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Optimal and Hierarchical Formation Control for UAV

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Abstract— **In this paper, optimal and hierarchical control concepts are investigated for cooperative formation flying of aircrafts. The airplanes are modeled as point mass and represented by double integrators. And all the planes are considered to be in a plane. For demonstration of the concepts, a task of forming a square from arbitrary initial conditions is presented to four airplanes. The final position that each airplane has to reach is unknown to them. The goal for the team is abstracted in the top layer.**

The system is modeled as a two layer hierarchical system in which the global information comes from the top layer. Following this global information, a square is formed in the bottom layer according to tracking of the top layer setting. Two cases are considered in this paper: the first is the time optimal square forming problem. The other is maintenance and movement of the formation. both are solved in this hierarchical structure. Numerical results from simulating these two cases are presented. From the simulation results, the effectiveness of the hierarchical concept for the UAV class of problems is demonstrated.

I. Introduction

UAV control has become a major area for control applications. In the last 2004 AIAA-Guidance, Navigation and Control conference [1], about one hundred papers were presented in this area. This kind of research is of great interest in military and civilian applications. Aerial surveillance/tracking, collision/obstacle avoidance, and formation flight are the current shot spots of UAV control [2]. Murry and others at California Institute of Technology have done some interesting work in formation flight using structural potential functions and model predictive control [3].

In this paper, a formation flight problem is considered. A case where four airplanes are required to form a square is treated with a hierarchical control framework [4, 5]. None of the airplanes in the formation is aware that a square is being formed. The upper level supplies the global goal required for the formation to form the square. The global goal is in terms of the separation distance between the two adjacent airplanes. The upper level dictates how the separation distance should change such that a square is formed. The airplanes at the lower level then cooperatively adjust their trajectories such that they track the global goal propagated from the top and form the square.

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II. Problem Setup

A. Bottom Layer Dynamics

All the kinematics of the i^{th} airplane can be considered as a double integrator with the dynamics described by

$$\begin{aligned} {}^i\dot{x}_1 &= {}^i\dot{x}_2 \\ {}^i\dot{x}_2 &= {}^iu_1 \\ {}^i\dot{y}_1 &= {}^i\dot{y}_2 \\ {}^i\dot{y}_2 &= {}^iu_2 \end{aligned} \tag{1}$$

where superscript $i \in [1,2,3,4]$ is the index for the airplanes, x_1 and y_1 are the displacements of the airplane in the x and y directions respectively. Similarly u_1 and u_2 are the forces acting on the airplanes in the x and y directions respectively.

The dynamics of the formation will then be

$${}^i\dot{\mathbf{x}}_{bot} = {}^iA_{bot}\,{}^i\mathbf{x}_{bot} + {}^iB_{bot}\,{}^i\mathbf{u}_{bot} \tag{2}$$

$${}^iA_{bot} = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix} \qquad {}^iB_{bot} = \begin{bmatrix} 0 & 0 \\ 1 & 0 \\ 0 & 0 \\ 0 & 1 \end{bmatrix} \tag{3}$$

So the whole system dynamic A_{bot} and B_{bot} will be diagonal matrices with ${}^iA_{bot}$ and ${}^iB_{bot}$ along the diagonal where i ranges from 1 to 4.

$$\dot{\mathbf{x}}_{bot} = A_{bot}\mathbf{x}_{bot} + B_{bot}\mathbf{u}_{bot} \tag{4}$$

$$\mathbf{x}_{bot} = \begin{bmatrix} {}^1x_1 & {}^1x_2 & {}^1y_1 & {}^1y_2 & \cdots \\ \cdots & {}^4x_1 & {}^4x_2 & {}^4y_1 & {}^4y_2 \end{bmatrix} \tag{5}$$

$$\mathbf{A}_{bot} = \begin{bmatrix} {}^1\mathbf{A}_{bot} & & & \\ & {}^2\mathbf{A}_{bot} & & \\ & & {}^3\mathbf{A}_{bot} & \\ & & & {}^4\mathbf{A}_{bot} \end{bmatrix}$$

$$\mathbf{B}_{bot} = \begin{bmatrix} {}^{1}\mathbf{B}_{bot} & & & \\ & {}^{2}\mathbf{B}_{bot} & & \\ & & {}^{3}\mathbf{B}_{bot} & \\ & & & {}^{4}\mathbf{B}_{bot} \end{bmatrix} \quad (6)$$

