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Active Vibration Control Using On-line Algebraic Identification of Harmonic Vibrations

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Abstract

In this paper is described the application of an on-line algebraic identification methodology for parameter and signal estimation in vibrating systems. The algebraic identification is employed to estimate the frequency and amplitude of exogenous vibrations affecting the mechanical system using only position measurements. The algebraic identification is combined with an adaptive-like sliding mode control scheme to asymptotically stabilize the system response and, simultaneously, cancel the harmonic vibrations. Numerical and experimental results show the dynamic and robust performance of the algebraic identification and the active vibration control scheme.

Keywords: Active vibration control, On-line algebraic identification, Sliding mode control.

1 Introduction

The identification of dynamical systems, involving explicitly the parameter identification, is a well-known process to develop or improve the mathematical description of a physical system by means of a proper use of experimental data. System identification methods provide the analytical tools, algorithms, computational programs and real-time implementation to get good approximations to an adequate model for analysis and control purposes. There exists a vast literature on the area, although most of the identification and estimation methods are essentially asymptotic, recursive or complex that lead to unrealistic implementations. See, e.g., Ljung [5], Soderstrom [6] and Sagara and Zhao [7, 8].

This paper deals with the application of an on-line algebraic identification approach to estimate the physical parameters on mass-spring-damper systems as well as the amplitude and excitation frequency of harmonic perturbations affecting directly the mechanical system. The algebraic identification is combined with an adaptive-like sliding mode control scheme to asymptotically stabilize the system response and, simultaneously, cancel the harmonic vibrations. The main virtue of the proposed identification and adaptive-like sliding mode control scheme for vibrating systems is that only measurements of the transient input/output behavior are used during the identification process, in contrast to the well-known persisting excitation condition and complex algorithms required by most of the traditional identification methods (Ljung [5] and Soderstrom [6]). The proposed results are strongly based on a theoretical framework on algebraic identification methods reported recently by Fliess and Sira-Ramírez [4].

2 Parameter identification

To illustrate the basic ideas of the algebraic approach to parameter identification proposed by Fliess and Sira-Ramírez [4], consider the following mathematical model in operational calculus terms of a simple one degree-of freedom mass-spring-damper system

$$\begin{aligned} & m\left(s^2x(s) - sx_0 - \dot{x}_0\right) + c\left(sx(s) - x_0\right) + kx(s) \\ & = u(s) \end{aligned} \tag{1}$$

where x denotes the displacement of the mass and u is a control input (force). Here $x_0 = x(t_0)$ and $\dot{x}_0 = \dot{x}(t_0)$ are unknown constants denoting the system initial conditions at $t_0 \geq 0$. The parameters to be identified are the mass m, the linear stiffness k and the linear viscous damping c.

In order to eliminate the dependence of the constant initial conditions, the equation (1) is differentiated twice with respect to the variable s, and the result is multiplied by s^{-2}, and next transformed into the time domain, resulting

$$\begin{aligned} & m\left[2\left(\int_{t_0}^{(2)} x\right) - 4\left(\int_{t_0}(\Delta t)\, x\right) + (\Delta t)^2 x\right] \\ & +c\left[-2\left(\int_{t_0}^{(2)}(\Delta t)\, x\right) + \left(\int_{t_0}(\Delta t)^2 x\right)\right] \\ & +k\left(\int_{t_0}^{(2)}(\Delta t)^2 x\right) = \left(\int_{t_0}^{(2)}(\Delta t)^2 u\right) \end{aligned} \tag{2}$$

where $\Delta t = t - t_0$ and $\left(\int_{t_0}^{(n)} \varphi(t)\right)$ are iterated integrals of the form $\int_{t_0}^{t}\int_{t_0}^{\sigma_1}\cdots\int_{t_0}^{\sigma_{n-1}} \varphi(\sigma_n)\, d\sigma_n \cdots d\sigma_1$, with $\left(\int_{t_0}\varphi(t)\right) = \int_{t_0}^{t}\varphi(\sigma)\, d\sigma$ and n a positive integer.

