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Flying in Formation Using a Pursuit Guidance Algorithm

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Abstract— **This paper describes a simple, straightforward algorithm for flying in formation of multiple Unmanned Air Vehicles (UAV's). In particular, we are interested in a formation with no communication between the vehicles. It is assumed that relative kinematics parameters are available to each UAV from an on-board passive sensor followed by estimation processes and a controller which may use visual information. The paper introduces a guidance algorithm, which is based on the theory of pursuit curves in conjunction with a velocity controller.**

I. INTRODUCTION

In this paper, we introduce a simple and applicable algorithm for flying in formation of multiple Unmanned Aerial Vehicles (UAV's) using only data from imaging sensors. We assume that one of the UAV's (labeled the *Leader*) follows a (possibly) unknown trajectory relative to which the *Followers* must track the Leader. In particular, we assume that there is no communication among the UAV's in the formation, and that each UAV only has available the local information from those UAV's which are in its given Field of View (FOV).

More specifically, the necessary local information is range, line of sight (LOS) angle and LOS rate which are estimated using computer vision algorithms integrated with estimation techniques (e.g., Kalman filtering). Such algorithms are described in [1] for a tracking problem, i.e. one Leader and one Follower. In this work, the proposed algorithm is based on two control loops. The first loop employs well-known Guidance Pursuit laws, i.e. Proportional Navigation and LOS Guidance [2] to keep the desired relative angle in the formation. These laws are used mainly in homing missiles. Unlike a missile whose (usual) mission is to hit a target, here the vehicles must keep a relative range as defined by the formation. Therefore, a second velocity controller is designed which depends on the error between the desired (pre-defined) range and the estimated range.

Much research has been performed in the last few years in the area of formation for multiple UAV's, robots, undersea vehicles, autonomous agents and more. Several papers deal with the vision only problem. In [3], the authors control the range between the vehicles in the formation, while avoiding obstacles using dynamic inversion and a Neural Network adaptive loop. In [4], an algorithm is proposed for controlling the relative position and orientation of robots while following a planned trajectory. This algorithm is based on feedback linearization of the relative kinematics, where the unknown state of the Leader is treated as an input.

In other related work, it is assumed that there is communication among the members of the formation, i.e. each vehicle in the formation knows the state of the other vehicles in the given formation. For example, the researchers in [5] describe control strategies based on the motion of their nearest neighbors.

Close formation for multiple aircraft has been examined in several papers. The formation is implemented mainly for drag reduction. Close formation flying causes various problems including aerodynamic coupling effects which are highly nonlinear. Among the papers which are concerned with such a formation scenario are [6] in which the authors use PID control, and [7] in which a linear quadratic regulator (LQR) controller is proposed for the tracking problem. In [8] an LQR outer loop is proposed, while in [9] sliding mode control for the outer loop and an adaptive dynamic inversion inner loop are employed for close formation flying. In the work of [10], the researchers consider the best position of the Follower relative to the Leader for drag reduction, using a peak-seeking controller. Finally in [11], we have an investigation of three dimensional close formation flying using a PID controller.

The references [12], [13] describe a cyclic pursuit strategy of a nonlinear multi-vehicle system. By changing the control gain, the shape of the trajectory is changed and the local stability of these formations is investigated.

The theory of pursuit is very old and goes back at least to the work of Pierre Bouguer in 1732. The basic mathematical problem can be stated as follows: given a point P which moves along a given curve, then the point Q describes a *pursuit curve* if Q points in the direction of P and the points P, Q move at uniform velocities. There is a huge literature devoted to this; see [14], [15], [16] and the references therein. In a certain sense, the present paper continues this line of research applied to a modern control problem.

The organization of the paper is as follows: In Section II we give a precise formulation of the formation problem. Section III describes a simple Leader-Follower scenario, which amounts to a classical tracking problem. Section IV extends this tracking methodology to multiple UAV formations, with simulation results given in Section V. The conclusion and statement of future research follows in Section VI.

