Multi-Layer Switching Control

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Abstract— **In this paper, adaptive control of systems using switching techniques is investigated. It is assumed that the plant model belongs to a known finite set of models. It is also assumed that a set of controllers which solve the robust servomechanism problem for the family of plant models and a set of simultaneous stabilizers for certain subsets of plant models are given. It is shown that by using the above set of controllers and simultaneous stabilizers and choosing a proper switching sequence, one can minimize the number of switchings to destabilizing controllers. This can significantly improve the transient response of the system, which is one of the common weak points in most switching control schemes. Simulation results show the effectiveness of the proposed method in improving the transient response.**

Keywords: Switching control, Adaptive control, Family of plants, Simultaneous stabilizer, Transient response.

I. INTRODUCTION

The control of a partially known plant has received considerable attention in the adaptive control literature. One of the relatively new lines of research in this area is switching control which was motivated to weaken the classical *a priori* information required in conventional adaptive control and can be traced back to [1]. During the past several years, switching control schemes have been developed to accomplish a wide variety of tasks which would not have been possible using traditional adaptive control methods [2], [3], [4], [5], [6], [7], [8], [9], [10].

Switching control of systems using family of plants was first introduced by Miller and Davison [4]. In this approach, it is assumed that a high performance controller is designed for each plant in the family. Then, by using a proper switching mechanism which monitors the norm of error in the output, the system locks onto a stabilizing controller after a number of switchings. One of the advantages of switching control compared to the conventional adaptive control is its effectiveness for highly uncertain systems. However, the main disadvantage of switching control is bad transient response in general. Several methods have been proposed to reduce the magnitude of the transient response [11], [12]. One of the main reasons for undesirable transient response in switching control systems is that in the transition from the initial controller to the final one, the system may switch to several destabilizing controllers.

In this paper, a method is proposed to improve the transient response of the switching control system by reducing the number of switchings to destabilizing controllers. The proposed method utilizes different layers of controllers with different properties. This is an extension of the switching method introduced in [4], which assumes that the set of plants models $\{\mathbf{P}_i : i = 1, 2, \ldots, p\}$ is given, and upper-bounds on the disturbance and reference input magnitudes are available. It is also assumed that the plant is controllable and observable. Switching in the system occurs when the norm of the error signal becomes greater than or equal to the corresponding upper-bound signal. In our proposed multi-layer scheme, $p-2$ layers of controllers are designed, where p denotes the number of models in the family of plants. Layer $k \in \{2, \ldots, p-2\}$ consists of a set of controllers which have the property that each one stabilizes k plants in the family and destabilizes the remaining $p-k$ plants. Layer 1 consists of a set of p controllers, where each one solves the robust servomechanism problem for one of the models in the family. The main difference between the previous switching methods and the proposed one is the additional controllers which represent layers $2, \ldots, p-2$ and are used to improve the transient response. Throughout this paper, the previous switching control methods will be referred to as single-layer switching.

This paper is organized as follows. The problem formulation is given in section II. A multi-layer switching system is proposed in section III and in section IV the method is compared to a single-layer counterpart.

II. PROBLEM FORMULATION

It is assumed that the current plant $\mathbf{P}(t)$ belongs to a known finite set of plant models given by

$$\forall t : \mathbf{P}(t) \in \Pi := \{\mathbf{P}_i : i = 1, 2, \ldots p\} \tag{1}$$

It is also assumed that each plant model in the above set is described by the following state-space equations

$$\dot{x} = A_i x + B_i u + E_i w \tag{2a}$$
$$y = C_i x + F_i w \tag{2b}$$

where $x(t) \in R^{n_i}$, $i \in \bar{\text{p}} = \{1, 2, \ldots, p\}$ is the state, $u(t) \in R^m$ is the control input, $y(t) \in R^r$ is the output, $y_{ref}(t) \in R^r$ is the reference input and $w(t) \in R^v$ is the disturbance signal. Reference input and disturbance are bounded piecewise continuous functions.

As in the previous works, it is assumed that for each $i \in \bar{\text{p}}$ there exists a high performance controller $\mathbf{K}_i$ of the

This work has been supported by the Natural Sciences and Engineering Research Council of Canada under grant RGPIN-262127-03.

form

$$\dot{z} = G_i z + H_i u + J_i y_{ref} \tag{3a}$$
$$u = K_i z + L_i y + M_i y_{ref} \tag{3b}$$

This set represents the first layer of controllers in our proposed multi-layer architecture and is denoted by Φ_1.

$$\Phi_1 = \{\mathbf{K}_i : i \in \bar{\mathrm{p}}\}, \quad N(\Phi_1) = N(p) \tag{4}$$

On the other hand, the set of controllers of layer k, $k = 2, \ldots, p$ is denoted by Φ_k, as follows

$$\Phi_k = \{\mathbf{K}_{i_1 i_2 \ldots i_k} | i_1, i_2, \ldots, i_k \in \bar{\mathrm{p}}\} \tag{5}$$

where $i_j, j = 1, \ldots, k$ are distinct integers and the indices of each controller represent the plants that can be stabilized by that controller, e.g. $\mathbf{K}_{i_1 i_2 \ldots i_k}$ "only" stabilizes plant models $\mathbf{P}_{i_1}, \mathbf{P}_{i_2}, \ldots, \mathbf{P}_{i_k}$, and destabilizes the other plants in the set Π.

