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Nonlinear Observer Design and Experimental Verification for Heat Exchangers during the Start-up Process

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***Abstract*—In this paper, a nonlinear observer design is proposed to estimate the two-phase section length in the evaporator during the start-up process for the use of oil monitoring of air conditioning systems. The dynamic system is non-autonomous due to the time-varying parameters such as heat transfer coefficients. The convergence of the observer is proved using contraction theory which is applicable to non-autonomous systems and the experiment testing demonstrates the accuracy and convergence of the nonlinear observer.**

I. Introduction

In HVAC systems, lubrication oil protects moving component in a compressor. Poor lubrication is one of the major causes of compressor failures for residential, commercial and automotive HVAC systems. According to the 2001 annual Field Survey Report of the Mobile Air Conditioning Society (MACS) Worldwide, more than 50% automotive air conditioning compressor failures are caused by improper lubrication. Therefore, monitoring of lubrication is a crucial problem for improving reliability and maintainability of HVAC systems. An oil circulation observer for estimating oil concentration and oil amount in refrigerant compressors was presented in [1]. The estimation of the distribution of refrigerant and oil in the air conditioning system is based on the phase lengths in the evaporator and condenser which are not available from sensor measurements. It is necessary to dynamically estimate some immeasurable variables based on available sensor measurements. A model-based heat exchanger observer was developed to estimate the phase length in [2] and this observer design was used in the oil circulation observer [1].

The focus in this paper is on the start-up process of the air conditioning system because a lot of compressor failures happen in this period. However, a lot of parameters in the heat exchanger models are not constants during the start-up process. For example, the heat transfer coefficients are functions of the refrigerant mass flow rate. In the start-up process, the mass flow rate changes in a wide range. Therefore, the heat transfer coefficients are time-varying. Other parameters such as inlet vapor quality, saturated refrigerant liquid and vapor enthalpies are not constants either. Therefore, this system is a non-autonomous system. As mentioned in [9], the contraction theory is applicable to non-autonomous systems. Thus, the contraction theory is used here to guarantee the convergence of the observer.

Another issue during the start-up process of the air conditioning system is that the phase conditions in heat exchangers are changing as shown in Fig.1. For example, before the start up of the compressor, there is significant amount of two-phase liquid and vapor refrigerant mixture in the evaporator. About several ten seconds after the start up, all the liquid initially stored in the evaporator is quickly pumped into the accumulator. After that, two-phase refrigerant from the expansion valve flows into the evaporator, and the liquid dry-out point will move from the inlet of the evaporator to the outlet of evaporator and then move to regular position depending on the operation condition. For different phase condition, the model of the evaporator is different. The convergence of the model-based observer depends on the correct selection of model.

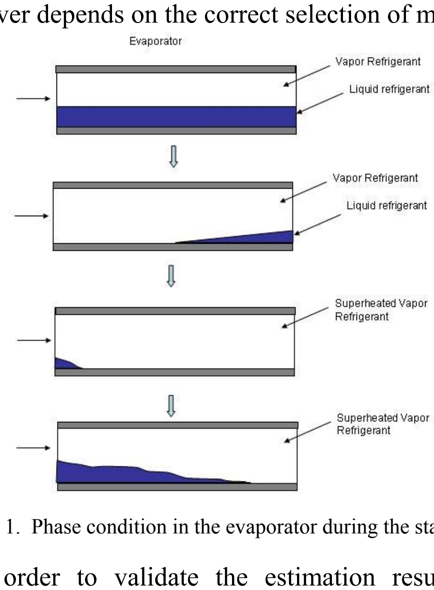

Fig. 1. Phase condition in the evaporator during the start-up process

In order to validate the estimation results from the observer, the experimental testing is conducted and the length of the two-phase section during the start-up process is obtained by analyzing the experiment data of refrigerant temperatures.

This paper is organized as follows. Section 2 presents experimental testing of the start-up process. Section 3 presents a nonlinear observer design for the start-up process and the analysis of convergence is carried out based on contraction theory. The comparison of the results is discussed in Section 4.

II. Experimental Testing of the Start-up Process

There is no reference about the lumped parameter models of the heat exchangers during the start-up process which can be used directly for the observer design. Therefore, start-up process data were experimentally obtained before designing an observer so that dynamic behavior characteristics of the start-up process can be better understood.

