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Output Tracking of Non Input Affine Systems using Extended Hammerstein Models

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Abstract— **Tracking is a very important issue for control and has received considerable treatment in the input affine framework. Unfortunately, many real systems exhibit a behavior which is clearly non input affine. To cope with this, an integral extension of the model can be used to produce an extended input affine system, hiding the input nonlinearity in the drift term of the new model. This approach, however, has several disadvantages. In this paper we consider a different approach, motivated by physical considerations and practical experiences, which is based on the transformation or approximation of the original system by a cascade of a nonlinear non input affine map and a following input affine nonlinear system. As the paper shows, well known results of tracking control can be easily extended to this class yielding the solution of the original problem. Simulation results are presented to confirm the validity of the proposed techniques.**

I. INTRODUCTION

TRACKING of an external reference signal with the output of the controlled system is a central subject in control engineering and has received accordingly extensive treatment, both for linear and nonlinear systems. For nonlinear systems, essential contributions have been given *e.g.* in [1] and [2], where also the fundamental problems associated with zero dynamics have been addressed. Perfect tracking implies system inversion, and is only solvable if the initial conditions and the parameters of the system are known exactly. As this does not often hold in practical life, a substantial effort has been devoted to design asymptotic tracking control rules which allow some degree of uncertainty, while keeping the tracking error below given limits and recover, in the ideal case, the performance of the perfect control (see [3] and [4] for this approach and [5],[6],[7] for others methods aimed at the same goal).

Another essential component of the tracking setup is the kind of signal to be tracked, the so called reference signal. Usually reference signals are assumed to be exogenous signals that can be described as the output of an external system. In case of non exogenous reference signals other techniques have been presented in the work of [8] or in a similar way in the work of [9]. The results proposed in [8] had been enhanced to improve robustness and to deal with time varying systems as it can be seen in [10] and [11].

Most work on tracking of nonlinear systems has been concerned with input affine systems

$$\begin{aligned} \dot{x} &= f(x) + g(x)u \\ y &= h(x) \end{aligned} \tag{1}$$

where $x \in R^n$, $u \in R^p$ and $y \in R^p$. In the sequel, f and g are assumed to be differentiable a sufficient number of times. Unfortunately, many systems are not input affine:

$$\begin{aligned} \dot{x} &= \tilde{f}(x,u) \\ y &= h(x) \end{aligned} \tag{2}$$

A common solution (see *e.g.* [12]) consists in extending the system by an integrator and restating the control problem in terms of the derivative of the physical control input, while the input is treated as a state of the extended system. This formally simple approach has some disadvantages, among which the order increase of the system or the additional phase change introduced by the integrator which may affect the robustness of the system. In its core, integral extension boils down to hide the non input affine nonlinearity in the extended state space.

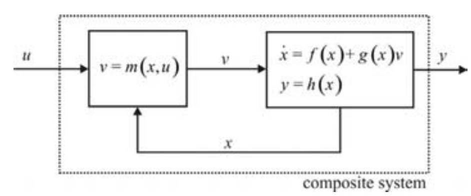

Fig. 1: Structure of the considered system class

Motivated by a practical experience in the engine control field [13], this paper proposes a different focus, that is to represent the original non input affine system by a special cascaded class (figure 1), which arises quite naturally by considering that many technical systems are indeed a chain of subsystems and that a cascaded representation can offer the advantage of simplicity while retaining the essential characteristics of the plant [14]. The proposed class consists of a static non input affine system followed by a dynamic input affine system, thus being in some sense a

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generalization of a Hammerstein model.

It is easy to see that the nonlinear systems included in this class are given by

$$\begin{aligned}\dot{x} &= f(x) + g(x)m(x,u) \\ y &= h(x).\end{aligned} \tag{3}$$

As it will be shown in the next section, on one side this class includes exactly a wide number of nonlinear non input affine systems, and, what is probably much more important for practical applications, offers a very good approximation framework for a much wider class. On the other side, as it will be shown in a later section, the standard methods developed for input affine systems can be easily extended to this class. Finally, some simulation examples will give an insight of the practical use of this class.

II. Derivation of Extended Hammerstein Systems

There are several ways to get a system of the form (3), the most natural one being first principle models, something which occurs quite often, *e.g.* in engine control or in general mechatronical systems. Notice, by the way, that classical Hammerstein systems are trivially included in this class. This approach is evident and will not be discussed here.

