2005 American Control Conference
June 8-10, 2005. Portland, OR, USA

A sequential design methodology for large-scale LBT systems

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Abstract— **H_2 and H_∞ based control design methodologies are widely used to deal with multivariable control problems. However, some special problems arise in the case of large-scale systems. On one hand, the complexity (due to the size of the models) of the related H_2 or H_∞ standard problems may be too high to be solved in a classical manner. On the other hand, structure constraints on the controller often exist (use of decentralized processors) and complicate the optimization problem. This paper focuses on the problem of Lower Block Triangular (LBT) H_2 controller design for LBT systems. Based on the principle of sequential design, an algorithm is proposed, attempting to solve iteratively the global H_2 problem. It is finally applied to a particular problem, the control of a platoon of vehicles, and compared with local strategies.**

I. INTRODUCTION

A physical system is called large-scale system when it presents a great number of inputs / outputs, and complex dynamics (great number of states), or / and if it is composed of distinct subsystems, more or less strongly interconnected. Some typical large-scale systems are: Power network [1], digital communication networks, manufacturing processes such as winding systems [2], automotive control [3].

Because of the structure constraints on the feedback or / and the complexity of the global problem, H_2 or H_∞ design strategies can rarely be applied directly to control such systems. In fact, optimal control problems under structural constraints are most often non convex one. Since the 90th, some algorithms using bilinear matrices inequalities (BMI) were proposed to formulate and solve such problem (see e.g. [4]-[5] and references therein). More recently, the notion of quadratic invariance for particular structure of the system and the controller has been used to reformulate the optimization problem as a convex one in the term of the Youla-Kucera parameter [6],[7]. Although both of these strategies are interesting ways to deal with the problem, they require a too heavy computational effort to be applied to large-scale systems.

In order not to manipulate the global model, this paper revisits the concept of sequential design of hierarchical controller for LBT systems, in which local controllers are design one after the other [8],[9]. Based on the decomposition property of the H_2-norm [10],[11], the proposed sequential design strategy attempt to minimize a global H_2-problem by solving iteratively low complexity subproblems associated to each subsystems. Moreover, Lyapunov and Riccati equations are preferred over LMI.

The paper is organized as follows: the principal notations and the position of the problem are first presented in section 2. In section 3, the sequential LQ-LQG control design introduced by Özgüner and Perkins [8], and Šiljak [9] is revisited in the H_2 framework. In order to reduce the global H_2 criterion, an iterative algorithm is proposed in section 4. It is then tested on the platoon of vehicles control problem, considered as a benchmark and compared with the solution obtained using the BMI framework.

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II. PROBLEM STATEMENT

A. Definitions and notations

Definition 1: A linear system S is said LBT if it can be represented by a state-space realization of the form (1), or equivalently a LBT transfer matrix.

$$S\begin{cases}\dot{x}(t)=Ax(t)+Bu(t)\\ y(t)=Cx(t)\end{cases} \tag{1}$$

where

$$A=\begin{bmatrix}A_{11} & & \\ \vdots & \ddots & \\ A_{N1} & \cdots & A_{NN}\end{bmatrix},\ B=\begin{bmatrix}B_{11} & & \\ \vdots & \ddots & \\ B_{N1} & \cdots & B_{NN}\end{bmatrix},\ C=\begin{bmatrix}C_{11} & & \\ \vdots & \ddots & \\ C_{N1} & \cdots & C_{NN}\end{bmatrix},$$

$x(t)=\begin{bmatrix}x_1^T & \cdots & x_N^T\end{bmatrix}^T\in\Re^n$, $u(t)=\begin{bmatrix}u_1^T & \cdots & u_N^T\end{bmatrix}^T\in\Re^m$, $y(t)=\begin{bmatrix}y_1^T & \cdots & y_N^T\end{bmatrix}^T\in\Re^p$ are the state, input, and output of S, $x_i(t)\in\Re^{n_i}$, $u_i(t)\in\Re^{m_i}$, and $y_i(t)\in\Re^{p_i}$, the ones of subsystem S_i, $i=1,\ldots,N$.

In what follows all subsystems S_i are supposed to be stabilizable and detectable.