Note that the dynamics of the airplane are decoupled from each other. Suppose the airplanes are asked to form a square such that all the airplanes have to reach the corners of the square at the same time. The final position that the airplanes have to take is unknown to them. Since their dynamics are decoupled, each airplane does not have any idea about the position of the other airplanes in the formation. The global goal (to form a square) is unknown to each of the airplanes. In such a formation, in the absence of a global goal, it is extremely difficult for the airplanes to form the square. Even if they do form a square successfully, the trajectories followed by the airplanes will be highly circuitous and far from optimal.

B. Top Layer Dynamics

As mentioned before, the top layer sets the system performance. Here we will set the top layer in an optimal style. And first of all, top layer dynamic is built as following:

Consider airplanes 1 and 2. Differentiating relative distance between them twice with respect to t,

$$^{12}\dot{d}x = {}^{1}\dot{x}_1 - {}^{2}\dot{x}_2 \quad (7)$$

$$^{12}\dot{d}y = {}^{1}\dot{y}_1 - {}^{2}\dot{y}_2 \quad (8)$$

$$^{12}\ddot{d}x = {}^{1}\ddot{x}_1 - {}^{2}\ddot{x}_2 \quad (9)$$

$$^{12}\ddot{d}y = {}^{1}\ddot{y}_1 - {}^{2}\ddot{y}_2 \quad (10)$$

$$^{12}\ddot{d}x = {}^{1}f_x - {}^{2}f_x = {}^{12}F_x \quad (11)$$

$$^{12}\ddot{d}y = {}^{1}f_y - {}^{2}f_y = {}^{12}F_y \quad (12)$$

Equations (11) and (12) describe the separation dynamics of the airplanes 1 and 2. It can be assumed that airplane 1 and 2 are connected by an imaginary string, referring to Fig 2.

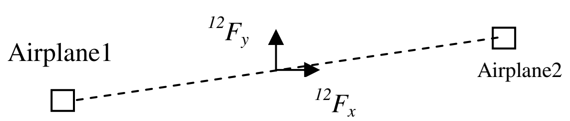

Fig. 1 Imaginary String between the Airplanes

The forces $^{12}F_x$ and $^{12}F_y$ are the net forces due to the two airplanes that acts at the center of the imaginary string. Fig. 1 shows the imaginary string.

The corresponding system matrices for the separation dynamics Equation (11) and (12) are

$$^{12}\mathbf{A}_{top} = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix} \qquad ^{12}\mathbf{B}_{top} = \begin{bmatrix} 0 & 0 \\ 1 & 0 \\ 0 & 0 \\ 0 & 1 \end{bmatrix} \quad (13)$$

The combined separation dynamics for all four airplanes can thus be written as 3

$$\dot{\mathbf{x}}_{top} = \mathbf{A}_{top}\mathbf{x}_{top} + \mathbf{B}_{top}\mathbf{u}_{top} \quad (14)$$

where A_{top} and B_{top} are block diagonal matrices such that

$$x_{top} = [{}^{12}x \quad {}^{12}\dot{x} \quad {}^{12}y \quad {}^{12}\dot{y} \quad {}^{23}x \quad {}^{23}x$$
$$^{23}y \quad {}^{23}\dot{y} \quad {}^{34}x \quad {}^{34}\dot{x} \quad {}^{34}y \quad {}^{34}\dot{y}]^T$$

$$u_{top} = \left[{}^{12}F_x \quad {}^{12}F_y \quad {}^{23}F_x \quad {}^{23}F_y \quad {}^{34}F_x \quad {}^{34}F_y\right]^T$$

$$\mathbf{A}_{top} = \begin{bmatrix} {}^{12}\mathbf{A}_{top} & & \\ & {}^{23}\mathbf{A}_{top} & \\ & & {}^{34}\mathbf{A}_{top} \end{bmatrix}$$

$$\mathbf{B}_{top} = \begin{bmatrix} {}^{12}\mathbf{B}_{top} & & \\ & {}^{23}\mathbf{B}_{top} & \\ & & {}^{34}\mathbf{B}_{top} \end{bmatrix} \quad (15)$$