However, models of the form (3) can be derived from systems in the form (2) by coordinate transformations, a trivial case being all sufficiently differentiable nonlinear systems with relative degree $r = n$ with respect to the chosen output.

It is easy to show that such a transformation exists under very simple conditions.

Lemma 1. *The general nonlinear system (2) can be structured as shown in figure 1 and described by the equations (3) if and only if there exists a solution of*

$$\tilde{f}(x,u) - \tilde{f}(x,0) = \left.\frac{\partial \tilde{f}(x,u)}{\partial u}\right|_{x,u=0} \cdot m(x,u) \tag{4}$$

for $m(x,u)$ which is a mapping of dimension $p \times 1$. The resulting system functions thus are:

$$\begin{aligned}f(x) &= \tilde{f}(x,0) \\ g(x) &= \left.\frac{\partial \tilde{f}(x,u)}{\partial u}\right|_{x,u=0}.\end{aligned} \tag{5}$$

Proof: The sufficiency of lemma 1 is easy to deduce by converting equation (4) and using the nomenclature of (5).

$$\begin{aligned}\dot{x} = \tilde{f}(x,u) &= \tilde{f}(x,0) + \left.\frac{\partial \tilde{f}(x,u)}{\partial u}\right|_{x,u=0} \cdot m(x,u) \\ &= f(x) + g(x) \cdot m(x,u)\end{aligned} \tag{6}$$

Necessity can be proven by using Taylor series expansion for the input dependency of the system as done in [15].

$$\dot{x} = \tilde{f}(x,0) + \sum_{l=1}^{\infty} \left(\frac{1}{l!} \cdot \left.\frac{\partial^l \tilde{f}(x,u)}{\partial u^l}\right|_{x,u=0} \cdot u^l \right) \tag{7}$$

It is clear that only if

$$\begin{aligned} span\left\{ \left.\frac{\partial \tilde{f}(x,u)}{\partial u}\right|_{x,u=0} \right\} &= span\left\{ \left.\frac{\partial^2 \tilde{f}(x,u)}{\partial u^2}\right|_{x,u=0} \right\} = \cdots \\ &= span\left\{ \left.\frac{\partial^l \tilde{f}(x,u)}{\partial u^l}\right|_{x,u=0} \right\} = \cdots \end{aligned} \tag{8}$$

spans a $n \times p$ subspace of R^n it is possible to extract the base of this subspace from the sum of equation (7). If the base is set equal to the first derivative of the system function with respect to the input equation (7) results in

$$\dot{x} = \tilde{f}(x,0) + \left.\frac{\partial \tilde{f}(x,u)}{\partial u}\right|_{x,u=0} \cdot \underbrace{\sum_{l=1}^{\infty} \left(\frac{1}{l!} \cdot \tilde{m}_l(x) \cdot u^l \right)}_{m(x,u)} \tag{9}$$

So if there is a solution of (4) there must be a $n \times p$ subspace and therefore a base $g(x)$.

♦

Remark: *Consider the system (3) developed by a transformation as discussed. It is possible to simplify the drift term of the system again if*

$$f(x,0) = \hat{f}(x) + g(x) \cdot \hat{m}(x) \tag{10}$$

has a solution for $\hat{m}(x)$. The static nonlinear function and the drift term thus get:

$$\begin{aligned} f(x) &= \hat{f}(x) \\ m(x,u) &= \sum_{l=1}^{\infty} \left(\frac{1}{l!} \cdot \tilde{m}_l(x) \cdot u^l \right) + \hat{m}(x) \end{aligned} \tag{11}$$

At first glance the system class seems to be restrictive. In particular, the dimensionality condition on the solution for $m(x,u)$ implies that one single map describes the non affine dependency of all the terms of the input map. However, from the practical point of view it turns out that a very large amount of systems belongs to this system class – *e.g.* drivelines with combustion engines, power trains driven by electrical machines such as lifts, mills or other systems like airplanes, helicopters and many mechatronical systems.

Notice that for a large number of cases the exact equivalence will not be given but approximate solutions will be required – for instance based on two time scales approaches.