2.1Experiment Set-up

The test equipment is a 2.8kW split type air conditioner for residential applications. A schematic diagram of the testing machine is shown in Fig. 2. The machine is equipped with variable speed compressor and electronic expansion valve.

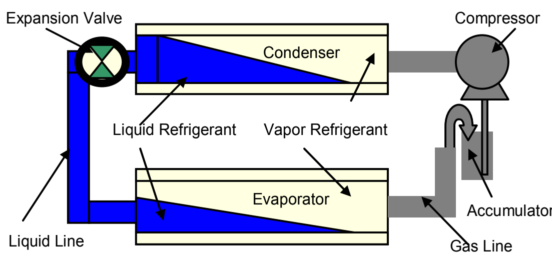

Fig. 2. A diagram of the testing machine

Fig.3 shows the positions of the eight thermo couples in the tube of the evaporator.

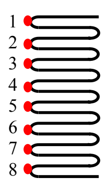

Fig. 3. Thermal couples in the evaporator tube

There are pressure sensors located at the inlet of the evaporator and gas line.

2.2Experiment Data

As mentioned previously, the phase condition in the heat exchangers and the time-varying parameters are two challenging issues for the nonlinear observer design of the start-up process. In this section, the experiment data is analyzed to obtain the phase condition in the evaporator. Based on this analysis, the appropriate models can be selected for the nonlinear observer design.

Fig.4 shows the relation of the temperature and enthalpy of water under certain pressure.

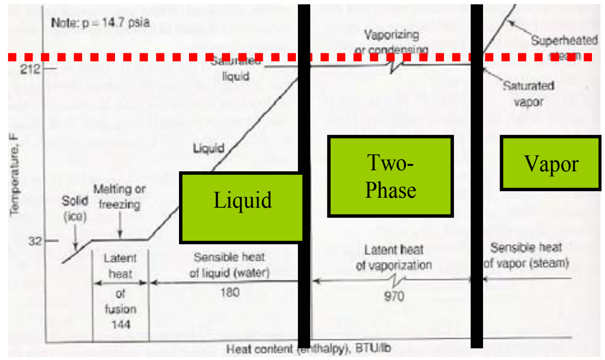

Fig. 4. Temperature-enthalpy change of water at 14.7 psia surrounding pressure [4]

It can be seen that under certain pressure, the saturation temperature is constant. The material is liquid if the temperature is below this saturation temperature. It is vapor if the temperature is above this temperature. This property is used to analyze the phase condition in the heat exchanger.

There are two tubes in this evaporator. Fig.5 and Fig.6 show the temperature measurements in the two tubes of the evaporator. The flow behaviors in these two tubes are a little bit different as shown in Fig. 5 and Fig.6.The third temperature sensor in tube 2 does not work. Therefore, there is no measurement available at that point.

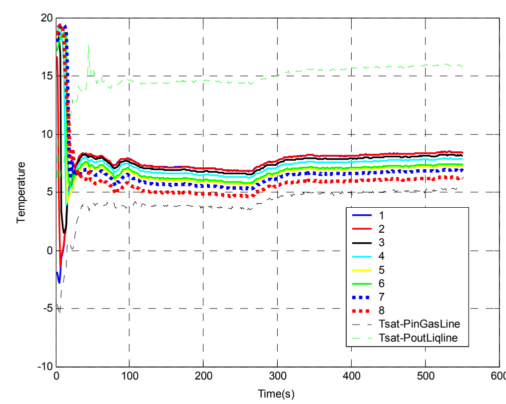

Fig. 5. Temperatures measured in tube 1 of the evaporator (^oC)

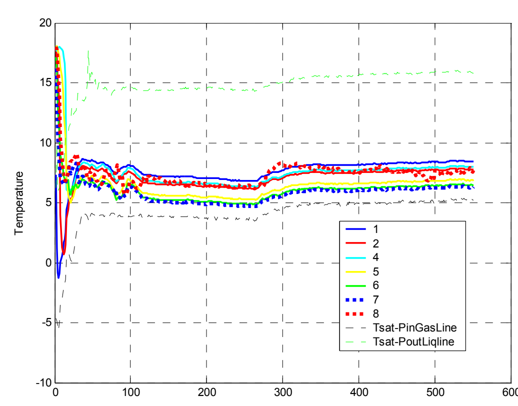

Fig. 6. Temperatures measured in tube 2 of the evaporator (^oC)