II. Problem Formulation

This section defines the formation problem we are interested in solving. A formation consists of $N+1$ planar UAV's, one of which is labeled the *Leader* F_0 with the others labeled *Followers,* $F_1, \ldots, F_N$. The Leader follows an unknown trajectory relative to which the Followers must track. The information available to a given Follower consists of measurements of its own state (configuration, velocity, and acceleration), and the image obtained from a fixed, forward-pointing, on-board, monocular camera system. It is assumed that using the visual information, each Follower can estimate the relative range, LOS angle, and LOS rate of the Leader and other Followers which are in its Field Of View (FOV). The estimation is performed by means of target tracking based image processing algorithms and an extended Kalman Filter developed in [1].

The formation structure is pre-defined, i.e. each Follower follows a chosen subset of the other UAVs lying in its FOV. The desired range and the desired relative (lead) angle to each UAV define the formation. We denote by $r^\star_{ik}$ and $\epsilon^\star_{ik}$ the desired range and the desired relative angle between F_i to F_k, respectively. Figure 1 describes a formation with 5 UAV's ($N=4$), where F_0 is the Leader, F_1 and F_2 follow the Leader, F_3 follows both F_1 and F_2, and F_4 follows both F_1 and F_3. The figure also depicts $r^\star_{41}$ and $r^\star_{43}$, the desired ranges between F_4 to F_1 and F_3, respectively, as well as $\epsilon^\star_{41}$ and $\epsilon^\star_{43}$ the desired relative angles between F_4 and F_1,F_3, respectively.

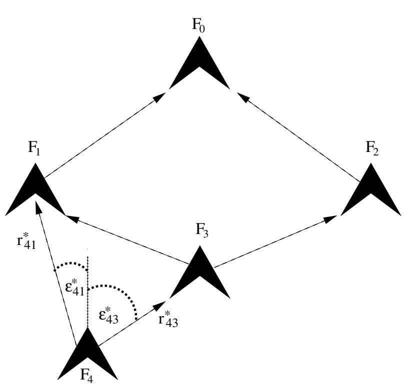

Fig. 1. Formation Definition

III. Tracking

Before considering the general case of multiple formations, the simpler case of just 2 UAV's ($N=1$) will be examined, one of which is labeled the Leader (F_0) and the other Follower (F_1). Thus we have a rather standard tracking problem. In Section IV below, the general case will be transformed to this tracking scenario.

In this simplified setting, the Leader follows an unknown trajectory relative to which the Follower must track. The desired range and the desired relative (lead) angle between the Leader and the Follower are denoted by $r^\star_1$ and $\epsilon^\star_1$, respectively as described in Figure 2.

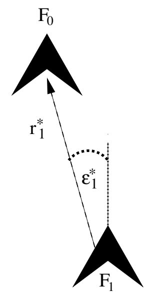

Fig. 2. Leader-Follower Formation Definition

The complete tracking closed loop system is summarized by the block diagram of Figure 3. It is assumed that each UAV has its own inner controller which receives acceleration commands. The Image Processing and Computer Vision block produces "measurements" to the Estimation block, which is based on an extended Kalman Filter (see [1] for more details). The Estimation block calculates the relative range, the line of sight (LOS), and the LOS rate between the Leader F_0 and the Follower F_1. These parameters are used by the Guidance block to produce commands for the vehicle's inner controller loop as described in the sequel.

A. Guidance Algorithm

The Guidance block receives inputs from the Estimation block, namely: the estimated relative range $\hat{r}_1$, the estimated relative angle $\hat{\epsilon}_1$, and the estimated LOS rate $\hat{\dot{\lambda}}_1$. In addition, it gets two predefined commands: the desired relative range r^*_1 and the desired relative (lead) angle ϵ^*_1. The outputs of the algorithm are the acceleration commands to the inner control block of the UAV. Control acceleration for the autonomous vehicle is decomposed into normal and tangential acceleration components, denoted as a^N_{c1} and a^T_{c1}, respectively.