The aggregation relation between the states of the top and the bottom level is

$$\mathbf{x}_{top} = \mathbf{C}_{agg}\mathbf{x}_{bot} \quad (16)$$

$$C_{agg} = \begin{bmatrix} C_1 & C_2 & & \\ & C_1 & C_2 & \\ & & C_1 & C_2 \end{bmatrix} \quad (17)$$

where:

$$C_1 = diag(1 \quad 1 \quad 1 \quad 1) \; C_2 = diag(-1 \quad -1 \quad -1 \quad -1) \quad (18)$$

Similarly relation between the control forces acting on the imaginary string at the upper level $\mathbf{u}_{top}$ and those at the bottom level $\mathbf{u}_{bot}$ is

$$\mathbf{u}_{top} = \mathbf{D}\mathbf{u}_{bot} \quad (19)$$

where

$$\mathbf{D}_{agg} = \begin{bmatrix} 1 & 0 & -1 & 0 & & & & \\ 0 & 1 & 0 & -1 & & & & \\ & & 1 & 0 & -1 & 0 & & \\ & & 0 & 1 & 0 & -1 & & \\ & & & & 1 & 0 & -1 & 0 \\ & & & & 0 & 1 & 0 & -1 \end{bmatrix} \quad (20)$$

C. Final Goal Expression

The goal of the controller is to make the four airplanes form a square. The first case will require the airplane to form a square in minimum time. The second is to let the four airplanes form a square and move long east 45 degree with constant velocity. Let the length of the square to be formed be 'L', the following final conditions must be satisfied:

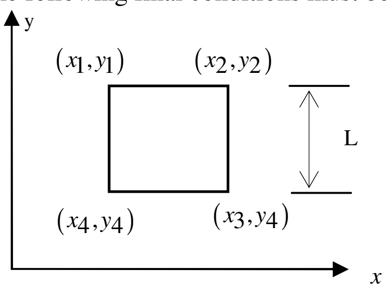

Fig. 2 Final Square of Length 'L'

That is:

$$ {}^{12}x_{final} = -L \quad {}^{12}y_{final} = 0 $$
$$ {}^{23}x_{final} = 0 \quad {}^{23}y_{final} = L $$
$$ {}^{34}x_{final} = L \quad {}^{34}y_{final} = 0 \tag{21} $$

III. CONTROL STRATEGY

A. Top Layer Dynamics in case of Minimum Time Square Forming

Set the top layer goal as a minimum time problem. And solve for the top layer control by minimizing the cost function

$$ J = \int_{t_0}^{t_f} \left(1+\frac{1}{2}\mathbf{u}^{\mathbf{T}}\mathbf{R}_{top}\mathbf{u}\right)dt \tag{22} $$

where t_f is the final time, $\mathbf{R}_{top}$ is a positive definite matrix Then Hamiltonian for this cost function is written as the following:

$$ H = 1 + \frac{1}{2}\mathbf{u}_{top}{}^{\mathbf{T}}\mathbf{R}_{top}\mathbf{u}_{top} + \lambda^{\mathbf{T}}\left(\mathbf{A}_{top}\mathbf{x}_{top}+\mathbf{B}_{top}\mathbf{u}_{top}\right) \tag{23} $$

And from optimal control theory [6], the following equations are obtained;

$$ \begin{cases} \dot{\mathbf{x}}_{top} = \mathbf{A}_{top}\mathbf{x}_{top} - \mathbf{B}_{top}\mathbf{R}_{top}^{-1}\mathbf{B}_{top}^{\mathbf{T}}\lambda \\ \dot{\lambda} = -\mathbf{A}_{top}^{\mathbf{T}}\lambda \\ H\left(t_f\right) = 0 \end{cases} \tag{24} $$

B. Top Layer Dynamics in case of Movement in Constant Velocity with Square Formed

Optimal cost is considered as an infinite time problem in this case, for no matter what the velocity is, once the square is formed, the top layer relative distance and relative velocity should be kept to maintain the square formation.