In order to analysis the phase condition in the evaporator, the measurement information of the pressure at the inlet of the gas line is transferred to the saturation temperature according to the refrigerant properties. If the pressure in the evaporator is the same everywhere, the saturation temperature of the refrigerant should be the same. Therefore, in the two-phase section of the evaporator, the temperature measurements should be the same. However, the saturation temperature along the tube should be decreasing and reaching the lowest point at the outlet of the evaporator due to the pressure drop in the evaporator.

As seen in Fig.5, the temperatures measured in tube 1 are decreasing consistently from the measurement point 1 to 8. The temperature at point 8 is close to but higher than the saturation temperature at the inlet of the gas line. It can be concluded that the refrigerant is in two-phase in tube 1. However, the temperatures measured in tube 2 are decreasing from measurement point 1 to 7 and increasing from measurement point 7 to 8. It can be concluded that tube 2 has two-phase section and superheated vapor section.

This experiment data not only tells us the phase condition in the heat exchanger, but also tells us the development process of the two-phase section in it. The bounds of the two-phase length in the heat exchanger can be estimated in the start-up process from the experiment data.

Fig.7 is a zoom in plot of the temperatures measured in tube 1 of the evaporator. The development of the two-phase section in tube 1 can be observed from the temperature measurements. At the start point, the temperature at point 1 is much lower than the temperature at point 2. It can be seen that in 8 seconds the temperature at the point 2 merges to the temperature at point 1. Only under saturated condition, the refrigerant temperature can be the same at different points of the heat exchanger because of the heat transfer with the tube. This means that the two-phase section reaches point 2 at that moment in tube 1. According to the observation from the data, the two-phase section reaches point 3 in 14 seconds, point 5 in 16 seconds, point 6 in 19 seconds and point 8 in 30 seconds in tube 1.

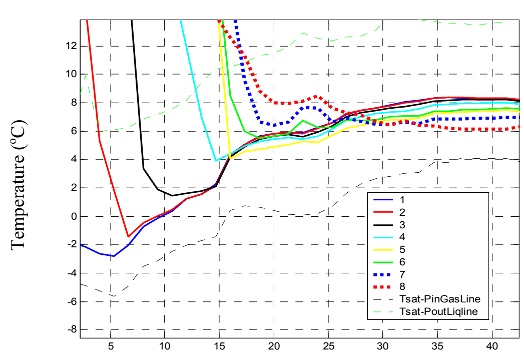

Fig. 7. Zoom in plot of temperatures measured in tube 1 of the evaporator (°C)

Fig.8 is a zoom in plot of the temperatures measured in tube 2 of the evaporator. Similar with tube 1, the two-phase section reaches point 2 in 11 seconds, point 6 in 22 seconds and point 7 in 40 seconds in tube 2.

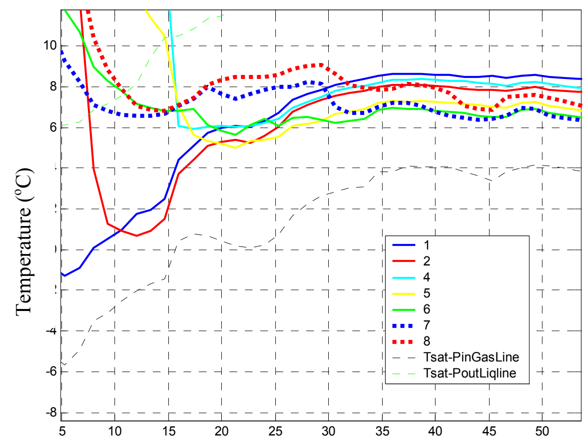

Fig. 8. Zoom in plot of temperatures measured in tube 2 of the evaporator (°C)

According to the information observed from the temperature experiment data and the position of the temperature sensors, the development of the two phase length can be estimated. As said above, the two-phase section reaches point 2 in 8 seconds in tube 1. However, in tube 2, the two-phase section reaches point 2 in 11 seconds. Therefore, at the 8th second, the two-phase section length in tube 1 is the distance from the inlet to point 2. In tube 2, the two-phase section is between point 1 and point 2. By adding these two lengths, we can get an upper bound and a lower bound of the two-phase section length at the 8th second. Similarly, the bounds of the two-phase section lengths at other time instants are obtained as shown in Fig.9.

