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H_∞ output feedback control for descriptor systems with delayed-state

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***Abstract*— Using the method of Linear matrix inequalities(LMI), this paper considers the H_∞ dynamic output feedback control for descriptor systems with delayed-state. The controller is one descriptor system without delay. Several equivalent sufficient conditions for the existence of one descriptor dynamic controller without impulsive models are given. Furthermore the explicit expression of the desired controller is obtained. Finally one example is given to show the validity of the proposed results.**

I. INTRODUCTION

Singular systems have comprehensive practical background such as power systems[1, 2], social economic systems[3], circuit systems[4], and so on. Great progress[5-7] has been made in the theory and its applications since 1970s. On the other hand, Control of delay systems has been a topic of recurring interest over the past decades since time-delays are often the main causes for instability and poor performance of systems and encountered frequently in various engineering systems. There have existed an extensive literature in this field [8]-[15]. In recent years some researchers have turned their attentions to singular time delay systems. For example, paper [16] studied singular LQ problem for discrete singular systems with multiple time delays. In paper [17], robust stability was analyzed for discrete singular time delay systems with uncertainty. For the continuous time systems, literatures [18,19] gave the numerical solution for such system. Paper [20] discussed the stability problem for linear delay differential algebraic equations, while paper [21] analyzed the stability problem for nonlinear delay differential algebraic equations. However in papers [20,21], no uncertainty appeared. Paper [22] considered robust stability and stabilization for singular time delay systems with uncertainty, while paper [23] studied guaranteed cost control for such systems.

In this note, we investigate the H_∞ control problem for linear singular systems with delay in state. Here we design one singular dynamic output feedback controller without impulsive modes. This paper first gives the sufficient condition, which ensures the closed loop is asymptotically stable and has H_∞-norm bound. Using the method of LMI, we give several equivalent sufficient condition for the existence of a controller without impulsive modes. Furthermore, the explicit expression of the desired controller is presented.

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II. PROBLEM SETTING

Let us consider the following descriptor system with delayed-state:

$$\begin{aligned} E\dot{x}(t) &= Ax(t) + A_2x(t-\tau) + B_1w(t) + B_2u(t) \\ z(t) &= C_1x(t) + D_{11}w(t) + D_{12}u(t) \\ y(t) &= C_2x(t) + D_{21}w(t) + D_{22}u(t) \end{aligned} \tag{1}$$

where, $x \in R^n$ is the descriptor variable, $u(t) \in R^m$ is the control input, $w \in R^q$ is the exogenous input, $X(t) \in R^s$ is the controlled output and $y(t) \in R^p$ is the measured output $E,\ A, A_\tau, B_1, B_2, C_1, C_2, D_{11}, D_{12}, D_{21}$ and D_{22} are constant matrices with appropriate sizes. The matrix E has rank $\gamma, (\gamma \leq n)$ without loss of generality, we assume that $D_{ij} = 0, i, j = 1, 2$. $\tau > 0$ is given constant.

The aim of the paper is to design a dynamic output feedback controller as follows:

$$\begin{aligned} E\dot{x}_k(t) &= A_kx_k(t) + B_ky(t) \\ u(t) &= C_kx_k(t) \end{aligned} \tag{2}$$

We hope that the resulting closed-loop

$$\begin{aligned} E_c\dot{x}_c(t) &= A_cx_c(t) + A_{c\tau}x_c(t-\tau) + B_cw(t) \\ z(t) &= C_cx_c(t) \end{aligned} \tag{3}$$

satisfies:
(1) zero-solution is asymptotically stable;
(2) $\|T_{zw}(s)\|_\infty \leq \gamma$. (4)
here,

$$\begin{gathered} E_c = \begin{bmatrix} E & 0 \\ 0 & E \end{bmatrix}, A_c = \begin{bmatrix} A & B_2C_k \\ B_kC_2 & A_k \end{bmatrix} \\ A_{c\tau} = \begin{bmatrix} A_\tau & 0 \\ 0 & 0 \end{bmatrix}, B_c = \begin{bmatrix} B_1 \\ 0 \end{bmatrix} \\ C_c = \begin{bmatrix} C_1 & 0 \end{bmatrix}, x_c(t) = \begin{bmatrix} x(t) \\ x_k(t) \end{bmatrix} \end{gathered} \tag{5}$$

and $\|T_{zw}(s)\| = C_c(sE_c - A_c - A_{c\tau}e^{-s\tau})^{-1}B_c$ is the transfer function of closed-loop, $\gamma > 0$ is given constant.

