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Adaptive partial state feedback control of the DC-to-DC Ćuk converter

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Abstract— **The problem of output voltage regulation by means of partial state and parameters information for the DC–to–DC Ćuk converter is addressed and solved. It is shown that the combination of a full information controller with a stable state and parameters estimator, designed using the immersion and invariance technique, yields a globally asymptotically stable closed–loop system. A few simulations complete the paper.**

Keywords: Power converters, Adaptive stabilization, Lyapunov Analysis.

I. INTRODUCTION

The DC–to–DC Ćuk converter is one of the most studied power converters. Goal of this converter is to invert the polarity of the input voltage and to step-up or step-down its absolute value. Its main application is in regulated DC power supplies, where an output voltage with *negative polarity* with respect to the common terminal of the input voltage is desired. Moreover, the use of inductors in the input and output loops induces very little input and output current ripple. This is a significant advantage over other inverting topologies such as the buck–boost and fly–back.

The output voltage regulation problem for the DC–to–DC Ćuk converter, operating in continuous conduction mode, has attracted the attention of many researchers. Besides its practical relevance, this system is an interesting theoretical case study because it is a switched device whose averaged dynamics can be described by a fourth order bilinear model with saturated input and highly uncertain parameters, namely the load resistance and the input voltage. Moreover, with respect to the available measured states, the system is non-minimum phase, hence standard output feedback stabilization tools for nonlinear minimum-phase systems are not applicable.

Nevertheless, the problem of output voltage regulation for the DC–to–DC Ćuk converter has been addressed and solved using various techniques. In particular, classical linear design tools have been used in [3], whereas the applicability of advanced nonlinear methods, such as feedback linearization [9], passivation [5], [11], sliding mode [12] and H_∞ design [4], has also been investigated. However, in spite of this intensive research activity, the problem of output voltage regulation with partial state and parameters information has not yet been solved.

Goal of this paper is to address and solve this problem. In particular, the problem is solved combining a full information controller, derived using simple energy considerations, in the spirit of the results in [5], [6], with a state and parameters estimator, which relies on the general results developed in [1].

It is assumed that only the input capacitor voltage and the output inductor current are measured. These are then used to construct, under suitable assumptions, asymptotic estimates for the input inductor current, the output voltage, the load resistance and the input voltage (assumed constant). The estimated states and parameters are then used to replace unmeasured quantities in the full information control law.

The rest of the paper is organized as follows. In Section II we present the average model of the DC–to–DC Ćuk converter and we give a precise definition of the control problem of interest. Section III is devoted to the design of a full information state feedback controller. We propose two possible solutions, the former requires only minimal information on the system, the latter needs the knowledge of (almost) all states and parameters. Section IV is devoted to the development of the estimation algorithm, whereas in Section V we propose and analyze two adaptive partial information feedback schemes. Finally, in Section VI we illustrate the theory with a few numerical simulations and in Section VII we present some concluding remarks.

II. THE MODEL OF THE DC–TO–DC ĆUK CONVERTER

The averaged model of the DC–to–DC Ćuk converter shown in Fig. 1 is given by the equations

$$\begin{aligned} L_1 \frac{d}{dt} i_1 &= -(1-u)\, v_2 + E \\ C_2 \frac{d}{dt} v_2 &= (1-u)\, i_1 + u\, i_3 \\ L_3 \frac{d}{dt} i_3 &= -u\, v_2 - v_4 \\ C_4 \frac{d}{dt} v_4 &= i_3 - G\, v_4 \end{aligned} \tag{1}$$

where i_1 and i_3 describe the currents in the inductances L_1 and L_3, respectively; v_2 and v_4 are the voltages across the capacitors C_2 and C_4, respectively. L_1, C_2, L_3 and C_4 are also used (with a slight abuse of notation) for the values of the capacitance and of the inductances, respectively. Finally, G represents the load admittance, E the value of the voltage source and u a continuous control signal, which represents

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the slew rate of a PWM circuit used to control the switch position in the converter.

It must be noted that, because of physical considerations, the state vector $x = [i_1,\, v_2,\, i_3,\, v_4]^\top$ is constrained in the set

$$\mathcal{D} = \mathbb{R}_{>0} \times \mathbb{R}_{>0} \times \mathbb{R}_{<0} \times \mathbb{R}_{<0}. \qquad (2)$$

As a result, in what follows we will implicitly assume that $x(t) \in D$ for all t.

Goal of the control is to regulate the voltage V_d across the load, which is equivalent to regulate the capacitor voltage v_4 to the constant value V_d.