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The above integral-type equation (2), after some more integrations, leads to the following *linear* system of equations

$$A(t)\theta = b(t) \tag{3}$$

where $\theta = \left[\widehat{m}, \widehat{c}, \widehat{k}\right]^T$ denotes the parameter vector to be identified and $A(t)$, $b(t)$ are 3×3 and 3×1 matrices. From the equation (3) can be concluded that the parameter vector θ is *algebraically identifiable* if, and only if, the trajectory of the dynamical system is *persistent* in the sense established by Fliess and Sira-Ramírez [4], that is, the trajectories of the system (1) satisfy the condition $\det A(t) \neq 0$. In general, this condition holds at least in a small time interval $(t_0, t_0+\delta]$, where δ is a positive and sufficiently small value. By solving the equations (3), we can determine the unknown system parameters.

2.1 Simulation and experimental results

The performance of the algebraic identifier of parameters (3) was evaluated by means of numerical simulations and experiments on an ECP^{TM} rectilinear plant with $m = 2.2685$ [kg], $c = 4.1241$ [Ns/m] and $k = 356.56$ [N/m]. Fig. 1 shows the numerical and experimental results using the algebraic identification for a step input $u = 4$ [N]. In the simulations is clear how the parameter identification is quickly performed and it is almost exact with respect to the real parameters. It is also evident the presence of singularities in the algebraic identifier, i.e., when the $\det A(t)$ is zero. However, the first singularity occurs after the identification has been finished. The experimental response is quite similar to the numerical simulation, resulting in the following parameters: $m = 2.25$ [kg], $c = 4.87$ [Ns/m], $k = 362$ [N/m], which represent good approximations for the real parameters. Nevertheless, the identification process starts with some irregular behavior, which we have attributed to several factors like neglected nonlinear effects (stiffness and friction), noise on the output measurements and especially the computational algorithms based upon a sampled-time system with fast sampling time $t_s = 0.000884$ s and numerical integrations based on trapezoidal rules. Some of these problems in the parameter estimation have been already analyzed by Sagara and Zhao [7].

3 Identification of Harmonic Vibrations

Consider again the mechanical system (1), with known parameters m, k and c, undergoes harmonic vibrations

$$m\left(s^2x(s) - sx_0 - \dot{x}_0\right) + c\left(sx(s) - x_0\right) + kx(s) \\ = u(s) + F_0\frac{\omega}{s^2+\omega^2} \tag{4}$$

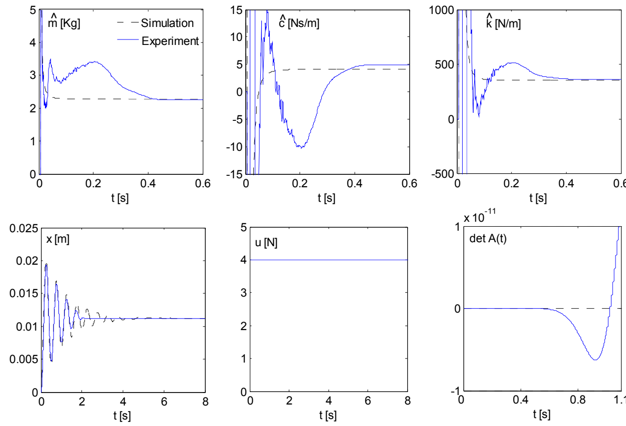

Figure 1: Numerical and experimental results of the online algebraic identification of the system parameters.

In this case we proceed to synthesize algebraic identifiers for the excitation frequency ω and amplitude F_0.

3.1 Identification of the excitation frequency ω

By multiplying (4) by $\left(s^2+\omega^2\right)$ is obtained

$$s^2\left(ms^2x + csx + kx - u\right) + \omega^2\left(ms^2x + csx + kx - u\right) \\ = F_0\omega + a_3s^3 + a_2s^2 + a_1s + a_0 \tag{5}$$

where a_i, $i = 0...3$, are unknown constants depending on the system initial conditions and the excitation frequency ω.

In order to eliminate the amplitude F_0 and the constants a_i, $i = 0...3$, we differentiate the equation (5) four times with respect to s and the result is multiplied by s^{-4}, and next transformed into the time domain, resulting

