2005 American Control Conference
June 8-10, 2005. Portland, OR, USA

Sliding Mode Fuzzy Gain Scheduling in Sampled Data Nonlinear Systems

José Manuel Andrade Da Silva (§), Pedro Antonio Teppa Garrán y José Jesús Ferrer Suárez

***Abstract*— In this paper we propose a design methodology that integrates sliding mode control, fuzzy control and gain scheduling, in an original and innovative manner, through the perspective of sampled data systems. We have called the technique *Sliding Mode Fuzzy Gain Scheduling.* The synthesis procedure systematically yields a control law. The results obtained demonstrate that this approach satisfactorily solves the problem of transition between the operating conditions of a sampled data nonlinear dynamic system.**

I. INTRODUCTION

Sliding Mode Control (SMC) is a robust control technique that can be applied to nonlinear systems with uncertainties and external disturbances. It consists of using switched or discontinuous control actions across a sliding surface. A sliding mode control system is insensitive to parametric uncertainty and external disturbances; however, one of its main disadvantages is the presence of high-frequency oscillations in the control signal, i.e. chattering.

The SMC theory was originally developed by Emelyanov [1], Utkin [2] and Itkis [3] for continuous time systems; however, SMC's discrete approach is important when implementing such a controller on a digital computer. Preference for controlling dynamic systems with digital rather than analog elements is mainly based on computer cost, combined with the advantage of the treatment of digital over analog signals. Various efforts have been made to develop the discrete sliding mode control theory. These include the work described by Dote and Hoft [4], Milosavljevic [5], Sarpturk *et al* [6], Furuta [7], Sira-Ramirez [8]. These research papers have shown that a discrete sliding mode control system cannot be obtained through a mere equivalence with the continuous sliding mode control.

The concept of Quasi Sliding Mode [5], or the term Pseudo Sliding Mode [9], is used to express the fact that the conditions for the existence of the sliding mode in continuous time systems do not necessarily guarantee the motion of a sampled data system to bring the state trajectory close to the sliding surface.

SMC theory has been combined with the theory of intelligent systems such as fuzzy logic, neural networks, probabilistic reasoning, genetic algorithms and chaos [10].

José Manuel Andrade Da Silva (§), Pedro Antonio Teppa Garrán and José Jesús Ferrer Suárez are with the Departamento de Procesos y Sistemas, Universidad Simón Bolívar, Apdo. 89000, Valle de Sartenejas, Baruta, Edo. Miranda, Venezuela. (e-mail (§): jandrade@usb.ve).

This combination seeks to reduce the problems or disadvantages connected with implementing Sliding Mode Control Systems (SMCS) by using the synergy between the different theories.

Ensuring the transition of a nonlinear system between operating points is a fundamental component of nonlinear systems control over a broad spectrum of operating conditions. This problem has been approached using various techniques, the main ones being: multiple model adaptive control [11], neural networks [12] and supervisory control [13].

This paper solves the problem of transition between the operating conditions of a sampled data nonlinear dynamic system, integrating the results of the theories of sliding mode control, fuzzy control and gain-scheduled control, originally as well as innovatively, through the perspective of sampled data systems. Hence, we have entitled it *Sliding Mode Fuzzy Gain Scheduling* (SMFGS) for sampled data systems.

The article is structured as follows: Sections II, III and IV present the theoretical bases of sliding mode control, fuzzy control and gain scheduling which is subsequently used in our hybrid technique. Section V introduces the problem of transition between operating conditions. Section VI shows the methodology proposed and Section VII presents the results obtained as a result of applying the methodology to a magnetic levitation system.

II. DISCRETE-TIME SLIDING MODE CONTROL

Sliding mode control theory has aroused interest among researchers due to its robust nature, decoupling high-dimension systems in a set of lower-dimension sub-systems, application to SISO and MIMO linear and nonlinear systems. This interest has focused fundamentally on continuous time systems, thus the theory for those systems has evolved significantly. However, few studies have been conducted in the case of discrete time systems.

Definition 1: *A switching function is a linear function* $\sigma_i : \Re^n \to \Re$ *of the form*

$$\sigma_i(\mathbf{x}) = \boldsymbol{\gamma}_i^T \mathbf{x} \quad , \quad i = 1, \dots, m \qquad (1)$$

where $\boldsymbol{\gamma}_i^T \in \Re^n$ *is a vector of constants to be determined. This vector corresponds to the i*th *row of the matrix* $\boldsymbol{\Gamma}$. *The matrix representation is:*

$$\boldsymbol{\sigma}(\mathbf{x}) = \begin{bmatrix} \sigma_1 & \sigma_2 & \cdots & \sigma_i & \cdots & \sigma_m \end{bmatrix}^T = \boldsymbol{\Gamma}\mathbf{x} \qquad (2)$$

with $\boldsymbol{\Gamma} \in \Re^{m \times n}$.