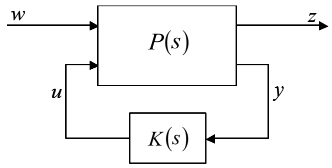

Fig. 1. H_2 standard problem

B. The LBT H_2 control problem

Consider Fig. 1; $P(s)$ is the standard model associated to the global system (1), and $K(s)$ is the researched dynamical controller (3). $P(s)$ is defined by

$$P(s)=\begin{bmatrix} P_{11}(s) & P_{12}(s) \\ P_{21}(s) & P_{22}(s) \end{bmatrix} \Leftrightarrow \begin{bmatrix} \dot{x} \\ z \\ y \end{bmatrix} = \left[\begin{array}{c|cc} A & B_1 & B_2 \\ \hline C_1 & D_{11} & D_{12} \\ C_2 & D_{21} & D_{22} \end{array}\right] \begin{bmatrix} x \\ w \\ u \end{bmatrix} \tag{2}$$

where $w(t)\in\Re^{m_w}$, $w=\begin{bmatrix} w_1^T & \cdots & w_N^T \end{bmatrix}^T$, $w_i(t)\in\Re^{m_{w_i}}$, $z(t)\in\Re^{p_z}$, $z=\begin{bmatrix} z_1^T & \cdots & z_N^T \end{bmatrix}^T$, $z_i(t)\in\Re^{p_{z_i}}$.

A is defined in (1), $B_2=B$, $C_2=C$, $D_{11}=0$, $D_{22}=0$,
$B_1=diag\left(B_1^{11},\ldots,B_1^{NN}\right)$, $C_1=diag\left(C_1^{11},\ldots,C_1^{NN}\right)$,
$D_{12}=diag\left(D_{12}^{11},\ldots,D_{12}^{NN}\right)$, $D_{21}=diag\left(D_{21}^{11},\ldots,D_{21}^{NN}\right)$.

The controller $K(s)$ is structured in the same way than the global system S (1), to be consistent with the information structure constraints.

$$K(s):=\left[\begin{array}{c|c} A_K & B_K \\ \hline C_K & D_K \end{array}\right] \Leftrightarrow \begin{bmatrix} u_1 \\ u_2 \\ \vdots \\ u_N \end{bmatrix} = \begin{bmatrix} K^1(s) & 0 & \cdots & 0 \\ K^2(s) & & 0\cdots 0 & \\ & \vdots & & \\ & K^N(s) & & \end{bmatrix} \begin{bmatrix} y_1 \\ y_2 \\ \vdots \\ y_N \end{bmatrix} \tag{3}$$

where $D_k=0$,

$$A_K=\begin{bmatrix} A_K^{11} & & \\ \vdots & \ddots & \\ A_K^{N1} & \cdots & A_K^{NN} \end{bmatrix},\ B_K=\begin{bmatrix} B_K^{11} & & \\ \vdots & \ddots & \\ B_K^{N1} & \cdots & B_K^{NN} \end{bmatrix},\ C_K=\begin{bmatrix} C_K^{11} & & \\ \vdots & \ddots & \\ C_K^{N1} & \cdots & C_K^{NN} \end{bmatrix}.$$

With $F_l(P,K)$ denoting the lower linear fractional transformation (LFT) on $P(s)$ and $K(s)$, we have

$$T_{zw}=F_l(P,K)\stackrel{\Delta}{=}P_{11}+P_{12}K(I-P_{22}K)^{-1}P_{21}. \tag{4}$$

Problem 1: LBT H_2 control problem

Under the classical assumptions [12], the LBT H_2 control problem considered here is defined as: " find the LBT controller given by (3) which *i*. internally stabilizes T_{zw}, *ii*. minimizes $J(K)=\|T_{zw}\|_2^2$ ".

Result 1: The closed-loop system is LBT

$$J(K)=\|T_{zw}\|_2^2=\left\|\begin{matrix} T_{z_1w_1} & 0 & 0 \\ \vdots & \ddots & 0 \\ T_{z_Nw_1} & \cdots & T_{z_Nw_N} \end{matrix}\right\|_2^2. \tag{5}$$

Proof: Obvious from (4), by taking into account the special structure of $P(s)$ (1) & (2) together with the LBT structure of the controller (3). □

Result 2: The H_2 criterion associated to the LBT H_2 control problem may be written as

$$J\left(K^1,\ldots,K^N\right)=\|T_{zw}\|_2^2=\sum_{i=1}^{N}\sum_{j=1}^{i}\left\|T_{z_iw_j}\right\|_2^2. \tag{6}$$

Proof: Obvious from result 1 and the H_2 norm properties. □

Synthesize the structured controller $K(s)$ as a whole leads to an H_2 optimization problem under structural constraints. As mentioned in the introduction, even if the strategies using BMI (e.g. [4],[5]) or the quadratic invariance property [6],[7] are efficient to deal with the structural constraints inherent in the large-scale systems, they are not tractable solution on a numerical point of view. BMI will be used however in this paper as a reference, for comparison.

