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# Distributed Architectures and Implementations of Observer Based Controllers for Performance Optimization

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*Abstract*— **In this paper we address the problem of synthesizing controllers that have to be implemented distributively and have structure. The effect of sub-controller to sub-controller communication noise on stability and performance is considered. Using an observer based controller parameterization, we provide suitable stabilizing sub-controller architectures that directly take into account the effect of communication noise on performance. In particular, the overall performance optimization can be cast as a convex problem in the Youla-Kucera parameter $Q$. Similar results hold for banded controller structures, i.e., when there is also a delay in the subsystem to subsystem communication.**

## I. INTRODUCTION

In large complex systems it is often desirable to implement the controller in a distributive manner. Such a task is becoming increasingly relevant because of the use of massively parallel systems that employ large numbers of sensors and actuators. For example, massively parallel cantilever based data storage and read out technologies hold the promise to revolutionize memory devices [3]. In such applications, the controller can not be realized at a single station due to the associated large computational load. This issue raises the important question on how to divide the control task into the various stations and characterize the effect of communication noise and uncertainty in the overall system performance. Such a distributed realization of an overall controller may also be necessary due to a priori imposed structural requirements such as decentralization (e.g., different actuator-sensor geographical locations in a networked control application).

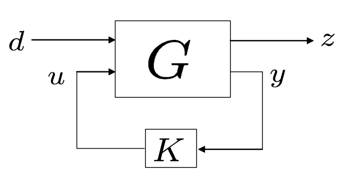


Fig. 1. $G - K$ framework.

In this paper, we utilize the general framework illustrated in Figure 1 where $G$ represents the generalized plant and $K$ represents the overall controller. Both $G$ and $K$ are assumed to be discrete time, linear and time invariant. The controller $K$ needs to be distributively implemented in some fashion that is dictated by some computational power constraints and/or an overall pre-specified structure such as, e.g., nested structure characterized by a block triangular structure of the transfer matrix $K$ [10] or, banded structure where neighboring subsystems interact with each other with one step delay [5]. For information on recent work on distributed control with and/or without structural constraints that lead to convex synthesis problems we refer to [1], [2], [6], [7], [8], [9]. Whether or not structural constraints are imposed, the question of distributive realization of a controller is an issue as many realizations/implementations are possible. For

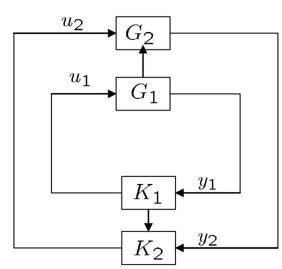


Fig. 2. Nested structured system

example, consider the case shown in Figure 2 where a triangular structure is imposed, such that location $K_2$ can obtain information from $K_1$ but there is no flow of information from $K_2$ to $K_1$. $K_1$ and $K_2$ can be obtained as parts of the overall controller $K$ which needs to be nested, i.e., of the form $K = \begin{bmatrix} K_{11} & 0 \\ K_{21} & K_{22} \end{bmatrix}$. Given a stabilizing $K$, there is a highly non-unique way to identify $K_1$ and $K_2$ as this depends on what signals are transmitted. For example, among the infinitely many, $K_1 = \begin{bmatrix} K_{11} \\ K_{21} \end{bmatrix}$ and $K_2 = \begin{bmatrix} I & K_{22} \end{bmatrix}$ or $K_1 = \begin{bmatrix} K_{11} \\ I \end{bmatrix}$ and $K_2 = \begin{bmatrix} K_{21} & K_{22} \end{bmatrix}$ are two possible functional identifications. Both are identical in the absence of sub-controller to sub-controller communication noise. However, different implementations will yield different performance with respect to communication noise that affects the transmission of information from $K_1$ to $K_2$. In [10] a particular architecture is described that leads to closed

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loop maps with noise as input, which are convex in the Youla-Kucera parameter $Q$. The architectural developments in [10] were based totally from a controller coprime factor representation perspective which relied on the underlying nested structure.

In this paper, we take a broader and more intuitive point of view that applies to both structured and unstructured $K$s. It relies primarily on the observer based parameterization of all stabilizing $K$. We first consider distributing an observer based controller and identify what are the signals needed to be transmitted via communication channels and how noise enters into the overall picture. We show that with the particular architectures provided, the effect of noise on the performance can be precisely characterized as a convex problem in the parameter $Q$. Based on this development, we also provide architectures with similar properties for banded controller structures.

The paper is organized as follows: in Section 2 we provide an observer based distributed controller architecture such that the input-output map from sub-controller to sub-controller communication noise to the regulated variables of interest is affine in $Q$. The optimal performance problem to design the controller in presence of sub-controller to sub-controller communication noise is formed in Section 3. In Section 4, we study the banded structure problem. We conclude in Section 5.