Definition 2: *Let the switching function be* $\sigma_i(\mathbf{x})$ *with* $i=1,\dots,m$. *A sliding surface* S_i *is defined as follows:*

$$S_i=\left\{\mathbf{x}\in\Re^n \middle| \sigma_i(\mathbf{x})=0\right\}\ ,\ i=1,2,\dots,m \tag{3}$$

where m *denotes the number of system inputs.*

The state trajectory of a sliding mode variable structure control system consists of two modes and a reaching condition; these concepts are defined below.

Definition 3: *The condition under which the state will move toward and reach a sliding surface is called a reaching condition.*

Definition 4: *The reaching mode is the system trajectory under reaching condition between a point in space state and a sliding surface, which occurs in finite time.*

The ability to guarantee the reaching of a sliding surface is a crucial element of a SMCS. This depends on the reaching condition. There are several approaches [14]; however, the principal one is the reaching law adopted in this paper. The Reaching Law determines the switching function dynamics, and the general form is found to be

$$\dot{\boldsymbol{\sigma}}(\mathbf{x})=-\mathbf{Q}\operatorname{sgn}(\boldsymbol{\sigma}(\mathbf{x}))-\mathbf{K}\mathbf{f}(\boldsymbol{\sigma}(\mathbf{x})) \tag{4}$$

with $\boldsymbol{\sigma}=[\sigma_1\ \cdots\ \sigma_i\ \cdots\ \sigma_m]^T$, $\mathbf{Q}\in\Re^{m\times m}$ a diagonal matrix, so that $q_{ii}>0$, and $\mathbf{K}\in\Re^{m\times m}$ a diagonal matrix with $k_{ii}>0$. Functions f_i satisfy the following condition

$$s_i f_i(s_i)>0 \quad \text{when } s_i\neq 0 \text{ with } i=1,\cdots,m \tag{5}$$

Definition 5: Let $S_i=\left\{\mathbf{x}\in\Re^n \middle| \sigma_i(\mathbf{x})=0\right\}$ *be a sliding surface. If, for any initial state* $\mathbf{x}_0\in S_i$, *we have* $\mathbf{x}\in S_i\ \ \forall\ \ t>t_0$, *then the system is in sliding mode.*

The control law in a SMCS can be designed using two strategies, the first based on the switching scheme chosen and the second on the use of structures that are pre-defined for the control law [14]. The first control strategy includes, among others, the control law for the free-order switching scheme obtained from the reaching law. If the appropriate design of the sliding surface is combined with a reaching condition in the control law design, the asymptotic stability of the system can be guaranteed [15].

The state trajectory of a discrete-time SMCS should have the following properties [16]:

(P1)-. From any initial state, the state trajectory will move monotonically in the direction of the switching surface and will cross it in a finite time.

(P2)-. Once the trajectory has crossed the switching surface the first time, it will cross the surface again at each successive sampling, resulting in a zigzag motion about the switching surface.

(P3)-. The magnitude of each zigzagging step will not increase so that the trajectory will remain within a specified band.

The above properties are used to define the Quasi-Sliding Mode and the Reaching Law for the design of a discrete-time control law.

Definition 6: *The movement of a discrete-time SMCS that satisfies with the assumptions P2 and P3 is called Quasi-Sliding Mode (QSM) and is defined by*

$$\left\{\mathbf{x}\middle| -\Delta<s(\mathbf{x})<+\Delta\right\} \tag{6}$$

where 2Δ *is the width of the band.*

The equivalent form of the reaching law (4) for discrete-time systems is the one proposed by Gao *et al.* [16] which is given as follows

$$\boldsymbol{\sigma}(k+1)-\boldsymbol{\sigma}(k)=-\mathbf{Q}T\operatorname{sgn}(\boldsymbol{\sigma}(k))-\mathbf{K}T\boldsymbol{\sigma}(k) \tag{7}$$

$$\mathbf{I}-\mathbf{K}T>0 \tag{8}$$

with $\mathbf{K}$, $\boldsymbol{\sigma}$ and $\mathbf{Q}$ are as in (4), and T is the sampling period.

The inequality (8) guarantees satisfaction of P1; whereas the term $\operatorname{sgn}(\boldsymbol{\sigma}(k))$ guarantees *P2* and *P3*.

Definition 7: *A SMC discrete-time system is said to meet the reaching condition if the resulting system has properties P1, P2 and P3.*

III. Fuzzy Control

A fuzzy controller is based on the theory of fuzzy sets and fuzzy logic proposed by Lotfi Zadeh [17] as a mathematical mode of representing the vagueness and inexactness that exists in language, real physical objects and how a human being thinks. Such a controller is formed by the following units: the Fuzzification Interface, the Data Base and Rules Base integrated Knowledge Base, the Inference Mechanism and, lastly, the Defuzzification Interface. Immediately we presente each unit in detail.

