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Decentralized Regulation for a Class of Large-Scale Networks with Saturation

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Abstract— **In this paper, we study a nonlinear large-scale network model with saturation constraints on both the control input and the state variables. We achieve buffer queue length regulation against unknown inter-node traffic interferences via a decentralized saturated control law. Motivated by physical characteristics of communication networks, two conditions regarding the inter-node traffic are discussed, namely a Lipschitz-type condition and a "PE" condition. Under these conditions, by using sliding-mode type controllers, the regulation error of every node converges asymptotically to zero for all feasible initial queue lengths. Our main (global) regulation result is based on an interesting extension of the popular Young's inequality to the case with saturation.**

Index Terms— **Congestion control, capacity constraints, buffer management, nonlinear control, asymptotic regulation.**

I. Introduction

The increasingly wide use of large-scale telecommunication networks has attracted great research interest. To address the network design problem, previous efforts resort to intuitive approaches since it is in general difficult to model and analyze large-scale networks rigorously. These approaches belong to trial-and-error methods and do not scale well since they usually handle specific settings.

To overcome the above-mentioned shortcomings, model-based schemes have been proposed to provide theoretic analysis for networking problems. On one hand, optimization theory has found successful applications in network topology design, optimal routing decision, and bandwidth management in large-scale networks, cf. [5][4][11]. On the other hand, linear/nonlinear analysis and control design tools have recently been applied to control traffic in ATM networks [15], stability analysis of congestion control schemes in TCP/IP networks [22][20][13][24], network performance analysis against delays [2][21][25][7][6][18] and power control in wireless networks [8].

In this paper, our attention is focused on applying nonlinear control theory to address the queue regulation of large-scale networks, especially when physical constraints and nonlinear disturbances are involved. So far this problem has not received enough attention. Among the existing literature, we mention the work most related to our subject. The authors of [19] studied a nonlinear buffer management model for a network node. The design objective is queue length regulation, due to its important impact on the quality of service for the "premium traffic" such as video, audio and teleconference transmissions. A nonlinear congestion control scheme is proposed based on robust adaptive control ideas, leading to a solution for bounded regulation against unknown, time-varying traffic interferences. It is interesting to investigate asymptotic queue regulation instead of bounded regulation, to improve the control performance. In addition, extending their robust adaptive control scheme to large-scale networks is far from trivial, in particular in the presence of saturation nonlinearities.

Based upon the model as used in [19], we have recently modified their control strategy and achieved asymptotic queue regulation for the single-node system in [9] and for cascaded systems in [10]. The performance limitation caused by capacity constraints is analyzed. Here we continue our task and take a step forward to the regulation of an interconnected network by using saturated controllers. Our ideas are inspired in part by recent advances in decentralized control for large-scale nonlinear systems [16].

We extend the model of a single network node in [19] to large-scale networks, where the inter-node traffic is considered as subsystem interconnection. The proposed decentralized sliding-mode controller achieves asymptotic queue regulation of each node. Explicit conditions on large-scale networks are identified for asymptotic regulation using decentralized saturated control law. The first condition is that the traffic satisfies a saturation-based Lipschitz-type condition. The second condition is that the traffic is "persistently exciting" (PE), see Assumption 1 for a precise definition.

The first contribution of this paper is that we achieve asymptotic regulation for each interconnected node as op-

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posed to the bounded regulation of a single-node system in [19]. The proposed decentralized control scheme is constructive and performance-oriented. We take advantage of the fact that the uncertain disturbance (unknown inter-node traffic) is separated into structured (vanishing) and unstructured (non-vanishing) uncertainties to enhance performance, as opposed to our previous results in [10] where the incoming traffic is treated as a non-vanishing disturbance. The second contribution of the paper is that we address the practical issue of saturation constraints on the control input and the state variables. In the current literature, the stabilization of linear systems under control input saturation has received considerable attention [14], while less results are known for general nonlinear systems. Under the aforementioned conditions, we solve the constrained regulation problem for the nonlinear network model.