Additional we need the following lemmas:

Lemma1.[10] There exists $X \in R^{n\times n}$ such that

$$\begin{gathered} E^TX = X^TE \geq 0 \\ A^TX + X^TA < 0 \end{gathered}$$

iff A pair (E, A) is admissible, i.e.,
(a) $det(sE - A) \neq 0$;
(b) $degree\{det(sE - A)\} = rankE$;

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(c) the finite eigenvalues of $det(sE-A)$ are all in open left plane.

Lemma2. The partitioned matrix $\begin{bmatrix} Q_1+W_1 & Q_2 \\ Q_2^T & Q_4 \end{bmatrix} < 0$, where Q_1, W_1 are all symmetric matrices, if symmetric matrix $W_2 \leq W_1$, then

$$\begin{bmatrix} Q_1+W_2 & Q_2 \\ Q_2^T & Q_4 \end{bmatrix} < 0$$

III. Controller design

First, we give a sufficient condition for the asymptotic stability of the closed-loop system (3).

Theorem 1. If there exist matrix $X_c \in R^{2n\times 2n}$ and positive matrix $Y \in R^{n\times n}$ such that:

$$E_c^T X_c = X_c^T E_c \geq 0 \quad (6a)$$

$$A_c^T X_c + X_c^T A_c + X_c^T A_{c\tau}\Phi Y^{-1}\Phi^T A_{c\tau}^T X_c + \Phi Y \Phi^T < 0 \quad (6b)$$

here, $\Phi = \begin{bmatrix} I_n \\ 0 \end{bmatrix}$, then the closed-loop system (3) is asymptotically stable.

The proof is similar as the corresponding proof in literature [22] or [23].

Remark1. Inequalities (6) are invariable in restricted system equivalence transformation.

Remark2. By Lemma 1, when $A_\tau = 0$ in the inequality (6b), the corresponding descriptor system (E_c, A_c) is admissible. Therefore, theorem 1 is an extension for lemma 1.

Theorem 2. If there exist matrix $X_c \in R^{2n\times 2n}$ and positive matrix $Y \in R^{n\times n}$ satisfy

$$E_c^T X_c = X_c^T E_c \geq 0 \quad (9a)$$

$$\begin{aligned} &A_c^T X_c + X_c^T A_c + X_c^T A_{c\tau}\Phi Y^{-1}\Phi^T A_{c\tau}^T X_c \\ &+\Phi Y \Phi^T + \gamma^{-1} X_c B_c B_c^T X_c + \gamma^{-1} C_c^T C_c < 0 \end{aligned} \quad (9b)$$

where $\Phi = \begin{bmatrix} I_n \\ 0 \end{bmatrix}$, then the closed-loop system satisfies condition (4).

Proof. By theorem 1, it is only needed to prove $\|T_{zw}(s)\|_\infty \leq \gamma$.

Using the method of literature [159-162], let

$$\begin{aligned} S = &-(A_c^T X_c + X_c^T A_c + X_c^T A_{c\tau}\Phi Y^{-1}\Phi^T A_{c\tau}^T X_c \\ &+\Phi Y \Phi^T + \gamma^{-1} X_c^T B_c B_c^T X_c + \gamma^{-1} C_c^T C_c) > 0 \end{aligned}$$

then

$$\begin{aligned} &-S - A_c^T X_c - X_c^T A_c - X_c^T A_{c\tau}\Phi Y^{-1}\Phi^T A_{c\tau}^T X_c \\ &-\Phi Y \Phi^T - \gamma^{-1} X_c^T B_c B_c^T X_c - \gamma^{-1} C_c^T C_c = 0 \end{aligned}$$