Note now that the steady-state relationship between the output voltage V_d and the duty cycle u is given by

$$V_d = \frac{u}{1-u} E.$$

Moreover, setting the duty cycle u to a constant value, the equilibria $(\bar{i}_1, \bar{v}_2, \bar{i}_3, \bar{v}_4)$ of system (1) verify the following relations

$$\bar{i}_1 = \tfrac{G}{E}\bar{v}_4^2, \quad \bar{v}_2 = -\left(\bar{v}_4 - E\right), \quad \bar{i}_3 = G\,\bar{v}_4. \qquad (3)$$

As a result, setting $\bar{v}_4 = -V_d$, yields

$$i_{1\star} = \frac{G\,V_d^2}{E},\; v_{2\star} = V_d + E,\; i_{3\star} = -G\,V_d,\; u_\star = \frac{V_d}{V_d+E}. \qquad (4)$$

In what follows we show that the considered control goal is achievable with a bounded control signal and with partial state and parameters information, *i.e.* under the following standing assumptions.

C1 v_2 and i_3 are the only measured states. Moreover, all parameters are constant and E and G are unknown.

C2 The control u is such that $u(t) \in (0,\, 1)$, for all t.

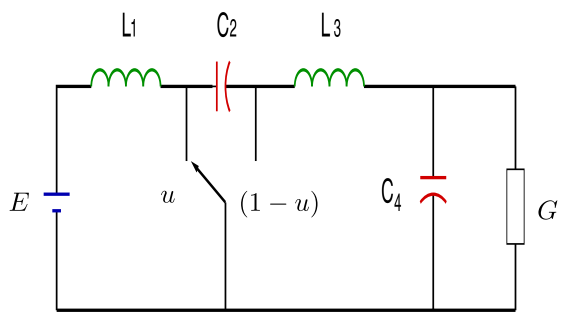

Fig. 1. DC–to–DC Ćuk converter circuit.

III. Full information controllers

In this section we present a full information controller yielding global asymptotic stability of the desired equilibrium.

Proposition 1: Consider the DC–to–DC Ćuk converter model (1) in closed–loop with the full information controller

$$u_{FI} = \lambda \frac{G\,V_d\,v_2 + E\,(i_3 - i_1)}{1 + \left[G\,V_d\,v_2 + E\,(i_3 - i_1)\right]^2} + u_\star \qquad (5)$$

where $\lambda = \lambda(x)$ is any non-negative function of x and such that

$$0 \leq \lambda < 2\min(u_\star, 1 - u_\star).$$

Then for any $V_d > 0$ the equilibrium

$$x_\star = [i_{1\star}, v_{2\star}, i_{3\star}, -V_d]^\top$$

is globally asymptotically stable. Moreover, the function

$$H_d(\tilde{x}) \;=\; \tfrac{1}{2}L_1\tilde{i}_1^2 + \tfrac{1}{2}C_2\tilde{v}_2^2 + \tfrac{1}{2}L_3\tilde{i}_3^2 + \tfrac{1}{2}C_4\tilde{v}_4^2, \qquad (6)$$

where $\tilde{x} = x - x_\star$, is a Lyapunov function for the closed–loop system. Finally, u_{FI} is such that $u_{FI}(t) \in (0, 1)$ for all t.

Proof: To begin with note that in terms of the *error variable* $\tilde{x}$ the DC–to–DC Ćuk converter dynamics (1) is described by the equations

$$\begin{array}{rcl} L_1\,\dfrac{d}{dt}\tilde{i}_1 &=& -(1-u)\,\tilde{v}_2 + (V_d + E)\,\tilde{u} \\ C_2\,\dfrac{d}{dt}\tilde{v}_2 &=& (1-u)\,\tilde{i}_1 + u\,\tilde{i}_3 - G\,V_d(1+\dfrac{V_d}{E})\,\tilde{u} \\ L_3\,\dfrac{d}{dt}\tilde{i}_3 &=& -u\,\tilde{v}_2 - \tilde{v}_4 - (V_d+E)\,\tilde{u} \\ C_4\,\dfrac{d}{dt}\tilde{v}_4 &=& \tilde{i}_3 - G\,\tilde{v}_4 \end{array} \qquad (7)$$

where $\tilde{u} = u - u_\star$.