Our work distinguishes from most previous robust control applications for large-scale networks. For example, the robust network flow control scheme in [3] is toward a linear network model. The robust control result for TCP/AQM network in [21] is local since it is based on linearization ideas and linear $\mathcal{H}_\infty$ control theory. The saturation constraint is not considered in [3][21]. We instead study a nonlinear large-scale network model, and achieve "global" constrained queue regulation, in the sense that the regulation error converges for all feasible initial conditions.

The rest of the paper is arranged as follows: in Section 2, we introduce a differential equation model for large-scale network, and formulate our design objectives with practical limitations in mind. In Section 3, some preliminary results, i.e., the boundedness of queues and useful inequalities are presented. Our main result on the global asymptotic regulation of large-scale networks is presented in Section 4. Our conclusion is summarized in Section 5.

II. Problem Formulation

In this section, we introduce our model for a large-scale network composed of n interconnected nodes. By "node", we refer to a router/switch in the network for the rest of the paper. For each node $i = 1, ..., n$, we use the following model to describe its dynamics, which is first introduced in [1]. Pitsillides $et\ al.$ continue to consider this model for the purpose of network performance evaluation and control under non-stationary conditions [19][23][12]. The model uses the conservation law to establish the buffer queue length dynamic equation of a network node:

$$\dot{x}_i = -\frac{x_i}{1+x_i} \cdot C_i + \lambda_i(t, x_1, ..., x_n), \quad (1)$$

$$x_i \in [0, x_{buffer}^{[i]}], \quad (2)$$

$$C_i \in \left[0, C_{server}^{[i]}\right], \quad (3)$$

where x_i denotes the queue length of the ith node for each $1 \le i \le n$. It is taken as the state variable. C_i represents the to-be-assigned capacity. It is chosen as the control input. These variables are subject to physical constraints (2)-(3). For node i, $x_{buffer}^{[i]}$ represents the buffer size and $C_{server}^{[i]}$ represents the maximum service capacity. In a large-scale network where the nodes are interconnected, the interfering traffic is affected by the activities of the interfering nodes, reflected in part by their queue lengths. We use a nonlinear function $\lambda_i : [0, \infty) \times \Re_+^n \to \Re_+$ to denote the average incoming traffic rate (to node i) from other connected nodes. By conservation law, the first term $\mu(x_i) = \frac{x_i}{1+x_i}$ in equation (1) represents the average traffic departure rate. $\mu(x_i)$ is chosen based on general principles for modelling traffic in communication networks [1]. It is verified through simulations [23] that $\mu(x_i)$ is indeed a valid choice to represent the average departure rate for a wide range of networks.

Let x_i^{ref} denote a desired reference queue length for node i. It should be chosen such that the switch/router is sufficiently utilized while preserving certain capability to handle instantaneous traffic bursts. In practice, an empty or extremely small steady-state queue usually leads to resource under-utilization and is thus undesired. It is assumed that each reference value $x_i^{ref}, \forall i \in [1, n]$, satisfies:

$$\underline{\epsilon} \le x_i^{ref} < x_{buffer}^{[i]},$$

with lower bound $\underline{\epsilon} > 0$ being an arbitrary positive value. Our control law does not bear singularity when $x_i^{ref} \to 0$. The assumption $x_i^{ref} \ge \underline{\epsilon}$ is due to physical considerations. The objective is to design decentralized regulators for each local node when x_i^{ref} is given and when the system is subject to uncertain traffic interferences from other nodes, control input (capacity) saturation and state constraints.

Denote $\bar{x}_i := x_i - x_i^{ref}$ the regulation error between the queue state and the reference value. In the rest of this paper, we consider a node as being under-utilized when the queue length of this node is below its reference value ($x_i < x_i^{ref}$). Otherwise, we consider the network node as sufficiently utilized ($x_i \ge x_i^{ref}$ or $\bar{x}_i \ge 0$).