The Fuzzification Interface transforms the crisp value of the input variable into a fuzzy set through of Fuzzifier operator as follows

$$\Phi_I=\mathit{Fuzzifier}(\varphi_c) \tag{9}$$

where Φ_I is a fuzzy input set and φ_c is the controller input variable.

The data base characterizes the rule base and the manipulation of data in the fuzzy controller, whereas the rules base determines the control action to be taken. The Inference Mechanism simulates human decision-making based on fuzzy concepts and infers control actions.

Finally, the Defuzzification Interface converts the fuzzy control action into a crisp action through of Defuzzifier operator as follows

$$f_c=\mathit{Defuzzifier}(\Phi_0) \tag{10}$$

where Φ_0 is the fuzzy set of the controller output and f_c is the crisp control action.

In the *TAKAGI-SUGENO* (TS) model [18][19], the consequent of fuzzy rules uses a linear combination of the fuzzy system's input variables.

Definition 8: *The* ith *fuzzy implication in the TS model is given by*

$$\begin{array}{l}\text{If } \varphi_{i1} \text{ is } \Phi_{i1} \text{ and } \cdots \text{ and } \varphi_{iq} \text{ is } \Phi_{iq} \\ \quad \text{Then } f = \alpha_{i1}\varphi_{i1} + \alpha_{i2}\varphi_{i2} + \cdots + \alpha_{iq}\varphi_{iq}\end{array} \tag{11}$$

with φ_{il} *measurable or estimated input variables,* Φ_{il} *constant fuzzy sets, the signal* f *is the controller output, and* $\alpha_{il} \in \Re$ *with* $i = 1,2,\cdots,r$ *and* $l = 1,2,\cdots,q$.

IV. Gain Scheduling

Gain scheduling is a classical technique that is widely used in the industry. It is a way of getting around the local limitation of the technique to linearize a system's nonlinear equations. The technique is essentially an intuitive and intensely heuristic approach which has gained strength and aroused interest among the scientific community owing to its success in a wide variety of control problems. The general procedure used for the technique has evolved as folllows:

1. Linearize the dynamic equations of the nonlinear system around a set of equilibrium points.
2. Determine a controller for each linear model through a conventional method of linear time-invariant systems (LTI).
3. Design a global control law which can consist of the interpolation of the parameters of the local controllers.
4. Ensure the stability and performance of the nonlinear system along the intermediate points, through intensive simulations.

This strategy has a number of disadvantages. First, it does not guarantee closed-loop stability. Additionally, proven know-how and knowledge of the system, are often essential for implementing this strategy. This last point means that this kind of method cannot be reproduced in a general context. A complete survey about Gain Scheduling can be found in [20].

V. Problem of Transition Between Operating Conditions

Consider a nonlinear dynamic system P that is affine in the control signal described $\forall\, t \geq 0$ by the equations

$$\dot{\mathbf{x}}(t) = \mathbf{g}(\mathbf{x}(t)) + \mathbf{h}(\mathbf{x}(t))\mathbf{u}(t) = \mathbf{f}(\mathbf{x}(t), \mathbf{u}(t)) \tag{12}$$

where $\mathbf{x}(t) \in X \subseteq \Re^{\mathbf{n}}$ is the state vector, X is an open set, $\mathbf{u}(t) \in U \subseteq \Re^m$ is the control vector, U is the set of all the admissible controls, therefore $\mathbf{u}(\cdot) \in U$ is a measurable function. The operating space of P is $\Psi = X \times U$ with an operating vector $\boldsymbol{\psi} = (\mathbf{x}, \mathbf{u}) \in \Psi$, hence the set of operating conditions is defined as

$$\Omega = \left\{ \boldsymbol{\psi} = \left(\mathbf{x}_{eq}^{(j)}, \mathbf{u}_{eq}^{(j)}\right) \in \Psi : \mathbf{f}\left(\mathbf{x}_{eq}^{(j)}, \mathbf{u}_{eq}^{(j)}\right) = \mathbf{0},\, j = 1,\ldots,p \right\} \tag{13}$$

The vectorial functions $\mathbf{g}$ and $\mathbf{h}$ are C^1 so they fulfill a *Lipschitz* condition in the vicinity of the operating points Ψ, which guarantees the existence and uniqueness of the solutions of (12).

Accordingly, we define scheduling that regulates transitions in Ω through the variable $\mathbf{x}_P(k)$.

Problem: Determine a control law $\mathbf{u}(k) \in U \subseteq \Re^m$ that guarantees the transitions of the nonlinear system P according to scheduling $\mathbf{x}_P(k)$ within the set of operating conditions Ω. Closed-loop stability between transitions and certain performance specifications must be ensured.

VI. Sliding Mode Fuzzy Gain Scheduling

The SMFGS control system is a sampled data system, so samplers and hold elements must be incorporated, e.g. Zero-Order-Hold (Z.O.H.), as shown in Figure 1. Block P represents the nonlinear plant defined through (12). We denote the controller by operator K and $S^{(j)}$ is the jth generator of sliding surfaces, with $j = 1,\ldots,p$.