The purpose of the input a^N_{c1} is to maintain the desired LOS angle between the Leader and Follower. The algorithm is based on standard Proportional Navigation and LOS guidance laws [2]. Since the Leader dynamics are unknown, the guidance law assumes a non-accelerating Leader. Using

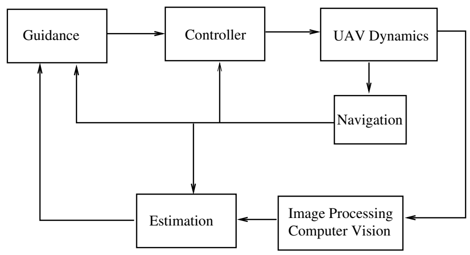

Fig. 3. System Block Diagram

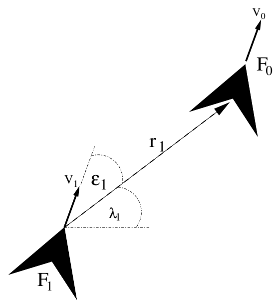

Fig. 4. System Block Diagram

the estimated values for the LOS and the LOS rate we define the normal acceleration command as

$$a_{c1}^N = NV_1\left(K_N\hat{\dot{\lambda}}_1 + (1-K_N)e_{\epsilon_1}\right). \quad (1)$$

Here N is the proportional navigation constant, K_N is a parameter in the range $[0,1]$, V_1 is the forward velocity of the Follower F_1, and $e_{\epsilon_1} \equiv \epsilon_1^* - \hat{\epsilon}_1$, where $\hat{\epsilon}_1$ and $\hat{\dot{\lambda}}_1$ are the estimates of the lead angle ϵ_1 and the LOS rate $\dot{\lambda}_1$, respectively. These terms are illustrated in Figure 4.

The role of a_{c1}^T is to maintain the desired relative range between the Leader and Follower. Consequently, a_{c1}^T is a function of the range error $e_{r_1} = r_1^* - \hat{r}_1$, where $\hat{r}_1$ is the estimated relative range between the Leader and the Follower. Any velocity or acceleration control loops designed to control the error signal should take into account the model of the Follower. Specifically, a very simple acceleration control loop is described in Figure 5, where V_1^{nom} is the nominal forward velocity of the Follower and V_1 is the measured velocity along the LOS. PI is a Proportional Integral controller and K_1 is a simple gain. The velocity command V_{c1} and the acceleration command a_{c1}^T are limited according to the Follower limitations.

IV. Formations of Multiple UAV's

In this section, the problem of multiple UAV's flying in formation is reduced to a tracking problem of the type described in Section III via a simple trick.

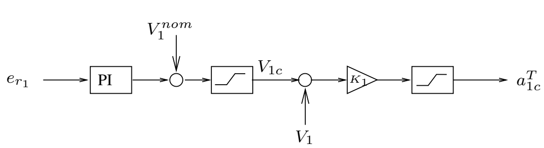

Fig. 5. Axial acceleration command control loop.

Referring to the general scenario of Figure 1, each UAV may follow several UAVs and may have multiple desired relative ranges and angles. The guidance algorithm of Section III receives only two tracking parameters: desired relative range and desired relative angle. The multiple relative range and angles specifying a formation may be combined to obtained single desired relative range angle for each UAV (similar to idea of averaging the pseudo-control in [3]) in the following manner:

$$r_i^* = \sum_{k=0}^{N} w_{ik} r_{ik}^*\,, \quad \text{and} \quad (2)$$

$$\epsilon_i^* = \sum_{k=0}^{N} w_{ik}\epsilon_{ik}^*, \quad (3)$$

where the weights w_{ik} are nonzero only if F_i follows F_k, and the weight satisfy $\sum_{k=0}^{N} w_{ik} = 1$.