A possible consequence of the cascading is the overall loss of controllability even under conditions for which the dynamic input affine subsystem is controllable. Indeed, it is well known (see [12]) that this subsystem (like any input affine system) is controllable if the space spanned by

$$span\left\{g(x), ad_{-f}g(x), ..., ad_{-f}{}^{n-1}g(x)\right\} \tag{12}$$

has full dimension $x \in X$, where X is the interesting subset of R^n. Controllability of the cascade system can be guaranteed by two additional conditions:

Lemma 2. *The cascade system (3) is locally controllable in the set* $x \in X$ *and* $u \in U$ *with* $X = \{x \in R^n \| \|x - x_0\| < \varepsilon\}$ *and* $U = \{u \in U \| |u - u_0| < \delta\}$ *if the dynamical subsystem is locally controllable in the set* $x \in X$ *and if the nonlinear function* $m(x,u)$ *fulfills the conditions*

$$\begin{aligned} &1) \quad m(x,u) \neq 0, \qquad \forall x \in X \wedge \forall u \neq 0 \\ &2) \quad span\left\{\frac{\partial m(x,u)}{\partial u}\right\} = R^p, \ \forall x \in X \wedge \forall u \in U \end{aligned} \tag{13}$$

for sufficiently small ε *and* δ.

Proof: The first condition is obvious. The second condition is shown by considering the input map of the input affine system. Without loss of generality, we assume that $g(x)$ has rank p in X (otherwise two or more different inputs could be combined). We consider now the Taylor series expansion of $m(x,u)$ in terms of u near to $u_0' \in U$.

$$m(x,u) = m(x,u_0') + \left.\frac{\partial m(x,u)}{\partial u}\right|_{x,u=u_0'} \cdot \Delta u + O(u^2) \tag{14}$$

With $\hat{f}(x) = f(x) + g(x) \cdot m(x,u_0')$ it is possible to rewrite the system for a sufficient small environment Δu of u_0'

$$\dot{x} \approx \hat{f}(x) + \underbrace{g(x) \cdot \left.\frac{\partial m(x,u)}{\partial u}\right|_{x,u=u_0'}}_{\hat{g}(x)} \cdot \Delta u \tag{15}$$

Clearly, if condition 2 of (13) is fulfilled the input map $\hat{g}(x)$ does not lose rank and the controllability property will be retained for the cascade system.

III. Output Regulation of the considered System Class

A. Problem statement

For simplicity the cascade system is considered separated in the two parts throughout the next sections. Thus the system is given by the differential equation and the output function

$$\begin{aligned} \dot{x} &= f(x) + g(x) \cdot v \\ y &= h_y(x) \end{aligned} \tag{16}$$

and the algebraic equation

$$v = m(x,u). \tag{17}$$

The reference signal is assumed to be generated by the neutrally stable exosystem

$$\begin{aligned} \dot{\omega} &= s(\omega) \\ y_r &= h_s(\omega). \end{aligned} \tag{18}$$

The linearization in ($x_0 = 0, u_0 = 0$)[1] of the cascade system is given by

$$\begin{aligned} \dot{x} &= \tilde{A} \cdot x + \tilde{B} \cdot u \\ y &= C_y \cdot x \end{aligned} \tag{19}$$

with:

$$\begin{aligned} A &= \left.\frac{\partial f(x)}{\partial x}\right|_0, \quad B = g(0), \quad C_y = \left.\frac{\partial h_y(x)}{\partial x}\right|_0 \\ P &= \left.\frac{\partial m(x,u)}{\partial x}\right|_{0,0}, \quad R = \left.\frac{\partial m(x,u)}{\partial u}\right|_{0,0} \\ \tilde{A} &= A + B \cdot P, \quad \tilde{B} = B \cdot R \end{aligned} \tag{20}$$

It is assumed that the cascade system is locally controllable and observable. The problem of output regulation can be split into two sub problems: solving the tracking control problem of the dynamical system and inverting the nonlinear function. These sub problems yield the conditions for the solution of the overall tracking problem in the theorems for the full information problem and the error feedback output regulation problem. Therefore the error signal

$$e = h_e(x,\omega) = y - y_r = h_y(x) - h_s(\omega) \tag{21}$$

is considered as output of the joined system ((16) and (18)).