III. The local /sequential H_2 procedure

The most common way to deal with structurally constrained control for LBT systems is the sequential strategy; the i^{th} controller is designed by taking into account the 1^{st} to the $(i-1)^{th}$ already implemented.

Let us give a generalization in the H_2 framework of the classical sequential L.Q. – L.Q.G. control design strategy developed by Özgüner and Perkins [8], and Šiljak [9]. The principle of this sequential methodology is the following : Each subsystem S_i has an associated cost function J_i to be minimized, independent of lower (hierarchically) subsystems. At the same time, the effects of already closed-loop higher-level subsystems are treated locally as known disturbances. Let us consider Fig. 2 where P^i is the standard model allowing to define the local H_2 criterion.

$$P^i\stackrel{\Delta}{=}\left[\begin{array}{c|c} P_{11}^i & P_{12}^i \\ \hline P_{21}^i & P_{22}^i \end{array}\right]=\left[\begin{array}{c|cc} P_{z_ir_{i-1}} & P_{z_iw_i} & P_{z_iu_i} \\ P_{y_i^1r_{i-1}} & P_{y_i^1w_i} & P_{y_i^1u_i} \\ \hline P_{r_ir_{i-1}} & P_{r_iw_i} & P_{r_iu_i} \end{array}\right] \tag{7}$$

P^1 and P^N are specific in that they are obtained by suppressing respectively the first column and the last row.

Let us denote $u_i^1=\begin{bmatrix} u_1^T & \cdots & u_i^T \end{bmatrix}^T$, $w_i^1=\begin{bmatrix} w_1^T & \cdots & w_i^T \end{bmatrix}^T$, $y_i^1=\begin{bmatrix} y_1^T & \cdots & y_i^T \end{bmatrix}^T$, $r_i=\begin{bmatrix} u_i^{1^T} & x_1^T & \cdots & x_i^T & y_i^{1^T} \end{bmatrix}^T$,

$$G^i\stackrel{\Delta}{=}\left[\begin{array}{cc|c} P_{z_ir_{i-1}} & P_{z_iw_i} & P_{z_iu_i} \\ \hline P_{y_i^1r_{i-1}} & P_{y_i^1w_i} & P_{y_i^1u_i} \end{array}\right]=\left[\begin{array}{c|c} G_{11}^i & G_{12}^i \\ \hline G_{21}^i & G_{22}^i \end{array}\right], \tag{8}$$

and generically $T_{\beta\alpha}$, the closed-loop transfer matrix from signal α to β; $T_{\beta_i\alpha}$ is the i^{th} row of $T_{\beta\alpha}$.

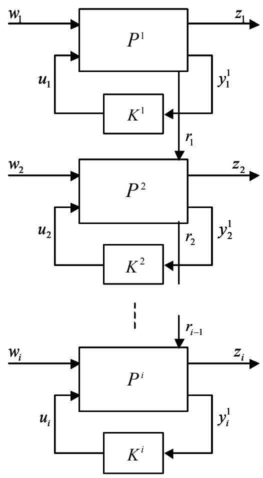

Fig. 2. local / sequential H_2 procedure

Let us now introduce the following results.

Result 3:

$$T_{z_i w} = T_{z_i w_i^1},\ T_{r_i w} = T_{r_i w_i^1} \tag{9}$$

and both depend only of $\left(K^j(s)\right)_{1\le j\le i}$.