$$N_1(t) - D_1(t)\,\omega^2 = 0 \tag{6}$$

where

$$\begin{aligned} N_1(t) &= \textstyle\int_{t_0}^{(4)}\left[24mx - 24c(\Delta t)x + (12kx-12u)(\Delta t)^2\right] \\ &+ \textstyle\int_{t_0}^{(3)}\left[-96m(\Delta t)x + 36c(\Delta t)^2x + 8(u-kx)(\Delta t)^3\right] \\ &+ \textstyle\int_{t_0}^{(2)}\left[72m(\Delta t)^2x - 12c(\Delta t)^3x + (kx-u)(\Delta t)^4\right] \\ &+ \textstyle\int_{t_0}\left[-16m(\Delta t)^3x + c(\Delta t)^4x\right] + m(\Delta t)^4x \\ D_1(t) &= \textstyle\int_{t_0}^{(4)}\left[-12m(\Delta t)^2x + 4c(\Delta t)^3x \right. \\ &\left. + (\Delta t)^4(u-kx)\right] + \textstyle\int_{t_0}^{(3)}\left[8m(\Delta t)^3x - c(\Delta t)^4x\right] \\ &- m\left(\textstyle\int_{t_0}^{(2)}(\Delta t)^4x\right) \end{aligned}$$

Therefore, when the condition $D_1(t) \neq 0$ be satisfied at least for a small time interval $(t_0, t_0+\delta_0]$, where δ_0 is a sufficiently small quantity, we can find from (6) a closed-form expression for the estimated excitation frequency

$$\omega_e = \sqrt{\frac{N_1(t)}{D_1(t)}}, \ \ \forall t \in (t_0, t_0 + \delta_0] \tag{7}$$

3.2 Identification of the amplitude F_0

To synthesize an algebraic identifier for the amplitude F_0, we first express the equation (4) as follows

$$\begin{aligned} & ms^2 x(s) + csx(s) + kx(s) \\ & = u(s) + f(s) + mx_0 s + m\dot{x}_0 + cx_0 \end{aligned} \tag{8}$$

Taking derivatives, twice, with respect to s and multiplying by s^{-2} makes possible to remove the dependence on the initial conditions. Next, the resulting equation is transformed back into the time domain, to get the following algebraic identifier

$$F_{0e} = \frac{N_2(t)}{D_2(t)}, \ \ \forall t \in (t_0 + \delta_0, t_0 + \delta_1] \tag{9}$$

where

$$\begin{aligned} N_2(t) &= m\left[2\left(\int_{t_0+\delta_0}^{(2)} x\right) - 4\left(\int_{t_0+\delta_0} (\Delta t)\, x\right) + (\Delta t)^2 x\right] \\ &\quad + c\left[-2\left(\int_{t_0+\delta_0}^{(2)} (\Delta t)\, x\right) + \left(\int_{t_0+\delta_0} (\Delta t)^2 x\right)\right] \\ &\quad + k\left(\int_{t_0+\delta_0}^{(2)} (\Delta t)^2 x\right) - \left(\int_{t_0+\delta_0}^{(2)} (\Delta t)^2 u\right) \\ D_2(t) &= \left(\int_{t_0+\delta_0}^{(2)} (\Delta t)^2 \sin\left[\omega_e(t_0+\delta_0)t\right]\right) \end{aligned}$$

In this case the system trajectory is persistent if, and only if, the condition $D_2(t) \neq 0$ is satisfied for all $t \in (t_0 + \delta_0, t_0 + \delta_1]$ with $\delta_1 > \delta_0 > 0$.
It is important to note that equation (9) still depends on the excitation frequency ω. Therefore, it is required to synchronize both algebraic identifiers for ω and F_0. This procedure is sequentially executed, first by running the identifier for ω and, after some small time interval with the estimation $\omega_e(t_0 + \delta_0)$ is then started the algebraic identifier for F_0.

3.3 Simulation and experimental results

In Fig. 2 are shown some numerical and experimental results using the algebraic identifiers for the excitation frequency ω and amplitude F_0 with $u \equiv 0$ and $f(t) = 4\sin 5t$ [N]. First of all it is started the identifier for ω, which takes about $t < 0.2$ s to get a good estimation. After the time interval $(0, 0.7]$ s, where $t_0 = 0$ s and $\delta_0 = 0.7$ s with an estimated value $\omega_e(t_0 + \delta_0) = 4.9$ [rad/s], it is activated the identifier for F_0.
In the experimental results the identification processes are slower with respect to the numerical simulations. Again, we assume that such incompatibility is caused by unmodelled dynamics and nonlinearities into the physical system, the numerical methods used in the computational algorithms and noise into the input-output measurements. Such obstacles, however, do not affect substantially the algebraic identification process, resulting in fast and good estimations $\omega_e = 4.9$ [rad/s] and $F_{0e} = 3.933$ [N].