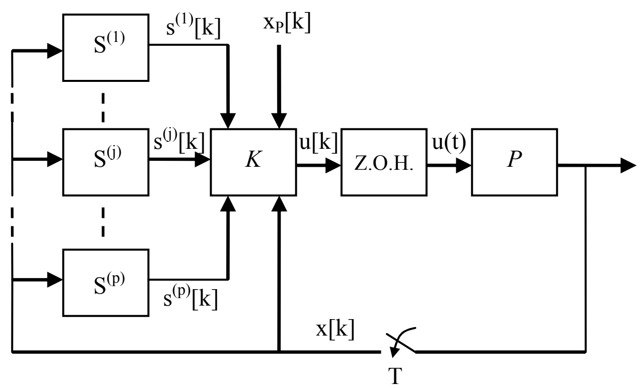

Fig. 1 Block diagram of the Sampled Data SMFGS Control System

As mentioned previously, the SMFGS control system is a sampled data system. Thus, we may conduct the analysis from two different perspectives: that of the plant, which considers everything as continuous, and that of the controller which sees everything as discrete. In this paper we shall adopt the controller's perspective, which proposes the requirement to discretize plant P.

The nonlinear system P (12) is represented within a polyhedral region through an uncertain linear model. The polyhedral region is constructed around the jth equilibrium point $\left(\mathbf{x}_{eq}^{(j)}, \mathbf{u}_{eq}^{(j)}\right)$ through the determination of the error between the nonlinear system and the first order approximation. This induces a symmetrical polyhedron whose size is δ with the equation

$$\Sigma_S = \left\{ x_i \in X : \left| x_i - x_{i_{eq}}^{(j)} \right| \leq \delta,\ i = 1,\cdots,n;\, j = 1,\cdots,p \right\} \tag{14}$$

The Discrete Uncertain Linear Model (DULM) is

$$\mathbf{x}(k+1) = \left(\mathbf{A}_0^{(j)} + \Delta\mathbf{A}^{(j)}\right)\hat{\mathbf{x}}^{(j)}(k) + \left(\mathbf{B}_0^{(j)} + \Delta\mathbf{B}^{(j)}\right)\hat{\mathbf{u}}^{(j)}(k) \tag{15}$$

with $\hat{\mathbf{x}}^{(j)}(k)=\mathbf{x}(k)-\mathbf{x}^{(j)}(k)$, $\hat{\mathbf{u}}^{(j)}(k)=\mathbf{u}(k)-\mathbf{u}^{(j)}(k)$, $\mathbf{A}_0^{(j)}$ and $\mathbf{B}_0^{(j)}$ are appropriately sized Jacobian matrices, $\Delta\mathbf{A}^{(j)}$ and $\Delta\mathbf{B}^{(j)}$ are matrices of uncertainties associated with the states and inputs, respectively. The uncertainties are bounded as follows

$$\rho_A^{(j)} = \max_i \left\|\Delta\mathbf{A}_i^{(j)}\right\|_2 \quad , \quad \rho_B^{(j)} = \max_l \left\|\Delta\mathbf{B}_l^{(j)}\right\|_2 \qquad (16)$$

with $i=1,\ldots,2^q$ and $l=1,\ldots,2^p$; where q is the number of nonlinear states and p the number of nonlinear states associated with the inputs..

The following result shows the sliding mode of the nonlinear system and an uncertain linear model to be identical. This result enables sliding mode linear control techniques to be used to design nonlinear systems of the type considered. More importantly, it means that stability of nonlinear system can be guaranteed when using the discrete uncertain linear model in the design of the controller.

Proposition 1 (Proposition of Identity between Discrete Sliding Modes): *Given the discretized nonlinear system P, the DULM (15) and a sliding surface described by*

$$\boldsymbol{\sigma}^{(j)}(\mathbf{x}(k)) = \boldsymbol{\Gamma}^{(j)}\left(\mathbf{x}(k)-\mathbf{x}_{eq}^{(j)}\right) \qquad (17)$$

with $\boldsymbol{\sigma}^{(j)} \in \Re^{m \times n}, j=1,\cdots,p$. *If the following assumptions are true*

$$\mathbf{g}(\mathbf{x}(k)) = \widetilde{\mathbf{A}}^{(j)}\left(\mathbf{x}(k)-\mathbf{x}_{eq}^{(j)}\right)+\widetilde{\mathbf{B}}^{(j)}\boldsymbol{\Theta}^{(j)}(\mathbf{x}(k)) \qquad (18)$$

$$\mathbf{h}(\mathbf{x}(k)) = \widetilde{\mathbf{B}}^{(j)}\left[\mathbf{I}+\boldsymbol{\Xi}^{(j)}(\mathbf{x}(k))\right] \qquad (19)$$

with $\widetilde{\mathbf{A}}^{(j)} = \mathbf{A}_0^{(j)} + \rho_A^{(j)}\mathbf{I}_A^*$, $\widetilde{\mathbf{B}}^{(j)} = \mathbf{B}_0^{(j)} + \rho_B^{(j)}\mathbf{I}_B^*$, $\boldsymbol{\Theta}^{(j)}(\mathbf{x}(k))$ *and* $\boldsymbol{\Xi}^{(j)}(\mathbf{x}(k))$, *appropriately sized.*

Therefore the identity between the sliding modes of the discretized nonlinear system P (12) and the DULM (15) is guaranteed.