Proof: Obvious from the LBT structure of T_{zw} (see result 1) and the definition of w_i^1. □

Result 4: Let $\overline{G}^i = G^i \begin{bmatrix} T_{r_{i-1}w} & 0 & 0 \\ 0 & I & 0 \\ 0 & 0 & I \end{bmatrix}$. (10)

Then $T_{z_i w} = F_l\left(\overline{G}^i, K^i\right)$ (11)

Proof: $F_l\left(\overline{G}^i, K^i\right) = F_l\left(G^i, K^i\right)\begin{bmatrix} T_{r_{i-1}w} & 0 \\ 0 & I \end{bmatrix}$, from the LFT definition. By definition of G^i, $F_l\left(G^i, K^i\right) = \begin{bmatrix} T_{z_i r_{i-1}} & T_{z_i w_i} \end{bmatrix}$ holds. It is clear from result 3 that $T_{z_i w} = T_{z_i w_i^1}$ and $T_{r_{i-1}w} = T_{r_{i-1}w_{i-1}^1}$. Moreover, $T_{z_i w_i^1} = \begin{bmatrix} T_{z_i w_{i-1}^1} & T_{z_i w_i} \end{bmatrix}$ results from the definition of w_i^1. Finally, $T_{z_i w_{i-1}^1}$ results from the series connection of $T_{z_i r_{i-1}}$ and $T_{r_{i-1}w_{i-1}^1}$ which leads to result 4. □

Definition 2: Let us define the local criterion:

$$J_i\left(K^1,\ldots,K^i\right) \triangleq \left\|T_{z_i w}\right\|_2^2 = \sum_{j=1}^{i}\left\|T_{z_i w_j}\right\|_2^2. \tag{12}$$

Result 5: The global criterion $J(K) = \|T_{zw}\|_2^2$ can be rewritten as

$$J(K) = J\left(K^1,\ldots,K^N\right) = \sum_{i=1}^{N} J_i\left(K^1,\ldots,K^i\right). \tag{13}$$

Proof: Obvious from def. 2 and the H_2 norm properties. □

The *local / sequential design procedure* based on local optimization H_2 problems may now be stated.

Algorithm 1:
Starting from $i=1$.

1. Find $K^{1*} = \operatorname{argmin}\left(\left\|F_l\left(G^1, K^1\right)\right\|_2^2\right)$.
2. $i = i+1$
3. Build[1] $\overline{G}^i = G^i \begin{bmatrix} T_{r_{i-1}w} & 0 & 0 \\ 0 & I & 0 \\ 0 & 0 & I \end{bmatrix}$.
4. Find $K^{i*} = \operatorname{argmin}\left(\left\|F_l\left(\overline{G}^i, K^i\right)\right\|_2^2\right)$.
5. while $i < N$, goto 2
6. End

Remark: To be applicable, some assumptions have naturally to be satisfied. Each $\overline{G}^i$ for example must be such that $\overline{G}_{12}^i$ and $\overline{G}_{21}^i$ have no zeros on the imaginary axis. $\overline{G}_{11}^i$ and $\overline{G}_{22}^i$ are assumed to be strictly proper. Moreover, if the state-space realization is used, classical assumptions on stabilizability, detectability and non-singularity have to be made.

Theorem 1: Under the assumptions introduced in the previous remark, the *local / sequential procedure* leads to the global controller $K^*(s)$ (given by (3) by replacing K^i by K^{i*} for each *i*) which has the following properties:

i. K^* stabilizes the global system,
ii. K^{i*} minimizes $J_i\left(K^{1*},\ldots,K^{i-1*},K^i\right)$,
iii. but K^* do not optimize the global criterion $J\left(K^1,\ldots,K^N\right)$.

Proof: i. The local stability reached by controllers K^{i*} imply the global stability by property of the LBT structure. *ii.* comes from the definition of J_i (12), assuming that $K^j = K^{j*}$, $j \ne i$. *iii.* It is clear that solving sequentially the problems K^{1*}=argmin($J_1(K^1)$) ,..., K^{i*}=argmin($J_i(K^{1*},\ldots, K^{i-1*},K^i)$) does not lead to the solution of the problem $\left(K^{1*},\ldots,K^{N*}\right) = \operatorname{argmin}\left(\sum_{i=1}^{N} J_i\left(K^1,\ldots,K^i\right)\right)$. □

The research of $K(s)$ which minimize the global criterion $J(K) = \|T_{zw}\|_2^2$ is considered in section 4.

[1] $T_{r_{i-1}w}$ depends only of the previously computed controllers $K^{1*},\ldots,K^{i-1*}$ (see result 3).