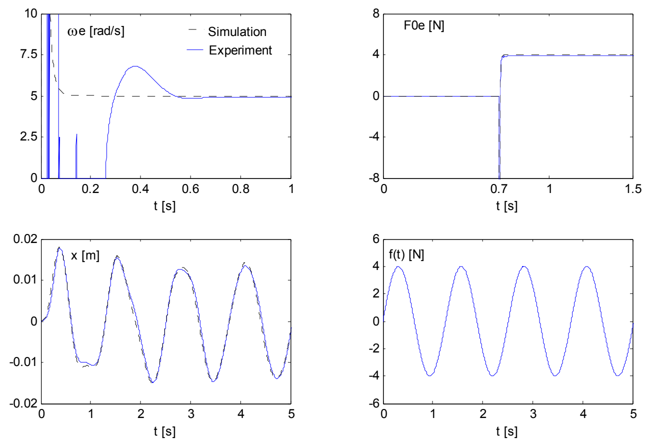

Figure 2: Numerical and experimental results using online identification for ω and F_0.

4 An Active Vibration Control Scheme

Consider the vibrating mechanical system shown in Fig. 3, which consists of an active vibration absorber (secondary system) coupled to the perturbed mechanical system (primary system). The generalized coordinates are the displacements of both masses, x_1 and x_2, respectively. In addition, u represents the (force) control input and f a harmonic perturbation. Here m_1, k_1 and c_1 denote mass, linear stiffness and linear viscous damping on the primary system; similarly, m_2, k_2 and c_2 denote mass, stiffness and viscous damping of the vibration absorber.
The mathematical model of the two degree-of-freedom system is described by two coupled ordinary differential equations

$$\begin{aligned} m_1\ddot{x}_1 + c_1\dot{x}_1 + k_1x_1 + k_2(x_1 - x_2) = f(t) \\ m_2\ddot{x}_2 + k_2(x_2 - x_1) = u(t) \end{aligned} \tag{10}$$

where $f(t) = F_0 \sin \omega t$. In order to simplify the analysis we have assumed that $c_1 \approx 0$ and $c_2 \equiv 0$.
Defining the state variables as $z_1 = x_1$, $z_2 = \dot{x}_1$, $z_3 = x_2$ and $z_4 = \dot{x}_2$, one obtains the following state space description

$$\begin{aligned} \dot{z}_1 &= z_2 \\ \dot{z}_2 &= -\tfrac{k_1+k_2}{m_1} z_1 - \tfrac{c_1}{m_1} z_2 + \tfrac{k_2}{m_1} z_3 + \tfrac{1}{m_1} f \\ \dot{z}_3 &= z_4 \\ \dot{z}_4 &= \tfrac{k_2}{m_2} z_1 - \tfrac{k_2}{m_2} z_3 + \tfrac{1}{m_2} u \\ y &= z_1 \end{aligned} \tag{11}$$

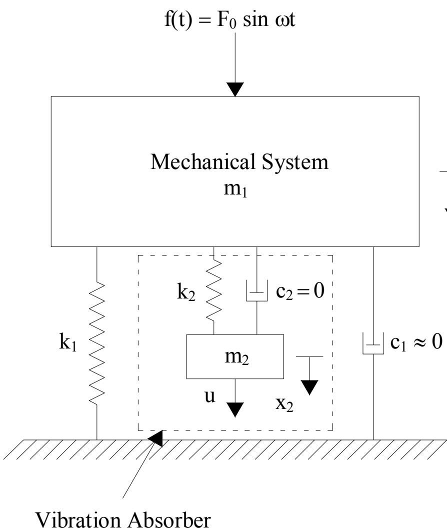

Figure 3: Schematic diagram of the vibrating mechanical system.

It is easy to verify that system (11) is completely controllable and observable, as well as marginally stable in case of $c_1 = 0$, $f = 0$ and $u = 0$ (asymptotically stable when $c_1 > 0$). Note that the disturbance decoupling problem of $f(t)$ using state feedback is not solvable because the output $y = z_1$ has relative degree 4 with respect to u and relative degree 2 with respect to f.
In what follows we will apply the algebraic identification method to estimate the harmonic force $f(t)$ and design an active vibration controller based on state feedback and feedforward information of $f(t)$.