According to the above definition, in each layer $k \in \bar{\mathrm{p}}$, there exist $N(\Phi_k) \times k$ combinations of stable closed-loop configurations. The total number of all stable configurations corresponding to all controller layers is given by

$$\phi = \sum_{k=1}^{p-2} N(\Phi_k) \times k$$

The closed-loop control law corresponding to the controller $\mathbf{K}_{i_1 i_2 \ldots i_k}$ can be written in the following form [4]

$$\tilde{u} = \tilde{K}_{i_1 i_2 \ldots i_k} \tilde{y} \tag{6}$$

which results in a stable system corresponding to the controllable and observable plant $\mathbf{P}_j : j \in \{i_1, i_2, \ldots i_k\}$ as follows

$$\dot{\tilde{x}} = \tilde{A}_j \tilde{x} + \tilde{B}_j \tilde{u} + \tilde{E}_j w \tag{7a}$$
$$\tilde{y} = \tilde{C}_j \tilde{x} + \tilde{D}_j y_{ref} + \tilde{F}_j w \tag{7b}$$

where

$$\tilde{x} = \begin{bmatrix} x \\ z \end{bmatrix}, \quad \tilde{u} = \begin{bmatrix} u \\ \dot{z} \end{bmatrix}, \quad \tilde{y} = \begin{bmatrix} y \\ z \\ y_{ref} \end{bmatrix} \tag{8}$$

and

$$\tilde{A}_j = \begin{bmatrix} A_j & 0 \\ 0 & 0 \end{bmatrix}, \tilde{B}_j = \begin{bmatrix} B_j & 0 \\ 0 & I \end{bmatrix}, \tilde{C}_j = \begin{bmatrix} C_j & 0 \\ 0 & I \\ 0 & 0 \end{bmatrix},$$

$$\tilde{D}_j = \begin{bmatrix} 0 \\ 0 \\ I \end{bmatrix}, \tilde{E}_j = \begin{bmatrix} E_j \\ 0 \end{bmatrix}, \tilde{F}_j = \begin{bmatrix} F_j \\ 0 \\ 0 \end{bmatrix},$$

$$\tilde{K}_{i_1 i_2 \ldots i_k} = \begin{bmatrix} L_{i_1 i_2 \ldots i_k} & K_{i_1 i_2 \ldots i_k} & M_{i_1 i_2 \ldots i_k} \\ H_{i_1 i_2 \ldots i_k} & G_{i_1 i_2 \ldots i_k} & J_{i_1 i_2 \ldots i_k} \end{bmatrix}$$

According to the above formulation, there are different layers of controllers which are used in the multi-layer switching method.

Definition 1: Throughout this paper, a switching to a destabilizing controller will be called an unstable switching.

It is desired now to find a switching path which consists of at most one unstable switching in general, between the controllers of different layers.

Figure 1 shows a family of 6 plant models and the architecture of controller layers, where the plant models are represented by black circles and controllers of layer 1, 2, 3 and 4 are represented by triangles, squares, pentagons and hexagons, respectively.

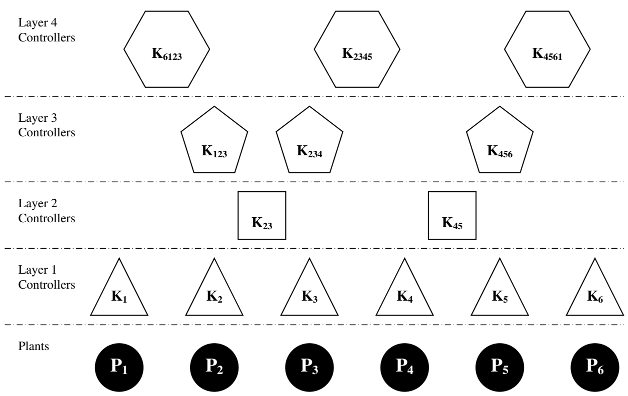

Fig. 1. Four layers of controllers for six plant models.

III. Main Result

One of the shortcomings of switching control methods is the bad transient response which is mainly contributed by switching to destabilizing controllers before the system locks onto the correct controller. When the plant dynamics change from one model to another one, the system may switch to a series of destabilizing controllers until it finds the correct controller. Addition of new layers of controllers can potentially reduce the number of "unstable switchings" as will be shown later.

Definition 2: Any controller whose indices include all but one of the indices of another controller is called a "parent" of that controller. For instance, $\mathbf{K}_{i_1 i_2 \ldots i_{k-1}}$ is a parent of $\mathbf{K}_{i_1 i_2 \ldots i_k}$. On the other hand, any controller in a layer other than layer 1 is called a "child" to its parent in the lower layer.

Definition 3: A "child-parent switching route" is a switching path from a controller in one of the higher layers to a controller in the first layer that consists of only child-parent controllers.