Note now that the function H_d is positive definite, radially unbounded and has a unique minimum for $\tilde{x} = 0$. Moreover, one has

$$\dot{H}_d = -\left(\frac{V_d+E}{E}\right)\left[G\,V_d\,v_2 + E\,(i_3 - i_1)\right]\,\tilde{u} - G\,\tilde{v}_4^2.$$

Thus, replacing the control law (5) in $\dot{H}_d$, *i.e.* setting $u = u_{FI}$ and hence $\tilde{u} = u_{FI} - u_\star$, yields

$$\begin{array}{l} \dot{H}_d = -G\,\tilde{v}_4^2 \\ \quad -\lambda\left(\dfrac{V_d+E}{E}\right)\dfrac{\left[G\,V_d\,v_2 + E\,(i_3-i_1)\right]^2}{1+\left[G\,V_d\,v_2 + E\,(i_3-i_1)\right]^2} \leq 0. \end{array} \qquad (8)$$

This implies that the closed–loop system is globally stable and that all its trajectories converge to the largest invariant set contained in

$$\mathcal{S} \triangleq \left\{x \in \mathcal{D} \mid \lambda\left[G\,V_d\,v_2 + E\,(i_3-i_1)\right]^2 + G\,\tilde{v}_4^2 = 0\right\}.$$

This implies that, on the set $\mathcal{S}$, $u = u_\star$, and that

$$\tilde{v}_4 = 0 \Rightarrow v_4 = -V_d \Rightarrow \dot{v}_4 = 0 \Rightarrow i_3 = -G\,V_d,$$

yielding

$$\dot{i}_3 = 0 \Rightarrow v_2 = E + V_d \Rightarrow \dot{v}_2 = 0.$$

Finally,

$$i_1 = \frac{G}{E}V_d^2 \Rightarrow \dot{i}_1 = 0,$$

which shows that the largest invariant set of the closed–loop system compatible with the conditions $\tilde{v}_4 = 0$ and $u = u_\star$ is the equilibrium point $x_\star$.

To complete the proof note that the control signal is the sum of two signals, $u_\star$ and

$$\lambda \frac{G\,V_d\,v_2 + E\,(i_3 - i_1)}{1 + [G\,V_d\,v_2 + E\,(i_3 - i_1)]^2},$$

which is bounded in modulo by $\lambda/2$. ■

As a simple consequence of Proposition 1 we give the following result.

Corollary 1: Consider the DC–to–DC Ćuk converter model (1) in closed–loop with the full information controller

$$u = u_\star = \frac{V_d}{E + V_d}. \tag{9}$$

Then, for any $V_d > 0$ the equilibrium

$$x_\star = [i_{1\star}, v_{2\star}, i_{3\star}, -V_d]^\top$$

is globally asymptotically stable.

The control law in Corollary 1 is indeed very simple, as it requires knowledge of E only. However, because of its simplicity the resulting performance may be unacceptable in practice.

IV. Adaptive observer

In this Section we present one of the main contributions of this paper, namely an estimation algorithm yielding, under suitable assumptions, asymptotic estimates for the unmeasured states and the unknown parameters.

Proposition 2: Consider the DC–to–DC Ćuk converter described by equations (1). Assume that the physical parameters L_1, C_2, L_3, C_4 are known.

Consider now the system

$$\begin{aligned}
\dot{\hat{E}} &= -L_1\,\Gamma_1 \Big[(1-\hat{u})\,(\hat{i}_1 + C_2\,\Gamma_2\,v_2) + \hat{u}\,i_3\Big] && (10)\\
\dot{\hat{i}}_1 &= \frac{1}{L_1}\Big[-(1-\hat{u})\,v_2 + (\hat{E} + L_1\,C_2\,\Gamma_1\,v_2)\Big] && (11)\\
&\quad -\Gamma_2\Big[(1-\hat{u})\,(\hat{i}_1 + C_2\,\Gamma_2\,v_2) + \hat{u}\,i_3\Big] &&\\
\dot{\hat{G}} &= \left(\frac{\hat{v}_4}{L_3} - \Gamma_3\,i_3\right)\Big[\hat{u}\,v_2 + (\hat{v}_4 - L_3\,\Gamma_3\,i_3)\Big] && (12)\\
&\quad -i_3\,\dot{\hat{v}}_4 &&\\
\dot{\hat{v}}_4 &= \frac{i_3 - (\hat{G} + \hat{v}_4\,i_3 - \frac{L_3\,\Gamma_3}{2}\,i_3^2)\,(\hat{v}_4 - L_3\,\Gamma_3\,i_3)}{C_4} && (13)\\
&\quad +\Gamma_3\Big[-\hat{u}\,v_2 - (\hat{v}_4 - L_3\,\Gamma_3\,i_3)\Big] &&
\end{aligned}$$

where $\hat{u}$ is any known signal such that $\hat{u}(t) \in (0,\, 1-\epsilon)$, for some $\epsilon \in (0,1)$.