4.1 Sliding mode control

Consider the perturbed system (11), where only the output $y = z_1$ and the input u are available for use on a sliding mode control scheme, under the temporary assumption that the excitation frequency ω is perfectly known. For simplicity, we also suppose that $c_1 = c_2 \equiv 0$. It is easy to verify that the output y satisfies the following input-output differential equation (see Beltrán *et al* [1])

$$y^{(6)} + d_1 y^{(4)} + d_2\ddot{y} + d_3 y = d_4\left(\ddot{u} + \omega^2 u\right) \tag{12}$$

where

$$\begin{array}{ll} d_1 = \frac{k_1+k_2}{m_1} + \frac{k_2}{m_2} + \omega^2 & d_3 = \frac{k_1 k_2}{m_1 m_2}\omega^2 \\ d_2 = \left(\frac{k_1+k_2}{m_1} + \frac{k_2}{m_2}\right)\omega^2 + \frac{k_1 k_2}{m_1 m_2} & d_4 = \frac{k_2}{m_1 m_2} \end{array}$$

By successive integrations of the equation (12), the time derivatives of the output variable y up to fifth order can be reconstructed by means of integral reconstruction. This is, they can be expressed in terms of the output y, the input u and iterated integrals of the input and the output variables. For more details on this topic see Fliess *et al* [3].
The integral input-output parameterization of the time derivatives of the output is given, modulo initial conditions, by

$$\begin{array}{l} \widehat{\dot{y}} = -d_1\int_{t_0} y + \int_{t_0}^{(3)}(d_4 u - d_2 y) + \int_{t_0}^{(5)}\left(d_4\omega^2 u - d_3 y\right) \\ \widehat{\ddot{y}} = -d_1 y + \int_{t_0}^{(2)}(d_4 u - d_2 y) + \int_{t_0}^{(4)}\left(d_4\omega^2 u - d_3 y\right) \\ \widehat{y}^{(3)} = -d_1\widehat{\dot{y}} + \int_{t_0}(d_4 u - d_2 y) + \int_{t_0}^{(3)}\left(d_4\omega^2 u - d_3 y\right) \\ \widehat{y}^{(4)} = -d_1\widehat{\ddot{y}} - d_2 y + \int_{t_0}^{(2)}\left(d_4\omega^2 u - d_3 y\right) + d_4 u \\ \widehat{y}^{(5)} = -d_1\widehat{y}^{(3)} - d_2\widehat{\dot{y}} + \int_{t_0}\left(d_4\omega^2 u - d_3 y\right) + d_4\dot{u} \end{array} \tag{13}$$

For non-zero initial conditions, these expressions differ from the actual values by at most a fourth order time polynomial of the form: $p_4 t^4 + p_3 t^3 + p_2 t^2 + p_1 t + p_0$, where p_i, $i = 0...4$, are all real constants depending on the unknown initial conditions.
A sliding surface can now be proposed as

$$\begin{array}{l} \widehat{\sigma} = \widehat{y^{(5)}} - (y^*)^{(5)} + \alpha_9\left[\widehat{y^{(4)}} - (y^*)^{(4)}\right] \\ +\alpha_8\left[\widehat{y^{(3)}} - (y^*)^{(3)}\right] + \alpha_7\left(\widehat{\ddot{y}} - \ddot{y}^*\right) \\ +\alpha_6\left(\widehat{\dot{y}} - \dot{y}^*\right) + \alpha_5 e + \alpha_4\int_{t_0} e + \alpha_3\int_{t_0}^{(2)} e \\ +\alpha_2\int_{t_0}^{(3)} e + \alpha_1\int_{t_0}^{(4)} e + \alpha_0\int_{t_0}^{(5)} e \end{array} \tag{14}$$

where $y^*(t)$ is a desired output reference trajectory and $e = y - y^*(t)$. Then, the ideal sliding condition $\widehat{\sigma} = 0$ results in a tenth order dynamics

$$\begin{array}{l} e^{(10)} + \alpha_9 e^{(9)} + \alpha_8 e^{(8)} + \alpha_7 e^{(7)} + \alpha_6 e^{(6)} + \alpha_5 e^{(5)} \\ +\alpha_4 e^{(4)} + \alpha_3 e^{(3)} + \alpha_2\ddot{e} + \alpha_1\dot{e} + \alpha_0 e = 0 \end{array} \tag{15}$$

which is completely independent of any initial conditions. Therefore, selecting the design parameters α_i, $i = 0, ..., 9$, such that the associated characteristic polynomial for (15) be *Hurwitz,* one guarantees that the error dynamics on the sliding surface $\widehat{\sigma} = 0$ be globally asymptotically stable. In addition, by forcing the sliding surface $\widehat{\sigma}$ to satisfy the discontinuous closed loop dynamics:

$$\dot{\widehat{\sigma}} = -W\,\mathrm{sign}\left(\widehat{\sigma}\right), \quad W > 0 \tag{16}$$

where "sign" stands for the *signum* function, one then gets the sliding mode control as a solution of the differential equation

$$\begin{array}{l} \ddot{u} + \omega^2 u = d_4^{-1} v + d_4^{-1}\left(d_1\widehat{y}^{(4)} + d_2\widehat{\ddot{y}} + d_3 y\right) \\ v = y^{*(6)} - \alpha_9\left[\widehat{y}^{(5)} - y^{*(5)}\right] \\ \quad - \alpha_8\left[\widehat{y}^{(4)} - y^{*(4)}\right] - \alpha_7\left[\widehat{y}^{(3)} - y^{*(3)}\right] \\ \quad - \alpha_6\left[\widehat{\ddot{y}} - \ddot{y}^*\right] - \alpha_5\left[\widehat{\dot{y}} - \dot{y}^*\right] - \alpha_4\left[y - y^*\right] \\ \quad - \alpha_3\xi_1 - \alpha_2\xi_2 - \alpha_1\xi_3 - \alpha_0\xi_4 - W\mathrm{sign}(\widehat{\sigma}) \end{array} \tag{17}$$

where $\alpha_5 = \alpha_9\left(d_2 - d_1^2\right) + \alpha_7 d_1$. This controller employs only measurements of $y = z_1$ and the excitation frequency ω.

4.1.1 Simulation and experimental results

Fig. 4 shows the numerical and experimental dynamic behavior of the sliding mode control scheme. In this case, the harmonic perturbation $f(t) = 0.515 \sin(17.7t)$ was applied to the mechanical system, which is close to the system's resonance. In the implementation of this controller, an error in the measurement of the excitation frequency was introduced intentionally, this is $\omega = 19$ [rad/s]. In spite of that, we can see how the active vibration absorber dissipates the vibrating energy from the primary system H_1 and allows the asymptotic output regulation about $\bar{y} = \bar{z}_1 = 5$ [mm]. The parameters for the ECP^{TM} rectilinear plant are $m_1 = 3.4316$ [kg], $c_1 = 1.11$ [Ns/m], $k_1 = 356.56$ [N/m], $m_2 = 2.745$ [kg], $c_2 = 3.8122$ [Ns/m], $k_2 = 356.56$ [N/m].

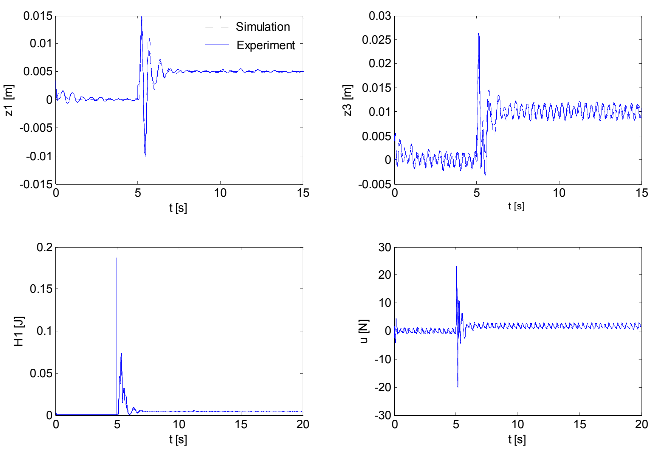

Figure 4: Numerical and experimental responses of the close-loop system.

4.2 Algebraic identification of harmonic vibrations

The main goal is the algebraic identification of the harmonic force $f(t)$, which will be obtained through similar procedures stated in previous sections using *only* measurements of the output $y = z_1$ and considering that the system parameters are perfectly known. It is easy to verify that the output y also satisfies the following input-output differential equation (see Beltrán *et al* [1]) writen in operational calculus terms

$$\begin{aligned}&m_1 s^4 y(s) + \left(k_1 + k_2 + \tfrac{m_1 k_2}{m_2}\right) s^2 y(s) + \tfrac{k_1 k_2}{m_2} y(s)\\&= \tfrac{k_2}{m_2} u(s) + \left(\tfrac{k_2}{m_2} - \omega^2\right) F_0 \tfrac{\omega}{s^2+\omega^2}\\&+ a_3 s^3 + a_2 s^2 + a_1 s + a_0\end{aligned} \tag{18}$$