IV. The global / sequential H_2 procedure

A. Algorithm

The main idea to be developed in this section consists, for a given set of initial subcontrollers, to modify sequentially each subcontroller in order to reduce the global H_2 criterion,

$$J(K) = J\left(K^1,\ldots,K^N\right) = \sum_{l=1}^{N} J_l\left(K^1,\ldots,K^l\right). \tag{14}$$

To do so, let us introduce some preliminary results. Starting from Fig. 2, let us denote:

$$\tilde{G}^i \triangleq \left[\begin{array}{c|c} \tilde{G}^i_{11} & \tilde{G}^i_{12} \\ \hline \tilde{G}^i_{21} & \tilde{G}^i_{22} \end{array}\right] = \left[\begin{array}{cc|c} P_{z_i r_{i-1}} & P_{z_i w_i} & P_{z_i u_i} \\ P_{r_i r_{i-1}} & P_{r_i w_i} & P_{r_i u_i} \\ \hline P_{y_i^1 r_{i-1}} & P_{y_i^1 w_i} & P_{y_i^1 u_i} \end{array}\right]. \tag{15}$$

Let also $z_N^i = \left[z_i^T \quad \cdots \quad z_N^T\right]^T$.

$\tilde{G}^1$ and $\tilde{G}^N$ are specific in that they are obtained by suppressing respectively the first column and the second row. In the same way than for result 4, and referring to figure 3, we have the following result.

Result 6: Let

$$\tilde{\tilde{G}}^i = \begin{bmatrix} I & 0 & 0 \\ 0 & T_{z_N^{i+1} r_i} & 0 \\ 0 & 0 & I \end{bmatrix} \tilde{G}^i \begin{bmatrix} T_{r_{i-1} w_{i-1}^1} & 0 & 0 \\ 0 & I & 0 \\ 0 & 0 & I \end{bmatrix}. \tag{16}$$

Then $T_{z_N^i w_i^1} = F_l\left(\tilde{\tilde{G}}^i, K^i\right)$ (17)

Proof: $F_l\left(\tilde{\tilde{G}}^i, K^i\right) = \begin{bmatrix} I & 0 \\ 0 & T_{z_N^{i+1} r_i} \end{bmatrix} F_l\left(\tilde{G}^i, K^i\right) \begin{bmatrix} T_{r_{i-1} w_{i-1}^1} & 0 \\ 0 & I \end{bmatrix}$ (18)

from the LFT definition. Furthermore, the equality $F_l\left(\tilde{G}^i, K^i\right) = \begin{bmatrix} T_{z_i r_{i-1}} & T_{z_i w_i} \\ T_{r_i r_{i-1}} & T_{r_i w_i} \end{bmatrix}$ holds. Series connection of the 3 terms in the right part of (18) leads then to $T_{z_N^i w_i^1}$. □

Let us now assume that $\left(\bar{K}^1,\ldots,\bar{K}^N\right)$ are admissible subcontrollers for the problem considered (i.e. they stabilize the global system). The question is: how to modify the i^{th} subcontroller in order to obtain:

$$J\left(\bar{K}^1,\ldots,K^i,\ldots,\bar{K}^N\right) < J\left(\bar{K}^1,\ldots,\bar{K}^i,\ldots,\bar{K}^N\right)? \tag{19}$$

After (14), the choice of K_i affects all subcriterions J_l for $l \geq i$. The way proposed here consists to modify $\bar{K}^i$ by K^i solving the problem 2.

Problem 2: "Find

$$K^{i*} = \underset{K^i}{\operatorname{argmin}}\left(\sum_{l=i}^{N} J_l\left(\bar{K}^1 \ldots, \bar{K}^{i-1}, K^i, \bar{K}^{i+1}, \ldots, \bar{K}^N\right)\right)". \tag{20}$$

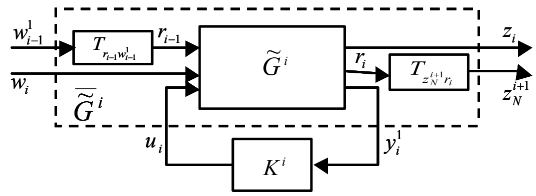

Fig. 3. The global / sequential H_2 procedure

Result 7: Let $K^j = \bar{K}^j$ for all j except $j = i$. Then problem 2 is equivalent to,

$$\text{"Find } K^{i*} = \underset{K^i}{\operatorname{argmin}}\left(\left\|F_l\left(\tilde{\tilde{G}}^i, K^i\right)\right\|_2^2\right)\text{"}. \tag{21}$$