## II. OBSERVER BASED DISTRIBUTED CONTROLLER ARCHITECTURE

We consider herein the case where the overall controller $K$ is unstructured and we are interested to design and distribute $K$, for simplicity, into two sub-controllers $K_1$ and $K_2$ (similar to the setup shown in Figure 2 but with the two way transmission between inner and outer nests). Generalizations to the case of $n$ sub-controllers follows similarly. In particular, let the state space realization of $G_{22}$, the part of $G$ in Figure 1 that maps the control input $u$ to the measured output $y$, be as $\left[\begin{array}{c|c} A & B \\ \hline C & 0 \end{array}\right]$ where $A, B,$ and $C$ are given by

$$\begin{aligned}
G_{22}:\begin{pmatrix} x_1^+ \\ x_2^+ \end{pmatrix} &= \begin{pmatrix} A_{11} & A_{12} \\ A_{21} & A_{22} \end{pmatrix}\begin{pmatrix} x_1 \\ x_2 \end{pmatrix} \\
&+ \begin{pmatrix} B_{11} & B_{12} \\ B_{21} & B_{22} \end{pmatrix}\begin{pmatrix} u_1 \\ u_2 \end{pmatrix} \\
\begin{pmatrix} y_1 \\ y_2 \end{pmatrix} &= \begin{pmatrix} C_{11} & 0 \\ 0 & C_{22} \end{pmatrix}\begin{pmatrix} x_1 \\ x_2 \end{pmatrix}.
\end{aligned}$$

That is, the plant can be considered as the interconnection of two subsystems $\{G_i\}_{i=1,2}$ with the corresponding local variables $x_i$, $u_i$ and $y_i$. The fact that we use diagonal $C$ and no feed-through terms is for simplicity in the expressions and does not change the development. Initially we proceed by providing sub-controller architectures for a basic observer based $K$ that will form the basis for parameterizing all $K$. To this end, assume that overall system $\left[\begin{array}{c|c} A & B \\ \hline C & 0 \end{array}\right]$ is stabilizable and detectable and let $F$ and $L$ be such that $A-BF$ and $A-LC$ have stable eigenvalues (Hurwitz). Then, an observer based (centrally) stabilizing controller $K$ is given by

$$\begin{aligned}
K:\begin{pmatrix} \bar{x}_1^+ \\ \bar{x}_2^+ \end{pmatrix} &= \begin{pmatrix} A_{11} & A_{12} \\ A_{21} & A_{22} \end{pmatrix}\begin{pmatrix} \bar{x}_1 \\ \bar{x}_2 \end{pmatrix} \\
&+ \begin{pmatrix} B_{11} & B_{12} \\ B_{21} & B_{22} \end{pmatrix}\begin{pmatrix} u_1 \\ u_2 \end{pmatrix} \\
&+ \begin{pmatrix} L_{11} & L_{12} \\ L_{21} & L_{22} \end{pmatrix}\begin{pmatrix} y_1-\bar{y}_1 \\ y_2-\bar{y}_2 \end{pmatrix} \\
\begin{pmatrix} \bar{y}_1 \\ \bar{y}_2 \end{pmatrix} &= \begin{pmatrix} C_{11} & 0 \\ 0 & C_{22} \end{pmatrix}\begin{pmatrix} \bar{x}_1 \\ \bar{x}_2 \end{pmatrix} \\
\begin{pmatrix} u_1 \\ u_2 \end{pmatrix} &= -\begin{pmatrix} F_{11} & F_{12} \\ F_{21} & F_{22} \end{pmatrix}\begin{pmatrix} \bar{x}_1 \\ \bar{x}_2 \end{pmatrix}.
\end{aligned}$$

Let, $x = (x_1', x_2')', u = (u_1', u_2')', y = (y_1', y_2')', \bar{x} = (\bar{x}_1', \bar{x}_2')', \bar{y} = (\bar{y}_1', \bar{y}_2')',$ and $e = y - \bar{y}$. This controller $K$ can now be split into two sub-controllers $K_1$ and $K_2$ in conformity with the underlying plant subsystems $G_1$ and $G_2$ as

$$\begin{aligned}
G_1: x_1^+ &= \begin{pmatrix} A_{11} & A_{12} \end{pmatrix}\begin{pmatrix} x_1 \\ x_2 \end{pmatrix} + B_{11}u_1 + B_{12}u_2 \\
y_1 &= C_{11}x_1 \\
K_1: \bar{x}_1^+ &= \begin{pmatrix} A_{11} & A_{12} \end{pmatrix}\begin{pmatrix} \bar{x}_1 \\ \bar{x}_2 \end{pmatrix} + B_{11}u_1 + B_{12}u_2 \\
&+ \begin{pmatrix} L_{11} & L_{12} \end{pmatrix}\begin{pmatrix} y_1-\bar{y}_1 \\ y_2-\bar{y}_2 \end{pmatrix} \\
\bar{y}_1 &= C_{11}\bar{x}_1 \\
u_1 &= -\begin{pmatrix} F_{11} & F_{12} \end{pmatrix}\begin{pmatrix} \bar{x}_1 \\ \bar{x}_2 \end{pmatrix},
\end{aligned}$$