1) Full information output regulation problem:

Given the nonlinear system with the neutrally stable exosystem (18) then find, if possible, a mapping $\alpha(x,\omega)$ such that:

1. the equilibrium $x = 0$ of

   $$\dot{x} = f(x) + g(x) \cdot m(x, \alpha(x,0))$$

   is asymptotically stable in first approximation,
2. there exists a neighborhood V of *(0,0)* such that, for each initial condition $(x(0), \omega(0)) \in V$ the solution

   $$\begin{aligned} \dot{x} &= f(x) + g(x) \cdot m(x, \alpha(x,\omega)) \\ \dot{\omega} &= s(\omega) \end{aligned}$$

   satisfies:

   $$\lim_{t \to \infty} h_e(x(t), \omega(t)) = 0$$

2) Feedback error output regulation problem:

Given a nonlinear system with the neutrally stable exosystem (18), find, if possible two mappings $\theta(\xi)$ and $\eta(\xi,e)$, such that

1. the equilibrium $(x,\xi) = (0,0)$ of

   $$\begin{aligned} \dot{x} &= f(x) + g(x) \cdot m(x, \theta(\xi)) \\ \dot{\xi} &= \eta(\xi, h_e(x,0)) \end{aligned} \tag{22}$$

   is asymptotically stable in its first approximation,

[1] Without loss of generality the equilibrium point is assumed to be at $x_0 = 0, u_0 = 0$

2. there exists a neighborhood V of $(0,0,0)$ such that, for each initial condition $(x(0),\xi(0),\omega(0))\in V$ the solution of

$$\begin{aligned}\dot{x} &= f(x)+g(x)m(x,\theta(\xi))\\ \dot{\xi} &= \eta(\xi,h_e(x,\omega))\\ \dot{\omega} &= s(\omega)\end{aligned} \tag{23}$$

satisfies:

$$\lim_{t\to\infty} h_e(x(t),\omega(t))=0 \tag{24}$$

B. Full Information Regulation Problem

To solve the full information regulation problem the theorem stated in [1] is extended to the actual problem, yielding:

Theorem 1: *Consider the system described by (16), (17) and an exosystem (18). There exists a controller, which solves the output regulation problem if and only if the system is controllable and there exist mappings* $\pi : W^0 \to R^n$ *and* $c : W^0 \to R^m$

$$\begin{aligned}x&=\pi(\omega), \quad \text{with} \quad \pi(0)=0\\ v&=c(\omega), \quad \text{with} \quad \mathrm{c}(0)=0\end{aligned}$$

such that

$$\begin{aligned}\frac{\partial\pi(\omega)}{\partial\omega}s(\omega)&=f(\pi(\omega))+g(\pi(\omega))\cdot c(\omega)\\ 0&=h_e(\pi(\omega),\omega)\end{aligned} \tag{25}$$

and if there exist a mapping

$$u=l(\omega), \quad \text{with} \quad l(0)=0$$

such that the static function fulfills

$$m(\pi(\omega),l(\omega))=c(\omega) \tag{26}$$

Proof: The proof of necessity of the theorem can be directly adopted from the literature [1, p. 396 ff]. For sufficiency, the property of exponential attractiveness of a center manifold is applied to the problem. If the closed loop system is stable, the manifold $x=\pi(\omega)$ is invariant under the control law $u=l(\omega)$ and is exponentially attractive, thus the existence of the manifold $x=\pi(\omega)$ and the input function $u=l(\omega)$ are sufficient to solve the output regulation problem. If the cascade system is not stable it may be stabilized by a feedback law:

$$u=\alpha(x,\omega) \tag{27}$$

Since the control law stabilizes the system in its equilibrium point, the system matrix of the linearized closed loop system

$$A+B\cdot(P+R\cdot K) \text{ with } K=\left.\frac{\partial\alpha(x,\omega)}{\partial x}\right|_{0,0} \tag{28}$$

has all its eigenvalues with negative real parts and thus since the system matrix of the linearization of the overall system (combined with the exosystem) has a block diagonal form, it is obvious that a center manifold exists which is locally exponentially attractive. As the center manifold is fixed by the dynamical part of the system (see Theorem 1) the control law must fulfill the (necessary) condition

$$\alpha(\pi(\omega),\omega)=l(\omega) \tag{29}$$

Thus a sufficient feedback control law to stabilize the system in its equilibrium point to ensure condition (29) is:

$$u=\alpha(x,\omega)=l(\omega)+K\cdot(\pi(\omega)-x) \tag{30}$$

Thus the existence of a solution of theorem 1 and the existence of a stabilizing control law which is ensured by the theorem is sufficient for a solution of the full information output regulation problem.