A. Multi-layer Switching Algorithm

Assume that the plant is stabilized by controller $\mathbf{K}_{i_1}$ in the first layer. Once a change in the model occurs, the new plant model is known to be one of the remaining $p-1$ models $\bar{\mathrm{p}} - \{i_1\} = \{i_2, i_3, \ldots, i_p\}$, which results in instability of the closed-loop system. The following assumption is made for the development of the proposed algorithm.

Assumption 1:
i) There exists a controller in layer $p-2$ which destabilizes two of plant models and stabilizes all other models.

ii) Each of the plant models is destabilized by at least one controller in layer $p-2$.
iii) There exists a child-parent route from any of the controllers in layer $p-2$ to at least one controller in the first layer.

Algorithm 1

1) Switch to a controller in layer $p-2$ which can stabilize all of the models except $\mathbf{P}_{i_1}$ and any one of the other models in the set. Let this controller be denoted by $\mathbf{K}_{i_3 i_4 \ldots i_p}$. If the closed-loop system becomes unstable, the actual plant model is identified to be $\mathbf{P}_{i_2}$. Switch to $\mathbf{K}_{i_2}$ and stop. Otherwise, go to the next step.
2) At this point, it is known that the actual plant model belongs to the set $\{\mathbf{P}_i | i = i_3, i_4, \ldots, i_p\}$. Switch to one of the parent controllers of $\mathbf{K}_{i_3 i_4 \ldots i_p}$ in layer $p-3$. Let this controller be denoted by $\mathbf{K}_{i_4 i_5 \ldots i_p}$. If the closed-loop system becomes unstable, the actual plant model is identified to be $\mathbf{P}_{i_3}$. Switch to $\mathbf{K}_{i_3}$ and stop. Otherwise, go to the next step.
3) At this point, it is known that the actual plant model belongs to the set $\{\mathbf{P}_i | i = i_4, i_5, \ldots, i_p\}$. Switch to one of the parent controllers of $\mathbf{K}_{i_4 i_5 \ldots i_p}$ in layer $p-4$. Let this controller be denoted by $\mathbf{K}_{i_5 i_6 \ldots i_p}$. If the closed-loop system becomes unstable, the actual plant model is identified to be $\mathbf{P}_{i_4}$. Switch to $\mathbf{K}_{i_4}$ and stop. Otherwise, go to the next step.
$\vdots$
p-2) At this point, it is known that the actual plant model belongs to the set $\{\mathbf{P}_i | i = i_{p-1}, i_p\}$. Switch to one of the two controllers $\mathbf{K}_{i_{p-1}}$ or $\mathbf{K}_{i_p}$. Assume that we switch to $\mathbf{K}_{i_{p-1}}$. If the closed-loop system becomes unstable, the actual plant model is identified to be $\mathbf{P}_{i_p}$; switch to $\mathbf{K}_{i_p}$ and stop. Otherwise, the actual plant model is $\mathbf{P}_{i_{p-1}}$; switch to $\mathbf{K}_{i_{p-1}}$ and stop.

It can be easily verified that using the switching sequence described in Algorithm 1, it is guaranteed that the system will eventually switch to the correct controller with at most one unstable switching, provided all required controllers in different layers exist.

Example 1: Assume that there is a family of 6 plants as shown in Figure 2. Initially, the actual plant model is $\mathbf{P}_4$ which is stabilized by controller $\mathbf{K}_4$. Assume that the plant model changes to $\mathbf{P}_2$ at time t_0 which is the new unknown plant. The system becomes unstable and it switches to $\mathbf{K}_{6123}$ which stabilizes $\mathbf{P}_1$, $\mathbf{P}_2$, $\mathbf{P}_3$, $\mathbf{P}_6$ and destabilizes $\mathbf{P}_4$ and $\mathbf{P}_5$. The system becomes stable and switches to $\mathbf{K}_{123}$ which is the parent of the previous controller $\mathbf{K}_{6123}$. The system remains stable and should switch to a parent of the current controller. The current controller has three parents $\mathbf{K}_{12}$, $\mathbf{K}_{13}$, and $\mathbf{K}_{23}$. Assume that the system switches to $\mathbf{K}_{23}$. The system becomes stable and should switch to one of the parents of $\mathbf{K}_{23}$. Assume that it switches to $\mathbf{K}_3$. This controller destabilizes the system and it is the only time the system becomes unstable. At this point, the new actual plant model is identified to be $\mathbf{P}_2$ and the system switches to $\mathbf{K}_2$.

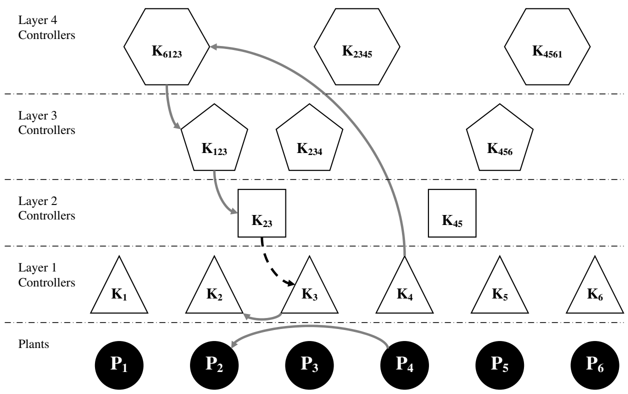

Fig. 2. Switching in four layers. Solid arrows represent stable switchings and dashed arrow denote an unstable switching.

