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Observers for a Special Class of Bilinear Systems: Design, Analysis, and Application

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Abstract

This paper presents full and reduced order observation strategies for a subclass of systems that are bilinear in the immeasurable states. Lyapunov analysis is carried out to derive the conditions for convergence of the observed states for all strategies. The full order observer is then utilized in the problem of bus voltage regulation effected by a STATCOM acting as a controlled reactive current source. Simulations results are presented to show efficacy of the observation strategy integrated with the nonlinear STATCOM controller with regard to bus voltage regulation in the presence of disturbances in load and uncertainty in system frequency.

1 Introduction

Bilinear systems have been seen to occur frequently in chemical reactor and fault diagnosis dynamics. Most of the work in bilinear systems has concentrated on bilinear systems up to output injection. In the early 1980s, [1] and [2] obtained necessary and sufficient conditions for the existence of bilinear observers for bilinear systems. In the late 1990s, a renewed interest was seen in bilinear systems. In [3], the design of a residual generator for robust fault detection in bilinear systems is considered utilizing methods based on a linear time-invariant observer up to output injection and the so-called Kalman-like observer. Martinez-Guerra [4] utilized Fliess' generalized observable canonical form and generalized controllable canonical form to derive an observer-based controller for bilinear systems. In [5], an output-feedback stabilizing controller for bilinear systems was proposed utilizing a periodic switching of the controller and the use of a dead-beat observer. In [6], a separation principle was posited for a class of dissipative systems with bilinearities.

The class of bilinear systems that we study in this paper are motivated by a control problem for designing the current reference for a shunt-connected Static Compensator (STATCOM) at a load bus in a power distribution system. The solution of the control problem in a Lyapunov framework leads us into observer design for a class of multi-output bilinear systems that are not transformable into the so-called nonlinear observable canonical form for which exponential observers exist [7]. The bilinearity that we deal with emanates from the multiplication of two immeasurable system states. However, our result does exploit the skew-symmetric structure of the system dynamics. It is important to note here that we do not attempt to solve the output feedback control problem which is common in literature; it is our belief that a modular control and estimator structure allows for flexibility in controller and estimator choices and is easier to tune and implement.

In this paper, we present three different observation strategies for a special class of bilinear systems that leads up to a form that is amenable to the application that motivated this bilinear problem definition. Specifically, we posit bilinear full and reduced order observers with feed-forward compensation in a Lyapunov based framework which facilitates proof of asymptotic stabilization of the observation errors. We then show how the distribution system problem can be rewritten into a multi-output state estimation problem (the order of which is 1 higher than the original problem) that has structure similar to our generic system. Next, the observation strategy for the distribution system is shown to be asymptotically stable in the sense that all estimation errors converge to the origin in the limit. Finally, simulation results are presented to show the performance of the observer integrated in the STATCOM control problem.

The rest of the paper is organized as follows. Section 2 describes the form of bilinear system and defines the estimation problem. Section 3 presents full and reduced order observation strategies along with detailed proofs of convergence. Section 4 presents ap-

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plication of the full order observer to the STATCOM control problem. Simulation results are presented in Section 5.

2 System Dynamics and Problem Statement

In this section, we motivate the design of various estimation strategies for a system that is motivateed by the STATCOM control problem, defined as follows

$$\begin{aligned} \dot{x}_1 &= f_1 + x_2 && (1)\\ \dot{x}_2 &= x_3 J_2 x_2 && (2)\\ \dot{x}_3 &= f_2 && (3)\\ y &= x_1 && (4) \end{aligned}$$

where $x_1(t), x_2(t) \in \Re^2$and $x_3(t) \in \Re$ denote system states, while $y(t) \in \Re^2$ denotes the measurable output vector. Additionally, $f_i \triangleq f(y(t), t)$ for $i = 1, 2$ denote measurable vectors the dimension of which is obvious from context, while J_2 is a skew-symmetric matrix defined as

$$J_2 = \begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix} \qquad (5)$$

We assume the boundedness of the system states $x_i(t)$ for $i = 1, 2, 3$ and the vectors $f_i(\cdot)$ for $i = 1, 2$. Our primary objective is to design an observation strategy for the unmeasurable state vector $x_2(t)$. In order to design the estimators, the following error definitions are set up

$$\tilde{x}_i = x_i - \hat{x}_i \qquad \text{for } i = 1, 2, 3 \qquad (6)$$

where $\hat{x}_i(t)$ denote the estimated values for the system states $x_i(t)$, while $\tilde{x}_i(t)$ denote the errors in the respective estimations. The challenge here lies in the fact that the immeasurable states $x_2(t)$ and $x_3(t)$ have a bilinear structure as can be seen in (2).