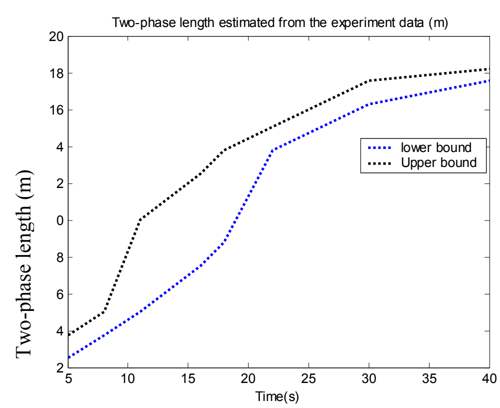

Fig. 9. Two-phase length estimated from the experiment data (m)

The two-phase length estimated from the experiment is not very accurate because it is estimated from the temperature measurements instead of direct measurement of the length. Only a bound of the two-phase length can be obtained from the experiment data due to the discrepancy of these two tubes in the evaporator. Actually, the temperature in the two-phase section is not constant due to the pressure drop. However, there is no direct measurement of the two-phase length. This is the first time that the two-phase length is estimated from experiment data and used for the comparison with the observer results.

III. Nonlinear Observer Design of the Start-up Process

According to the experiment data analysis in section 2, for the evaporator, one tube is in two-phase condition, the other one has two-phase and superheated vapor sections. The mechanism of this phenomenon is not clear yet. It may result from different mass flow rates in these two tubes. However, there are no exact measurements of the mass flow rates in these two tubes. These two tubes have to be considered as a whole which have superheated and two-phase sections. Therefore, the model presented in [2] is still applicable for this case. The state space representation for the low evaporator model is given below

$$\begin{pmatrix}\dot{T}_e\\ \dot{T}_w\\ \dot{l}\end{pmatrix} = \begin{pmatrix} \frac{\pi D_i\alpha_i}{kh_{\lg}} l(T_w-T_e)+\frac{x_o}{k}\dot{m}_{in}-\frac{1}{k}\dot{m}_{out}\\ \frac{1}{(C_p\rho A)_e}(\pi D_o\alpha_o(T_a-T_w)-\pi D_i\alpha_i(T_w-T_e))\\ \frac{1}{\rho_l(1-\bar{\gamma})A}(-\frac{\pi D_i\alpha_i l(T_w-T_e)}{h_{\lg}}+\dot{m}_{in}(1-x_o))\end{pmatrix} \quad (1)$$

T_e is the evaporating temperature, l is the length of the two-phase section. T_w is the wall temperature of the tube. T_a is the room air temperature. $\dot{m}_{in}$ and $\dot{m}_{out}$ are the inlet and outlet refrigerant mass flow rates, respectively.

The nonlinear observer design presented in [2] is used here by some modification. Fig. 10 shows the nonlinear observer structure for the evaporator with the experiment measurements in the start-up process. The heat transfer coefficients, latent heat and inlet vapor quality are time varying as show in Fig.10. Therefore, the observer dynamics is non-autonomous. The contraction theory [9] can be used to non-autonomous case and is used here to guarantee the convergence. However, some treatment is needed due to the time-varying parameters in order to guarantee the contraction of the observer dynamics.