Equations (2), (3) imply that each UAV follows a virtual UAV according to the averaged quantities r_i^* and ϵ_i^*. In this manner the problem of multiple UAV formation flying is reduced to that of the tracking problem described in Section III. The resulting guidance algorithm for each UAV is a modification of Equations (1) and Figure 5. For each Follower F_i the normal acceleration command is

$$a_{ci}^N = NV_i\left(K_N\hat{\dot{\lambda}}_i + (1-K_N)e_{\epsilon_i}\right), \quad (4)$$

with the same parameters defined for Equation (1), and where $e_{\epsilon_i} \equiv \epsilon_i^* - \hat{\epsilon}_i$ uses the averaged values

$$\hat{\epsilon}_i = \sum_{k=0}^{N} w_{ik}\hat{\epsilon}_{ik} \quad (5)$$

and

$$\hat{\dot{\lambda}}_i = \sum_{k=0}^{N} w_{ik}\hat{\dot{\lambda}}_{ik}. \quad (6)$$

Figure 5 is modified in a similar way where $e_{r_i} = r_i^* - \hat{r}_i$ and

$$\hat{r}_i = \sum_{k=0}^{N} w_{ik}\hat{r}_{ik}. \quad (7)$$

V. Simulation Results

This section presents a simulation of the formation control technique. Consider the specific of five planar UAV's ($N = 4$), each UAV modeled by the equations of motion

$$\begin{aligned} \dot{x}_i &= V_i \cos\psi_i\,, \\ \dot{y}_i &= V_i \sin\psi_i\,, \\ \dot{\psi}_i &= a_i^N / V_i\,, \\ \dot{V}_i &= a_i^T\,, \end{aligned} \quad (8)$$

where ψ_i is the heading angle, and

$$\begin{aligned} a_i^N &= H_i^N(s)a_{ci}^N\,, \quad \text{and} \\ a_i^T &= H_i^T(s)a_{ci}^T \end{aligned} \quad (9)$$

are accelerations normal and tangent to the heading angle, respectively. The functions $H_i^N(s)$ and $H_i^T(s)$ are transfer functions from the acceleration commands a_{ci}^N and a_{ci}^T to the actual accelerations.

In our scenario, the Leader flies a squared trajectory, while the Followers are controlled to maintain the formation defined by Figure 1. Figure 6 depicts the trajectory flown and also chronicles the guidance commands for each Follower. Figure 7 describes the trajectory parameters of the Leader and the four Followers.

To illustrate the results more clearly, we present that part of the trajectory during the first 20 seconds and during a turn, in Figures 8 and 9. Figure 8 shows that there is no problem of convergence to the formation if the initial conditions are "reasonably close" to the formation definition. Admittedly this is difficult to quantify formally, on the other hand our simulations do indicate robustness to various initial conditions. Table I describes the initial conditions of the Leader and the 4 Followers. In addition, it describes the formation definition.

UAV	Initial Condition				Formation Definition	
	x_0 $[m]$	y_0 $[m]$	v_0 $[m/s]$	ψ_0 $[deg]$	x_F $[m]$	y_F $[m]$
L	0	0	40	0	0	0
F_1	-50	-50	40	0	-50	-8
F_2	-70	30	30	20	-50	8
F_3	-100	0	45	10	-70	0
F_4	-150	20	40	0	-100	-4

TABLE I

SIMULATION INITIAL CONDITION AND FORMATION DEFINITION.

From Figure 7, one can see that the Followers follow the same trajectory as the Leader and they keep the formation at steady state. During turns of the Leader, the formation is kept but with some error because of the limits on the normal acceleration (the normal acceleration was limited to $10[m/s^2]$ - Figure 9(b)), and because of the delay in the plant. As stated above the acceleration commands pass through transfer functions $H_i^N(s)$,$H_i^T(s)$ to create the actual acceleration. These transfer functions are modeled in the simulation to be of second order with $\omega = 1[Hz]$ and $\xi = 0.7$. The delay that is caused by the plant dynamics propagates through the level of the formation, so the response of F_4 which follows F_3 and F_2 is delayed with respect to the other Followers, as can be seen in Figure 9. After each turn the formation converges to a steady state as the defined by the desired formation.

VI. CONCLUSIONS

In this paper, a simple and applicable guidance law is introduced to maintain the relative range and orientation of multiple UAV formations. The algorithm is based on well-known guidance laws which are used in missile homing algorithms [2]. Unlike missiles whose purpose is to hit a target, the formation algorithm controls the range between the vehicles by a simple velocity controller. This controller depends on the error between the desired and the estimated range. We gave some simulation results in order to demonstrate the behavior of the algorithm.