$$\begin{aligned}
G_2: x_2^+ &= \begin{pmatrix} A_{21} & A_{22} \end{pmatrix}\begin{pmatrix} x_1 \\ x_2 \end{pmatrix} + B_{21}u_1 + B_{22}u_2 \\
y_2 &= C_{22}x_2 \\
K_2: \bar{x}_2^+ &= \begin{pmatrix} A_{21} & A_{22} \end{pmatrix}\begin{pmatrix} \bar{x}_1 \\ \bar{x}_2 \end{pmatrix} + B_{21}u_1 + B_{22}u_2 \\
&+ \begin{pmatrix} L_{21} & L_{22} \end{pmatrix}\begin{pmatrix} y_1-\bar{y}_1 \\ y_2-\bar{y}_2 \end{pmatrix} \\
\bar{y}_2 &= C_{22}\bar{x}_2 \\
u_2 &= -\begin{pmatrix} F_{21} & F_{22} \end{pmatrix}\begin{pmatrix} \bar{x}_1 \\ \bar{x}_2 \end{pmatrix}.
\end{aligned}$$

In the above realization of $K$, $K_1$ needs three external signals $y_2$, $\bar{x}_2$ and $u_2$ from $K_2$. This can be achieved by transmitting only two signals $y_2$ and $\bar{x}_2$ from $K_2$ to $K_1$ because $u_2$ can then be computed at $K_1$ using the received $\bar{x}_2$. Similarly, $K_2$ will need $y_1$ and $\bar{x}_1$ from $K_1$. Thus, if this information is passed to each other, the system will be closed loop stable as the centralized $K$ is recovered. In the presence of communication noise between $K_1$ and $K_2$, the closed loop stability is still preserved. Indeed, let the noise affecting the transmitted $\bar{x}_1, y_1, \bar{x}_2,$ and $y_2$ signals be $w_{\bar{x}1}, w_{y1}, w_{\bar{x}2},$ and $w_{y2}$, respectively. Let the received signals be $\bar{x}_1^r = \bar{x}_1 + w_{\bar{x}1}, y_1^r = y_1 + w_{y1}, \bar{x}_2^r = \bar{x}_2 + w_{\bar{x}2},$ and $y_2^r = y_2 + w_{y2}$. Control signals and error signals can be computed using local signals and received external signals. Then, the state dynamics for $K_1$ and $K_2$ are given by:

$$\begin{aligned}
\bar{x}_1^+ &= (A_1 - L_1 C - B_1 F)\bar{x} + L_1 C x \\
&+ (A_{12} - L_{12}C_{22} - B_{11}F_{12} - B_{12}F_{22})w_{\bar{x}2} + L_{12}w_{y2} \\
\bar{x}_2^+ &= (A_2 - L_2 C - B_2 F)\bar{x} + L_2 C x \\
&+ (A_{21} - L_{21}C_{11} - B_{21}F_{11} - B_{22}F_{21})w_{\bar{x}1} + L_{21}w_{y1}
\end{aligned}$$

where in the above we use the notation $M_i$ to denote the $i^{th}$ row of a matrix $M$. Thus, the closed loop state space equation with noise as input, and state as $(x, \bar{x})$ is

$$\begin{aligned}
\begin{pmatrix} x^+ \\ \bar{x}^+ \end{pmatrix} &= \begin{pmatrix} A & -BF \\ LC & A - LC - BF \end{pmatrix} \begin{pmatrix} x \\ \bar{x} \end{pmatrix} \\
&+ \begin{pmatrix} 0 \\ B_w \end{pmatrix} w
\end{aligned}$$

where $w := (w'_{\bar{x}1}, w'_{y1}, w'_{u1}, w'_{\bar{x}2}, w'_{y2}, w'_{u2})'$ and $B_w = (B'_{w1} B'_{w2})'$ with $B_{w1} =$

$$\begin{pmatrix} 0 & 0 & 0 & (A_{12} - B_{11}F_{12} - B_{12}F_{22} - L_{12}C_{22}) & L_{12} & 0 \end{pmatrix}$$

and $B_{w2} =$

$$\begin{pmatrix} (A_{21} - B_{22}F_{21} - B_{21}F_{11} - L_{21}C_{11}) & L_{21} & 0 & 0 & 0 & 0 \end{pmatrix}.$$

Thus, the closed loop is stable in the presence of noise $w$. In above definition of $w$, $w_{ui}$'s are transmission errors in transmitting $u_i$'s. In this case, contrary to the case that follows, there is no need to transmit $u_i$'s (since they can readily be computed using the state information). This is indicated by having the corresponding blocks in $B_w$ equal to 0.