Proof: By construction of $\tilde{\tilde{G}}^i$. □

Result 8: Let K^{i*} be defined by (21). Then the following inequality holds,

$$J\left(\bar{K}^1,\ldots,K^{i*},\ldots,\bar{K}^N\right) < J\left(\bar{K}^1,\ldots,\bar{K}^i,\ldots,\bar{K}^N\right). \tag{22}$$

Proof: The result may be derived from (13) and (20). □

The *global / sequential design procedure* based on an iterative optimization of the global H_2 criterion $J(K) = \|T_{zw}\|_2^2$ may now be stated.

Algorithm 2:

Starting from $i = 1$, $k = 0$.

0. Initialization : Find a stabilizing controller $K^{(0)} = \left(K^{1(0)},\ldots,K^{N(0)}\right)$.
1. $k = k + 1$
   1.1 For $i = 1$ to N

   $$\text{Build}^2\ \tilde{\tilde{G}}^{i(k)} = \begin{bmatrix} I & 0 & 0 \\ 0 & T^{(k-1)}_{z_N^{i+1} r_i} & 0 \\ 0 & 0 & I \end{bmatrix} \tilde{G}^i \begin{bmatrix} T^{(k)}_{r_{i-1} w_{i-1}^1} & 0 & 0 \\ 0 & I & 0 \\ 0 & 0 & I \end{bmatrix}.$$

   $$\text{Find } K^{i(k)} = \operatorname{argmin}\left(\left\|F_l\left(\tilde{\tilde{G}}^{i(k)}, K^i\right)\right\|_2^2\right).$$

   1.2 End
2. If $\left|J\left(K^{(k-1)}\right) - J\left(K^{(k)}\right)\right| < \varepsilon$ then stop, else goto 1
3. End

Remark 1: As for algorithm 1, classical H_2 control problem assumptions on $\tilde{\tilde{G}}^{i(k)}$ must be satisfied.

Remark 2: The controller K^* obtained thanks to algorithm 1 can be used to initialize algorithm 2.

[2] $T^{(k-1)}_{z_N^{i+1} r_i}$ and $T^{(k)}_{r_{i-1} w_{i-1}^1}$ depends on the subcontrollers previously computed. For $i = 1$, $T^{(k)}_{r_{i-1} w_{i-1}^1} = I$.

Theorem 2: Under the classical assumptions on $\overline{\widetilde{G}}^{i(k)}$, algorithm 2, associated to the global criterion $J(K)=\|T_{zw}\|_2^2$, converges monotonically.

Proof: It is clear from result 8 that the sequence of $\alpha_i = J\left(K^{1(k)},\ldots,K^{i},\ldots,K^{N(k-1)}\right)$ is decreasing as i increases. It then follows from the Bolzano – Weierstrass theorem that the algorithm converges monotonically. □

Remark 1: There is no guaranty to reach the optimum controller K_{opt} for the global criterion $J(K)=\|T_{zw}\|_2^2$. However *(i)* the global criterion is decreasing. *(ii)* if K_{opt} is used as initialization point, then the algorithm will stay at this optimal point.

Remark 2: If only one of the local controllers is modified, the criterion will increase. However, it is not possible to conclude to the local optimality, regarding simultaneously all the K^i.

B. Synthesis of controller $K^{i(k)}$

Synthesis of $K^{i(k)}$ implies the manipulation of the global closed-loop standard model $\overline{\widetilde{G}}^{i(k)}$ represented in Fig. 3 (replacing $T_{r_{i-1}w_{i-1}^1}$, $T_{z_N^{i+1}r_i}$, and K^i by $T_{r_{i-1}w_{i-1}^1}^{(k)}$, $T_{z_N^{i+1}r_i}^{(k-1)}$, and $K^{i(k)}$). This standard model may therefore be of a high order, leading to numerical difficulties.

Two ways to simplify the problem can be distinguished; model reduction and decomposition of the synthesis of the dynamic controller $K^{i(k)}$. 1/ Model reduction will consist in reducing both the transfer matrices $T_{r_{i-1}w_{i-1}^1}^{(k)}$ and $T_{z_N^{i+1}r_i}^{(k-1)}$. It can be done thanks to methodologies using balanced truncations or singular perturbations [13]. One can expect a significant order reduction, without important degradation, as the hierarchically distant subsystems should act weakly on S_i.