The above equality add and subtract $E_c^T X_c wj$, $X_c^T A_{c\tau} e^{-jw\tau}$ and $A_{c\tau}^T X_c e^{jw\tau}$, using the condition (9), we get

$$\begin{aligned} &(-jwE_c^T - A_c^T - A_{c\tau}^T e^{jw\tau})X_c + X_c^T(jwE_c - A_c \\ &-A_{c\tau}e^{-jw\tau}) - \gamma^{-1}X_c^T B_c B_c^T X_c - \gamma^{-1}C_c^T C_c \\ &-S - (X_c^T A_{c\tau}\Phi Y^{-1}\Phi^T A_{c\tau}^T X_c - A_{c\tau}^T X_c e^{jw\tau} \\ &-X_c^T A_{c\tau} e^{-jw\tau} + \Phi Y \Phi^T) = 0 \end{aligned}$$

and

$$\begin{aligned} &(X_c^T A_{c\tau}\Phi Y^{-1}\Phi^{-1} A_{c\tau}^T X_c - A_{c\tau}^T X_c e^{jw\tau} \\ &-X_c^T A_{c\tau} e^{-jw\tau} + \Phi Y \Phi^T) \\ &= (\Phi^T A_{c\tau}^T X_c e^{jw\tau} - Y\Phi^T)^* Y^{-1} (\Phi^T A_{c\tau}^T X_c e^{jw\tau} - Y\Phi^T) \\ &\triangleq W(jw) \geq 0 \end{aligned}$$

Write $X(jw) = (jwE_c - A_c - A_{c\tau}e^{-jw\tau})^{-1}$, then $T_{zw}(s) = C_c X(jw) B_c$. Premultiply equality (10) with $(X(jw)B_c)^*$, and postmultiply the resulting with $X(jw)B_c$, obtain

$$\begin{aligned} &\gamma^{-1} T_{zw}^* T_{zw} \\ &= B_c^T X_c X(jw) B_c + (B_c^T X_c X(jw) B_c)^* \\ &\quad -\gamma^{-1}(B_c^T X_c X(jw) B_c)^* (B_c^T X_c X(jw) B_c)^* \\ &\quad -(X(jw)B_c)^*(W(jw)+S)X(jw)B_c \end{aligned}$$

Because $W(jw) + S > 0$, have

$$\begin{aligned} &\gamma I - \gamma^{-1} T_{zw}^* T_{zw} \\ &\geq \gamma I - B_c^T X_c X(jw) B_c - (B_c^T X_c X(jw) B_c)^* \\ &\quad -\gamma^{-1}(B_c^T X_c X(jw) B_c)^* (B_c^T X_c X(jw) B_c) \\ &= (\gamma^{\frac{1}{2}} I - \gamma^{-\frac{1}{2}} B_c^T X_c X(jw) B_c)^* (\gamma^{\frac{1}{2}} I - \\ &\quad \gamma^{-\frac{1}{2}} B_c X_c X(jw) B_c) \\ &\geq 0 \end{aligned}$$

Theorem $\|T_{zw}(s)\|_\infty \leq \gamma$. The proof is completed.

Using Schur complement, inequality (9b) can be rewritten as

$$\begin{aligned} &\Gamma(\gamma, X_c, Y, A_c, A_{c\tau}, B_c, C_c) \\ &= \begin{bmatrix} A_c^T X_c + X_c^T A_c + \Phi Y \Phi^T & X_c^T A_{c\tau}\Phi & X_c^T B_c & C_c^T \\ \Phi^T A_{c\tau}^T X_c & -Y & 0 & 0 \\ B_c^T X_c & 0 & -\gamma I & 0 \\ C_c & 0 & 0 & -\gamma I \end{bmatrix} \\ &< 0 \end{aligned} \quad (11)$$

From Theorem 2, to design controller (2), to solve A_k, B_k, C_k is to solve inequalities (9a) and (11). It is difficult to solve quadratic inequality (11). Next, this paper introduce a nonlinear transformation which will change equivalently quadratic inequality to a new linear inequality. Therefore, assume inequalities (9a) and (11) have solution X_c, and let

$$X_c = \begin{bmatrix} X & X_2 \\ X_3 & X_4 \end{bmatrix}, X \in R^{n\times n}, X_i \in R^{n\times n}, i = 2,3,4 \quad (12)$$