Next, we proceed by looking at all possible stabilizing $K$ using this basic observer structure and the Youla-Kucera parameter and provide an architecture for each sub-controller. As it is well known [11], we can parameterize all internally (centrally) stabilizing controllers $K$ by adding a stable transfer function $Q$ between the error signal of observer $e$ and the control input $u$. Let, $Q$ be given by $\begin{pmatrix} Q_1 \\ Q_2 \end{pmatrix} = \begin{pmatrix} Q_{11} & Q_{12} \\ Q_{21} & Q_{22} \end{pmatrix}$ with $Q_i = \left[\begin{array}{c|c} A_{Qi} & B_{Qi} \\ \hline C_{Qi} & D_{Qi} \end{array}\right]$. Implementing $Q_1$ at $K_1$ and $Q_2$ at $K_2$, we can write the following state equations for the two sub-controllers:

$$\begin{aligned}
K_1 : \bar{x}_1^+ &= (A_1 - B_1 F)\begin{pmatrix} \bar{x}_1 \\ \bar{x}_2 \end{pmatrix} + B_1 \begin{pmatrix} v_1 \\ v_2 \end{pmatrix} \\
&+ L_1 \begin{pmatrix} e_1 \\ e_2 \end{pmatrix} \\
\bar{y}_1 &= C_{11}\bar{x}_1 \\
u_1 &= -F_1 \begin{pmatrix} \bar{x}_1 \\ \bar{x}_2 \end{pmatrix} + v_1 \\
e_1 &= y_1 - C_{11}\bar{x}_1 \\
x_{Q1}^+ &= A_{Q1}x_{Q1} + B_{Q1}\begin{pmatrix} e_1 \\ e_2 \end{pmatrix} \\
v_1 &= C_{Q1}x_{Q1} + D_{Q1}\begin{pmatrix} e_1 \\ e_2 \end{pmatrix},
\end{aligned}$$

$$\begin{aligned}
K_2 : \bar{x}_2^+ &= (A_2 - B_2 F)\begin{pmatrix} \bar{x}_1 \\ \bar{x}_2 \end{pmatrix} + B_2 \begin{pmatrix} v_1 \\ v_2 \end{pmatrix} \\
&+ L_2 \begin{pmatrix} e_1 \\ e_2 \end{pmatrix} \\
\bar{y}_2 &= C_{22}\bar{x}_2 \\
u_2 &= -F_2 \begin{pmatrix} \bar{x}_1 \\ \bar{x}_2 \end{pmatrix} + v_2 \\
e_2 &= y_2 - C_{22}\bar{x}_2 \\
x_{Q2}^+ &= A_{Q2}x_{Q2} + B_{Q2}\begin{pmatrix} e_1 \\ e_2 \end{pmatrix} \\
v_2 &= C_{Q2}x_{Q2} + D_{Q2}\begin{pmatrix} e_1 \\ e_2 \end{pmatrix}.
\end{aligned}$$

Compared to the case where $Q = 0$, here we need to transmit one more signal $u_2$ from $K_2$ to $K_1$. Similarly, $K_2$ needs $u_1$ from $K_1$. Stability in the case where there is no communication noise follows from the fact that $K$ is recovered. Note that the fact that the overall $Q$ may not be minimally realized does not generate undetectable and/or non-stabilizable modes as it is a stable system.

Closed loop stability is preserved in the presence of noise. Indeed, let the received signals at $K_2$ be $\bar{x}_1^r = \bar{x}_1 + w_{\bar{x}1}, y_1^r = y_1 + w_{y1}, u_1^r = u_1 + w_{u1}$, and at $K_1$ be $\bar{x}_2^r = \bar{x}_2 + w_{\bar{x}2}, y_2^r = y_2 + w_{y2}$ and $u_2^r = u_2 + w_{u2}$. Let, the generated signals at $K_2$ be $v_1^r = v_1 + w_{u1} + F_{11}w_{\bar{x}_1}$, and $e_1^r = e_1 + w_{y1} - C_{11}w_{\bar{x}_1}$, and at $K_1$ be $v_2^r = v_2 + w_{u2} + F_{22}w_{\bar{x}2}$, and $e_2^r = e_2 + w_{y2} - C_{22}w_{\bar{x}2}$. We can write the state space equation for the nominal controller $K_w$ with $y, w$ and $v$ as input signals and $u, e$ as output signals as given by

$$\begin{aligned}
K_w : \bar{x}^+ &= (A - BF - LC)\bar{x} + Bv + Ly + Sw \\
u &= -F\bar{x} + v + Rw \\
e &= -C\bar{x} + y
\end{aligned}$$

where $S = (S'_1\ S'_2)'$ and $R = (R'_1\ R'_2)'$ where

$$S_1 = \begin{pmatrix} 0 & 0 & 0 & (A_{12} - B_{11}F_{12} - L_{12}C_{22}) & L_{12} & B_{12} \end{pmatrix},$$