Then there exist positive constants Γ_1, Γ_2 such that for any $\Gamma_3 > 0$ and any initial condition $[\hat{E}(0),\ \hat{i}_1(0),\ \hat{G}(0), \hat{v}_2(0)]$ the following hold:

$$\begin{aligned}
\lim_{t\to\infty}(\hat{E} + L_1\,C_2\,\Gamma_1\,v_2 - E) &= 0\\
\lim_{t\to\infty}(\hat{i}_1 + C_2\,\Gamma_2\,v_2 - i_1) &= 0,
\end{aligned} \tag{14}$$

exponentially, and

$$\begin{aligned}
(\hat{G} + \hat{v}_4\,i_3 - \tfrac{1}{2}L_3\,\Gamma_3\,i_3^2 - G) &\in \mathcal{L}_\infty\\
(\hat{v}_4 - L_3\,\Gamma_3\,i_3 - v_4) &\in \mathcal{L}_\infty.
\end{aligned} \tag{15}$$

Moreover, if $v_4 \in \mathcal{L}_\infty$ and $v_4 < -\sigma$, for some $\sigma > 0$, then

$$\begin{aligned}
\lim_{t\to\infty}(\hat{G} + \hat{v}_4\,i_3 - \tfrac{1}{2}L_3\,\Gamma_3\,i_3^2 - G) &= 0,\\
\lim_{t\to\infty}(\hat{v}_4 - L_3\,\Gamma_3\,i_3 - v_4) &= 0,
\end{aligned} \tag{16}$$

exponentially.

Proof: To begin with define the estimation errors

$$\begin{aligned}
z_1 &= \hat{E} + L_1\,C_2\,\Gamma_1\,v_2 - E\\
z_2 &= \hat{i}_1 + C_2\,\Gamma_2\,v_2 - i_1\\
z_3 &= \hat{G} + \hat{v}_4\,i_3 - \tfrac{1}{2}L_3\,\Gamma_3\,i_3^2 - G\\
z_4 &= \hat{v}_4 - L_3\,\Gamma_3\,i_3 - v_4.
\end{aligned} \tag{17}$$

Straightforward computations show that the estimation errors satisfy the equations

$$\begin{aligned}
\dot{z}_1 &= -L_1\,\Gamma_1\,(1-\hat{u})\,z_2\\
\dot{z}_2 &= \tfrac{1}{L_1}\,z_1 - \Gamma_2\,(1-\hat{u})\,z_2\\
\dot{z}_3 &= \tfrac{1}{L_3}\Big(\hat{v}_4 - L_3\,\Gamma_3\,i_3\Big)\,z_4\\
\dot{z}_4 &= -\tfrac{1}{C_4}\,(\hat{v}_4 - L_3\,\Gamma_3\,i_3)\,z_3 - \Big(\tfrac{G}{C_4} + \Gamma_3\Big)\,z_4.
\end{aligned} \tag{18}$$

Equations (18) can be regarded as two decoupled subsystems, one with state (z_1, z_2) and one with state (z_3, z_4). Therefore, to establish the claims, we consider these two subsystem separately.

For, rewrite the (z_1, z_2) subsystem as the feedback interconnection of the system

$$\begin{aligned}
\begin{bmatrix} \dot{z}_1 \\ \dot{z}_2 \end{bmatrix} &= \begin{bmatrix} 0 & -L_1\,\Gamma_1\,\bar{d} \\ \frac{1}{L_1} & -\Gamma_2\,\bar{d} \end{bmatrix} \begin{bmatrix} z_1 \\ z_2 \end{bmatrix} + \begin{bmatrix} L_1\,\Gamma_1\bar{d} \\ \Gamma_2\bar{d} \end{bmatrix} w\\
\bar{y} &= \begin{bmatrix} 0 & 1 \end{bmatrix} \begin{bmatrix} z_1 \\ z_2 \end{bmatrix}
\end{aligned} \tag{19}$$

with the static nonlinearity

$$w = -\frac{\delta d}{\bar{d}}\,\bar{y},$$

where $\bar{d} = \frac{1}{2}(1+\epsilon)$, $\delta d = d - \bar{d}$, and $d = 1 - \hat{u}$. We now study this interconnected system using the small gain theorem. To this end note that, since $\Gamma_1 > 0$ and $\Gamma_2 > 0$, system (19) is (globally) asymptotically stable. Moreover, its H_∞ norm is given by