2/ Simplification by decomposition of the synthesis of $K^{i(k)}$ is made possible by using the state-space formulation. Application of the separation principle will permit to obtain $K^{i(k)}$ in a state feedback – observer form. Referring to [8],[9], the decomposition of the synthesis of state feedback and observer gains $K_{sf}^{i(k)}$ and $K_{ob}^{i(k)}$ is possible considering the decomposition results in the theorem 1 in [8], or in [9] (L.Q. – L.Q.G. cases). The reformulation in the H_2 case is omitted by lack of place.

Briefly stated, considering for instance the state feedback gain $K_{sf}^{i(k)}$, it can be design as $\begin{bmatrix} K_{sf1}^{i(k)} & K_{sf2}^{i(k)} \end{bmatrix}$ by solving sequentially reduced order Sylvester and Riccati equations.

V. Case Study

A. Control of a platoon of vehicles

In order to appreciate the efficiency of the proposed algorithm, the speed control of a platoon of vehicles is now considered (*e.g.* [3],[8]). Briefly stated, the problem consists to keep the platoon of vehicles with a constant velocity and constant intra platoon separations. A platoon of vehicles is indeed a potential large-scale system being composed of interconnected subsystems. A structured controller is a natural solution to meet the technological constraints (local embedded controllers).

Consider a set of N vehicles moving in a straight line as illustrated in fig. 4. The variables parameterizing the model of the i^{th} vehicle are; $h_i(t)$, $v_i(t)$, m_i, $u_i(t)$, $-\alpha_i v_i(t)$ which are respectively the position of the i^{th} vehicle at time t, its velocity, its masse, the force applied, and the drag force action. All the states are assumed to be measured. By considering the following deviations, $x_1 = v_d - v_1$, $x_i^1 = v_d - v_i$, and $x_i^2 = \Delta h_d - \Delta h_i$, $i = 2,\ldots,N$, where v_d and Δh_d are respectively the desired velocity and intra platoon separation, a state-space representation of a platoon of N vehicles can be defined as (27) [3].

To focus on the methodology, it is assumed from now (as in [3]) that $\alpha_i = 1$, $m_i = 1$, and $N = 3$.

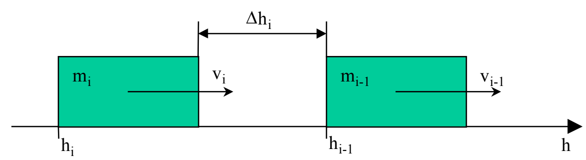

Fig. 4. Vehicles moving in string

$$
S\left\{
\begin{aligned}
\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2^1 \\ \dot{x}_2^2 \\ \vdots \\ \dot{x}_N^1 \\ \dot{x}_N^2 \end{bmatrix}
&=
\begin{bmatrix}
-\alpha_1/m_1 & 0 & 0 & \ldots & 0 & 0 \\
0 & -\alpha_2/m_2 & 0 & \ldots & 0 & 0 \\
1 & -1 & 0 & \ldots & 0 & 0 \\
\vdots & \vdots & \vdots & \ddots & \vdots & \vdots \\
0 & 0 & 0 & \ldots & -\alpha_N/m_N & 0 \\
0 & 1 & 0 & \ldots & -1 & 0
\end{bmatrix}
\begin{bmatrix} x_1 \\ x_2^1 \\ x_2^2 \\ \vdots \\ x_N^1 \\ x_N^2 \end{bmatrix}
+
\begin{bmatrix}
1/m_1 & 0 & \ldots & 0 \\
0 & 1/m_2 & \ldots & 0 \\
0 & 0 & \ldots & 0 \\
\vdots & \vdots & \ddots & \vdots \\
0 & 0 & \ldots & 1/m_N \\
0 & 0 & \ldots & 0
\end{bmatrix}
\begin{bmatrix} u_1 \\ u_2 \\ \vdots \\ u_N \end{bmatrix} \\
\begin{bmatrix} y_1 \\ y_2^1 \\ y_2^2 \\ \vdots \\ y_N^1 \\ y_N^2 \end{bmatrix}
&=
\begin{bmatrix}
1 & 0 & 0 & \ldots & 0 & 0 \\
0 & 1 & 0 & \ldots & 0 & 0 \\
0 & 0 & 1 & \ldots & 0 & 0 \\
\vdots & \vdots & \vdots & \ddots & \vdots & \vdots \\
0 & 0 & 0 & \ldots & 1 & 0 \\
0 & 0 & 0 & \ldots & 0 & 1
\end{bmatrix}
\begin{bmatrix} x_1 \\ x_2^1 \\ x_2^2 \\ \vdots \\ x_N^1 \\ x_N^2 \end{bmatrix}
\end{aligned}
\right. \quad (27)
$$