The proof of proposition 1 consists of determining the difference equation associated to the dynamics of sliding mode for the discretized nonlinear system P as for (15), using in both cases the hypotheses (18) and (19).

The following result is used to design the control law.

Proposition 2: Given a nonlinear system P (12) , the set of operating conditions Ω (13), the DULM (15), and the scheduling variable $\mathbf{x}_P(k)$. *If*

(a)-. $\mathbf{I}-\mathbf{K}T>0$

(b)-. Proposition 1 is confirmed

(c)-. The matrix $\boldsymbol{\Gamma}^{(j)}\widetilde{\mathbf{B}}^{(j)}$ *is non- singular*

Then, the control law

$$\mathbf{u}^{(j)}(k) = -\left[\boldsymbol{\Gamma}^{(j)}\widetilde{\mathbf{B}}^{(j)}\right]^{-1}\left[\boldsymbol{\Gamma}^{(j)}\left(\widetilde{\mathbf{A}}^{(j)}-\mathbf{I}\right)\left(\mathbf{x}^{(j)}(k)-\mathbf{x}_{eq}^{(j)}\right)+ +\mathbf{Q}^{(j)}T\,\mathrm{sgn}\left(\boldsymbol{\sigma}^{(j)}(k)\right)+\mathbf{K}^{(j)}T\boldsymbol{\sigma}^{(j)}(k)\right]+\mathbf{u}_{eq}^{(j)} \qquad (20)$$

with $\widetilde{\mathbf{A}}^{(j)} = \mathbf{A}_0^{(j)} + \rho_A^{(j)}\mathbf{I}_A^*$ *y* $\widetilde{\mathbf{B}}^{(j)} = \mathbf{B}_0^{(j)} + \rho_B^{(j)}\mathbf{I}_B^*$ *guarantees:*

(i)-. Reaching the sliding surface according to the free-order switching scheme corresponding to each operating condition as a function of scheduling variable $\mathbf{x}_P(k)$.

(ii)-. Global Asymptotic Stability.

The proof of proposition 2 is immediate when using (15), the first difference of (17) and the reaching law for discrete-time systems (7).

Matrices $\mathbf{I}_A^*$, $\mathbf{I}_B^*$ are binary matrices whose structure is linked to the application.

The design methodology proposed is condensed in the algorithm in Figure 2.

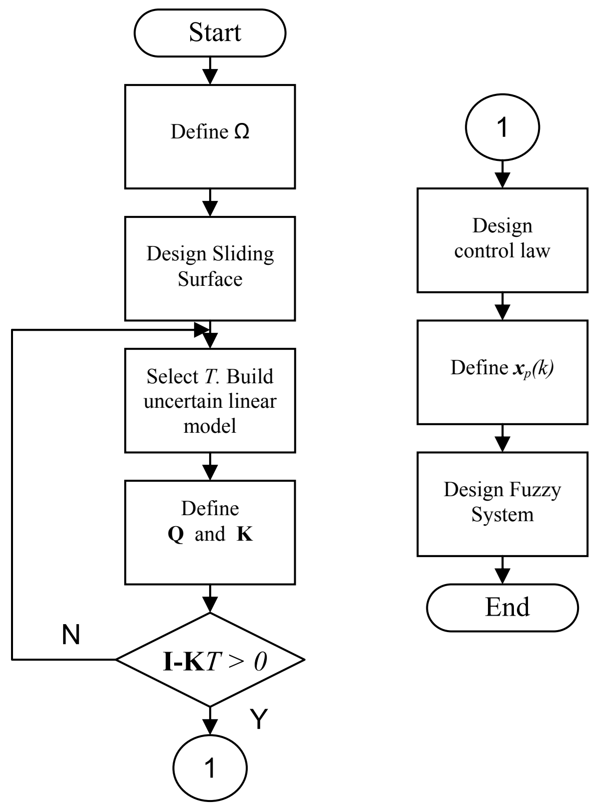

Fig. 2 Sampled Data SMFGS algorithm

Table 1 contains the template for constructing the fuzzy system rules base.