3 Observation Strategies

In this Section, we present three different observation schemes. We begin our analysis with the simplifying assumption that $x_3(t)$ is known and then extend our estimation strategy to include the immeasurable state $x_3(t)$.

3.1 Observer 1: Measurable $x_3(t)$

In this section, we design a 4^{th} order observation strategy based on the Lyapunov equality $A_0^T P + P A_0 = -Q$ and facilitated by the skew-symmetric nature of J_2. We posit the following observer system

$$\begin{aligned} \dot{\hat{x}}_1 &= f_1 + \hat{x}_2 + k_1 \tilde{x}_1 - x_3 J_2 \tilde{x}_1 && (7)\\ \dot{\hat{x}}_2 &= x_3 J_2 \hat{x}_2 + k_2 \tilde{x}_1 && (8) \end{aligned}$$

where $k_1, k_2 \in \Re$ are positive estimator gains that are chosen to ensure that the matrix $A_0 = \begin{pmatrix} -k_1 I_2 & I_2 \\ -k_2 I_2 & 0_2 \end{pmatrix} \in \Re^{4\times4}$ is Hurwitz[1]. After utilizing the system dynamics of (1), (2), and the error definitions of (6), the closed-loop error dynamics for $\tilde{x}_1(t)$ and $\tilde{x}_2(t)$ are obtained as

$$\begin{aligned} \dot{\tilde{x}}_1 &= -k_1 \tilde{x}_1 + \tilde{x}_2 + x_3 J_2 \tilde{x}_1 && (9)\\ \dot{\tilde{x}}_2 &= -k_2 \tilde{x}_1 + x_3 J_2 \tilde{x}_2 && (10) \end{aligned}$$

The closed-loop error systems of above can be written into the following compact matrix notation

$$\dot{z} = A_0 z + B(t) z \qquad (11)$$

where A_0 is a Hurwitz matrix that has been previously designed, $z(t) = \begin{bmatrix} \tilde{x}_1(t) & \tilde{x}_2(t) \end{bmatrix}^T$ denotes a composite error vector, while $B(t) = \begin{pmatrix} x_3(t) J_2 & 0_2 \\ 0_2 & x_3(t) J_2 \end{pmatrix} \in \Re^{4\times4}$ is a skew-symmetric matrix. Motivated by our subsequent stability analysis, we define a real symmetric positive definite matrix Q as follows

$$Q = k_q I_4 \qquad (12)$$

where k_q is a positive scalar and $I_4 \in \Re^{4\times4}$ is the identity matrix. Given that the matrix A_0 is Hurwitz and Q is symmetric p.d., we can find a symmetric positive definite matrix P such that

$$P = \int_0^\infty \exp\left(A_0^T t\right) Q \exp\left(A_0 t\right). \qquad (13)$$

From (13) as well as the particular choice for Q and the special structure of A_0, the matrix P is symmetric p.d. in the special form $P = \begin{pmatrix} aI_2 & -bI_2 \\ -bI_2 & cI_2 \end{pmatrix}$. This structure of P ensures that $B(t)^T P + PB(t) \equiv 0$. We now define a nonnegative function $V_1(t)$ as follows

$$V_1 = z^T P z \qquad (14)$$

Differentiation of (14) along the closed-loop system trajectories of (11) and utilizing the fact that $B(t)^T P + PB(t) \equiv 0$ and (13) leads to the following expression

$$\dot{V}_1 = -z^T Q z \qquad (15)$$

From (14) and the negative definiteness of $\dot{V}_1(t)$, it is easy to verify global asymptotic (exponential) stability for the observation error system.

[1] I_2 and 0_2 denote the 2×2 identity and zero matrices, respectively.