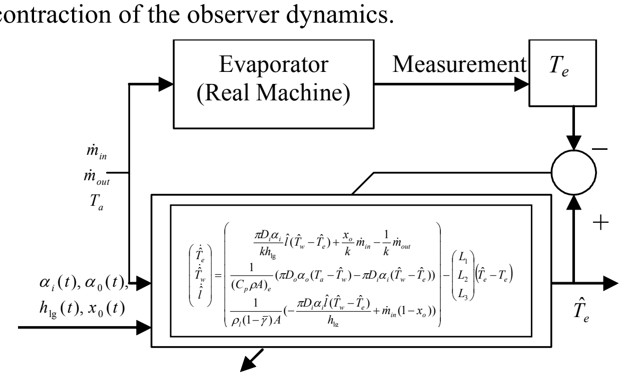

Fig. 10. Two-phase length estimated from the experiment data (m)

Some part of the convergence analysis method presented in [2] is repeated here. The following is the dynamics of the nonlinear observer.

$$\begin{pmatrix}\dot{\hat{T}}_e\\ \dot{\hat{T}}_w\\ \dot{\hat{l}}\end{pmatrix} = \begin{pmatrix} \frac{\pi D_i\alpha_i}{kh_{\lg}} \hat{l}(\hat{T}_w-\hat{T}_e)+\frac{x_o}{k}\dot{m}_{in}-\frac{1}{k}\dot{m}_{out}\\ \frac{1}{(C_p\rho A)_e}(\pi D_o\alpha_o(T_a-\hat{T}_w)-\pi D_i\alpha_i(\hat{T}_w-\hat{T}_e))\\ \frac{1}{\rho_l(1-\bar{\gamma})A}(-\frac{\pi D_i\alpha_i \hat{l}(\hat{T}_w-\hat{T}_e)}{h_{\lg}}+\dot{m}_{in}(1-x_o))\end{pmatrix} - \begin{pmatrix}L_1\\ L_2\\ L_3\end{pmatrix}(\hat{T}_e-T_e) \quad (2)$$

The contraction theory is used to guarantee the estimated state variables in the observer will converge to the actual states in the plant. It says the system $\dot{\underline{x}} = f(\underline{x},t)$ is said to be contracting if $\partial f/\partial x$ is uniformly negative definite. All system trajectories then converge exponentially to a single trajectory, with convergence rate $|\lambda_{\max}|$, where $\lambda_{\max}$ is the largest eigenvalue of the symmetric part of $\partial f/\partial x$. Therefore, if the actual states are particular solutions of the observer and the observer dynamics is contracting, then all the trajectories of the observer will converge to the particular solutions which are the actual states.

From the proposed observer dynamics, if $\hat{T}_e$ is equal to T_e, the observer dynamics is the same with the system dynamics. Therefore, the actual states which are the solutions of this set of equations are particular solutions of the observer dynamics. If the symmetric part of the Jacobian matrix of the observer dynamics is uniformly negative definite, then the trajectory of the observer dynamics will converge to the particular solution. It means the observed states are the same with the actual states.

The Jacobian matrix for the observer system is as follows

$$\frac{\partial f}{\partial x} = \begin{pmatrix} -\frac{B_3}{kh_{\lg}}\hat{l}-L_1 & \frac{B_3}{kh_{\lg}}\hat{l} & \frac{B_3}{kh_{\lg}}(\hat{T}_w-\hat{T}_e)\\ \frac{B_3}{B_1}-L_2 & -\frac{B_2+B_3}{B_1} & 0\\ \frac{B_3}{B_4h_{\lg}}\hat{l}-L_3 & -\frac{B_3}{B_4h_{\lg}}\hat{l} & -\frac{B_3}{B_4h_{\lg}}(\hat{T}_w-\hat{T}_e)\end{pmatrix} \quad (3)$$

where

$$B_1 = (C_p\rho A)_e$$
$$B_2 = \pi D_o\alpha_o$$
$$B_3 = \pi D_i\alpha_i$$
$$B_4 = \rho_l(1-\bar{\gamma})A$$

As mentioned before, the latent heat $h_{lg}(t)$ and the heat transfer coefficients $\alpha_i(t), \alpha_o(t)$ are time-varying. According to contraction theory, if the eigenvalues of the symmetric part of the Jacobian matrix are all negative, then the system is contracting. By looking at the characteristic equation $\lambda^3 + a_1\lambda^2 + a_2\lambda + a_3 = 0$, if $a_1 > 0$ and $a_1a_2 - a_3 > 0$, then the eigenvalues are all negative. This gives us a criterion to guarantee the convergence of the nonlinear observer. The criterion is that these two conditions $a_1 > 0$ and $a_1a_2 - a_3 > 0$ have to be satisfied at any point in certain domain with given observer gains L_1, L_2 and L_3. If they are no satisfied, the observer gains have to be changed.