$$S_2 = \begin{pmatrix} (A_{21} - B_{22}F_{21} - L_{21}C_{11}) & L_{21} & B_{21} & 0 & 0 & 0 \end{pmatrix},$$

$$R_1 = \begin{pmatrix} 0 & 0 & 0 & -F_{12} & 0 & 0 \end{pmatrix},$$

$$R_2 = \begin{pmatrix} -F_{21} & 0 & 0 & 0 & 0 & 0 \end{pmatrix}.$$

The state space equation of Youla-Kucera parameter $Q$ can also be written as a function of $e$ and $w$ as given by

$$\begin{aligned}
Q_w : x_Q^+ &= \begin{pmatrix} A_{Q1} & 0 \\ 0 & A_{Q2} \end{pmatrix} x_Q + \begin{pmatrix} B_{Q1} \\ B_{Q2} \end{pmatrix} e + \begin{pmatrix} B_{Q1}W_1 \\ B_{Q2}W_2 \end{pmatrix} w \\
v &= \begin{pmatrix} C_{Q1} & 0 \\ 0 & C_{Q2} \end{pmatrix} x_Q + \begin{pmatrix} D_{Q1} \\ D_{Q2} \end{pmatrix} e + \begin{pmatrix} D_{Q1}W_1 \\ D_{Q2}W_2 \end{pmatrix} w
\end{aligned}$$

where

$$W_1 = \begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 \\ -C_{22} & I & 0 & 0 & 0 & 0 \end{pmatrix},$$

$$\text{and } W_2 = \begin{pmatrix} 0 & 0 & 0 & -C_{11} & I & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}.$$

Let, $S_Q = \begin{pmatrix} B_{Q1}W_1 \\ B_{Q2}W_2 \end{pmatrix}$ and $R_Q = \begin{pmatrix} D_{Q1}W_1 \\ D_{Q2}W_2 \end{pmatrix}$.

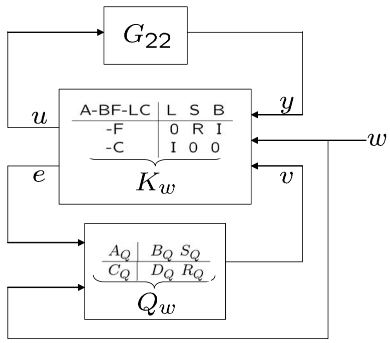


Fig. 3. Observer based distributed controller in LFT form, with communication noise as external input

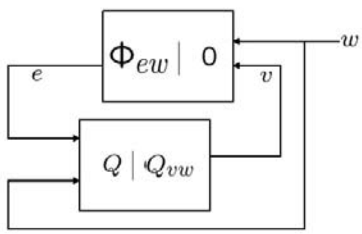


Fig. 4. Redrawing Figure 3 in terms of transfer functions, $\Phi_{ew}$ is the closed loop map from $w$ to $e$

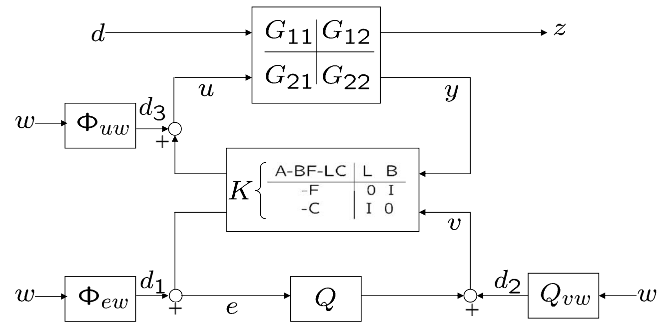


Fig. 5. Simplified block diagram of parameterized observer based controller with filtered noise as exogenous inputs

We can now draw an LFT form of the closed loop map of the observer based controller with $w$ as an additional input as in Figure 3. Note, that if $Q = 0$, the closed loop map is stabilized by the nominal $K_w$. This implies that in the case $Q = 0$, the closed loop map $\Phi_{ew}$ from $w$ to $e$ is stable. The closed loop map from $v$ to $e$ is 0. Thus, it is sufficient to check the internal stability of the simplified interconnection in Figure 4. $Q = \begin{pmatrix} Q_1 \\ Q_2 \end{pmatrix}$ is a stable map from $e$ to $v$ and therefore the map $Q_{vw} = \begin{pmatrix} Q_1 W_1 \\ Q_2 W_2 \end{pmatrix}$ from $w$ to $v$ is also stable ($W_1$ and $W_2$ are fixed matrices) as it is evident from the state equation of $Q_w$. Hence, it follows from Figure 4 that the map from $w$ to any internal signal of the interconnection is stable. That is, the closed loop is internally stable.