$$\gamma(\mu) = \frac{1}{\sqrt{1 + 2\,\mu^2\,\sqrt{1 + \frac{2}{\mu}} - 2\,\mu^2 - 2\,\mu}},$$

where

$$\mu = \frac{\Gamma_1}{\Gamma_2^2\,\bar{d}}.$$

Note now that

$$\left|\frac{\delta d}{\bar{d}}\right| < \frac{1-\epsilon}{1+\epsilon},$$

therefore the closed–loop system will be (exponentially) stable provided

$$\gamma(\mu)\,\frac{1-\epsilon}{1+\epsilon} < 1. \tag{20}$$

Hence, to complete the proof of the claim we need to show that for any $\epsilon \in (0,1)$ it is possible to select $\Gamma_1 > 0$ and $\Gamma_2 > 0$ such that (20) holds. For, note that $\gamma(\mu) \geq 1$ for any $\mu \geq 0$, and that (20) is equivalent to

$$\epsilon > \frac{\gamma(\mu)-1}{\gamma(\mu)+1}.$$

Observe now that the range of the function

$$\frac{\gamma(\mu)-1}{\gamma(\mu)+1}$$

is the open set $(0,1)$. Therefore, for any $\epsilon \in (0,1)$ there is a $\mu > 0$, and hence $\Gamma_1 > 0$ and $\Gamma_2 > 0$, such that condition(20) holds.

Consider now the (z_3, z_4) subsystem and the Lyapunov function

$$V_{z_{34}} = \frac{1}{2} L_3 z_3^2 + \frac{1}{2} C_4 z_4^2.$$

A simple calculation shows that

$$\dot{V}_{z_{34}} = -C_4 \left(\frac{G}{C_4} + \Gamma_3 \right) z_4^2 \leq 0$$

and this proves that condition (15) holds. Finally, if $v_4 \in \mathcal{L}_\infty$ and $v_4 < -\sigma$ for some $\sigma > 0$, we conclude (invoking standard results on stability of skew-symmetric systems and the notion of persistence of excitation, see *e.g.* [10], [7]) that the origin of the (z_3, z_4) subsystem is (globally) exponentially stable. ∎

Remark 1: The condition $v_4 \in \mathcal{L}_\infty$ and $v_4 < -\sigma$ for some $\sigma > 0$, which guarantees exponential convergence of the (z_3, z_4) subsystem can be relaxed as detailed in [8].

V. Partial state feedback controllers

In this section we present two asymptotically stabilizing partial information dynamic control laws for the DC–to–DC Ćuk converter. These are obtained combining the full information controllers (9) and (5) with the estimation algorithm presented in Proposition 2.

Proposition 3: Let $\epsilon \in (0,1)$. For any V_d such that

$$0 < V_d < \frac{\epsilon - 1}{\epsilon} E \tag{21}$$

the DC to DC Ćuk converter described by equations (1) in closed–loop with the dynamic controller

$$u = \text{sat}_{[0,\,1-\epsilon]} \left(\frac{V_d}{\bar{E} + V_d} \right) \tag{22}$$

with[1] $\bar{E} = \hat{E} + L_1 C_2 \Gamma_1 v_2$ and $\hat{E}$ computed from equations (10)–(11), has a globally asymptotically stable equilibrium at (4).

Proof: To begin with note that the closed–loop dynamics can be written, in the x and z coordinates, as

$$\dot{x} = A(u_\star) x + b E + \varphi(z_1) D x \tag{23}$$

where $A(u_\star)$, b and D are constant matrices and

$$\varphi(z_1) = \text{sat}_{[0,\,1-\epsilon]} \left(\frac{V_d}{E + z_1 + V_d} \right) - u_\star$$

Note now that system (23) with $\varphi(z_1) = 0$ is globally exponentially stable. Moreover, as $\varphi(0) = 0$, $|\varphi(z_1)| < \kappa_0$ for some $0 < \kappa_0 < 1$, and z_1 is exponentially converging to zero, then (see *e.g.* [2]) system (23) is asymptotically stable. Hence the claim. ∎

Remark 2: The control law in Proposition 3 requires only partial knowledge on the systems parameters, as it does not use explicitly the parameters L_3 and C_4.