From inequality (9b), we know $detX_c \neq 0$. Without loss of generality, we assume that the matrices X, X_2, X_3, X_4 are all nonsingular. If this is not satisfied, we can take some modification similar as in literature [10]. Write

$$\begin{aligned} &T_1 = \begin{bmatrix} I & 0 \\ 0 & XX_3^{-1} \end{bmatrix}, T_2 = \begin{bmatrix} I & 0 \\ 0 & X_2^{-1}X \end{bmatrix} \\ &T_3 = \begin{bmatrix} T_2^T & & & \\ & I_n & & \\ & & I_q & \\ & & & I_s \end{bmatrix} \end{aligned} \quad (13)$$

have

$$\bar{X}_c = T_1 X_c T_2 = \begin{bmatrix} X & X \\ X & X X_3^{-1} X_4 X_2^{-1} X \end{bmatrix} \tag{14}$$

$$\begin{aligned} \bar{E}_c = T_1^{-T} E_c T_2 &= \begin{bmatrix} E & 0 \\ 0 & X^{-T} X_3^T E X_2^{-1} X \end{bmatrix} \\ &\overset{(9a)}{=} \begin{bmatrix} E & 0 \\ 0 & E \end{bmatrix} \end{aligned} \tag{15a}$$

$$\begin{aligned} \bar{A}_c &= T_1^{-T} A_c T_2 \\ &= \begin{bmatrix} A & B C_k X_2^{-1} X \\ X^{-T} X_3^T B_k C_2 & X^{-T} X_3^T A_k X_2^{-1} X \end{bmatrix} \\ &\triangleq \begin{bmatrix} A & B_2 \bar{C}_k \\ \bar{B}_k C_2 & \bar{A}_k \end{bmatrix} \end{aligned} \tag{15b}$$

$$\begin{aligned} \bar{B}_c &= T_1^{-T} B_c = \begin{bmatrix} B_1 \\ 0 \end{bmatrix} \\ \bar{C}_c &= C_c T_2 = \begin{bmatrix} C_1 & 0 \end{bmatrix} \end{aligned} \tag{15c}$$

$$\bar{A}_{c\tau} = T_1^{-T} A_{c\tau} T_2 = A_{c\tau} = \begin{bmatrix} A_\tau & 0 \\ 0 & 0 \end{bmatrix} \tag{15d}$$

where,

$$\begin{aligned} &\bar{A}_k = X^{-T} X_3^T A_k X_2^{-1} X, \bar{B}_k = X^{-T} X_3^T B_k, \\ &\bar{C}_k = C_k X_2^{-1} X \end{aligned} \tag{15e}$$

Because closed-loop $(\bar{E}_c, \bar{A}_c, \bar{A}_{c\tau}, \bar{B}_c, \bar{C}_c)$ and $(E_c, A_c, A_{c\tau}, B_c, C_c)$ are algebraically equivalent, they have the same transfer function. Furthermore,

$$\begin{aligned} \bar{X}_c^T \bar{E}_c &= T_2^T X_c^T T_1^T T_1^{-T} E_c T_2 = T_2^T X_c^T E_c T_2 \\ &= T_2^T E_c^T X_c T_2 = \bar{E}_c \bar{X}_c^T > 0 \end{aligned} \tag{16a}$$

and

$$\begin{aligned} &\Gamma(\gamma, \bar{X}_c, Y, \bar{A}_c, \bar{A}_{c\tau}, \bar{B}_c, \bar{C}_c) \\ &= T_3 \Gamma(\gamma, X_c, Y, A_c, A_{c\tau}, B_c, C_c) T_3^T \end{aligned} \tag{16b}$$

Using Theorem 2, closed-loop $(\bar{E}_c, \bar{A}_c, \bar{A}_{c\tau}, \bar{B}_c, \bar{C}_c)$ and $(E_c, A_c, A_{c\tau}, B_c, C_c)$ simultaneously satisfy condition (4).