## III. The Performance Problem

A block diagram of the parameterized observer based distributed controller for the generalized plant $G$ as realized in the previous section, with sub-controller to sub-controller noise $w$ is shown in Figure 5. In this figure, the effect of $w$ on the system is represented by the inputs $d_1 = \Phi_{ew} w$, $d_2 = Q_{vw} w$ and $d_3 = \Phi_{uw} w$ where, as with $\Phi_{ew}$, $\Phi_{uw}$ is the map from $w$ to $u$ when $Q = 0$.

Let $z$ be the regulated variable and $\Phi$ be the resulting closed loop transfer function from $(d', w')'$ to $z$ in Figure 5. Now, referring to the equivalent Figure 6, let $H : d \mapsto z, V : d \mapsto e, \tilde{H} : d_3 \mapsto z, \tilde{V} : d_3 \mapsto e$ and $-U : v \mapsto z$ when $Q = 0$. Then, the closed loop map from $d$ to $z$ is $H - UQV$, from $d_1$ to $z$ is $-UQ$, from $d_2$ to $z$ is $-U$ and from $d_3$ to $z$ is $\tilde{H} - UQ\tilde{V}$. Thus, the closed loop map from $w$ to $z$ is $\tilde{H}\Phi_{uw} - UQ\tilde{V}\Phi_{uw} - UQ\Phi_{ew} - UQ_{vw}$. Note that $\Phi_{ew}$ and $\Phi_{uw}$ are stable and independent of $Q$ while $Q_{vw}$ is linear in $Q$. Thus, $w \mapsto z$ is affine in $Q$. The resulting

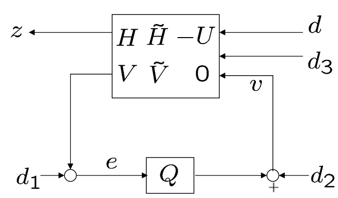


Fig. 6. Equivalent performance problem

optimal performance problem can be written as:

$$\mu = \inf_{K-\text{stabilizing}} \|\Phi(K)\| = \inf_{\text{stable } Q} \|\Phi(Q)\|$$

$$= \inf_{\text{stable } Q} \|H - UQV \quad \tilde{H}\Phi_{uw} - U(Q(\tilde{V}\Phi_{uw} + \Phi_{ew}) + Q_{vw})\|$$

The above convex optimization problem can be formulated as a problem with structural contraint on parameter $Q$. Let, $\bar{Q}$ be defined as $\bar{Q} = \text{diag}(Q_1, Q_2)$. The above problem can be rewritten in terms of $\bar{Q}$ as:

$$\mu = \underbrace{\inf}_{\substack{\text{stable } \bar{Q} \\ \text{block diag } \bar{Q}}} \|[H_1 - U_1\bar{Q}V_1,\ H_2 - U_2\bar{Q}V_2]\|$$

where $H_1 = H$, $H_2 = \tilde{H}\Phi_{uw}$, $U_1 = U_2 = U$, $V_1 = [V' \ V']'$ and $V_2 = [[(\tilde{V}\Phi_{uw} - \Phi_{ew})' \quad (\tilde{V}\Phi_{uw} + \Phi_{ew})']' -$

$[W_1' \ W_2']'$]. Above problem can be further written as

$$\mu = \underbrace{\inf}_{\text{stable } Z, Z = \text{diag}(\bar{Q}, \bar{Q}), \text{block diag } \bar{Q}} \|T_1 - T_2 Z T_3\|$$

where $T_1 = [H_1 \ H_2]$, $T_2 = \text{diag}(U_1, U_2)$ and $T_3 = [V_1' \ V_2']'$.

Thus, utilizing the architectures previously described, we can precisely characterize the effect of the communication noise in a convex manner using a structured parameter. The underlying structured controller synthesis problem can be efficiently solved using the method described in [4], [6]. However, it should be noted that this characterization depends on the information signals that are transmitted between the sub-controllers which in turn depend on $F$ and $L$ of the observer based controller (note that $\Phi_{ew}$, $\Phi_{uw}$ depend on the parameters $F$ and $L$). Hence, this suggests a $F-L-Q$ iteration to further improve the overall performance if necessary. This problem may not in general lead to a convex formulation.