where a_i, $i = 0...3$, denote unknown constants depending on the unknown system initial conditions. Now, equation (18) is multiplied by $\left(s^2 + \omega^2\right)$, leading to

$$\begin{aligned}&\left(s^2+\omega^2\right)\left[\left(s^4 y(s) + \tfrac{k_2}{m_2} s^2 y(s)\right) m_1 +\right.\\&\left.\left(s^2 y(s) + \tfrac{k_2}{m_2} y(s)\right) k_1 + k_2 s^2 y(s)\right]\\&= \tfrac{k_2}{m_2}\left(s^2+\omega^2\right) u(s) + \left(\tfrac{k_2}{m_2} - \omega^2\right) F_0 \omega\\&+ \left(s^2+\omega^2\right)\left(a_3 s^3 + a_2 s^2 + a_1 s + a_0\right)\end{aligned} \tag{19}$$

This equation is differentiated six times with respect to s in order to cancel the constants a_i and the unknown amplitude F_0. The resulting equation is then multiplied by s^{-6}, and next transformed into the time domain, to get

$$\begin{aligned}&\left[a_{11}(t) + \omega^2 a_{12}(t)\right] m_1 + \left[a_{12}(t) + \omega^2 b_{12}(t)\right] k_1\\&= c_1(t) + \omega^2 d_1(t)\end{aligned} \tag{20}$$

where

$$\begin{aligned}a_{11}(t) &= m_2 g_{11}(t) + k_2 g_{12}(t)\\a_{12}(t) &= m_2 g_{12}(t) + k_2 g_{13}(t)\\b_{12}(t) &= m_2 g_{13}(t) + k_2 \left(\int_{t_0}^{(6)} t^6 z_1\right)\\c_1(t) &= k_2 g_{14}(t) - k_2 m_2 g_{12}(t)\\d_1(t) &= k_2 \left(\int_{t_0}^{(6)} t^6 u\right) - k_2 m_2 g_{13}(t)\end{aligned}$$

with

$$\begin{aligned}g_{11}(t) =&\ 720\left(\int_{t_0}^{(6)} y\right) - 4320\left(\int_{t_0}^{(5)} (\Delta t)\, y\right)\\&+5400\left(\int_{t_0}^{(4)} (\Delta t)^2 y\right) - 2400\left(\int_{t_0}^{(3)} (\Delta t)^3 y\right)\\&+450\left(\int_{t_0}^{(2)} (\Delta t)^4 y\right) - 36\left(\int_{t_0} (\Delta t)^5 y\right) + (\Delta t)^6 y\\g_{12}(t) =&\ 360\left(\int_{t_0}^{(6)} (\Delta t)^2 y\right) - 480\left(\int_{t_0}^{(5)} (\Delta t)^3 y\right)\\&+180\left(\int_{t_0}^{(4)} (\Delta t)^4 y\right) - 24\left(\int_{t_0}^{(3)} (\Delta t)^5 y\right)\\&+\left(\int_{t_0}^{(2)} (\Delta t)^6 y\right)\\g_{13}(t) =&\ 30\left(\int_{t_0}^{(6)} (\Delta t)^4 y\right) - 12\left(\int_{t_0}^{(5)} (\Delta t)^5 y\right)\\&+\left(\int_{t_0}^{(4)} (\Delta t)^6 y\right)\\g_{14}(t) =&\ 30\left(\int_{t_0}^{(6)} (\Delta t)^4 u\right) - 12\left(\int_{t_0}^{(5)} (\Delta t)^5 u\right)\\&+\left(\int_{t_0}^{(4)} (\Delta t)^6 u\right)\end{aligned}$$

Finally, solving for the excitation frequency ω in (20) leads to the following on-line identifier

$$\omega_e^2 = \frac{N_1(t)}{D_1(t)} = \frac{c_1(t) - a_{11}(t) m_1 - a_{12}(t) k_1}{a_{12}(t) m_1 + b_{12}(t) k_1 - d_1(t)} \tag{21}$$