3.2 Observer 2: Measurable $x_3(t)$

In this section, we show how it is possible to reduce the 4^{th} order observation strategy of 3.1 to a 2^{nd} order system via the use of a feed-forward component. We posit the following implementable form of the observer

$$\hat{x}_2 = p_1 + k_3 y \tag{16}$$
$$\dot{p}_1 = -k_3 f_1 - k_3 \hat{x}_2 + x_3 J_2 \hat{x}_2 \tag{17}$$

where $k_3 \in \Re$ is a constant positive estimator gain and $p_1(t) \in \Re$ is an auxiliary measurable signal. After differentiating (16) and utilizing (1) and (17), we obtain the following expression

$$\dot{\hat{x}}_2 = k_3 \tilde{x}_2 + x_3 J_2 \hat{x}_2 \tag{18}$$

The closed loop dynamics for the observation error $\tilde{x}_2(t)$ are obtained as follows

$$\dot{\tilde{x}}_2 = -k_3 \tilde{x}_2 + x_3 J_2 \tilde{x}_2 \tag{19}$$

To prove that the observation errors converge to zero, a positive definite Lyapunov function $V_2(t) \triangleq \|\tilde{x}_2\|^2$ is differentiated along the trajectories of (19) in order to obtain $\dot{V}_2 = -2k_3 \|\tilde{x}_2\|^2 < 0$. Again, it is easy to obtain global asymptotic (exponential) stability for the observation error system.

Remark 1 *It is to be noted here that the gain k_3 is chosen to be a scalar for ease of exposition. One could easily replace k_3 with a Hurwitz matrix; the proof of stability then follows along the lines of Section 3.1.*

3.3 Observer 3: Immeasurable $x_3(t)$

The estimators of Sections 3.1 and 3.2 assume knowledge of the state $x_3(t)$. For the observer of (16) and (17), a high gain k_3 with a best guess estimate for $x_3(t)$ drives the observation errors $\tilde{x}_2(t)$ into a residual set about the origin the size of which can be attenuated by making k_3 larger. However, a large gain makes the observation strategy very impractical in the presence of any measurement noise in the output $y(t)$. In this Section, we extend our estimation strategy to develop a 5^{th} order observer in order to account for the unmeasurable state $x_3(t)$. The following estimation strategy is proposed

$$\dot{\hat{x}}_1 = f_1 + \hat{x}_2 + k_4 \tilde{x}_1 \tag{20}$$
$$\dot{\hat{x}}_2 = \hat{x}_3 J_2 \hat{x}_2 + \tilde{x}_1 \tag{21}$$
$$\hat{x}_3 = k_5 \left(\tilde{x}_1^T J_2 \hat{x}_2 + p_2 \right) \tag{22}$$
$$\dot{p}_2 = k_4 \tilde{x}_1^T J_2 \hat{x}_2 + \hat{x}_3 \tilde{x}_1^T \hat{x}_2 + k_5^{-1} f_2 \tag{23}$$

where $k_4, k_5 \in \Re$ are positive observer gains and $p_2(t) \in \Re$ is an auxiliary measurable signal. The closed loop error systems for $\tilde{x}_1(t)$ and $\tilde{x}_2(t)$ are obtained as

$$\dot{\tilde{x}}_1 = \tilde{x}_2 - k_4 \tilde{x}_1 \tag{24}$$
$$\dot{\tilde{x}}_2 = x_3 J_2 \tilde{x}_2 + \tilde{x}_3 J_2 \hat{x}_2 - \tilde{x}_1 \tag{25}$$

where we have utilized (1), (2), (20), (21), and the error definitions of (6). To obtain the closed loop error dynamics for $\tilde{x}_3(t)$, we take the time derivative of (22) and then utilize (21), (23), and (24) to obtain the following expression

$$\dot{\hat{x}}_3 = k_5 \tilde{x}_2^T J_2 \hat{x}_2 + f_2 \tag{26}$$

where the following identities have been utilized: $J_2^T J_2 = I_2$ and $J_2^2 = -I_2$. Given the above expression, the state equation (3) and the error definition of (6), the closed loop dynamics for $\tilde{x}_3(t)$ can be obtained as

$$\dot{\tilde{x}}_3 = -k_5 \tilde{x}_2^T J_2 \hat{x}_2 \tag{27}$$

Towards proving stability, we state and prove the following Theorem.