Noticing

$$\begin{aligned} &\begin{bmatrix} I & 0 \\ -X_3 X^{-1} & I \end{bmatrix} \begin{bmatrix} X & X_2 \\ X_3 & X_4 \end{bmatrix} \begin{bmatrix} I & -X^{-1} X_2 \\ 0 & I \end{bmatrix} \\ &= \begin{bmatrix} X & 0 \\ 0 & X_4 - X_3 X^{-1} X_2 \end{bmatrix} \end{aligned} \tag{17}$$

set $X_4 - X_3 X^{-1} X_2$ is nonsingular. Let

$$\begin{aligned} &X X_3^{-1} X_4 X_2^{-1} X - X \\ &= X X_3^{-1} (X_4 - X_3 X^{-1} X_2) X_2^{-1} X = S^{-1} \end{aligned} \tag{18}$$

then rewrite $\bar{X}_c$ as

$$\bar{X}_c = \begin{bmatrix} X & X \\ X & S^{-1} + X \end{bmatrix} \tag{19}$$

Write

$$T^T = S + X^{-1}$$

the

$$\bar{X}^{-1} = \begin{bmatrix} T^T & -S \\ -S & S \end{bmatrix}$$

Comparing the coefficient matrices of closed-loop $(\bar{E}_c, \bar{A}_c, \bar{A}_{c\tau}, \bar{B}_c, \bar{C}_c)$ and $(E_c, A_c, A_{c\tau}, B_c, C_c)$, we see the difference between them is the controller parameters $(\bar{A}_k, \bar{B}_k, \bar{C}_k)$ and (A_k, B_k, C_k). Therefore, without generality, from now on we directly see $(E_c, A_c, A_{c\tau}, B_c, C_c, A_k, B_k, C_k)$ and X_c in equalities (2), (3) and (12) as $(\bar{E}_c, \bar{A}_c, \bar{A}_{c\tau}, \bar{B}_c, \bar{C}_c, \bar{A}_k, \bar{B}_k, \bar{C}_k)$ and $\bar{X}_c$ in equalities (15) and (19).

Write

$$T_4 = \begin{bmatrix} T & -S^T \\ I & 0 \end{bmatrix}, T_5 = \begin{bmatrix} T_4 & & & \\ & I_n & & \\ & & I_q & \\ & & & I_s \end{bmatrix} \tag{20}$$

From

$$T_4 X_c^T E_c T_4^T = T_4 E_c^T X_c T_4^T \geq 0 \tag{21}$$

get:

$$\begin{aligned} &\begin{bmatrix} E & 0 \\ 0 & E^T \end{bmatrix} \begin{bmatrix} T^T & I \\ I & X \end{bmatrix} \\ &= \begin{bmatrix} T & I \\ I & X^T \end{bmatrix} \begin{bmatrix} E^T & 0 \\ 0 & E \end{bmatrix} \geq 0 \end{aligned} \tag{22}$$

And from

$$T_5 \Gamma(\gamma, X_c, Y, A_c, A_{c\tau}, B_c, C_c) T_5^T < 0 \tag{23}$$

get:

$$\begin{aligned} &\alpha(\gamma, W_B, W_C, W, X, Y, T, L) \\ &= \begin{bmatrix} \Omega_1 & L & A_\tau & B_1 & T C_1^T \\ L^T & \Omega_2 & X^T A_\tau & X^T B_1 & C_1^T \\ A_\tau^T & A_\tau^T X & -Y & 0 & 0 \\ B_1^T & B_1^T X & 0 & -\gamma I & 0 \\ C_1 T^T & C_1 & 0 & 0 & -\gamma I \end{bmatrix} < 0 \end{aligned} \tag{24}$$

where

$$\begin{cases} W = T Y T^T, W_c = C_k S, W_B = X^T B_k \\ L = A + T A^T X - W_C^T B_2^T X + T C_2^T W_B^T \\ \qquad -S^T A_k^T X + T Y \\ \Omega_1 = A T^T + T A^T - B_2 W_c - W_c^T B_2^T + W \\ \Omega_2 = Y + X^T A + A^T X + W_B C_2 + C_2^T W_B^T \end{cases}$$

Study the inequalities (22),(24) and equality (25), it is obvious that equality (25) is a nonlinear transformation which change the matrices $A_k, B_k, C_k, Y, X, T, S$ to W_B, W_C, W, Y and L. Inequalities (22) and (24) are linear about unknown matrices X, Y, T, W_B, W_C, W and L. Therefore, the problem to solve controller parameter A_k, B_k, C_k is changed to solve linear inequalities (22) and (24) to get X, Y, T, W_B, W_C, W and L, and from these matrices obtain A_k, B_k, C_k according to equality (25).