TABLE 1
RULES BASE OF THE CONTROLLER FOR EACH OPERATING CONDITION

If $\boldsymbol{\sigma}^{(j)}$ is *ZE* Then $\mathbf{u}(k)=\mathbf{u}_{EQ}(k)$
If $\boldsymbol{\sigma}^{(j)}$ is *N* Then $\mathbf{u}(k) = -\left[\boldsymbol{\Gamma}\widetilde{\mathbf{B}}\right]^{-1}\left[\ \boldsymbol{\Gamma}\left(\widetilde{\mathbf{A}}-\mathbf{I}\right)\left(\mathbf{x}(k)-\mathbf{x}_{eq}\right)+ +\mathbf{K}T\boldsymbol{\sigma}^{(j)}(k)-\mathbf{Q}T\mathrm{sgn}\left(\boldsymbol{\sigma}^{(j)}(k)\right)\ \right]+\mathbf{u}_{eq}$
If $\boldsymbol{\sigma}^{(j)}$ is *P* Then $\mathbf{u}(k) = -\left[\boldsymbol{\Gamma}\widetilde{\mathbf{B}}\right]^{-1}\left[\ \boldsymbol{\Gamma}\left(\widetilde{\mathbf{A}}-\mathbf{I}\right)\left(\mathbf{x}(k)-\mathbf{x}_{eq}\right)+ +\mathbf{K}T\boldsymbol{\sigma}^{(j)}(k)+\mathbf{Q}T\mathrm{sgn}\left(\boldsymbol{\sigma}^{(j)}(k)\right)\ \right]+\mathbf{u}_{eq}$

where $\mathbf{u}_{EQ}(k)$ is the equivalent control law.

Figures 3 and 4 show the fuzy sets associated with sliding surface and scheduling variable.

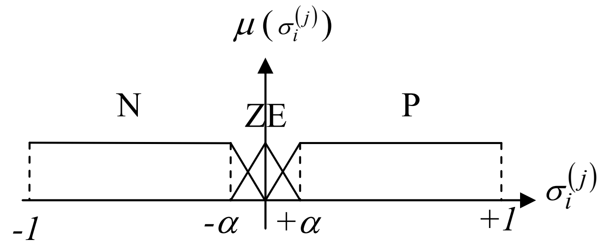

Fig. 3 Fuzzy sets associated with each sliding surface

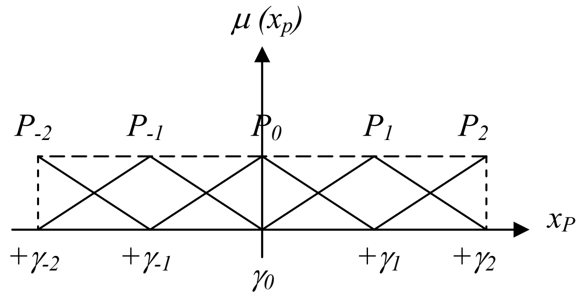

Fig. 4 Typical fuzzy sets associated with the scheduling variable

VII. Application: Magnetic Levitation System

The mathematical model for the magnetic levitation system consists of four mismatched SISO systems. Each SISO system is modeled by the following second order differential equation [21]:

$$m_L \ddot{x}(t) = m_L g - \frac{\mu_0 acN^2}{2}\left(\frac{i(t)}{x(t)}\right)^2 \qquad (21)$$

where m_L is ¼ of the total mass of the vehicle, μ_0 is the permeability of free space constant, a and c correspond to the dimensions of the electro magnet, N is the number of turns, $x(t)$ position of the mass with respect to the magnet and $i(t)$ is the current. Table 2 contains the values of each parameter of the system which are used in the simulation.

TABLE 2
Value of the Parameters of the Magnetic Levitation System

Nominal Position	0.005 [m]
Current in Stationary State	1.7124 [A]
Dimensions	$a = 0.03$ [m] , $c = 0.064$ [m]
Number of Turns	1300
Permeability of free space	$4\pi \times 10^{-7}$ [H/m]
Mass of ¼ of the Vehicle	24.4 [kg]

Two operating points are considered:

$$\Omega = \{ ([0.005 \;\; 0]^T, 2.9323), ([0.008 \;\; 0]^T, 7.5063)\} \quad (22)$$

therefore only points P_0 and P_1 are defined with $\gamma_0 = 0$ for $x_1(t) = 0.005\ m$ and $\gamma_0 = 1$ for $x_2(t) = 0.008\ m$.

The scheduling for the transition is defined as $t = 0.10$ sec . For the construction of the polyhedral region, $\delta = 0.001$ is used. $T = 0.001\ sec$ is selected.

The sliding surfaces are defined as:

$$\sigma^{(j)}(x(k)) = 13\left(x_1(k) - x_{1eq}^{(j)}\right) + x_2(k) \quad \text{with } j = 1,2 \qquad (23)$$

For the fuzzy sets of both sliding surfaces $\alpha = \pm 0.003$ is selected.