Theorem 1 *The closed-loop systems of (24) and (25) are globally asymptotically stable in the sense that $\lim_{t\to\infty} \tilde{x}_1(t), \tilde{x}_2(t) = 0$. Additionally, under the assumption that $\lim_{t\to\infty} x_2(t) \neq 0$, it can be shown that $\lim_{t\to\infty} \tilde{x}_3(t) = 0$.*

Proof: We define a nonnegative function $V_3(t) \in \Re$ as follows

$$V_3 = \frac{1}{2}\tilde{x}_1^T \tilde{x}_1 + \frac{1}{2}\tilde{x}_2^T \tilde{x}_2 + \frac{1}{2} k_5^{-1} \tilde{x}_3^2 \tag{28}$$

After differentiating (28) along the closed-loop expressions of (24), (25), and (27), we obtain

$$\begin{aligned} \dot{V}_3 &= \tilde{x}_1^T (\tilde{x}_2 - k_4 \tilde{x}_1) + \tilde{x}_2^T (x_3 J_2 \tilde{x}_2 - \tilde{x}_3 J_2 \hat{x}_2 - \tilde{x}_1) \\ &\quad - \tilde{x}_3 \tilde{x}_2^T J_2 \hat{x}_2 \\ &= -k_4 \tilde{x}_1^T \tilde{x}_1 \end{aligned} \tag{29}$$

From (28) and (29), it is easy to see that $\tilde{x}_1(t) \in \mathcal{L}_2 \cap \mathcal{L}_\infty$ while $\tilde{x}_2(t), \tilde{x}_3(t) \in \mathcal{L}_\infty$. Since $x_i(t) \in \mathcal{L}_\infty$, we can assert that $\hat{x}_i(t) \in \mathcal{L}_\infty \ \forall\, i = 1, 2, 3$. From the previous assertions as well as (24), (25), and (27), it can be seen that $\dot{\tilde{x}}_i(t) \in \mathcal{L}_\infty \ \forall\, i = 1, 2, 3$. We can now use Barbalat's Lemma [8] to prove that $\lim_{t\to\infty} \tilde{x}_1(t) = 0$. After taking the derivative of (24) and utilizing (24) and (25), we obtain the following expression

$$\ddot{\tilde{x}}_1 = x_3 J_2 \tilde{x}_2 + \tilde{x}_3 J_2 \hat{x}_2 - \tilde{x}_1 - k_4 \tilde{x}_2 + k_4^2 \tilde{x}_1 \tag{30}$$

It is easy to see that all the signals on the right hand side of (30) are bounded which implies that

$\ddot{\tilde{x}}_1\ (t) \in \mathcal{L}_\infty$; hence, $\dot{\tilde{x}}_1\ (t)$ is uniformly continuous. Given the fact that $\tilde{x}_1(t) \in \mathcal{L}_\infty$, it is possible to make the following assertion

$$\lim_{t\to\infty} \int_0^t \dot{\tilde{x}}_1\ (\tau) d\tau = \tilde{x}_1(t) - \tilde{x}_1(0) < \infty$$

From the integral form of Barbalat's Lemma [8], we can now state that $\lim_{t\to\infty} \dot{\tilde{x}}_1\ (t) = 0$. This implies from (24) that $\lim_{t\to\infty} \tilde{x}_2(t) = 0$. By differentiating (25) and utilizing previous boundedness assertions for all terms on the right hand side of the resulting expression, it is easy to see that $\ddot{\tilde{x}}_2\ (t) \in \mathcal{L}_\infty$ which implies that $\dot{\tilde{x}}_2\ (t)$ is uniformly continuous. Again, we can assert from the boundedness of $\tilde{x}_2\ (t)$ that $\lim_{t\to\infty} \int_0^t \dot{\tilde{x}}_2\ (\tau) d\tau < \infty$. Integral form of Barbalat's Lemma then implies that $\lim_{t\to\infty} \dot{\tilde{x}}_2\ (t) = 0$. An inspection of (25) and an application of the error definition of (6) now reveals that $\lim_{t\to\infty} \tilde{x}_3 J_2 x_2 = 0$ which implies that $\lim_{t\to\infty} \tilde{x}_3\ (t) = 0$ if it can be shown that $x_2\ (t)$ does not go to zero in the limit. Hence, we have proved 1. □

4 STATCOM Control

In this Section, we begin by describing the system model for a shunt-connected STATCOM on a distribution system which in turn motivates the state estimation problem. Next, we describe how we can rewrite the system dynamics for our application into a form that has the structure of (1)-(4). Finally, we succinctly describe how one can modify the control strategy presented in [9] in order to couple the estimation and control strategies.