Our underlying concept is based on the theory of pursuit curves and which has been applied in many different areas including robotics. However, we have come at this from a very different direction, namely guidance and control.

The situation described here is of course a great simplification in which we assume that we have perfect measurements. We are now explicitly including estimation (via the extended Kalman filter) and visual information in our set-up. The model we have also assumes we are flying at the same altitude in "clear air," i.e., there are no obstructions or occlusions. The key extension on which we are working is a full three dimensional model. This will also demand more sophisticated image processing and tracking algorithms for multiple objects in possibly cluttered environments such as the dynamic snakes described in [17].

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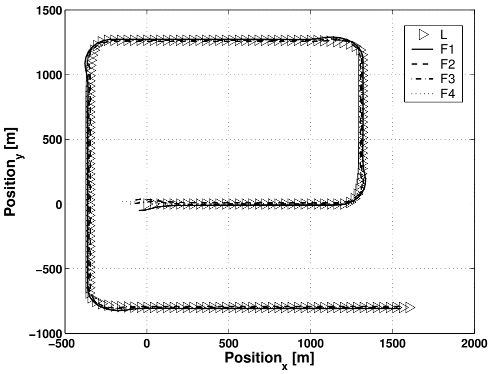
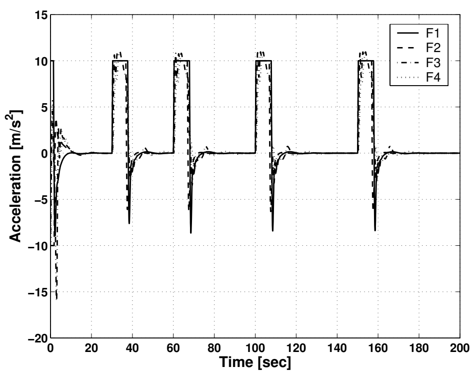
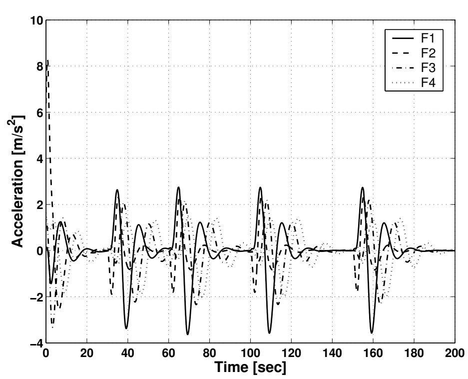


(a) Planar Trajectory

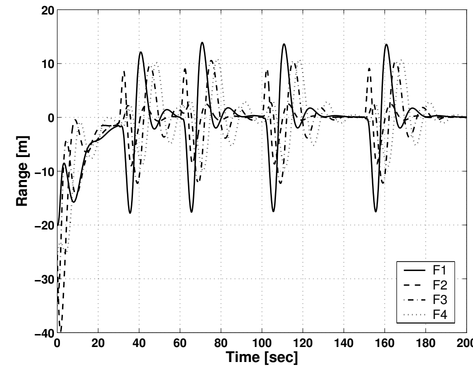


(b) Follower Normal Acceleration Command vs. Time.

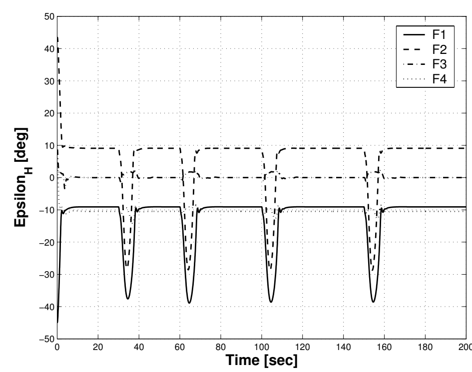


(c) Follower Tangential Acceleration Command vs. Time.