Lemma 3. *If condition 2 of lemma 2 holds in a sufficient small environment of* $X=\{x\in R^n \,\|\, \|x-x_0\|<\varepsilon\}$ *and* $U=\{u\in U \,\|\, u-u_0|<\delta\}$ *then it is always possible for a sufficient small* W^0 *to find a unique mapping*

$$l(\omega)=\alpha(\pi(\omega),\omega) \tag{31}$$

such that (26) is fulfilled.

Proof: The proof can easily be accomplished by applying the implicit function theorem. Therefore the following implicit function is defined.

$$F(\omega,u)=m(\pi(\omega),u)-c(\omega)=0 \tag{32}$$

From the implicit function theorem it is known that if

$$rank\left(\frac{\partial F(\omega,u)}{\partial u}\right)=p \tag{33}$$

then there exists a mapping $u=l(\omega)$ such that

$$F(\omega,l(\omega))=0 \tag{34}$$

With

$$\frac{\partial F(\omega,u)}{\partial u}=\frac{\partial m(\pi(\omega),u)}{\partial u} \tag{35}$$

and condition 2 of lemma 2 the existence of the mapping $u=l(\omega)$ is guaranteed.

♦

The restriction of W^0 of lemma 3 may be very strong. It can be considered as a limitation of possible reference trajectories for systems concerning this class.

C. Error feedback output regulation problem

Starting from the solution of the standard error feedback regulation problem for the dynamical part of the system it is possible to find a solution of the problem for the cascade system.

Theorem 2: *The error feedback output regulation problem is solvable if and only if there exist mappings* $x=\pi(\omega)$ *and* $v=c(\omega)$ *with* $\pi(0)=0$ *and* $c(0)=0$ *both defined in a neighborhood of the origin, satisfying the conditions*

$$\begin{aligned}\frac{\partial\pi(\omega)}{\partial\omega}s(\omega)&=f(\pi(\omega))+g(\pi(\omega))\cdot c(\omega)\\ 0&=h_e(\pi(\omega),\omega)\end{aligned} \tag{36}$$

for all $\omega \in W^0$ (sufficient small region around the origin) and such that the autonomous system (exosystem) with output $c(\omega)$ and $\pi(\omega)$ can be immersed via the function $\xi = \sigma(\omega)$ into a system

$$\left.\begin{aligned} \dot\omega &= s(\omega) \\ v &= c(\omega) \\ x &= \pi(\omega) \end{aligned}\right\} \rightarrow \xi = \sigma(\omega) \rightarrow \begin{cases} \dot\xi = \varphi(\xi) \\ v = \gamma(\xi) \\ x = \tilde\pi(\xi) \end{cases} \tag{37}$$

defined on a neighborhood of the origin, in which $\varphi(0) = 0$ and $\gamma(0) = 0$, whose linear approximation is observable and there exists a mapping $u = l(\xi)$ with $l(0) = 0$ such that

$$m(\tilde\pi(\xi), l(\xi)) = \gamma(\xi) \tag{38}$$

where $\tilde\pi(\xi)$ is the transformed error zeroing manifold and the linear approximation of the closed loop system

$$\begin{aligned} \dot x &= f(x) + g(x)\cdot(m(\tilde\pi(\xi), l(\xi) + u_{st})) \\ \dot\xi &= \varphi(\xi) + N \cdot h_e(x,\omega) \end{aligned} \tag{39}$$

has a stabilizable and detectable equilibrium at $(x,\xi) = (0,0)$ where u_{st} is the control and ω the reference input.