Using Lemma 2, we have the main result of this paper.

Theorem 3. (1) The following statement (a), (b), (c) are equivalent.

(a) There exists controller (2), which makes the closed-loop system satisfies inequalities (9).

(b) There exist matrices $(W_B, W_C, \bar{W}, X, Y, T, L)$, here, $Y > 0, \bar{W} \geq 0, \bar{W} \geq W = TYT^T$ and $X, \begin{bmatrix} T^T & I \\ I & X \end{bmatrix}$ are nonsingular, which satisfy inequalities (22) and (24);

(c) There exist matrices $(W_B, W_C, \bar{W}, X, Y, T)$, here, $Y > 0, \bar{W} \geq 0, \bar{W} \geq W = TYT^T$ and $X, \begin{bmatrix} T^T & I \\ I & X \end{bmatrix}$ are nonsingular, which satisfy (22) and the following LMIs:

$$M_1 = \begin{bmatrix} \Pi_1 & A_\tau & B_1 & TC_1^T \\ A_\tau^T & -Y & 0 & 0 \\ B_1^T & 0 & -\gamma I & 0 \\ C_1 T^T & 0 & 0 & -\gamma I \end{bmatrix} < 0 \qquad (26a)$$

$$M_2 = \begin{bmatrix} \Pi_2 & X^T A_\tau & X^T B_1 & C_1^T \\ A_\tau^T X & -Y & 0 & 0 \\ B_1^T X & 0 & -\gamma I & 0 \\ C_1 & 0 & 0 & -\gamma I \end{bmatrix} < 0 \qquad (26b)$$

where $\Pi_1 = AT^T + TA^T - B_2 W_c - W_c^T B_2^T + \bar{W}$, $\Pi_2 = Y + X^T A + A^T X + W_B C_2 + C_2^T W_B^T$

(2) If inequalities (22) and (26) have solution $(W_B, W_C, \bar{W}, X, Y, T)$, here, $Y > 0, \bar{W} \geq 0, \bar{W} \geq W = TYT^T$ and $X, \begin{bmatrix} T^T & I \\ I & X \end{bmatrix}$ nonsingular, then the problem of H_∞ dynamic output feedback has one solution:

$$\begin{cases} B_k = X^{-T} W_B, C_k = W_C S^{-1} \\ A_k = (AT^T + A_\tau Y^{-1} A_\tau^T + \gamma^{-1} B_1 B_1^T) S^{-1} \\ \quad -B_2 C_k + B_k C_2 + X^{-T} A^T S^{-1} \\ \quad +X^{-T}(Y + \gamma^{-1} C_1^T C_1)(I + X^{-1} S^{-1}) \end{cases} \qquad (27)$$

Proof. The equivalence about (a), (b) has been discussed in the above. Regarding the nonsingularity of $\begin{bmatrix} T^T & I \\ I & X \end{bmatrix}$, notice that:

$$\begin{bmatrix} I & -X^{-1} \\ 0 & I \end{bmatrix} \begin{bmatrix} T^T & I \\ I & X \end{bmatrix} \begin{bmatrix} I & 0 \\ -X^{-1} & I \end{bmatrix} = \begin{bmatrix} T^T - X^{-1} & 0 \\ 0 & X \end{bmatrix},$$
$$S = T^T - X^{-1}$$

It is obvious that (b) implies (c). Next prove that (b) can be obtained from (c). Take

$$T_6 = \begin{bmatrix} I & 0 & A_\tau Y^{-1} & \gamma^{-1} B_1 & \gamma^{-1} TC_1^T \\ 0 & I & 0 & 0 & 0 \\ 0 & 0 & I & 0 & 0 \\ 0 & 0 & 0 & I & 0 \\ 0 & 0 & 0 & 0 & I \end{bmatrix}$$