Before presenting the main result on asymptotic regulation, we will first address the boundedness of queues (or equivalently, how to guard against buffer overflow), then introduce useful inequalities for proving asymptotic regulation.

III. Preliminary Results

The following control law is applied through out the rest of this paper, which we will justify later. Since it is unnecessary to assign additional capacity when the node is under-utilized, we take $C_i = 0$ when $x_i < x_i^{ref}$. We focus mainly on the controller design when $x_i \ge x_i^{ref}$. For all $i = 1, ..., n$,

$$C_i(x_i) = \max\left\{0, C_{server}^{[i]} \cdot sat\left[\alpha \bar{x}_i + \beta_i sgn\left(\bar{x}_i\right)\right]\right\}, \quad (4)$$

where α_i, β_i are design parameters to be determined. "sat" is the commonly used saturation function defined as $sat(y) = \min\{|y|, 1\}sgn(y)$, where "$sgn$" is the standard signum function.

A. Global boundedness of queues.

Lemma 1: Consider the closed-loop system composed of (1) and (4). Select

$$\alpha_i \geq \frac{1-\beta_i}{x_{buffer}^{[i]} - x_i^{ref}},\ \beta_i \geq 0$$

and suppose for all $x_i \in [0, x_{buffer}^{[i]}]$, $i = 1, ..., n$,

$$\lambda_i(t, x_1, ..., x_n) \leq \mu_i < \frac{x_{buffer}^{[i]}}{1 + x_{buffer}^{[i]}} C_{server}^{[i]},$$

where $\mu_i > 0$ is a constant. Then, for all $x_i(0) \in [0, x_{buffer}^{[i]}]$, $x_i(t) \in [0, x_{buffer}^{[i]}], \forall t \geq 0$.

Proof: The lemma can be proved by considering the direction of the flow $\dot{x}_i$ on $[0, x_{buffer}^{[i]}]$. For all $x_i \in [0, x_i^{ref}]$, it holds that $C_i(x_i) = 0$ and

$$\dot{x}_i = \lambda_i \geq 0;$$

For all $x_i \in [x_{buffer}^{[i]} - \epsilon, x_{buffer}^{[i]}]$ where $\epsilon > 0$,

$$C_i \geq C_{server}^{[i]} sat\left\{(1-\beta_i)\frac{x_{buffer}^{[i]} - x_i^{ref} - \epsilon}{x_{buffer}^{[i]} - x_i^{ref}} + \beta_i\right\}.$$

When ϵ is small enough, it holds,

$$\dot{x}_i \leq -\frac{x_{buffer}^{[i]} - \epsilon}{1 + x_{buffer}^{[i]} - \epsilon} C_i + \lambda_i < 0.$$

It follows that $x_i(t)$ is constrained to $[0, x_{buffer}^{[i]}], \forall t \geq 0$ if $x_i(0) \in [0, x_{buffer}^{[i]}]$. ∎

B. Useful inequalities.

The technical lemma presented here is crucial for achieving decentralized asymptotic regulation using saturated controller. It should also be mentioned that the inequalities can be seen as extensions of the well-known Young's inequality and thus are of independent interest.

Lemma 2: For any integer $n \geq 2$ and any $a_{ij} \geq 0, i, j = 1, ..., n$, the following inequality holds:

$$\sum_{i=1}^{n} \sum_{j=1, j\neq i}^{n} a_{ij} \left| z_i sat(z_j) \right| \leq \sum_{i=1}^{n} b_{n,i} z_i sat(z_i) \qquad (5)$$

where $b_{n,i}$ is defined as

$$b_{n,i} = \sum_{j=1, j\neq i}^{n} \max\left\{a_{ij}, a_{ji}\right\}, i = 1, ..., n. \qquad (6)$$

The proof of Lemma 2 is given in the Appendix. As a simple application of the lemma, consider the special case with two variables z_1, z_2 and when $a_{12} = a_{21} = 1$, we arrive at the following corollary.