4.1 System Model and Estimation Problem

We begin by describing the system dynamics on a distribution system assuming that the shunt-connected STATCOM acts as a controlled reactive current source [9, 10]. Our simplified model for a load supplied on a power distribution system consists of: (a) the source modeled as an infinite bus ($v_{s,abc}$), (b) the distribution line represented by an inductive impedance (R_s, L_s), (c) the load modeled by a resistance[2] (R_L), (d) a STATCOM in parallel with the load modeled as an ideal current source, and (e) a coupling capacitor[3] (C_c). It is assumed that the source, load, and the STATCOM are balanced three-phase systems. One phase of the model is shown in Figure 1 . Under the assumption that zero sequence components are not present, the system dynamics in the familiar two phase $x-y$ system are given by following differential equations [10]

$$\begin{aligned} L_s \dot{i}_{s,xy} &= -R_s i_{s,xy} - v_{L,xy} + v_{s,xy} && (31) \\ C_c \dot{v}_{L,xy} &= -g_L v_{L,xy} + i_{s,xy} + i_{SC,xy} && (32) \end{aligned}$$

where the complex number notation $\lambda_{xy} \triangleq \lambda_x + j\lambda_y$, and $g_L = 1/R_L$ is the load conductance. After invoking the global invertible rotational transform

$$\lambda_{dq} = \lambda_d + j\lambda_q \triangleq \exp(-j\theta)\, \lambda_{xy}$$

we can obtain the equivalent circuits corresponding to the real (d-axis) and imaginary (q-axis) components of this equation. Now, choosing $\theta = \arctan(v_{Ly}/v_{Lx})$, the system of (31)-(32) can be represented in the following equivalent two phase $d-q$ system [10]

$$\begin{aligned} C_c \dot{v}_{Ld} &= -g_L v_{Ld} + i_{sd} + i_{SCd} && (33) \\ L_s \dot{i}_{sd} &= -v_{Ld} - R_s i_{sd} + \omega L_s i_{sq} + V_s \cos\alpha && (34) \\ L_s \dot{i}_{sq} &= -R_s i_{sq} - \omega L_s i_{sd} - V_s \sin\alpha && (35) \\ \dot{\alpha} &= \omega - \omega_s && (36) \\ \omega &= \frac{i_{sq} + i_{SCq}}{C_c v_{Ld}} && (37) \end{aligned}$$

where $v_{Ld}(t), i_{sd}(t), i_{sq}(t)$, and $\alpha(t) \in \Re$ are the states of the distribution-side system, $\omega(t) \triangleq d\theta(t)/dt, V_s$ denotes the constant magnitude of the infinite bus voltage, while ω_s represents the constant frequency of the infinite bus voltage. Since $v_{Lq}(t) \equiv 0$ by the above choice for $\theta(t)$, $v_{Ld}(t)$ represents the instantaneous magnitude of the load phase voltages, while $i_{SCq}(t) \in \Re$ denotes the reactive current supplied by the STATCOM and is considered to be the control input to the system. If parasitic losses are ignored, a STATCOM only supplies reactive power so $i_{SCd} \equiv 0$ is a readily justifiable simplifying assumption in (33). We remark here that $v_{Ld}(t), i_{sd}(t)$, and

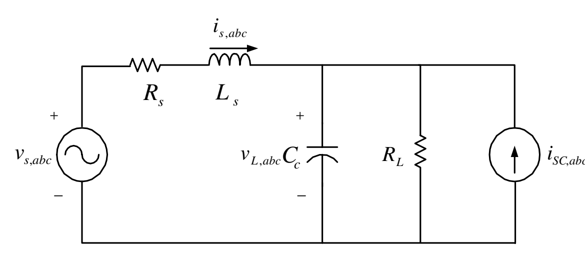

Figure 1: One Phase of the Distribution System

$i_{sq}(t)$ are measurable voltage and current signals at the load bus but the state $\alpha(t)$ (which is the angle between the load and the source voltage vectors) is unavailable for measurement; however, the Lyapunov

[2] As described in [9], the assumption of a purely resistive load allows for simplicity of analysis and does not involve any loss of generality.