Proposition 4: Let $\epsilon \in (0,1)$. Consider the DC–to–DC Ćuk converter model described by equations (1) in closed–loop with the dynamic controller

$$u = \text{sat}_{[0,\,1-\epsilon]} \left(\frac{\bar{G} V_d v_2 + \bar{E}(i_3 - \bar{i}_1)}{1 + \left[\bar{G} V_d v_2 + \bar{E}(i_3 - \bar{i}_1) \right]^2} + \frac{V_d}{\bar{E} + V_d} \right) \tag{24}$$

where

$$\begin{aligned} \bar{E} &= \hat{E} + L_1 C_2 \Gamma_1 v_2 \\ \bar{G} &= \hat{G} + \hat{v}_4 i_3 - \tfrac{1}{2} L_3 \Gamma_3 i_3^3 \\ \bar{i}_1 &= \hat{i}_1 + C_2 \Gamma_2 v_2 \end{aligned}$$

and $\hat{E}$, $\hat{G}$, $\hat{i}_1$ are computed from equations (10)–(11) and (12)–(13).

Assume that $v_4 \in \mathcal{L}_\infty$ and $v_4 < -\sigma$ for some $\sigma > 0$.

Then, for any V_d such that condition (21) holds, all trajectories of the closed–loop system are bounded and are such that

$$\lim_{t \to \infty} x(t) = x_\star.$$

Proof: To begin with note that, under the stated assumptions, the state and parameters estimation errors converge exponentially to zero. Note now that the closed–loop system can be described by the equations (7) perturbed by an additive term which is bounded by

$$\kappa \|x\| |u - u_{FI}|,$$

for some positive constant κ, and it is such that

$$\lim_{t \to \infty} |u - u_{FI}| = 0,$$

exponentially. Consider now the Lyapunov function $H_d(\tilde{x})$ in (6) and note that

$$\dot{H}_d \leq \kappa \| \frac{\partial H_d}{\partial x} \| \|x\| |u - u_{FI}|,$$

hence

$$\dot{H}_d \leq \tilde{\kappa} \|\tilde{x}\| \|x\| |u - u_{FI}| \leq \hat{\kappa}(H_d + \rho) |u - u_{FI}| \tag{25}$$

for some positive constants $\tilde{\kappa}$, $\hat{\kappa}$ and ρ. Equation (25) implies that, along the trajectories of the system in closed–loop with the partial information controller, H_d is bounded. As a result $\|x\|$ is bounded and, by standard properties of cascaded systems, x converges to $x_\star$, and this completes the proof. ∎

[1] To take into account that $E \geq 0$ we could define $\bar{E} = \max(0, \hat{E} + L_1 C_2 \Gamma_1 v_2)$. The forthcoming discussion will simply require minor modifications.

VI. Simulation Results

Numerical simulations were carried out to assess the performance of the proposed controllers. The values of the parameters have been taken from [4], *i.e.* $L_1 = 10\,\text{mH}$, $C_2 = 22.0\,\mu\text{F}$, $L_3 = 10\,\text{mH}$ and $C_4 = 22.9\,\mu\text{F}$. Moreover, it is assumed that the nominal values for the load admittance and the input voltage are $G_N = 0.0447\,S$ and $E_N = 12\,\text{V}$, respectively. The initial conditions for all simulations are set to $x(0) = \begin{bmatrix} 0.5 & 10 & -1 & -12 \end{bmatrix}^\top$. In all simulations we assume that the initial set point for the output voltage is $-5\,\text{V}$, and then this is changed at $t = 0.075$ to $-35\,\text{V}$.

Figures 2 and 3 show the performance of the full information controllers (9) and (5) (with $\lambda = 1$), respectively. As can be observed, the output voltage converges to the desired value.

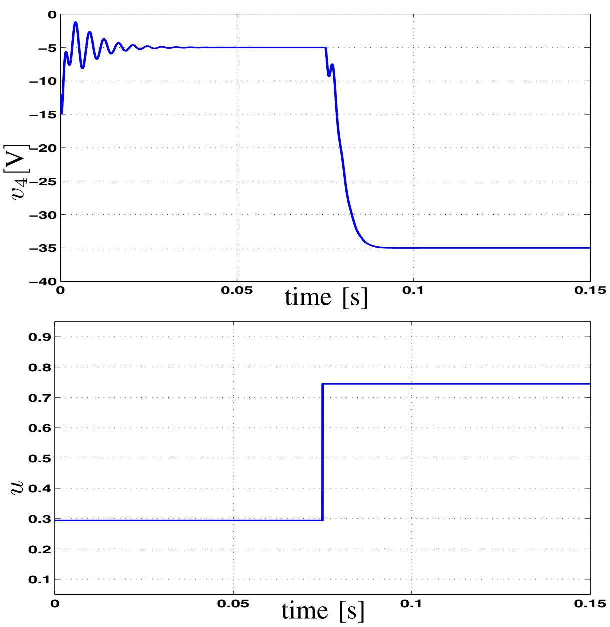

Fig. 2. Output voltage v_4 (top) and control input (9) (bottom).