Corollary 1: For any real z_1 and z_2,

$$|z_1 sat(z_2)| + |z_2 sat(z_1)| \leq z_1 sat(z_1) + z_2 sat(z_2).$$

We can consider the above corollary as an extension of the well-known Young's inequality

$$2|z_1 z_2| \leq z_1^2 + z_2^2.$$

IV. Main Result

The following hypothesis is motivated by physical characteristics of networks, and is proposed for asymptotic regulation of large-scale networks.

Assumption 1: $\forall i = 1, ..., n$, $\lambda_i(t, x_1, ..., x_n)$ satisfies the following Lipschitz-type condition[1]:

$$\left|\lambda_i(t, x_1, ..., x_n) - \lambda_i(t, x_1^{ref}, ..., x_n^{ref})\right| \leq \sum_{j=1, j\neq i}^{n} \gamma_{ij} sat\left\{\frac{|\bar{x}_j|}{\Delta_j}\right\}, \qquad (7)$$

$$\lambda_i(t, x_1^{ref}, ..., x_n^{ref}) \leq \upsilon_i,\ \forall t \geq 0 \qquad (8)$$

where $\bar{x}_j = x_j - x_j^{ref}$. υ_i, Δ_j and γ_{ij} are positive constants. For all $i = 1, ..., n$,

$$c_i \doteq \sum_{j=1, j\neq i}^{n} \max\{\gamma_{ij}, \gamma_{ji}\} \leq (1-\epsilon_i) C_{server}^{[i]} \frac{x_i^{ref}}{1 + x_i^{ref}}, \quad (9)$$

$$\upsilon_i \leq \epsilon_i \frac{C_{server}^{[i]} x_i^{ref}}{1 + x_i^{ref}}, \quad \epsilon_i \in (0, 1). \qquad (10)$$

Furthermore, for each i and for every fixed $t_0 \geq 0$, the following inequality holds where $X(t) = [x_1(t), ..., x_n(t)]^T$:

$$x_i^{ref} < \int_{t_0}^{\infty} \lambda_i(t, X(t)) dt \leq \infty. \qquad (11)$$

Remark 1: The function "sat" in (7) is adopted here to highlight the impact of capacity constraint (of each local node) on the interfering traffic.

Except for limited service capacity, other physical factors also affect the interference intensity among nodes, such as the distance between nodes i, j, the power constraint (in wireless network) and the connectivity condition. We use constant coefficient $\gamma_{ij} \geq 0$ to denote the impact of these physical factors on the disturbance traffic from node j to node i.

Remark 2: (11) is a "PE" (persistent excitation) requirement. Under this condition, any network node that is underutilized ($x_i < x_i^{ref}$) will, in the long run, achieve sufficient utilization. Namely, its queue size will accumulate due to

[1]Technically, the R.H.S. can be relaxed as $\sum_{j=1}^{n} \gamma_{ij} sat(\cdot)$. From the physical meaning of this problem, we only consider the case for $j \neq i$, which means the upper bound of the interfering traffics from other nodes to node i is not dependent on the state of node i.

incoming traffic from other nodes, until x_i reaches the reference value ($x_i \geq x_i^{ref}$). Thus a network with part of its nodes under-utilized can be considered as in a transient process. From the previous analysis, under the control law (4), it is guaranteed that the queue length of every node meets the requirement (2), or in other words, no buffer overflows will occur. Thus we can ignore such transient process and consider only the situation when the network is sufficiently utilized, i.e., every i-th node satisfies $x_i(t_0) \geq x_i^{ref}$ for some $t_0 \geq 0$. Next, we show that asymptotic queue regulation is achieved for all $x_i(t_0) \in [x_i^{ref}, x_{buffer}^{[i]}]$.