B. Structure of layers

A multi-layer structure can contain several controllers for a family of plants. It is not necessary to have all these controllers in order to reduce the number of unstable switchings to less than or equal to one. Assume that the conditions of Assumption 1 are met. Then, once the plant model changes within the known set of plant models, it is guaranteed that the system will switch to the correct controller after at most one unstable switching.

Remark 1: It is to be noted that the number of all possible controllers for layer k that can stabilize k plant models and destabilize the remaining $p-k$ models is equal to $\frac{p!}{k!(p-k)!}$. Thus, the total number of all possible controllers in the proposed multi-layer structure is equal to $\sum_{k=1}^{p-2} \frac{p!}{k!(p-k)!}$. More specifically, the number of all possible controllers for layer $p-2$ is equal to $\frac{p(p-1)}{2}$. However, it can be easily verified that only p controllers for layer 1 and fix$(\frac{p+1}{2})$ controllers for other layers would suffice, where fix$(.)$ represents the nearest integer towards zero. Since designing simultaneous stabilizers for layers $2, 3, \ldots, p-2$ can be difficult in practice, one may design at most $(p-3)$fix$(\frac{p+1}{2})$ simultaneous stabilizers.

Example 2: Assume that a family of seven plants $\Pi = \{\mathbf{P}_1, \mathbf{P}_2, \mathbf{P}_3, \mathbf{P}_4, \mathbf{P}_5, \mathbf{P}_6, \mathbf{P}_7\}$ is given. The proposed multi-layer architecture consists of five layers. There can be $\frac{7\times 6}{2} = 21$ controllers on the highest layer theoretically but only 4 of them are necessary. The conditions of Assumption 1 can be satisfied in different ways. For instance, one can choose the set of controllers $\{\mathbf{K}_{12345}, \mathbf{K}_{34567}, \mathbf{K}_{56712}, \mathbf{K}_{71234}\}$ to represent Φ_5. The next layers should then be designed in a way that child-parent routes exist from all controllers of Φ_5 to those of layer 1. $\Phi_4 = \{\mathbf{K}_{2345}, \mathbf{K}_{3456}, \mathbf{K}_{6712}, \mathbf{K}_{7123}\}$ is one of the several choices for layer four. Since the pair $\mathbf{K}_{2345}$ and $\mathbf{K}_{3456}$ and the pair $\mathbf{K}_{6712}$ and $\mathbf{K}_{7123}$ have a common parent, a smart choice for controllers of the third layer would be $\Phi_3 = \{\mathbf{K}_{345}, \mathbf{K}_{712}\}$. Layer two can then be chosen as $\Phi_2 = \{\mathbf{K}_{34}, \mathbf{K}_{71}\}$. Finally, the first layer should have seven controllers as in a single-layer architecture.

C. Switching Mechanism

The switching instants will be obtained by using the same approach as in [4]. The method consists of two phases. First, a bound on initial condition is obtained and then the desired controller is found by switching between different controllers. Let the 2-norm of a vector $x \in R^n$ be denoted by $\|x\|$. Similarly, for a matrix $A \in R^{n\times m}$, $\|A\|$ will denote the corresponding induced norm of A.

1) Finding a Bound on Initial Condition: The following result from Lemma 1 in [4] provides an upper-bound for the initial condition with $u(.) = 0$

$$\|x(0)\|^2 \leq \alpha_{i_1} \int_0^T \|y(\tau)\|^2 d\tau + \alpha_{i_2}\tilde{b} \tag{9}$$

where

$$W_i = \int_0^T e^{A_i'\tau} C' C e^{A_i\tau} d\tau,$$
α_{i_3} = the smallest singular value of W_i,
$$\alpha_{i_1} = 2/\alpha_{i_3},$$
$$\alpha_{i_2} = (2/\alpha_{i_3}) \int_0^T \left[\ \int_0^t \|C_i e^{A_i(t-\tau)} E_i\| d\tau + \|F_i\| \ \right]^2 dt$$
$\tilde{b}$ = upper-bound on norm of disturbance.

With $\tilde{u}(t) = 0$ for $t \in [0, T]$ and $z(0) = 0$, find

$$\theta := \int_0^T \|y(\tau)\| d\tau$$

and define the following ϕ upper-bound signals for all stable closed-loop configurations:

$$\begin{aligned}\dot{r}_{j,i_1i_2\dots i_k}(t) &= \lambda_j r_{j,i_1i_2\dots i_k}(t) \\ &+ \gamma_{(j,i_1i_2\dots i_k)_2} \|\tilde{K}_{i_1i_2\dots,i_k}(\tilde{y} - \tilde{D}_j y_{ref})\| \\ &+ \gamma_{(j,i_1i_2\dots i_k)_3}\tilde{b}\ , \quad t \in [0,T] \\ r_{j,i_1i_2\dots i_k}(0) &= 0\end{aligned} \tag{10}$$

where from Lemma 2 of [4] there exist $\lambda_{j,i_1i_2\dots i_k} < 0$ and $\gamma_{(j,i_1i_2\dots i_k)_1} > 0$ such that

$$\|e^{(\tilde{A}_j + \tilde{B}_j \tilde{K}_{i_1i_2\dots i_k}\tilde{C}_j)t}\| \leq \gamma_{(j,i_1i_2\dots i_k)_1} e^{\lambda_{j,i_1i_2\dots i_k}t} \tag{11}$$

and

$$\gamma_{(j,i_1i_2\dots i_k)_2} = \gamma_{(j,i_1i_2\dots i_k)_1} \|\tilde{B}_j\| \tag{12a}$$
$$\gamma_{(j,i_1i_2\dots i_k)_3} = \gamma_{(j,i_1i_2\dots i_k)_1} \|\tilde{E}_j + \tilde{B}_j \tilde{K}_{i_1i_2\dots i_k}\tilde{F}_j\| \tag{12b}$$