$$ J = \frac{1}{2}\int_{t_0}^{\infty}\left(\mathbf{x}^{\mathbf{T}}\mathbf{Q}_{top}\mathbf{x}+\mathbf{u}^{\mathbf{T}}\mathbf{R}_{top}\mathbf{u}\right)dt \tag{25} $$

And correspondingly,

$$ \dot{\mathbf{x}}_{top} = \mathbf{A}_{top}\mathbf{x}_{top} - \mathbf{B}_{top}\mathbf{R}_{top}^{-1}\mathbf{B}_{top}^{\mathbf{T}}\mathbf{P}\mathbf{x}_{top} \tag{26} $$

where **P** satisfies the Riccati equation

$$ \mathbf{PA} + \mathbf{A}^{\mathbf{T}}\mathbf{P} - \mathbf{PBR}^{-1}\mathbf{BP} + \mathbf{Q} = \mathbf{0} \tag{27} $$

Here, the desired top layer trajectory and top layer control are generated through the optimal control method, and in the next section, bottom controller will be designed to have this goal trajectory propagated form the top layer to bottom layer.

C. Bottom Layer Dynamics in case of Minimum Time Square Forming

Cost function in the bottom layer is to minimize the tracking cost in the bottom layer trajectory. And choose the following cost function at the bottom layer:

$$ J_{bot} = \frac{1}{2}\int_0^{t_f}\left[\begin{array}{l}\left(x_{top}-C_{agg}x_{bot}\right)^T Q_{bot}\left(x_{top}-C_{agg}x_{bot}\right) \\ +\left(u_{top}-D_{agg}u_{bot}\right)^T R_{bot}\left(u_{top}-D_{agg}u_{bot}\right)\end{array}\right] \tag{28} $$

subject to

$$ \dot{\mathbf{x}}_{bot} = \mathbf{A}_{bot}\mathbf{x}_{bot} + \mathbf{B}_{bot}\mathbf{u}_{bot} \tag{29} $$

According to optimal control theory [6],

$$ u_{bot} = -D_{agg}{}^T R_{bot}{}^{-1} D_{agg}\left(B_{bot}{}^T\left(Px_{bot}+h\right)-D_{agg}{}^T R_{bot}u_{top}\right) \tag{30} $$

where p and h come from the following differential equations:

$$ \dot{p} = -pA_{bot} - A_{bot}{}^T p - C_{agg}{}^T Q_{bot}C_{agg} + pB_{bot}D_{agg}{}^T R_{bot}{}^{-1}D_{agg}B_{bot}{}^T p \tag{31} $$

$$ \dot{h} = -A_{bot}{}^T h + pB_{bot}\left(D_{agg}{}^T R_{bot}{}^{-1}D_{agg}\right)^{-1}B_{bot}{}^T h - pB_{bot}\left(D_{agg}{}^T R_{bot}{}^{-1}D_{agg}\right)^{-1}\left(D_{agg}\right)^T R_{bot}{}^T u_{top} + C_{agg}{}^T Q_{bot}x_{top} \tag{32} $$

Since there is no constraint on the final states, the corresponding boundary conditions are obtained:

$$ p\left(t_f\right) = 0 \text{, and } h\left(t_f\right) = 0 \tag{33} $$

D. Bottom Layer Dynamics in case of Movement in Constant Velocity with Square Formed

In this case, objective function is

$$J_{bot} = \frac{1}{2}\int_{t_0}^{t_f}\left[\begin{array}{l}\left(x_{top}-C_{agg}x_{bot}\right)^T Q_{bot}\left(x_{top}-C_{agg}x_{bot}\right)+\\ \left(u_{top}-Du_{bot}\right)^T R_{bot}\left(u_{top}-Du_{bot}\right)+(Mx_{bot}-V)^T S(Mx_{bot}-V)\end{array}\right] \tag{34}$$

where $(Mx_{bot}-V)^T S(Mx_{bot}-V)$ is a constraint on the bottom velocity. V is a vector of desired velocity. In simulation, it is chosen as a constant vector, $[1\ \ 1\ \ 1\ \ 1\ \ 1\ \ 1\ \ 1\ \ 1]^T$.