To test the controllers of Propositions 3 and 4 we study the same control goal as above, *i.e.* output voltage regulation at $-5\,\text{V}$ for $t < 0.075$, and at $-35\,\text{V}$, for $t \geq 0.075$. Moreover, we consider two step changes of the load admittance and the input voltage: at $t = 0.05$ the load admittance G is decreased to $0.022\,S$ and the input voltage E is decreased to $10\,\text{V}$; at $t = 0.10$ the load admittance G is increased to $0.066\,S$ and the input voltage E is increased to $14\,\text{V}$.

The performance of the partial information controller (22) are shown in Figures 4 and 5. These display time histories of the output voltage and the control signal and of the estimation errors for the input voltage and the input current, respectively.

Finally, numerical simulations for the partial information controller (24), with $\lambda = 1$, are shown in Figures 6 to 8. These display time histories of the output voltage and the control signal, of the estimation errors for the input voltage and the input current, and of the estimation errors for the load and the output capacitor voltage, respectively.

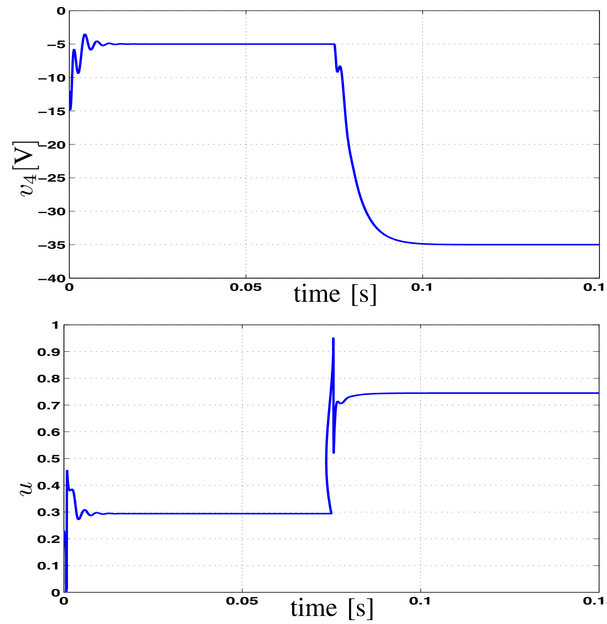

Fig. 3. Output voltage v_4 (top) and control input (5) with $\lambda = 1$ (bottom).

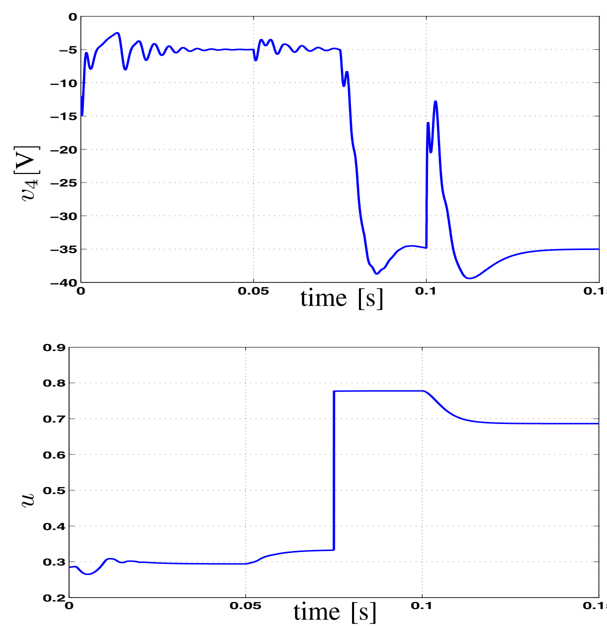

Fig. 4. Output voltage v_4 (top) and control input (22) (bottom).

Comparing the proposed simulations it is interesting to observe that the inclusion of a nonzero value of λ in the full information controller (5), and in its partial information counterpart, improves the transient behaviour, *i.e.* reduces oscillations and speed up convergence.