[3] See [?] for details on inclusion of C_c.

based large signal control strategy designed in [9] relies on measurements of $\alpha(t)$. Even if $\alpha(t_0)$ is known, it is practically impossible to reliably estimate $\alpha(t)$ from (36) because the system frequency ω_s exhibits small variations about its nominal 60 [Hz] value. An inspection of (36) reveals that even small offsets in the system frequency can lead to accumulative error in $\alpha(t)$. This motivates us to design a robust observation strategy for the unmeasurable state $\alpha(t)$ given measurements of $v_{Ld}(t), i_{sd}(t)$, and $i_{sq}(t)$ while accounting for uncertainty in and estimating the system frequency ω_s.

4.2 Application of the Observation Strategy

In this section, we will demonstrate how we can modify and extend the observer design of Section 3.3 for the distribution system dynamics of (33)-(36). To elucidate further, the following definitions are set up

$$
\begin{aligned}
\beta &= \begin{bmatrix} -V_s \sin\alpha & V_s \cos\alpha \end{bmatrix}^T \\
v_L &= \begin{bmatrix} v_{Ld} & 0 \end{bmatrix}^T \\
i_s &= \begin{bmatrix} i_{sq} & i_{sd} \end{bmatrix}^T, \ \Lambda = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}
\end{aligned}
\tag{38}
$$

where the vector dimensions are obvious from context. Given the definitions above, the system dynamics of (33)-(36) can be represented as

$$
\begin{aligned}
C_c \dot{v}_L &= -g_L v_L + \Lambda i_s & (39)\\
L_s \dot{i}_s &= -J_2 v_L - (R_s I_2 - \omega L_s J_2)\, i_s + \beta & (40)\\
\dot{\beta} &= (\omega - \omega_s) J_2 \beta & (41)\\
\dot{\omega}_s &= 0 & (42)
\end{aligned}
$$

The signals $v_L(t), i_s(t) \in \Re^2$ are clearly measurable, $\beta(t) \in \Re^2$ is an unmeasurable state, the system frequency ω_s is an unknown parameter, while $\omega(t) \in \Re^1$ is an auxiliary control input signal (previously defined in (37) and clearly related to the control signal $i_{SCq}(t)$ through a static transformation). Motivated by our desire to obviate the need for a direct cancellation of the $J_2 v_L(t)$ term in (40), we have pre-appended the system of (40)-(42) with the dynamics of $v_L(t)$. Additionally, since $\omega(t)$ is known and ω_s is an unknown constant, our generic estimation problem of Section 3.3 breaks down into a *state estimation* problem for $\beta(t)$ and a *parameter identification* problem for ω_s. In order to motivate the design of the observer/identifier, we set up the following error definitions

$$
\begin{array}{ll}
\tilde{v}_L = v_L - \hat{v}_L & \tilde{i}_s = i_s - \hat{i}_s \\
\tilde{\beta} = \beta - \hat{\beta} & \tilde{\omega}_s = \omega_s - \hat{\omega}_s
\end{array}
\tag{43}
$$

where $\hat{v}_L(t), \hat{i}_s(t), \hat{\beta}(t) \in \Re^2, \hat{\omega}_s(t) \in \Re^1$ denote the yet to be designed estimates of the load voltage, distribution current, source voltage vector $\beta(t)$, and system frequency, respectively, while $\tilde{v}_L(t)$, $\tilde{i}_s(t), \tilde{\beta}(t) \in \Re^2, \tilde{\omega}_s(t) \in \Re^1$ denote the corresponding estimation errors. Motivated by the structure of the systems dynamics of (40)-(42) and the estimation strategy of Section 3.3, we propose the following observer/identifier scheme