$$M = diag\left(M_0 \quad M_0 \quad M_0 \quad M_0\right)$$

$$M_0 = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \tag{35}$$

Accordingly,

$$u_{bot} = -D_{agg}{}^T R_{bot}{}^T D_{agg}\left(B_{bot}{}^{-1}(Px_{bot}+h)-D_{agg}{}^T R_{bot}u_{top}\right) \tag{36}$$

where p and h come from the following differential equations:

$$\dot{p} = -pA_{bot} - A_{bot}{}^T p - C_{agg}{}^T Q_{bot}C_{agg} - M^T SM + pB_{bot}D_{agg}{}^T\left(D_{agg}{}^T R_{bot}{}^T D_{agg}\right)^{-1} D_{agg}B_{bot}{}^T p \tag{37}$$

$$\dot{h} = -A_{bot}{}^T h + pB_{bot}\left(D_{agg}{}^T R_{bot}{}^{-1}D_{agg}\right)^{-1} B_{bot}{}^T h - pB_{bot}\left(D_{agg}{}^T R_{bot}{}^{-1}D_{agg}\right)^{-1}\left(D_{agg}\right)^T R_{bot}{}^T u_{top} + C_{agg}{}^T Q_{bot}x_{top} + M^T SM \tag{38}$$

Again, since there is no constraint on the final states, the corresponding boundary conditions are as following:

$$p\left(t_f\right) = 0 \text{, and } h\left(t_f\right) = 0 \tag{39}$$

IV. Simulation

In this section, numerical results from a four-airplane formation flying problem using the hierarchical control concepts developed in this study are presented.

When the initial separation distances (in non-dimensional units) between the airplanes are assumed as $(-3,4),(2,7),(5,-1),(-2,0)$, and a square of length 2 units will be formed in minimum time. Movements of the four airplanes are presented in Fig. 3. And the planes moves from square points to the star points.

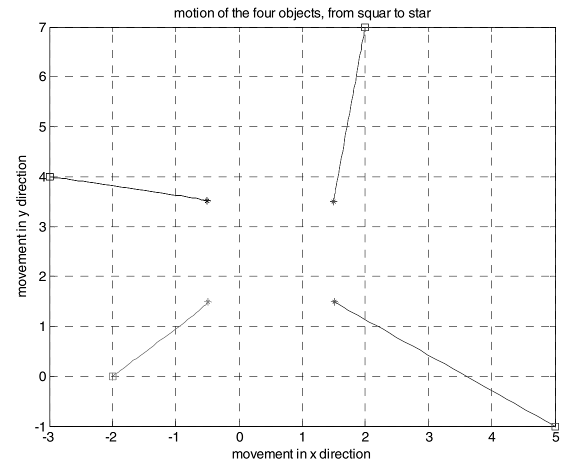

Fig. 3 Minimum Time Squaring Forming, Movement from Squares to Stars

And correspondingly, the top layer and bottom layer states dynamics are as following:

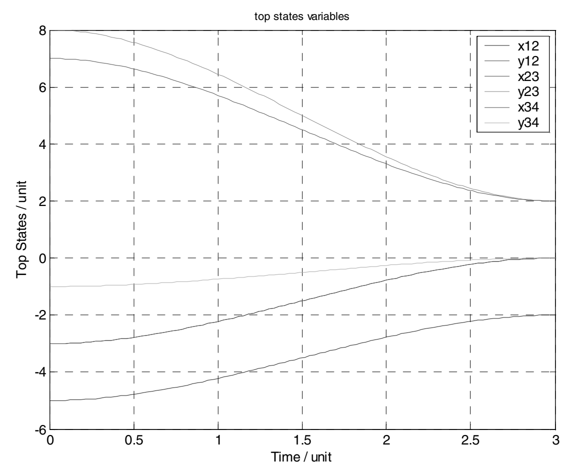

Fig. 4 Histories of Top Layer States

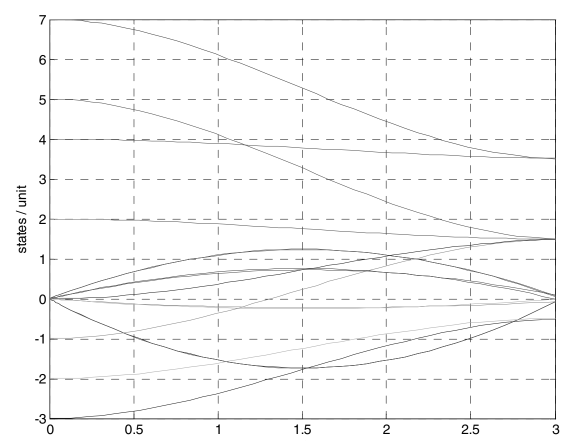

Fig. 5 Histories of Bottom Layer States

From resulting trajectories in Fig. 1, it can be easily found that the minimum time control is achieved near the geometric center of the initial points, instead of around one of the starting point. The results agree with intution.