By choosing the observer gains L_1=5, L_2=3, L_3=1, for constant B_1, B_2, B_3 and B_4 with the parameters choosing under the normal condition, the domain of $\hat{l}$ and $\hat{T}_w - \hat{T}_e$ in which a_1 >0 and $a_1 a_2 - a_3$ >0 are satisfied is the green parts as shown in Fig.11.

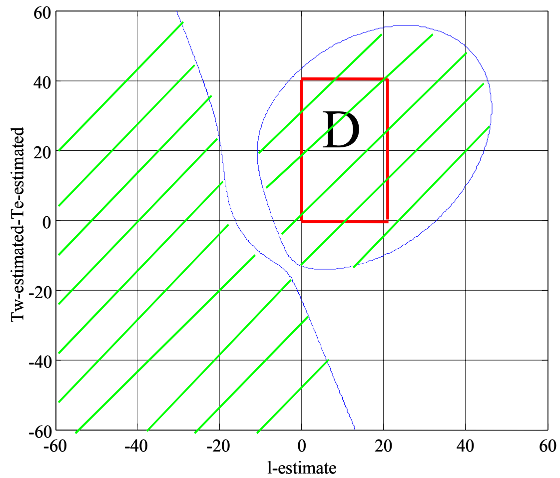

Fig. 11. Convergence domain for L_1=5, L_2=3, L_3=1

Therefore, it can be concluded that for any $x \in D$, where $D = \{0 < x_1 < L,\ 0 < x_2 < 40\}$ as shown in Fig. 11, the observer is converging. Here, $x_1 = \hat{l}$, $x_2 = \hat{T}_w - \hat{T}_e$ and L is the total length of the evaporator.

However, $h_{lg}(t)$, $B_1(t)$, $B_2(t)$, $B_3(t)$ and $B_4(t)$, are time-varying during the start-up process. The convergence domain obtained above may change due to the time-varying parameters. However, the uniformly negative definiteness of the Jacobian matrix is still a sufficient condition to the convergence of the observer as mentioned in the contraction theory. It is not feasible to find the analytical results of the contraction condition to this time-varying Jacobian matrix. Therefore, at each time step of the simulation of the observer dynamics, these two conditions a_1 >0 and $a_1 a_2 - a_3$ >0 have to be checked to guarantee the convergence of the observer for the start-up process.

Fig.12 and Fig.13 indicate that the observer gains used for the evaporator satisfy the two conditions at each time step. Therefore, the observer dynamics is contracting and the observer states are converging to the actual states.

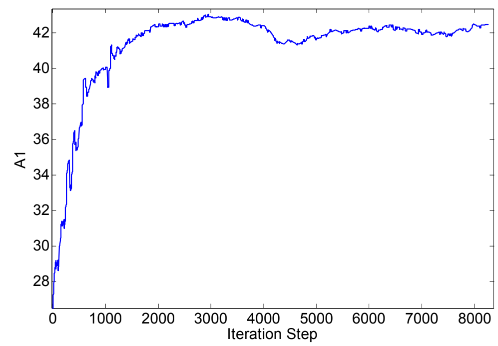

Fig. 12. A1=a_1 for the evaporator observer at each time step

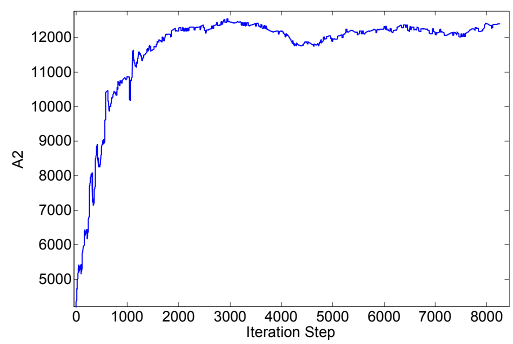

Fig. 13. A2=$a_1 a_2 - a_3$ for the evaporator observer at each time

IV. Estimation and Experiment Results Comparison

The initial condition such as the two-phase length of the heat exchangers cannot be measured. It cannot be derived from the steady state equations of the heat exchanger because the heat exchanger is not in steady state during the start-up process. Therefore, the simulator of the heat exchanger does not work because of the lack of the initial condition. Hence, the observer results are compared with the experiment results obtained from section 2.