The control laws are obtained by using proposition 2 with the following values:

$$Q^{(1)} = 150 \ , \ K^{(1)} = 75 \ \text{y} \ Q^{(2)} = 100 \ , \ K^{(2)} = 50 \qquad (24)$$

Figures 5 and 6 show the time response associated with each system state variable, *i.e.* position $x_1(t)$ and velocity $x_2(t)$ with initial condition $[0.01 \;\; 0]^T$. Both figures show the stable transition from operating point $[0.005 \;\; 0]^T$ to operating point $[0.008 \;\; 0]^T$. The system response is very fast and the steady-state error is zero. Figure 7 illustrates the sampled-data control signal. It also shows that the signal has no high-frequency oscillations.

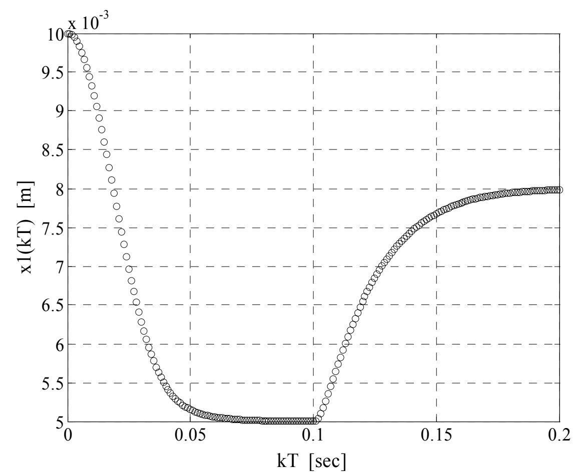

Fig. 5 Time Response $x_1(k)$

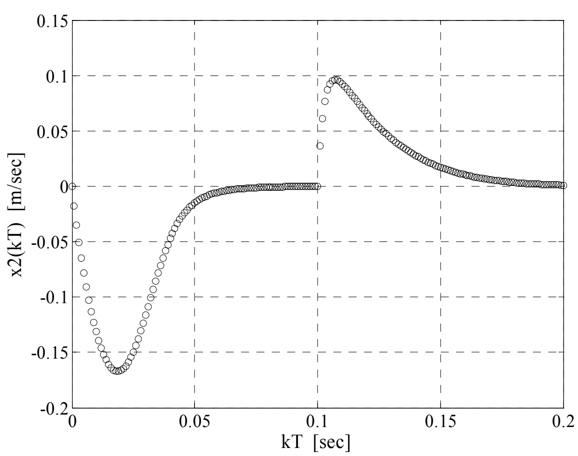

Fig. 6 Time Response $x_2(k)$

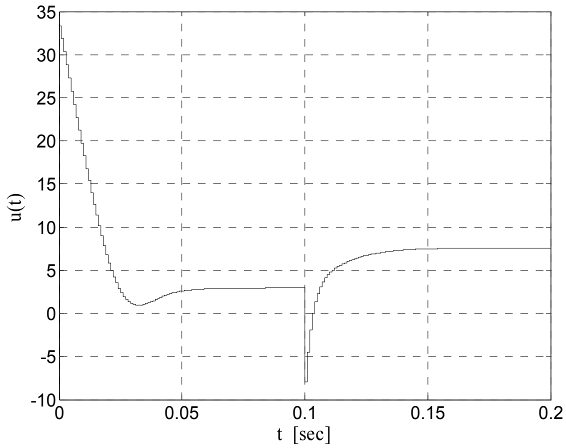

Fig. 7 Sampled-Data Control Signal $u(t)$

The surfaces $\sigma^{(1)}(x(k))$ and $\sigma^{(2)}(x(k))$ can be observed in the time intervals associated with their corresponding operating point in Figures 8 and 9 respectively.