Define

$$\mu_j = [\alpha_{j_1}\theta + \alpha_{j_2}\tilde{b}^2]^{\frac{1}{2}}$$

Assuming that $\|w(t)\| \leq \tilde{b}$, if the actual plant model is $\mathbf{P}_j$, it follows from (9) that $\|x(0)\| \leq \mu_j$.

2) Searching for the correct Controller: In this phase control action is applied and the upper-bound signal introduced in [4] is given by

$$\begin{aligned}&\dot{r}_{j,i_1i_2\dots i_1}(t) = \lambda_j r_{j,i_1i_2\dots i_k}(t) \\ &+ \gamma_{(j,i_1i_2\dots i_k)_2}\|\tilde{u}(t) - \tilde{K}_{i_1i_2\dots,i_k}(\tilde{y}(t) - \tilde{D}_j y_{ref})\| \\ &+ \gamma_{(j,i_1i_2\dots i_k)_3}\tilde{b}\end{aligned} \tag{13}$$

with initial condition

$$\begin{aligned}r_{j,i_1i_2\dots i_k}(T^+) &= r_{j,i_1i_2\dots i_k}(T) \\ &+ \gamma_{(j,i_1i_2\dots i_k)_1} e^{\lambda_j r_{j,i_1i_2\dots i_k}T} \mu_j\end{aligned} \tag{14}$$

Each closed-loop controller-plant pair has an upper-bound signal which is a function of the norm of the error. It is often desired to use a smooth error signal by applying a filter as follows

$$\dot{\tilde{r}} = \tilde{\lambda}\tilde{r}(t) + (\lambda - \tilde{\lambda})\|\tilde{y}(t) - \tilde{D}y_{ref}\|, \quad \tilde{r}(T) = 0 \tag{15}$$

where $\tilde{\lambda} < \min\{\lambda_i : i \in \bar{\mathrm{p}}\}$.

Each time the filtered error signal meets the upper-bound signal corresponding to the current controller $\mathbf{K}_{i_1i_2\dots i_k}$ and plant $\mathbf{P}_j$ ($j \in \{i_1, i_2, \dots, i_k\}$), instability is detected. In other words, the system will switch to another candidate controller when

$$\tilde{r}(t) = \|\tilde{C}_j\| r_{j,i_1i_2\dots i_k}(t) + \|\tilde{F}_j\|\tilde{b} + \varepsilon\ , \tag{16}$$

where ε is an arbitrary positive value [4].

The switching sequence of Algorithm 1 requires that the system switches from the higher layer controllers to the lower layer ones even if the system is stabilized in a higher layer. Unlike unstable switchings, stable switchings cannot be identified through the upper-bound signals. In order to detect stability, a sufficiently long time-interval will be used such that if the norm of error does not meet the upper-bound signal, the system is stable. This time-interval can be obtained by considering worse case scenario associated with initial conditions, reference input and disturbance signal. It can also be obtained experimentally. This time duration will be referred to as safety time and will be denoted by t_d.

Lemma 1: The controller $\mathbf{K}_{i_1i_2\dots i_k}$ destabilizes the system iff the filtered error signal meets any of the upper-bound signals corresponding to $\mathbf{P}_{i_1}, \mathbf{P}_{i_2}, \dots \mathbf{P}_{i_k}$ and controller $\mathbf{K}_{i_1i_2\dots i_k}$.

Proof of Lemma 1: Suppose that the current controller is $\mathbf{K}_{i_1i_2\dots i_k}$ which can stabilize both $\mathbf{P}_{i_1}$ and $\mathbf{P}_{i_2}$. For simplicity and with no loss of generality, it will be assumed that $\tilde{C}_i : i \in \bar{\mathrm{p}}$ has a unit norm. The upper-bound signals $r_{i_1,i_1i_2\dots i_k}$ and $r_{i_2,i_1i_2\dots i_k}$ are used as in [4] and are obtained from (10) and (13). Two new upper-bound signals are defined as follows

$$\begin{aligned}&\dot{r}'_{i_1,i_1i_2\dots i_k}(t) = \lambda_{i_1i_2\dots i_k} r'_{i_1,i_1i_2\dots i_k}(t) \\ &+ \gamma_{(i_1i_2\dots i_k)_2}\|\tilde{u}(t) - \tilde{K}_{i_1i_2\dots i_k}(\tilde{y}(t) - \tilde{D}_j y_{ref}(t))\| \\ &+ \gamma_{(i_1i_2\dots i_k)_3}\tilde{b},\end{aligned} \tag{17}$$