$$
\begin{aligned}
C_c \dot{\hat{v}}_L &= -g_L \hat{v}_L + \Lambda \hat{i}_s + k_v \tilde{v}_L & (44)\\
L_s \dot{\hat{i}}_s &= -J_2 \hat{v}_L - (R_s I_2 - \omega L_s J_2)\, \hat{i}_s + \hat{\beta} \\
&\quad + k_i \tilde{i}_s & (45)\\
\dot{\hat{\beta}} &= (\omega - \hat{\omega}_s) J_2 \hat{\beta} + \tilde{i}_s & (46)\\
\hat{\omega}_s &= -k_\omega \left(\tilde{i}_s^T J_2 \hat{\beta} + p \right) & (47)\\
\dot{p} &= \left(L_s^{-1} (R_s + k_i)\, \tilde{i}_s^T J_2 - \hat{\omega}_s \tilde{i}_s^T \right. \\
&\quad \left. + L_s^{-1} \tilde{v}_L^T \right) \hat{\beta} & (48)
\end{aligned}
$$

where $k_v, k_i, k_\omega \in \Re$ are constant, positive observer gains and $p(t) \in \Re$ is an auxiliary measurable signal. Given the error definitions of (43), the system dynamics of (39)-(41), and the observation strategy of (44)-(46), the closed-loop observation dynamics are obtained as

$$
\begin{aligned}
C_c \dot{\tilde{v}}_L &= -(g_L + k_v)\, \tilde{v}_L + \Lambda \tilde{i}_s & (49)\\
L_s \dot{\tilde{i}}_s &= -J_2 \tilde{v}_L - ((R_s + k_i)\, I_2 - \omega L_s J_2)\, \tilde{i}_s \\
&\quad + \tilde{\beta} & (50)\\
\dot{\tilde{\beta}} &= (\omega - \omega_s) J_2 \tilde{\beta} - \tilde{\omega}_s J_2 \hat{\beta} - \tilde{i}_s & (51)
\end{aligned}
$$

For obtaining the closed loop dynamics of $\tilde{\omega}_s(t)$, we differentiate (47) along (46), (48), and (50) to yield

$$
\dot{\tilde{\omega}}_s = k_\omega L_s^{-1} \tilde{\beta}^T J_2 \hat{\beta} \tag{52}
$$

where we have utilized (42). To prove that all the errors converge asymptotically to zero, we define the following non-negative function

$$
V = \frac{1}{2} C_c \tilde{v}_L^T \tilde{v}_L + \frac{1}{2} L_s \tilde{i}_s^T \tilde{i}_s + \frac{1}{2L_s} \tilde{\beta}^T \tilde{\beta} + \frac{1}{2} k_\omega^{-1} \tilde{\omega}_s^2 \tag{53}
$$

Differentiating (53) along the dynamics of (49)-(51) and (52), we obtain the following expression

$$
\dot{V} = -(g_L + k_v C_c)\, \tilde{v}_L^T \tilde{v}_L - (R_s + k_i L_s)\, \tilde{i}_s^T \tilde{i}_s \tag{54}
$$

Following arguments similar to those in the proof of Theorem 1, it is easy to see that: (a) all signals in the observation strategy are bounded for all time, and (b) $\lim_{t\to\infty} \tilde{v}_L(t), \tilde{i}_s(t), \tilde{\beta}(t) = 0$. By the first definition of (38), it is easy to see that $\lim_{t\to\infty} \beta(t) \neq 0$. Again, an application of Theorem 1 implies that $\lim_{t\to\infty} \tilde{\omega}_s(t) = 0$. □

Remark 2 *Given $\hat{\beta}(t)$, an estimate for $\alpha(t)$ for practical implementation in the control scheme can be obtained readily as follows*

$$
\hat{\alpha}(t) = atan2\left(-\hat{\beta}_1(t), \hat{\beta}_2(t) \right) \tag{55}
$$