Proof: As it is well known an error feedback controller solves the problem of output regulation (error is equal to zero), if there exist some mappings $x = \pi(\omega)$, $\xi = \sigma(\omega)$ with $\pi(0) = 0$ and $\sigma(0) = 0$ such that

$$\begin{aligned} \frac{\partial \pi(\omega)}{\partial\omega} s(\omega) &= f(\pi(\omega)) + g(\pi(\omega))\cdot\theta(\sigma(\omega)) \\ \frac{\partial \sigma(\omega)}{\partial\omega} s(\omega) &= \eta(\sigma(\omega), 0) \\ 0 &= h_e(\pi(\omega), \omega) \end{aligned} \tag{40}$$

are satisfied. See [2] for further details. For the considered system class this is possible only if (38) at $\xi = \sigma(\omega)$ with

$$c(\omega) = \theta(\sigma(\omega)),\quad \gamma(\xi) = \theta(\xi),\quad \varphi(\xi) = \eta(\xi, 0) \tag{41}$$

is guaranteed. The second equation of (40) considers the controllers internal dynamics. It is easy to see that this equation in combination with (41) is nothing else than the condition for immersing the autonomous system with the output $c(\omega)$ via $\xi = \sigma(\omega)$ into the controller dynamics equation (23). Thus this part of the proof is equal to the literature and the existence of such an immersion is a necessary condition. In this case not only the output of the autonomous system must agree with the output of the immersed system, also the manifold $x = \pi(\omega)$ must be mapped into the state space of the internal model such that $x = \tilde\pi(\xi)|_{\xi=\sigma(\omega)} = \pi(\omega)$ since it is necessary for inverting the nonlinear mapping $m(x,u)$. Thus if the condition of equation (38) is satisfied and the tracking error is zero, it is obvious that the controllers output signal is exactly the needed plant input to cause the state remaining on the error zeroing manifold $x = \pi(\omega)$.

In the next step it has to be proven that if the closed system is stabilizable and detectable, it is possible to find a stabilizing controller that makes the emerging (and invariant) manifold attractive. The linear approximation of the closed loop system (39) in the equilibrium point $(0,0)$ using the notation of (20) is

$$\begin{aligned} \begin{pmatrix} \dot x \\ \dot\xi \end{pmatrix} &= \begin{pmatrix} A & B\cdot P\cdot\tilde\Pi + B\cdot R\cdot\tilde\Gamma \\ N\cdot C_y & \Phi \end{pmatrix}\cdot\begin{pmatrix} x \\ \xi \end{pmatrix} + \\ &+ \begin{pmatrix} B\cdot R \\ 0 \end{pmatrix}\cdot u_{st} - \begin{pmatrix} 0 \\ N \end{pmatrix}\cdot y_r \\ y &= \begin{pmatrix} C & 0 \end{pmatrix} \end{aligned} \tag{42}$$

where

$$\tilde\Pi = \left.\frac{\partial\tilde\pi(\xi)}{\partial\xi}\right|_0,\quad \tilde\Gamma = \left.\frac{\partial l(\xi)}{\partial\xi}\right|_0,\quad \Phi = \left.\frac{\partial\varphi(\xi)}{\partial\xi}\right|_0. \tag{43}$$

N has to be chosen such that the system is stabilizable. If the system is stabilizable and detectable than it is possible to find a linear controller

$$\begin{aligned} \dot\xi_{LC} &= K\cdot\xi_{LC} + L\cdot e \\ u_{st} &= M\cdot\xi_{LC} \end{aligned} \tag{44}$$

consisting of *K, L, M* such that the matrix

$$\begin{pmatrix} \begin{pmatrix} A & B\cdot P\cdot\tilde\Pi + B\cdot R\cdot\tilde\Gamma \\ N\cdot C & \Phi \end{pmatrix} & \begin{pmatrix} B\cdot R\cdot M \\ 0 \end{pmatrix} \\ L\cdot\begin{pmatrix} C & 0 \end{pmatrix} & K \end{pmatrix} \tag{45}$$

has all its eigenvalues with negative real parts. Thus a sufficient controller in a sufficient small environment which solves the tracking problem is:

$$\begin{aligned} \dot\xi_{LC} &= K\cdot\xi_{LC} + L\cdot e \\ \dot\xi_{IM} &= \varphi(\xi_{IM}) + N\cdot e \\ u &= l(\xi_{IM}) + M\cdot\xi_{LC} \end{aligned} \tag{46}$$

Using this controller the resulting center manifold

$$M_C = \left\{ (x, \xi_{IM}, \xi_{LC}, \omega) : \begin{pmatrix} x = \pi(\omega) \\ \xi_{IM} = \tau(\omega) \\ \xi_{LC} = 0 \end{pmatrix} \right\} \tag{47}$$

is exponential attractive in sufficient small environment of the equilibrium point.