This estimation is valid if, and only if, the condition $D_1(t) \neq 0$ holds in a sufficiently small time interval $(t_0, t_0 + \delta_0]$ with $\delta_0 > 0$.
By using the same procedure to estimate the amplitude F_0 in the section 2, we obtain the following on-line identifier for the amplitude

$$F_{0e} = \frac{N_2(t)}{D_2(t)} \tag{22}$$

where

$$
\begin{aligned}
&N_2(t) = m_1 P_1(t) + \left(k_1 + k_2 + \tfrac{m_1 k_2}{m_2}\right) P_2(t) \\
&+ \tfrac{k_1 k_2}{m_2} \left(\int_{t_0+\delta_0}^{(4)} (\Delta t)^4 y\right) - \tfrac{k_2}{m_2} \left(\int_{t_0+\delta_0}^{(4)} (\Delta t)^4 u\right) \\
&D_2(t) = \left(\tfrac{k_2}{m_2} - \omega_e^2\right) \left(\int_{t_0+\delta_0}^{(4)} (\Delta t)^4 \sin\left[\omega_e (t_0+\delta_0) t\right]\right) \\
&P_1(t) = 24 \left(\int_{t_0}^{(4)} y\right) - 96 \left(\int_{t_0}^{(3)} (\Delta t) y\right) \\
&+72 \left(\int_{t_0}^{(2)} (\Delta t)^2 y\right) - 16 \left(\int_{t_0} (\Delta t)^3 y\right) + (\Delta t)^4 y \\
&P_2(t) = 12 \left(\int_{t_0}^{(4)} (\Delta t)^2 y\right) - 8 \left(\int_{t_0}^{(3)} (\Delta t)^3 y\right) \\
&+ \left(\int_{t_0}^{(2)} (\Delta t)^4 y\right)
\end{aligned}
$$

At this point we assume that the excitation frequency has been previously estimated, during a small time interval $(t_0, t_0 + \delta_0]$, using (21). After the time $t = t_0 + \delta_0$ it is started the on-line identifier for the amplitude. Such an estimation is valid as far as the system trajectory be persistent, that is, if the condition $D_2(t) \neq 0$ holds for a sufficiently small time interval $[t_0 + \delta_0, t_0 + \delta_1]$ with $\delta_1 > \delta_0 > 0$.

4.3 Simulation results

Fig. 5 shows the identification process of the resonant harmonic vibrations $f(t) = 2\sin(8.0109t)$ [N] and the dynamic behavior of the adaptive-like control scheme (17), which starts using the nominal value $\omega = 10$ [rad/s]. We can see that the resonant vibrations are asymptotically and actively cancelled from the displacement of the primary system z_1. A desired reference trajectory was considered for regulating the evolution of the output variable toward the desired equilibrium $\bar{y} = \bar{z}_1 = 0.01$ [m], which is given by a Bezier type polynomial in time. The parameters for the ECP^{TM} rectilinear plant are $m_1 = 10$ [kg], $k_1 = 1000$ [N/m], $m_2 = 2$ [kg] and $k_2 = 200$ [N/m].

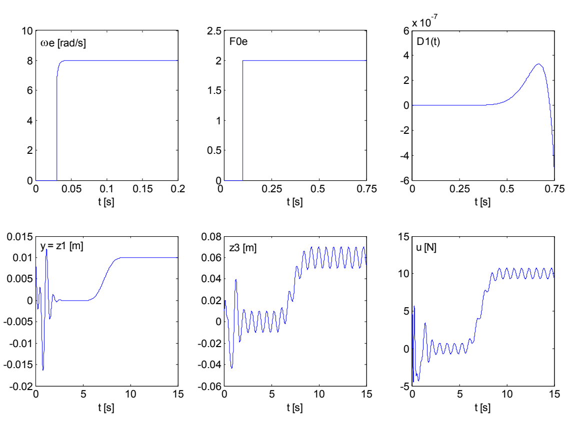

Figure 5: Controlled system responses and identification of frequency and amplitude of $f(t) = F_0 \sin \omega t$.

5 CONCLUSIONS

In this paper we have described the application of a novel algebraic identification approach for parameter and signal estimation in vibrating systems. This approach is quite promising, in the sense that only input-output information is needed to get precise and fast parameter and signal estimations. This fact was exploited in the formulation of an active vibration control scheme based on sliding modes. Since this active controller requires measurements of the excitation frequency of the harmonic vibrations, the algebraic identification is combined to get an adaptive-like controller. The adaptive-like control scheme results quite precise, fast and robust against parameter uncertainty and variations on the excitation frequency and amplitude of the exogenous perturbations. Further work is being conducted to extend the application to nonlinear vibrating systems.

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