B. Global criterion

To construct the global criterion $J(K)=\|T_{zw}\|_2^2$, the standard model $P(s)$ (2) is define referring to [8] by $B_1^T = [10\ \ 0\ \ 0\ \ 0\ \ 0\ \ 0]$, $C_1 = \begin{bmatrix} C_1^* & 0 \end{bmatrix}^T$, $D_{12} = \begin{bmatrix} 0 & D_{12}^* \end{bmatrix}^T$ with $C_1^* = diag\left(\sqrt{3}, I_2, I_2\right)$, $D_{12}^* = I_3$, $D_{21} = 0$, and $D_{21} = 0$.

Remark: Some local H_2 problems to be solved are singulars. However, these problems have been solved using the results of [14],[15].

C. Results

The following structured controllers have been compared; K^*, the static controller obtained by the local / sequential H_2 procedure, K_{opt}, the static controller optimized thanks to a BMI algorithm (heuristic in [5] was used), and lastly $K^\circ(s)$, a dynamic controller obtained thanks to algorithm 2. The global criterion $\|T_{zw}\|_2$ obtained with each of them are compared. The results are summed up in Table I. Table II demonstrates the evolution of the criterion through the iterations of algorithm 2.

As expected, the criterion decreases monotonically through the iterations. More iterations would have been necessary to exhibit definitely the convergence. The differences observed between K_{opt} and $K^\circ(s)$ comes from the fact that if K_{opt} is probably the best *static* controller, $K^\circ(s)$ is a *dynamic* controller, hence with more degree of freedom.

Table I. Criterions for several structured controllers

Controllers	K^*	K_{opt}	$K^\circ(s)$
$\|T_{zw}\|_2$	12.135	11.676	11.589

Table II. Evolutions of the criterion with iterations

$K^\circ(s)=\left(K^{1(k)},K^{2(k)},K^{3(k)}\right)$	$\|T_{zw}\|_2$
$\left(K^{1(0)},K^{2(0)},K^{3(0)}\right)=K^*$	12.135
$\left(K^{1(1)},K^{2(0)},K^{3(0)}\right)$	11.676
$\left(K^{1(1)},K^{2(1)},K^{3(0)}\right)$	11.650
$\left(K^{1(1)},K^{2(1)},K^{3(1)}\right)$	11.624
$\left(K^{1(2)},K^{2(1)},K^{3(1)}\right)$	11.589

VI. Conclusion

The H_2-norm has many appealing properties [10]-[11]. In particular, the square of the H_2-norm of a block transfer matrix is equal to the sum of the square of the H_2-norm of each block. Relying on this property, a sequential strategy to solve a global H_2-problem has been considered.

Structured H_2 controller design requires to solve an optimization problem involving a linear objective criterion under matrices inequalities constraints [5],[7]. This optimization problem is not tractable in the general case of large-scale systems.

This paper has considered the particular case of LBT large-scale systems. In that case, the subsystems may be considered one after another through a sequential design. Each subproblem consists then to minimize a local H_2 problem of low complexity. If this strategy insures the stability of the global LBT system, it is however suboptimal with regard to the global criterion.

The work in this paper highlights the comparison between the “classical” sequential H_2 and the structured H_2 design. Moreover, it extends the applicability of the sequential strategy to the case where a global H_2 criterion has to be minimized. A state observer feedback form may be used at each step, allowing to reduce the numerical complexity of each elementary problem. The convergence of the algorithm proposed has been proved, and illustrated on the platoon of vehicles control problem. Model reduction is included as a part of the algorithm to prevent a rapid increase of the controller degree.

Based on the results of this paper, future works will look for an adaptation of existing H_2-based control design methodologies [10] to the case of large-scale LBT systems. Its extension to non-LBT system will be also considered to be finally applied to the problem of winding systems [2].

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