343.

[7] D. Nesic and L. Grune, Lyapunov based continuous-time nonlinear controller redesign for sampled-data implementation, *Automatica*, Vol. 41, pp. 1143-1156, 2005.

[8] D. Nesic and L. Grune, Receding horizon control approach to sampled-data implementation of continuous-time controller, *Systems and Control Letters*, Vol. 55, pp. 660-672, 2006.

[9] D. Nesic and A. R. Teel, Stabilization of sampled-data nonlinear systems via backstepping on their Euler approximate model, *Automatica*, Vol. 42, pp. 1801-1808, 2006.

[10] D. Nesic, A. R. Teel and P. V. Kokotovic, Sufficent conditions for stabilization of sampled-data nonlinear systems via discrete-time approximation, *Systems and Control Letters*, Vol. 38, pp. 259-270, 1999.

[11] D. Nesic, A. R. Teel and E. D. Sontag, Formulas relating KL stability estimates of discrete-time and sampled-data nonlinear systems, *Systems and Control Letters*, Vol. 38, pp. 49-60, 1999.

[12] T. Perez, *Ship Motion Control*, Springer, 2005.

[13] K. Y. Pettersen and H. Nijmeijer, Output feedback tracking control for ships, In Nijmeijer and Fossen (Eds) *New Directions in Nonlinear Observer Design*, Springer, 1999, pp. 311-331.

[14] K. Y. Pettersen and T. I. Fossen, Underactuated dynamic positioning of a ship - Experimental results, *IEEE Trans. Contr. Syst. Technol.*, vol. 8, 2000, pp. 856-863.