Theorem 1: Consider the interconnected system defined by (1)-(3). Suppose that the conditions stated in Lemma 1 hold. In addition, if

$$\alpha_i \geq \max\left\{\frac{1-\beta_i}{x_{buffer}^{[i]} - x_i^{ref}}, \frac{1-\epsilon_i}{\Delta_i}\right\}, \beta_i \geq \frac{\upsilon_i(1+x_i^{ref})}{x_i^{ref} C_{server}^{[i]}}.$$

and if the interfering traffic rate λ_i satisfies Assumption 1, then the queue length x_i of every ith node converges to x_i^{ref} asymptotically, under the control law (4), for all $x_i(0) \in [0, x_{buffer}^{[i]}]$.

Proof: The global bounded-ness of queues have been established by Lemma 1. Namely, for any $x_i(0) \in [0, x_{buffer}^{[i]}]$, $x_i(t) \in [0, x_{buffer}^{[i]}], \forall t \geq 0$. We now proceed to prove that our control law (4) achieves asymptotic regulation for the large-scale network.

By the PE condition (11), there exists some $t_0 \geq 0$ such that $x_i(t_0) \geq x_i^{ref}, \ \forall i = 1, ..., n$. It follows that $x_i(t) \geq x_i^{ref}, i = 1, ..., n, \forall t \geq t_0$, by observing (1) and (3).

Consider the function $V = \frac{1}{2}\sum_{i=1}^{n} \frac{\bar{x}_i^2}{\Delta_i}$. In the sequel, we introduce $z_i = \frac{\bar{x}_i}{\Delta_i}$ for notational convenience. We now calculate the time derivative of V along the trajectories of the closed-loop system on $[t_0, \infty)$. Note that $\dot{V} = \sum_{i=1}^{n} z_i \dot{\bar{x}}_i$, $\dot{\bar{x}}_i = \dot{x}_i$ since $\bar{x}_i = x_i - x_i^{ref}$ and x_i^{ref} is a constant.

For all $x_i \in [x_i^{ref}, x_{buffer}^{[i]}]$,

$$\begin{aligned} C_i &= C_{server}^{[i]} sat\left[\alpha_i \bar{x}_i + \beta_i sgn(\bar{x}_i)\right] \\ &\geq (1-\epsilon_i) C_{server}^{[i]} sat\left(\frac{\bar{x}_i}{\Delta_i}\right) + \epsilon_i C_{server}^{[i]} sat\left(\frac{\beta_i sgn(\bar{x}_i)}{\epsilon_i}\right). \end{aligned}$$

Indeed, it suffices to apply the algebraic inequality

$$sat\{(1-\epsilon)y_1 + \epsilon y_2\} \geq (1-\epsilon) sat\{y_1\} + \epsilon sat\{y_2\},$$

with constants $y_1, y_2 \geq 0$ and $\epsilon \in (0, 1)$.

For all $x_i \in [x_i^{ref}, x_{buffer}^{[i]}]$, applying (7), (8), it follows that

$$\lambda_i(t, x_1, ..., x_n) \leq \sum_{j=1, j\neq i}^{n} \gamma_{ij} sat\left\{\frac{\bar{x}_j}{\Delta_j}\right\} + \upsilon_i. \quad (12)$$