## IV. Banded Structures

In this section, we consider banded structures for the controller $K$ and provide distributed architectures for its implementation. The banded system structure is characterized by one step delay in subsystem interactions as depicted in Figure 7. As it turns out all banded controllers can be appropriately parameterized in terms of the Youla-Kucera parameter $Q$ which has banded structure [5]. Herein we provide an observer based parameterization that leads to a way of realizing the structured controller distributively similar to previous section. To this end, consider the overall

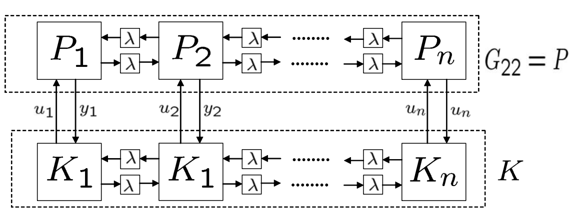


Fig. 7. Banded structure system.

system $P$ composed of subsystems $P_i$ with dynamics of the form

$$\begin{aligned} x_i(t+1) &= \sum_{j=1}^{n} A_{ij} x_j(t - |i-j|) + \sum_{j=1}^{n} B_{ij} u_j(t - |i-j|) \\ y_i(t) &= \sum_{j=1}^{n} C_{ij} x_j(t - |i-j|) + \sum_{j=1}^{n} D_{ij} u_j(t - |i-j|) \\ x_i^+(t) &:= x_i(t+1) \end{aligned}$$

where $x_i$ is the local state, $u_i$ is the local control, and $y_i$ is the local measurement variables. That is, each subsystem interacts with each neighbor with one step delay. Using a $\lambda$-transform representation and writing $x_i^+$ for $\lambda^{-1} x_i$, we can rewrite the $i$th subsystem dynamics as

$$\begin{aligned} x_i^+ &= \sum_{j=1}^{n} A_{ij} \lambda^{|i-j|} x_j + \sum_{j=1}^{n} B_{ij} \lambda^{|i-j|} u_j \\ y_i &= \sum_{j=1}^{n} C_{ij} \lambda^{|i-j|} x_j + \sum_{j=1}^{n} D_{ij} \lambda^{|i-j|} u_j. \end{aligned}$$

The overall system dynamics can be compactly described as

$$\begin{aligned} x^+ &= A(\lambda) x + B(\lambda) u \\ y &= C(\lambda) x + D(\lambda) u \end{aligned} \tag{1}$$

where

$$\begin{aligned} A(\lambda) &= \begin{pmatrix} A_{11} & \cdots & \lambda^{n-1} A_{1n} \\ \vdots & & \vdots \\ \lambda^{n-1} A_{n1} & \cdots & A_{nn} \end{pmatrix}, \\ B(\lambda) &= \begin{pmatrix} B_{11} & \cdots & \lambda^{n-1} B_{1n} \\ \vdots & & \vdots \\ \lambda^{n-1} B_{n1} & \cdots & B_{nn} \end{pmatrix}, \\ C(\lambda) &= \begin{pmatrix} C_{11} & \cdots & \lambda^{n-1} C_{1n} \\ \vdots & & \vdots \\ \lambda^{n-1} C_{n1} & \cdots & C_{nn} \end{pmatrix}, \\ D(\lambda) &= \begin{pmatrix} D_{11} & \cdots & \lambda^{n-1} D_{1n} \\ \vdots & & \vdots \\ \lambda^{n-1} D_{n1} & \cdots & D_{nn} \end{pmatrix}, \end{aligned}$$

$$\begin{aligned} x &= (x_1', \ldots, x_n')', u = (u_1', \ldots, u_n')', \\ y &= (y_1', \ldots, y_n')'. \end{aligned}$$

We note that in the representation in Equation 1 the matrices involved correspond to banded structures. We assume that the overall system is distributively stabilizable by a controller $K$ that is banded too, i.e., it consists of sub-controllers $K_i$ that pass information to their neighbors with one step delay. The system $P$ can be brought into a standard state-space form by introducing the state variable

$$\chi = (x' \ \ \lambda x' \cdots \lambda^{n-1} x' \ \lambda u' \ \lambda^2 u' \cdots \lambda^{n-1} u')'$$

to obtain

$$\begin{aligned} \chi^+ &= \overline{A} \chi + \overline{B} u \\ y &= \overline{C} \chi + \overline{D} u \end{aligned} \tag{2}$$

where $\overline{A}$, $\overline{B}$, $\overline{C}$, $\overline{D}$ are constant matrices independent of $\lambda$.

We will assume now that this is stabilizable in the following sense: there exists a constant gain $\overline{F}$ such that

$$u = -\overline{F} \chi$$

stabilizes the system (i.e., $\overline{A} - \overline{BF}$ Hurwitz) while it respects the sub-controller to sub-controller information exchange pattern. That is to say that $u_i$ should depend only on a subset of $\chi$ given by $\left\{\lambda^{|i-j|} x_j\right\}_{j=1}^{n}$, $\left\{\lambda^{|i-j|} u_j\right\}_{j=1}^{n}$ i.e.,

$$\begin{aligned} u = \overline{F}_{0,x} x + \overline{F}_{1,x} \lambda x + \cdots + \overline{F}_{n-1,x} \lambda^{n-1} x + \overline{F}_{1,u} \lambda u \\ + \cdots + \overline{F}_{n-1,u} \lambda^{n-1} u \end{aligned}$$