$$\dot{r}'_{i_2,i_1i_2\ldots i_k}(t) = \lambda_{i_1i_2\ldots i_k} r'_{i_2,i_1i_2\ldots i_k}(t) + \gamma_{(i_1i_2\ldots i_k)_2}\|\tilde{u}(t) - \tilde{K}_{i_1i_2\ldots i_k}(\tilde{y}(t) - \tilde{D}_j y_{ref}(t))\| + \gamma_{(i_1i_2\ldots i_k)_3}\tilde{b} \quad (18)$$

where

$$\gamma_{(i_1i_2\ldots i_k)_1} = \max(\gamma_{(i_1,i_1i_2\ldots i_k)_1}, \gamma_{(i_2,i_1i_2\ldots i_k)_1}), \quad (19a)$$
$$\gamma_{(i_1i_2\ldots i_k)_2} = \max(\gamma_{(i_1,i_1i_2\ldots i_k)_2}, \gamma_{(i_2,i_1i_2\ldots i_k)_2}), \quad (19b)$$
$$\gamma_{(i_1i_2\ldots i_k)_3} = \max(\gamma_{(i_1,i_1i_2\ldots i_k)_3}, \gamma_{(i_2,i_1i_2\ldots i_k)_3}), \quad (19c)$$
$$\lambda_{i_1i_2\ldots i_k} = \max(\lambda_{i_1,i_1i_2\ldots i_k}, \lambda_{i_2,i_1i_2\ldots i_k}). \quad (19d)$$

Since (11) holds for the new upper-bound signals, either $r_{i_1,i_1i_2\ldots i_k}$ or $r'_{i_1,i_1i_2\ldots i_k}$ can be chosen as the upper-bound signal for $\mathbf{P}_{i_1}$. A similar discussion can be made for the upper-bound signals of plant $\mathbf{P}_{i_2}$. In other words , there exist time instants $\tau_2 > \tau_1$ and $\tau_4 > \tau_3$ such that

$$\begin{aligned}\tilde{r}(\tau_1) = r_{i_1,i_1i_2\ldots i_k}(\tau_1) + \|\tilde{F}_{i_1}\|\tilde{b} + \varepsilon \Leftrightarrow \\ \tilde{r}(\tau_2) = r'_{i_1,i_1i_2\ldots i_k}(\tau_2) + \|\tilde{F}_{i_1}\|\tilde{b} + \varepsilon\end{aligned} \quad (20)$$

$$\begin{aligned}\tilde{r}(\tau_3) = r_{i_2,i_1i_2\ldots i_k}(\tau_3) + \|\tilde{F}_{i_2}\|\tilde{b} + \varepsilon \Leftrightarrow \\ \tilde{r}(\tau_4) = r'_{i_2,i_1i_2\ldots i_k}(\tau_4) + \|\tilde{F}_{i_2}\|\tilde{b} + \varepsilon\end{aligned} \quad (21)$$

Subtracting (18) from (17) results in:

$$r'_{i_1,i_1,i_1i_2\ldots i_k} - r'_{i_2,i_1,i_1i_2\ldots i_k} = k_0 e^{\lambda_{i_1i_2\cdots i_k} t} \quad (22)$$

where $k_0 > 0$ for $r'_{i_1,i_1,i_1i_2\ldots i_k} > r'_{i_2,i_1,i_1i_2\ldots i_k}$. It follows from (20) that if $\tilde{r}(\tau_2) = r'_{i_2,i_1i_2\ldots i_k}(\tau_2) + \|\tilde{F}_{i_2}\|\tilde{b} + \varepsilon$

$$\forall\varepsilon > 0 : \exists\varepsilon' = \varepsilon + k_0 e^{\lambda_{i_1i_2\cdots i_k}\tau_4} + \|\tilde{F}_{i_1} - \tilde{F}_{i_2}\|\tilde{b} > 0 \quad (23)$$

and $\exists\tau_5 > \tau_4$ such that

$$\begin{aligned}\tilde{r}(\tau_5) &= r'_{i_2,i_1,i_1i_2\ldots i_k}(\tau_5) + \|\tilde{F}_{i_2}\|\tilde{b} + \varepsilon' \\ &= r'_{i_2,i_1,i_1i_2\ldots i_k}(\tau_5) + k_0 e^{\lambda_{i_1i_2\cdots i_k}\tau_4} + \|\tilde{F}_{i_2}\|\tilde{b} \\ &+ \|\tilde{F}_{i_1} - \tilde{F}_{i_2}\|\tilde{b} + \varepsilon\end{aligned} \quad (24)$$

since $\|\tilde{F}_{i_2}\| + \|\tilde{F}_{i_1} - \tilde{F}_{i_2}\| \geq \|\tilde{F}_{i_1}\|$ and $e^{\lambda_{i_1i_2\cdots i_k}\tau_4} \geq e^{\lambda_{i_1i_2\cdots i_k}\tau_5}$, then

$$\begin{aligned}\tilde{r}(\tau_5) &\geq r'_{i_2,i_1,i_1i_2\ldots i_k}(\tau_5) + k_0 e^{\lambda_{i_1i_2\cdots i_k}\tau_5} + \|\tilde{F}_{i_1}\|\tilde{b} + \varepsilon \\ &= r'_{i_1,i_1,i_1i_2\ldots i_k}(\tau_5) + \|\tilde{F}_{i_1}\|\tilde{b} + \varepsilon\end{aligned} \quad (25)$$