With such observation, it holds

$$\begin{aligned} \dot{V} =& \sum_{i=1}^{n} z_i \Big(-\frac{x_i}{1+x_i} C_i + \lambda_i\Big) \\ \leq& \sum_{i=1}^{n} z_i \Bigg\{ -\frac{x_i^{ref}}{1+x_i^{ref}} \Big[(1-\epsilon_i) C_{server}^{[i]} sat\left(\frac{\bar{x}_i}{\Delta_i}\right) \\ & + \epsilon_i C_{server}^{[i]} sat\Big(\frac{\beta_i sgn(\bar{x}_i)}{\epsilon_i}\Big)\Big]\Bigg\} \\ & + \sum_{i=1}^{n} z_i \Big(\sum_{j=1, j\neq i}^{n} \gamma_{ij} sat\Big\{\frac{\bar{x}_j}{\Delta_j}\Big\} + \upsilon_i\Big) \\ \leq& \sum_{i=1}^{n} -(1-\epsilon_i) \frac{x_i^{ref}}{1+x_i^{ref}} C_{server}^{[i]} z_i sat(z_i) \\ & + \sum_{i=1}^{n} z_i \sum_{j=1, j\neq i}^{n} \gamma_{ij} sat(z_j) \\ & - \sum_{i=1}^{n} \left(\frac{C_{server}^{[i]} x_i^{ref}}{1+x_i^{ref}} \epsilon_i sat\left(\frac{\beta_i}{\epsilon_i}\right) - \upsilon_i \right) |z_i|. \end{aligned}$$

By application of Lemma 2, we obtain:

$$\begin{aligned} \dot{V} \leq& \sum_{i=1}^{n} -\left((1-\epsilon_i) \frac{x_i^{ref}}{1+x_i^{ref}} C_{server}^{[i]} - c_i \right) z_i sat(z_i) \\ & - \sum_{i=1}^{n} \left(\frac{C_{server}^{[i]} x_i^{ref}}{1+x_i^{ref}} \epsilon_i sat\left(\frac{\beta_i}{\epsilon_i}\right) - \upsilon_i \right) |z_i|. \end{aligned}$$

By applying (9) and (10), it can be shown that $\dot{V}$ is negative definite with reference to $[z_1, ..., z_n]^T$. By Barbălat's Lemma [17], we conclude that

$$\lim_{t\to\infty} \sum_{i=1}^{n} |\bar{x}_i(t)| = 0.$$

Thus for all feasible initial states, the system trajectories converge to $\Big\{X \in \Re^n \mid x_i = x_i^{ref}, \forall i = 1, ..., n\Big\}$ asymptotically, while obeying (2) at the same time. ■

Simulations in Figure 1 shows the response of system composed of three interconnected nodes. The parameters of the three nodes and their respective controller parameters are shown in Table I. The interfering traffics λ_is are modelled by sine waves. $\gamma_{12} = 0.54, \gamma_{13} = 0.6, \gamma_{21} = 0.45, \gamma_{23} = 0.85, \gamma_{31} = 0.55, \gamma_{32} = 0.76$. The three nodes achieve asymptotic regulation as shown in the Figure 1.

Remark 3: For all $x_i(0) \in [0, x_{buffer}^{[i]}], i = 1, ..., n$, it is shown that no buffer overflow can happen and in addition, $x_i(t) \to x_i^{ref}$ as $t \to \infty$. In this sense, global asymptotic regulation is achieved for large-scale networks.

Remark 4: It is of interest to note that our control design is scalable, since by tuning the control law parameters, we can accommodate new nodes appended to the existing network.

	node 1	node 2	node 3
$x_i(t_0)$	30	30	40
x_i^{ref}	6	4	10
$C_{server}^{[i]}$	5	10	8
Δ_i	5	5	8
β_i	1.2	2.8	1.85
ϵ_i	0.28	0.35	0.25

TABLE I

SIMULATION PARAMETERS FOR A THREE-NODE NETWORK.