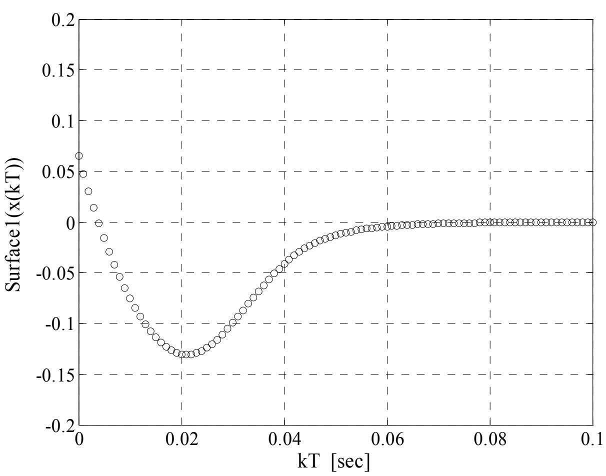

Fig. 8 Sliding surface $\sigma^{(1)}(x(k))$

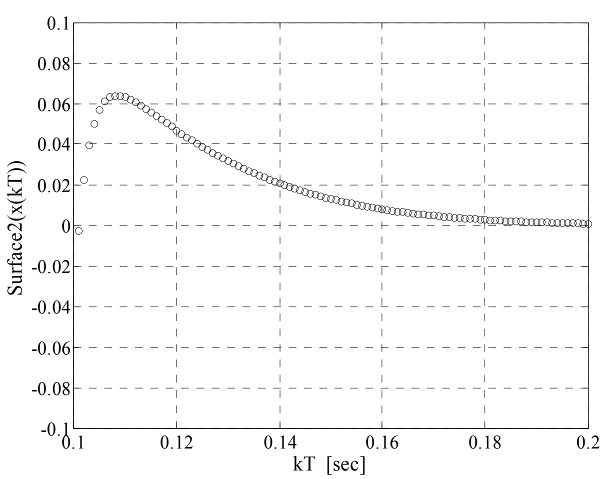

Fig. 9 Sliding surface $\sigma^{(2)}(x(k))$

VIII. CONCLUSIONS

A hybrid control formalism was developed that allows the synthesis of controllers for sampled data nonlinear dynamic systems. The technique proposed integrates elements of sliding mode control, fuzzy control and gain scheduling theories, in a successful and original manner, through a *Sliding Mode Fuzzy Gain Scheduling* strategy for sampled data systems. A stable transition between operating points in a nonlinear system of the type mentioned is guaranteed. The methodology was presented through a systematic algorithm based on the proposition of identity between sliding modes and the proposition to design a control law using an uncertain linear model for each operating point.

REFERENCES

[1] S. Y. Emelyanov. *Variable structure control systems*. Moscow: Nauka. (in Russian), 1967.

[2] V. I. Utkin. *Sliding modes and their application in variable structure systems*. Moscow: Nauka. (in Russian), 1974.

[3] U. Itkis. *Control systems of variable structure*. New York, Wiley, 1976.

[4] Y. Dote & R. G. Hoft. Microprocessor based sliding mode controller for a dc motor drives. Presented at the Ind. Applicat. Soc. Annu. Meeting, Cincinnati, OH, 1980.

[5] D. Milosavljevic. General conditions for the existence of a quasi-sliding mode on the switching hyperplane in discrete variable structure systems. *Automat. and Remote Control*, 46, 307-314, 1985.

[6] S. Z. Sarpturk, Y. Istefanopulos and O. Kaynak. On the stability of discrete-time sliding mode control systems. *IEEE on Transactions Automatic Control*, 32 (10), 930-932, 1987.

[7] K. Furuta. Sliding mode control of a discrete system. *Systems and Control Letters*, vol. 14, 145-152, 1990.

[8] H. Sira-Ramirez. Non-linear discrete variable structure systems in quasi-sliding mode. *Int. Journal of Control*, 54 (5), 1171-1187, 1991.

[9] X. Yu. *Digital variable structure control with pseudo-sliding modes. Variable structure and Lyapunov Control – (Lecture Notes in Control and Information Science 193).* Alan S. I. (ed), Springer-Verlag London Limited, 1994.

[10] O. Kaynak, E. Kemalettin and M. Ertugrul. The fusion of computationally intelligent methodologies and sliding-mode control – A survey. *IEEE Trans. Ind. Elec.,* 48 (1), 4-17, 2001.

[11] W. Bequette and K. Schott. Control of chemical reactors using multiple model adaptive control. In IFAC, Denmark, 1995.

[12] J. Suykens, B. De Moor and J. Vandewalle. Static and dynamic stabilizing neural controllers, applicable to transition between equilibrium points. Katholieke Universiteit Leuven – Departement Elecktrotechniek, ESAT-SISTA/TR 1993-03, 1993.

[13] M. Pivoso and A. Kosanovich. A dynamical supervisor strategy for multi-products processes. *Comp and Chem Eng*, 21, 149-154, 1997.

[14] J.Y. Hung, W. Gao and J.C. Hung. Variable structure control: A survey. *IEEE Trans. Ind. Elec.,* 40 (1), 2-22, 1993.

[15] G.-C. Hwang and S.-C. Lin. A stability approach to fuzzy control design for nonlinear systems. *Fuzzy Sets and Systems*, 48, 279-287, 1992.

[16] W. Gao, Y. Wang and A. Homaifa. Discrete-time variable structure control systems. *IEEE Trans. Ind. Elec.,* 42 (2), 117-122, 1995.

[17] L. A. Zadeh. Fuzzy Sets. *Information and Control*, 8, 338-353, 1965.

[18] T. Takagi and M. Sugeno. Fuzzy Identification of systems and its applications to modelling and control. *IEEE Transactions on Systems, Man and Cybernetics,* SMC-15 (1), 116-132, 1985.

[19] M. Sugeno and G. T. Kang. Structure identification of fuzzy model. *Fuzzy Sets and Systems*, 28, 15-33., 1988.

[20] Rugh, W. J. & Shamma, J. S. (2000). Research on gain scheduling. *Automatica*, 36, 1401-1425.

[21] A. Bittar and R. Moura. "H_2 and H_∞ Control for MagLev Vehicles". IEEE Control Systems, pp. 18-25, August 1998.