VII. Conclusions

The problem of output voltage regulation for the DC-to-DC Ćuk converter has been addressed and solved by

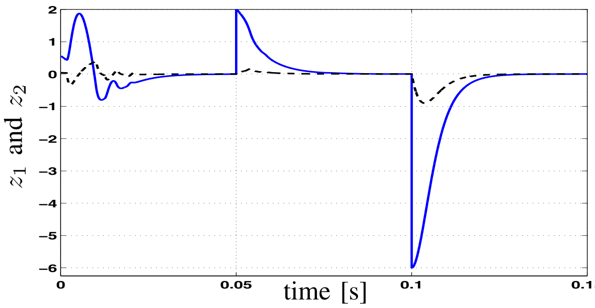

Fig. 5. Input voltage estimation error (continuous line) and input current estimation error (dashed line) for $\Gamma_1 = 280270$, $\Gamma_2 = 2000$, $\epsilon = 0.05$ and $\mu = 0.045$.

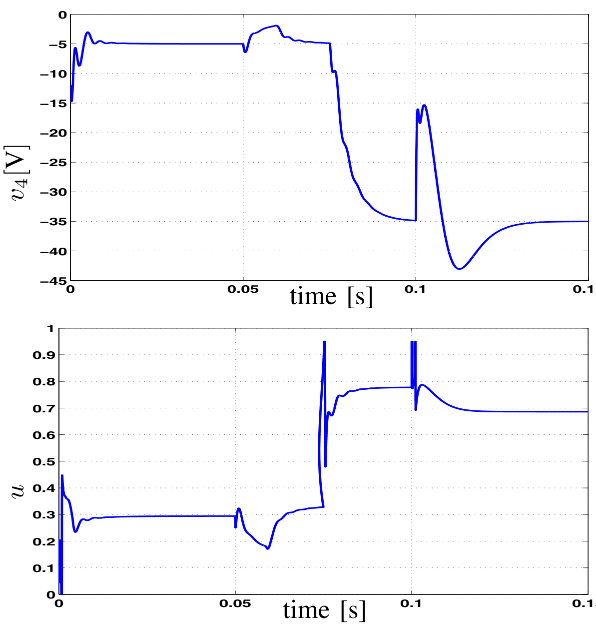

Fig. 6. Output voltage v_4 (top) and control input (24) with $\lambda = 1$ (bottom).

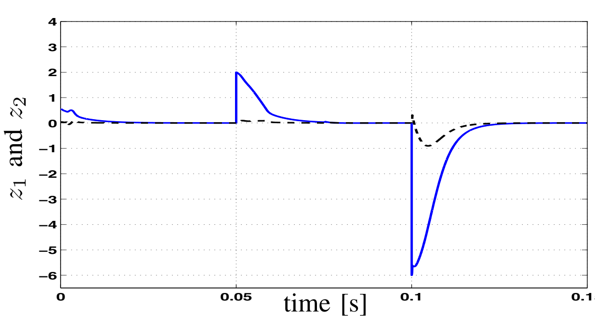

Fig. 7. Input voltage estimation error (continuous line) and input inductor current estimation error (dashed line) for $\Gamma_1 = 280270$, $\Gamma_2 = 2000$, $\epsilon = 0.05$ and $\mu = 0.045$.

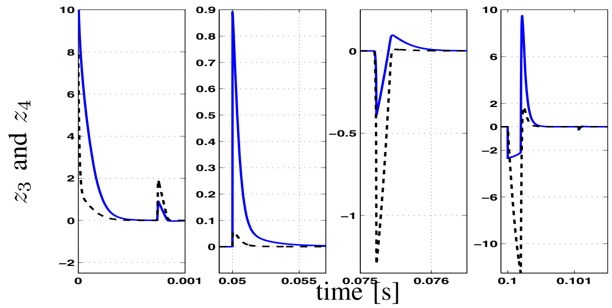

Fig. 8. Details of the load admittance estimation error (continuous line) and output capacitor voltage estimation error (dashed line) for $\Gamma_3 = 84588$. Note the different y-scales of the figures.

means of full information and partial information (dynamic) control laws, which are based on a certainty equivalence point of view. An adaptive observer for parameters and state estimation has been proposed. The design of the observer is independent from the desing of the stabilizing controller, hence the observer can be used in *conjunction* with any full information control law. Simulations have been proposed to illustrate the properties of the closed–loop system.

A few issues are left open in the present paper. First, the proposed controllers do not leave the set $\mathcal{D}$ defined in (2) invariant. Second, the exponential convergence of the estimation errors of the adaptive observers requires boundedness and persistency of excitation of v_4, hence further work is needed to relax (or avoid) these assumptions.

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