The following Fig. 6 is the movement with constant velocity in square format. The initial points start also from $(-3,4),(2,7),(5,-1),(-2,0)$. Corresponding top layer states and bottom layer states are plotted in Fig. 7 and Fig. 8.

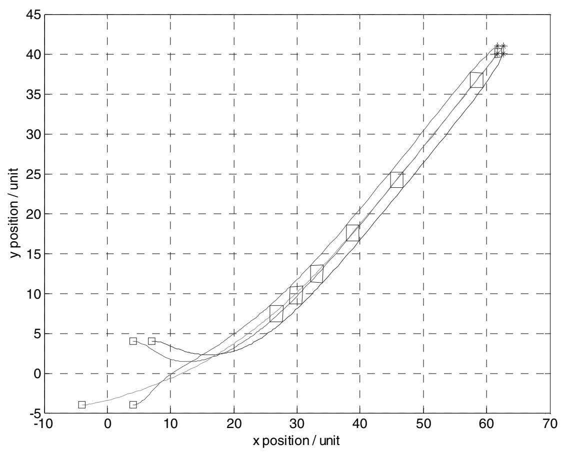

Fig. 6 Movement with Constant Velocity with Square Formed

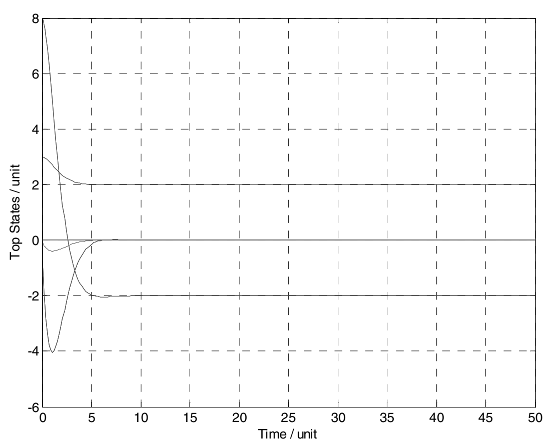

Fig. 7 Histories of Top Layer States

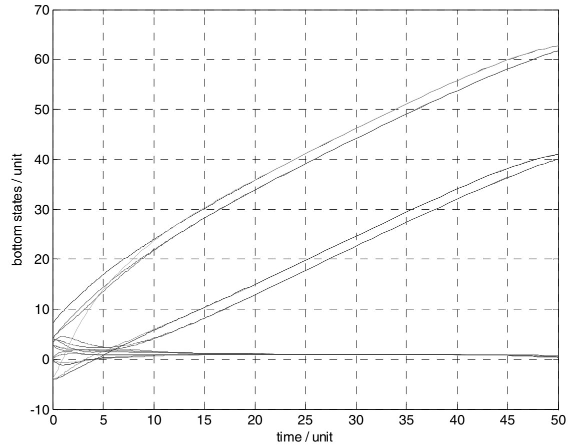

Fig. 8 Histories of Bottom Layer States

From the simulation results, we can observe that the mission is completed in both cases through the hierarchical structure in an optimal way.

V. Conclusions And Future Work

In this paper a cooperative control strategy was developed for formation flying of UAVs. A case four airplanes forming a square was discussed. Using a hierarchical model, the upper level evaluates the trajectory of the 'X' and 'Y' separation distances between the two adjacent airplanes. This trajectory is then passed to the group of airplanes at the lower level. The airplanes cooperatively decide on their individual trajectories such that two adjacent airplanes follow the separation distance trajectory as dictated by the upper level. But in this research, nonlinear dynamics of the airplanes are not included. And also obstacle avoidance in optimal sense in considered. Research is under way to include those elements.

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