Fig.14 to Fig.16 present the results of the evaporator observer. It can be seen in Fig.14 that the observed evaporating temperature converges to the experiment measurement very quickly. Fig.15 and Fig.16 indicate that superheated section occupies most part of the evaporator at the very beginning of the start-up process. With the development of the two-phase section, the most part of the evaporator is occupied by two-phase refrigerant. Fig.15 shows the comparison of the observed two-phase length with the estimated two-phase length from the temperature experiment data as shown in Fig.9. The development trend of the two-phase length matches very well with the experiment data.

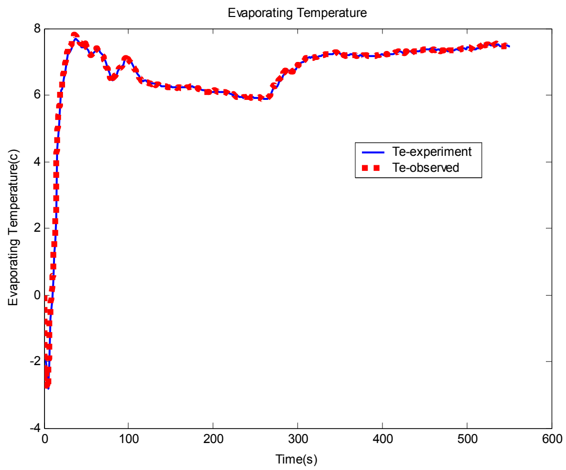

Fig. 14. Comparison of the evaporating temperature

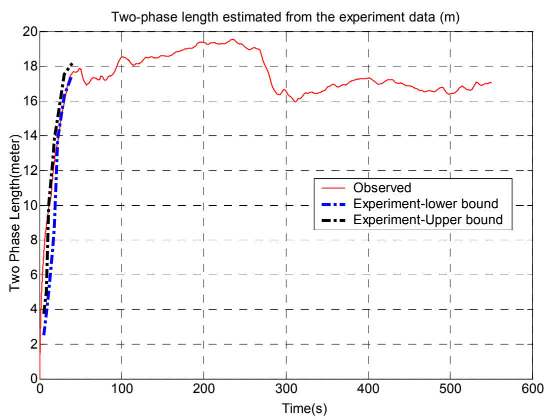

Fig. 15. Comparison of the observed two-phase length in the evaporator with experiment

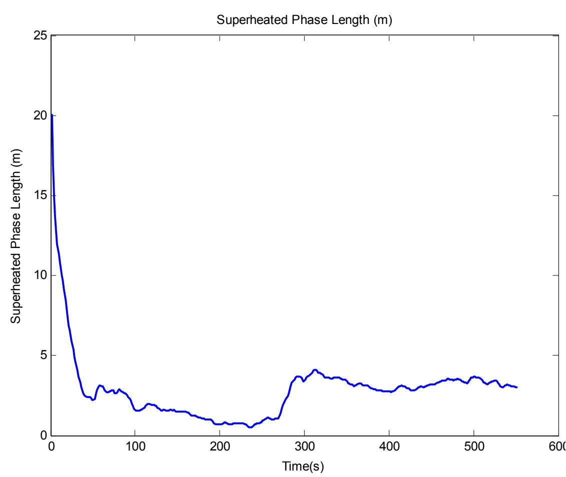

Fig. 16. Observed superheated phase length in the evaporator

V. Conclusions

The experiment data of the start-up process of the test machines is investigated. The phase conditions during the start-up process in the heat exchangers are obtained and used for the selection of heat exchanger models. The two-phase length in the evaporator is obtained in the start-up process from the experiment data for the first time. The nonlinear observer design for the heat exchangers during the start-up process is proposed. The contraction theory is used to guarantee the convergence of this non-autonomous system. The observer performance is validated by the experiment data. The results of the heat exchanger observer will be used in the investigation of the refrigerant and lubrication oil distribution for the air conditioning system.

VI. Acknowledgement

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