It follows from (23) and (25) that

$$\begin{aligned}\tilde{r}(\tau_3) = r'_{i_2,i_1i_2\ldots i_k}(\tau_3) + \|\tilde{F}_{i_2}\|\tilde{b} + \varepsilon \\ \Rightarrow \exists\tau_6 < \tau_5 : \tilde{r}(\tau_6) = r'_{i_1,i_1i_2\ldots i_k}(\tau_6) + \|\tilde{F}_{i_1}\|\tilde{b} + \varepsilon\end{aligned} \quad (26)$$

Substituting $\tau_2 = \tau_6$ into (20) results in

$$\begin{aligned}\tilde{r}(\tau_6) = r_{i_2,i_1i_2\ldots i_k}(\tau_6) + \|\tilde{F}_{i_2}\|\tilde{b} + \varepsilon \Rightarrow \\ \tilde{r}(\tau_1) = r_{i_1,i_1i_2\ldots i_k}(\tau_1) + \|\tilde{F}_{i_1}\|\tilde{b} + \varepsilon\end{aligned} \quad (27)$$

This implies that if the filtered upper-bound signal meets the smaller boundary signal corresponding to the closed-loop pair $(\mathbf{K}_{i_1i_2\ldots i_k}, \mathbf{P}_{i_2})$, it will definitely meet the other boundary signal corresponding to the closed-loop pair $(\mathbf{K}_{i_1i_2\ldots i_k}, \mathbf{P}_{i_1})$. Define t_i to be $\min(\tau_1, \tau_3)$. According to Lemma 1, for the higher layer controllers only the smallest upper-bound signal associated with each controller and its corresponding plant models needs to be compared to the filtered signal as it results in smaller time instants.

Theorem 1: Consider the system (2). Using the switching sequence of Algorithm 1, and the switching instants $t_s = \min(t_{i-1}+t_d, t_i)$ where t_i represents the time instants given in Lemma 1 ($t_0 := 0$) and t_d is the safety time, the system will eventually switch to the correct controller with no more than one unstable switching.

Proof of Theorem 1: The proof follows immediately from Lemma 1 and the results of Theorem 1 in [4].

IV. Numerical Example

Example 3: Consider the following unstable non-minimum phase plant model used in [9] and [13] :

$$\mathbf{P} = \lambda\frac{s-1}{(s-2)(s+1)}, \quad 1 < \lambda(t) < 6$$

A family of four plant models $\mathbf{P}_i = \{\mathbf{P}_1, \mathbf{P}_2, \mathbf{P}_3, \mathbf{P}_4\}$ is then considered as follows

$$\mathbf{P}_1 = \frac{s-1}{(s-2)(s+1)}, \ \mathbf{P}_2 = 2\mathbf{P}_1, \ \mathbf{P}_3 = 4\mathbf{P}_1, \ \mathbf{P}_4 = 6\mathbf{P}_1$$

The high-performance controllers of first layer are obtained as follows

$$\mathbf{K}_1 = \frac{448s^2 + 450s - 18}{31s(s-9)},$$
$$\mathbf{K}_2 = \frac{1}{2}\times\mathbf{K}_1, \ \mathbf{K}_3 = \frac{1}{4}\times\mathbf{K}_1, \ \mathbf{K}_4 = \frac{1}{6}\times\mathbf{K}_1.$$

The second layer consists of three controllers which are designed using the simultaneous stabilization method presented in [14]

$$\mathbf{K}_{12} = \frac{45.31s^4 + 325.5s^3 + 853.2s^2 + 1184s + 610.3}{s^4 - 29.81s^3 - 237.55s^2 - 515.6s - 670.1}$$
$$\mathbf{K}_{23} = \frac{22.65s^4 + 162.7s^3 + 426.7s^2 + 591.8s + 305.1}{s^4 - 29.81s^3 - 237.5s^2 - 515.6s - 670.1}$$
$$\mathbf{K}_{34} = \frac{0.2758s^4 + 53.1s^3 + 224.35s^2 + 425s + 253.6}{s^4 + 14.4s^3 - 124.4s^2 - 470.9s - 872.1}$$