♦

IV. Simulation Example and Result

The solution of the error feedback output regulation problem is applied to the following nonlinear system

$$\begin{aligned} \dot x_1 &= -\mu\cdot x_1 + x_2 \\ \dot x_2 &= (x_1 - 1)\cdot u^2 + (x_2 + 3 - (x_2 + 3)\cdot x_1)\cdot u \\ &\quad + x_1^2 + \left(x_2 + x_2^2\right)\cdot x_1 - x_2 + x_3 - x_2^2 \\ \dot x_3 &= (1 - x_1)\cdot u^2 + ((x_2 + 3)\cdot x_1 - x_2 - 3)\cdot u \\ &\quad + \left(x_2 + x_2^2\right)\cdot x_1 - 3\cdot x_3 - x_2 + x_2^2 \\ y &= x_1 \end{aligned} \tag{48}$$

The initial state of the system is zero at the start of the simulation. The signal to be tracked is

$$y_d = a\cdot\sin(b\cdot t) \tag{49}$$

It is easy to see that the cascade system is stable and controllable. The reference trajectory can be calculated by the exosystem

$$\left.\begin{array}{l}\dot{\omega}_1 = b\cdot\omega_2\\ \dot{\omega}_2 = -b\cdot\omega_1\end{array}\right\} \text{ with } \left\{\begin{array}{l}\omega_1(0)=0\\ \omega_2(0)=1\end{array}\right.$$
$$y_d = a\cdot\omega_1$$

After transforming the system as stated in lemma 1 into the structure of figure 1 and calculating the normal form of the system the dynamical part gets

$$\begin{aligned}\dot{z}_1 &= z_2\\ \dot{z}_2 &= -z_2\cdot(3+\mu)+z_1\cdot(z_1-3\cdot\mu)+z_3-(1-z_1)\cdot v\\ \dot{z}_3 &= -2\cdot z_3+z_1^2\end{aligned} \tag{50}$$

and the nonlinear static function is

$$\begin{aligned}m(z,u) &= z_2+\mu\cdot z_1+(z_2+\mu\cdot z_1)^2\\ &\quad +(z_2+\mu\cdot z_1+3)\cdot u-u^2\end{aligned} \tag{51}$$

For this system the output error regulation problem can be solved following theorem 2. As the nonlinear function (51) is of second order in u, there are two solutions. To choose the right solution, an additional condition is used. Therefore the equilibrium point condition is used *i.e.*:

$$m(z_0,u_0)=v_0 \tag{52}$$

and in this example ($z_0=0,\ u_0=0$)

$$m(0,0)=0 \tag{53}$$

With this condition the solution is unique and the results (using $a=0.1,\ b=1,\ \mu=2$) are shown in figure 2:

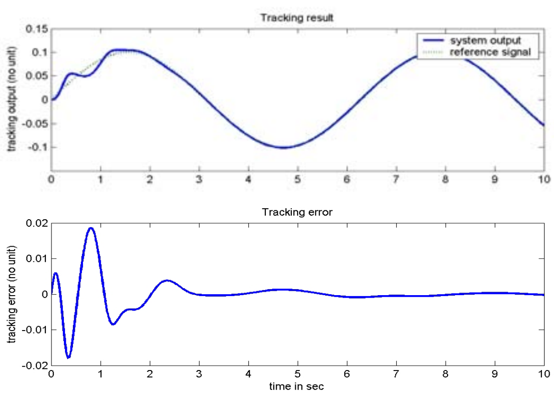

Fig. 2: Tracking result of the error feedback output tracking problem.

V. Conclusion and Outlook

This paper presents conditions to solve the tracking problem of both the full information problem and the error feedback regulation problem of a special system class. The system class is defined by a structure that often occurs in practical applications. Transforming a general nonlinear system into a system structure as it is discussed in this paper yields in an easier solvable tracking problem. A special property of this system structure is that non input affine systems can be treated as systems with input affine property.

Considering practical applications the nonlinear map may often be non-smooth or not analytically describable. To apply the presented control technique, the nonlinear map has to be approximated by a smooth and analytical map. Thus approximation errors are necessary to be considered. Approximation errors of the nonlinear map can be seen as input disturbances acting on the dynamical part of the system. Thus especially the robustness property of the control structure with respect to disturbances acting in the same direction as the input will be an important topic of future work.

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