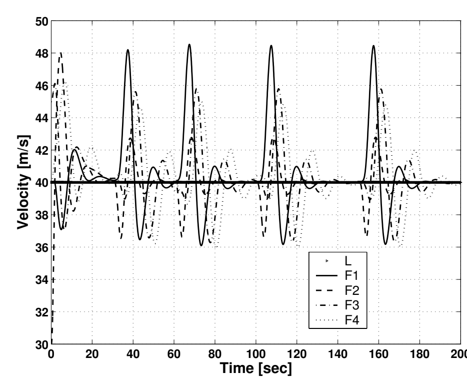
Fig. 6. Trajectory and Guidance Commands



(a) Range Error vs. Time.

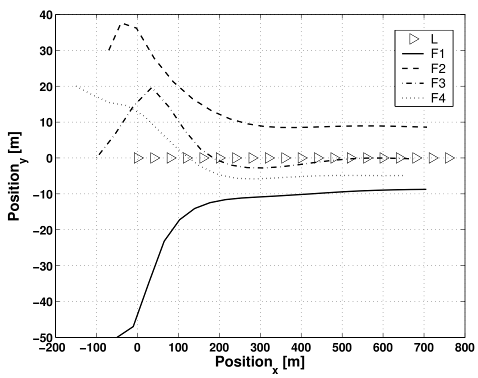


(b) Relative angle vs. Time.

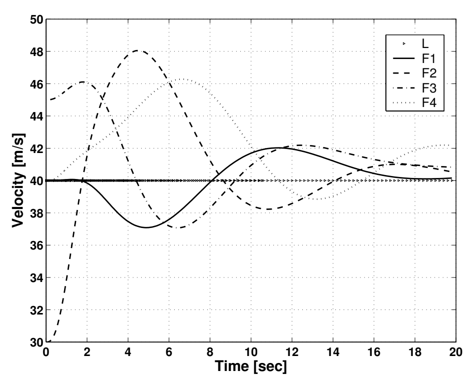


(c) Velocity vs. Time.

Fig. 7. Trajectory Parameters

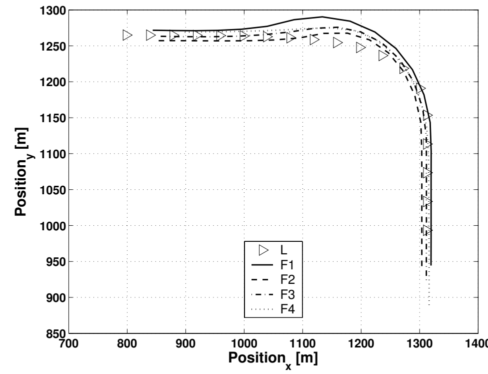


(a) Planar Trajectory

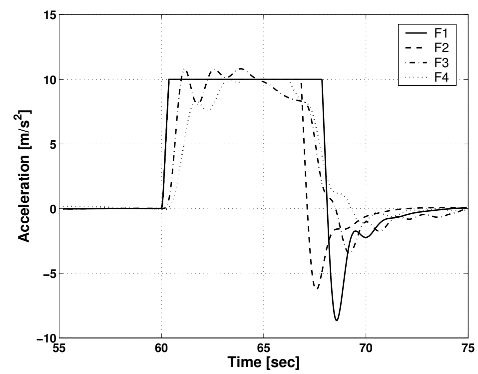


(b) Follower Velocity vs. Time.

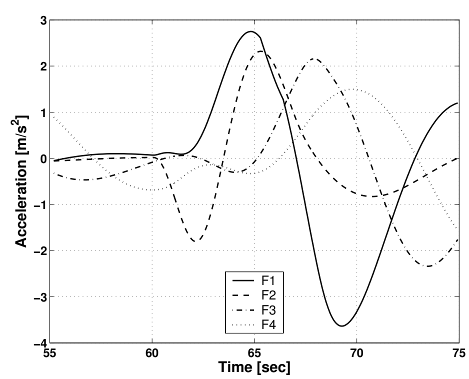
Fig. 8. Trajectory Parameters During The First 20 Seconds



(a) Planar Trajectory



(b) Follower Normal Acceleration Command vs. Time.



(c) Follower Tangential Acceleration Command vs. Time.

Fig. 9. Trajectory Parameter During a Turn