Then

$$T_6 \alpha(\gamma, W_B, W_c, W, X, Y, T, L) T_6^T = T_6 \begin{bmatrix} \Omega_1 & L & A_\tau & B_1 & TC_1^T \\ L^T & \Omega_2 & X^T A_\tau & X^T B_1 & C_1^T \\ A_\tau^T & A_\tau^T X & -Y & 0 & 0 \\ B_1^T & B_1^T X & 0 & -\gamma I & 0 \\ C_1 T^T & C_1 & 0 & 0 & -\gamma I \end{bmatrix} T_6^T = \left[\begin{array}{c|cccc} K & F & 0 & 0 & 0 \\ \hline G & & & & \\ 0 & & H & & \\ 0 & & & & \\ 0 & & & & \end{array}\right]$$

here,

$$K = AT^T + TA^T - B_2 W_c - W_c^T B_2^T + W + A_\tau Y^{-1} A_\tau^T + \gamma^{-1} B_1 B_1^T + \gamma^{-1} TC_1^T C_1 T^T$$
$$F = L + A_\tau Y^{-1} A_\tau^T X + \gamma^{-1} B_1 B_1^T X + \gamma^{-1} TC_1^T C_1$$
$$G = F^T$$
$$H = M_2$$

From (25), (26), (27), obtain

$$K < 0, F = 0, H < 0$$

So inequality (24) holds. The proof is completed.

Further, from the above $\{E, A_k, B_k, C_k\}$, we get a proper controller as follows: Based on a singular-value decomposition of E, we represent E and A_k by

$$\hat{E} = U_E \begin{bmatrix} \Sigma & 0 \\ 0 & 0 \end{bmatrix} V_E^T, \hat{A}_k = U_E \begin{bmatrix} \hat{A}_{11} & \hat{A}_{12} \\ \hat{A}_{21} & \hat{A}_{22} \end{bmatrix} V_E^T$$

where $\Sigma > 0$, if $\hat{A}_{22}$ is nonsingular, the controller is regular and has no impulsive modes. If $\hat{A}_{22}$ is singular we redefine $\hat{A}$ as

$$A_k^* = U_E \begin{bmatrix} \hat{A}_{11} & \hat{A}_{12} \\ \hat{A}_{21} & \hat{A}_{22} + \mu I \end{bmatrix} V_E^T$$

where $\mu > 0$ is a scalar such that $\hat{A}_{22} + \mu I$ is nonsingular and such that the new E, A_k^*, B_k.

Example. Design H_∞ output feedback controller for system (1) with the following data:

$$E = \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}, \quad A = \begin{bmatrix} -2 & 3 \\ -1 & -1 \end{bmatrix}, A_h = \begin{bmatrix} 1 & 0 \\ 0.2 & 1 \end{bmatrix},$$
$$B_1 = \begin{bmatrix} 0.1 \\ 0.2 \end{bmatrix}, \quad B_2 = \begin{bmatrix} 0 & 1 \\ 0.1 & 0.2 \end{bmatrix},$$
$$C_1 = \begin{bmatrix} 0 & 0.5 \end{bmatrix}, C_2 = \begin{bmatrix} 3 & 1 \end{bmatrix},$$

When take $\gamma = 1$, solve LMIs (22),(26a),and(26b), get controller (2) from (27), where

$$A_k = \begin{bmatrix} -1.9387 & -.8060 \\ -55.0352 & -0.9950 \end{bmatrix}, B_k = \begin{bmatrix} -0.1236 \\ -9.5649 \end{bmatrix},$$
$$C_k = \begin{bmatrix} 5.1546 & -2.7035 \\ -0.3268 & 0.1023 \end{bmatrix}.$$

IV. CONCLUSIONS AND FUTURE WORKS

The paper has discussed the H_∞ output feedback control for descriptor systems with delayed-state. Using the method of LMI, several equivalent sufficient conditions have been obtained, which ensure that the closed-loop system is zero-solution asymptotically stable and satisfies H_∞ norm bound. Furthermore we give the controller's design, and the controller has no impulsive modes.

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