It can be easily verified that controller $\mathbf{K}_{12}$ stabilizes the plant models $\mathbf{P}_1$, $\mathbf{P}_2$ and destabilizes $\mathbf{P}_3$, $\mathbf{P}_4$. Similarly, $(\mathbf{K}_{23}, \mathbf{P}_2)$, $(\mathbf{K}_{23}, \mathbf{P}_3)$, $(\mathbf{K}_{34}, \mathbf{P}_3)$ and $(\mathbf{K}_{34}, \mathbf{P}_4)$ are stable closed-loop pairs while $(\mathbf{K}_{23}, \mathbf{P}_1)$, $(\mathbf{K}_{23}, \mathbf{P}_4)$, $(\mathbf{K}_{34}, \mathbf{P}_1)$ and $(\mathbf{K}_{34}, \mathbf{P}_2)$ are unstable pairs. Assume that initially the actual plant model is $\mathbf{P}_1$ and at some point of time it changes to $\mathbf{P}_4$. In the single-layer approach the system will switch from $\mathbf{K}_1$ to $\mathbf{K}_2$, then to $\mathbf{K}_3$, and finally to $\mathbf{K}_4$. The first two switching instants are unstable. In the multi-layer approach the system will switch from $\mathbf{K}_1$ to $\mathbf{K}_{23}$, and then to $\mathbf{K}_4$. The only unstable switching corresponds to $\mathbf{K}_{23}$. Figures 3 and 4 show the closed-loop simulation results using the proposed multi-layer algorithm and the single-layer method of [4], respectively. These figures show a 90% reduction in the magnitude of the transient response using the proposed multi-layer algorithm.

Now assume that the plant model changes from $\mathbf{P}_4$ to $\mathbf{P}_3$. The single-layer method switches from $\mathbf{K}_4$ to $\mathbf{K}_1$, then to $\mathbf{K}_2$, and finally to $\mathbf{K}_3$. Two unstable switchings occur using single-layer method. However, the proposed multi-layer algorithm will switch from $\mathbf{K}_4$ to $\mathbf{K}_{23}$. The system becomes stable and then switches to $\mathbf{K}_3$. It is to be noted that in this case no unstable switching occurs using multi-layer algorithm. It can be verified that the maximum amplitude of the transient response is 4.02. It is much smaller than that of the single-layer method of [4] which is 1357.9.

V. Conclusion

In this paper, a switching control algorithm for a family of known plant models is proposed which has a good transient response compared to the existing methods. The algorithm uses different layers of controllers, where layer 1 consists of one high-performance controller for each plant model and other layers consist of simultaneous stabilizers for certain subsets of family of plant models. The proposed algorithm gives a sequence of controllers of different layers for switching and can be used with any switching scheme. In this paper, the switching scheme of [4] is considered. It is guaranteed that while the system searches for the correct controller in the first layer, it switches to at most one destabilizing controller and all other controllers in the given sequence stabilize the system. The system switches to the next controller if the current controller destabilizes the system, or the current controller stabilizes the system but does not belong to layer 1. The closed-loop system is identified to be unstable if the norm of the error hits its corresponding upper-bound, and is identified to be stable if it does not hit the upper-bound for a sufficiently long time. The simulation results show significant improvement in the magnitude of the transient response compared to that of the single layer method proposed in [4].

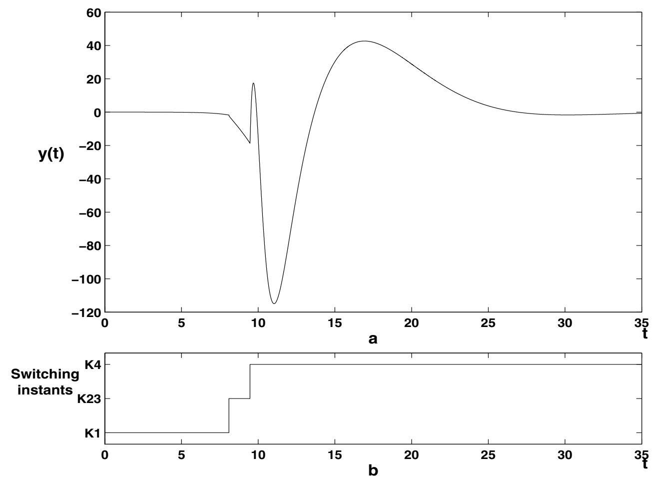

Fig. 3. Closed-loop simulation results for Example 3, using the multi-layer scheme, when the plant models change from P_1 to P_4. (a) Output signal; (b) switching instants.

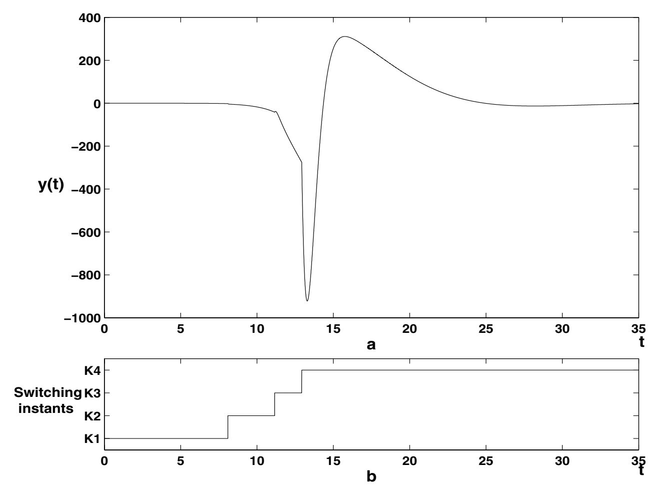

Fig. 4. Closed-loop simulation results for Example 3, using the single-layer scheme of [4], when the plant models change from P_1 to P_4. (a) Output signal; (b) switching instants.

Acknowledgement

The authors gratefully acknowledge the useful comments of Shauheen Zahirazami in development of the algorithm.

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