with $F_{i,x}$, $F_{i,u}$ being $(2i+1)$-diagonal matrices. More compactly, one can represent the above as

$$u = \overline{F}_x(\lambda)x + \overline{F}_u(\lambda)u$$

where $\overline{F}_x(\lambda)$, $\overline{F}_u(\lambda)$ are banded structures. Since addition, multiplication and inversion preserve the banded structure, solving for $u$ leads to

$$u = F(\lambda)x$$

with $F(\lambda)$ being also a banded map of the form $F(\lambda) = \begin{pmatrix} F_{11}(\lambda) & \lambda F_{12}(\lambda) & \cdots & \lambda^{n-1}F_{1n}(\lambda) \\ \vdots & \vdots & \vdots & \vdots \\ \lambda^{n-1}F_{n1}(\lambda) & \lambda^{n-2}F_{n2} & \cdots & F_{nn}(\lambda) \end{pmatrix}$. It should be clear that for a given $\overline{F}$ there is a unique $F(\lambda)$ to associate with .

As with stabilizability, we assume that the system is detectable in the following sense: There exists a constant gain $\overline{L}$ such that an observer

$$\overline{\chi}^{+} = \overline{A}\overline{\chi} + \overline{L}(y-\bar{y}), \quad \bar{y} = \overline{C}\overline{\chi} \tag{3}$$

has stable state error estimate dynamics (i.e., $\overline{A} - \overline{LC}$ is Hurwitz) with $\bar{x}_i^{+}$ depending only on a subset of available measurements that is consistent with the information pattern, i.e., on $\left\{\lambda^{|i-j|}y_j\right\}_{j=1}^{n}$. Thus, if $\overline{L} = (L'_{0,\bar{x}} \quad L'_{\bar{x}/\overline{\chi}})'$ the constraint is that $L_{0,\bar{x}}$ is diagonal which in turn leads, after simple manipulations in Equation 3, to a compact representation for the above observer

$$\begin{aligned} \bar{x}^{+} &= A(\lambda)\bar{x} + L(\lambda)(y-\bar{y}) \\ \bar{y} &= C(\lambda)\bar{x} \end{aligned}$$

where $L(\lambda)$ is a banded system $L(\lambda) = \begin{pmatrix} L_{11}(\lambda) & \cdots & \lambda^{n-1}L_{1n}(\lambda) \\ \vdots & & \vdots \\ \lambda^{n-1}L_{n1}(\lambda) & \cdots & L_{nn}(\lambda) \end{pmatrix}$. Again we note that there is a unique $L(\lambda)$ for a given $\overline{L}$. Thus, all stabilizing controllers can be parameterized as

$$\begin{aligned} \bar{x}^{+} &= A(\lambda)\bar{x} + B(\lambda)u + L(\lambda)(y-\bar{y}) \\ \bar{y} &= C(\lambda)\bar{x} + D(\lambda)u \\ u &= -F(\lambda)\bar{x} + v \end{aligned}$$

where $v = Q(\lambda)e$, $e = y - \bar{y}$ and $Q$ is stable and banded. The parameter $Q$ being banded is a necessary and sufficient condition for the controller $K$ to be banded. This follows as $A(\lambda)$, $B(\lambda)$, $L(\lambda)$, $C(\lambda)$, $D(\lambda)$, $F(\lambda)$ are banded and thus, using the standard formulas to obtain a factorization of $P$ from the above observer based controller leads to coprime factors that are banded [5].

The above observer-based parameterization also provides a way of implementing distributively the controller $K$. Namely, each sub-controller $K_i$ can be obtained in a compact form as

$$\begin{aligned} \bar{x}_i^{+} &= \sum_{j=1}^{n} A_{ij}\lambda^{|i-j|}\bar{x}_j + \sum_{j=1}^{n} B_{ij}\lambda^{|i-j|}u_j + L_i(\lambda)(y-\bar{y}) \\ \bar{y}_i &= \sum_{j=1}^{n} C_{ij}\lambda^{|i-j|}\bar{x}_j + \sum_{j=1}^{n} D_{ij}\lambda^{|i-j|}u_j \\ u_i &= -F_i(\lambda)\bar{x} + v_i \\ v_i &= Q_i(\lambda)(y-\bar{y}) \end{aligned}$$

which is similar to the previous section. The effect of communication noise can be captured in a similar manner leading to convex in $Q$ problems.

## V. Conclusion

In this paper we considered how to design and distribute a controller to various stations (sub-controllers) in the presence of noise in the sub-controller to sub-controller communication. Using an observer based controller parameterization, we provided suitable stabilizing sub-controller architectures that directly take into account the effect of communication noise on performance. In particular, the overall performance optimization was cast as a convex problem in the Youla-Kucera parameter $Q$. It was also indicated that similar results hold for banded controller structures, i.e., when there is a delay, in addition to noise, in the subsystem to subsystem communication.

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