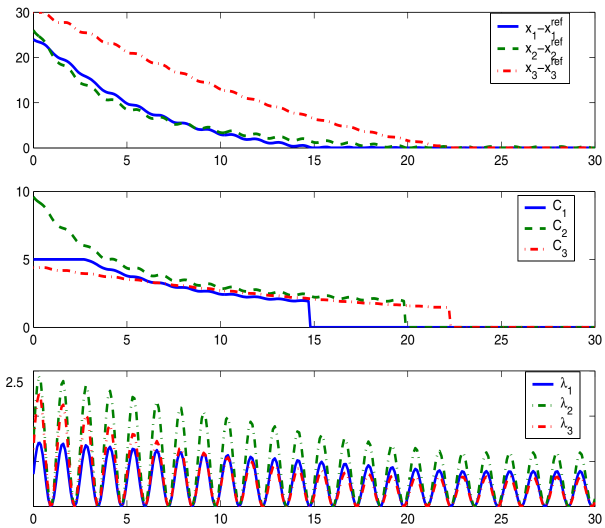

Fig. 1. Asymptotic regulation of three interconnected nodes

V. CONCLUSIONS

Through rigorous theoretic analysis, we show that our decentralized sliding mode control law achieves "global" (for all feasible initial queue lengths) asymptotic regulation of every node for large-scale networks subject to unknown time-varying inter-node traffic. Physical constraints on the control input and state variable are addressed. Our controller uses "high-gain" ideas and thus improves the performance of the closed-loop system as compared with our previous work [9] where "low-gain" feedback is used. It should be stressed that by decomposing the uncertain interconnections into structured (vanishing) and unstructured (non-vanishing) disturbances, we achieve performance improvement as compared with our previous work [10], where the unknown traffic is taken only as a unstructured disturbance. The inequalities proposed in Section III-B are themselves interesting and can be seen as extensions of the widely used Young's inequality. Simulations are presented which confirm the validity of our control law.

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APPENDIX–PROOF OF LEMMA 2

First notice that, for any real z_1 and z_2,

$$(f(z_1) - f(z_2))\,(g(z_1) - g(z_2)) \geq 0,$$

where $f(\cdot)$ and $g(\cdot)$ are odd and non-decreasing functions. Thus

$$f(z_1)g(z_1) + f(z_2)g(z_2) \geq f(z_1)g(z_2) + f(z_2)g(z_1).$$

Notice that

$$|f(z_1)g(z_2) + f(z_2)g(z_1)| = |f(z_1)g(z_2)| + |f(z_2)g(z_1)|.$$

It follows

$$|f(z_1)g(z_2)| + |f(z_2)g(z_1)| \leq f(z_1)g(z_1) + f(z_2)g(z_2).$$

The above facts can be applied to show

$$\begin{aligned}
\sum_{i=1}^{n}\sum_{j\neq i}^{n} & a_{ij} z_i sat(z_j) \\
&= \sum_{j=2}^{n} \left[a_{1j} z_1 sat(z_j) + a_{j1} z_j sat(z_1)\right] \\
&\quad + \sum_{j=3}^{n} \left[a_{2j} z_2 sat(z_j) + a_{j2} z_j sat(z_2)\right] \\
&\quad \vdots \\
&\quad + a_{n-1,n} z_{n-1} sat(z_n) + a_{n,n-1} z_n sat(z_{n-1}) \\
&\leq \sum_{j=2}^{n} \max\{a_{1j}, a_{j1}\} \left[z_1 sat(z_1) + z_j sat(z_j)\right] \\
&\quad + \sum_{j=3}^{n} \max\{a_{2j}, a_{j2}\} \left[z_2 sat(z_2) + z_j sat(z_j)\right] \\
&\quad \vdots \\
&\quad + \max\{a_{n-1,n}, a_{n,n-1}\}[z_{n-1} sat(z_{n-1}) + z_n sat(z_n)] \\
&= \sum_{j\neq 1}^{n} \max\{a_{1j}, a_{j1}\} z_1 sat(z_1) \\
&\quad + \sum_{j\neq 2}^{n} \max\{a_{2j}, a_{j2}\} z_2 sat(z_2) \\
&\quad \vdots \\
&\quad + \sum_{j\neq n}^{n} \max\{a_{nj}, a_{jn}\} z_n sat(z_n) \\
&= \sum_{i=1}^{n} b_{n,i} z_i sat(z_i).
\end{aligned}$$