4.3 Integrated Controller Structure

To achieve the regulation of the phase voltage $v_{Ld}(t)$ to a setpoint, a nonlinear control strategy based upon a novel nonlinear coordinate transformation in conjunction with a gradient based load conductance identifier was presented in [9]. As stated earlier this control strategy relied on an unmeasurable state $\alpha(t)$. Motivated by the desire to utilize the estimate of the angle between the load and the source vectors $\hat{\alpha}(t)$ (generated via (55)), this control input is redesigned as

$$i_{SCq} = -i_{sq} + C_c v_{Ld}\left(\hat{\Omega}_s + \frac{\partial \alpha^*}{\partial \hat{g}_L}\,\dot{\hat{g}}_L + \bar{u}_a\right) \quad (56)$$

where $\hat{\Omega}_s(t) \in \Re$ denotes a yet to be designed dynamic estimator for the system frequency, while the remainder of the variables are generated as follows:

$$\dot{\hat{g}}_L = -\frac{k_g v_{Ld}^* \zeta}{f} \quad (57)$$

$$\bar{u}_a = -\frac{k_\alpha \tilde{\alpha}}{\left[\lambda - L_s i_{sd}^* \frac{\bar{\eta}}{f} - L_s i_{sq}^* \frac{\bar{\gamma}}{f}\right]} \quad (58)$$

$$\begin{array}{ll} \zeta = v_{Ld} - v_{Ld}^* & \tilde{\alpha} = \hat{\alpha} - \alpha^* \\ \bar{\eta} = \eta - i_{sq}^* \tilde{\alpha} & \bar{\gamma} = \gamma + i_{sd}^* \tilde{\alpha} \end{array} \quad (59)$$

where $k_\alpha, k_g \in \Re$ are constant positive gains, $i_{sd}^*\left(v_{Ld}^*, \hat{g}_L(t)\right)$, $i_{sq}^*\left(v_{Ld}^*, \hat{g}_L(t)\right)$, and $\alpha^*\left(v_{Ld}^*, \hat{g}_L(t)\right) \in \Re$ are tracking trajectories for the signals $i_{sd}(t), i_{sq}(t)$, and $\alpha(t)$ and denote quasi-steady solutions for the system of (33)-(37), $\lambda \in \Re$ is a constant positive design parameter defined as $\lambda > \sup_t \left|L_s\left(i_{sd}^* + i_{sq}^*\right)\right|$, while $f(t) \in \Re$ is a positive function defined as $f = 1 + \frac{1}{2}C_c\zeta^2 + \frac{1}{2}L_s\bar{\eta}^2 + \frac{1}{2}L_s\bar{\gamma}^2$.

In order for our controller to act satisfactorily in the event of small variations in the system frequency $\omega_s(t)$, a dynamic estimator for the system frequency is designed as

$$\dot{\hat{\Omega}}_s = -k_\Omega \tilde{\alpha} \quad (60)$$

where $k_\Omega \in \Re$ is a constant positive estimation gain. To aid the stability analysis of the redesigned control, we define $\tilde{g}_L = g_L - \hat{g}_L$ and $\tilde{\Omega}_s = \omega_s - \hat{\Omega}_s$. In [9], the stability analysis was carried out by the use of a novel Lyapunov function; we present a slight modification of that Lyapunov function in order to prove the stability of the redesigned control. The new function is defined as

$$V_p = \ln f + \frac{1}{2}\lambda\tilde{\alpha}^2 + \frac{1}{2}k_g^{-1}\tilde{g}_L^2 + \frac{1}{2}k_\Omega^{-1}\tilde{\Omega}_s^2 \quad (61)$$

Differentiating (61) results in the following expression

$$\dot{V}_p \leq -\delta_6\zeta^2 - \delta_7\bar{\eta}^2 - \delta_8\bar{\gamma}^2 - \bar{k}_4\tilde{\alpha}^2 \quad (62)$$

where $\delta_6, \delta_7, \delta_8$ and $\bar{k}_4 \in \Re$ are positive constants of analysis the complete definitions of which can be found in [9]. From (61) and (62), we can utilize the arguments presented in [9] to prove asymptotic stability of the closed-loop system in the sense that $\lim_{t\to\infty} \zeta(t), \bar{\eta}(t), \bar{\gamma}(t), \tilde{\alpha}(t), \tilde{g}_L(t) = 0$. From assertions following (54), it can be proved that $\lim_{t\to\infty} \hat{\alpha}(t) = \alpha(t)$; similarly, from assertions following (62), it can be shown that $\lim_{t\to\infty} \hat{\alpha}(t) = \alpha^*(t)$. It then immediately follows that $\lim_{t\to\infty} \alpha(t) = \alpha^*(t)$.

5 Simulation Results

Available upon request.

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