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Evaluating UAV Flock Mission Performance Using Dudek's Taxonomy

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Abstract— **We use Dudek's taxonomy in order to investigate the performance of a group of autonomous UAVs cooperating in mission execution against a group of enemy agents acting in an unknown environment. We show that increasing the number of UAVs in the group proves to be beneficial as it allows the group to react to more enemy events. We also show that using communication helps creating better cooperation between the flock members; however, using infinite communication range or infinite communication bandwidth results in considerable computational complexity. We conclude that it may be sufficient to use finite-bandwidth communication, keeping the computational complexity constant with the number of UAVs in the group, thus allowing the group to be scalable to large numbers of UAVs. We use flocking behavior to control the movement of the UAVs when searching, as the flocking helps the UAVs to disperse in the environment and improve the detection probability of the flock. We show that using flocking improves the group performance only if the group is capable of mission task collaboration.**

I. INTRODUCTION

The problem of design, development and control of multi-agent systems has been studied in recent years for many applications. In particular, the use of systems consisted of multiple autonomous robots or UAVs has been proposed in recent years in order to meet the requirements of complex missions [1]. The field of UAVs has been an evolving research and development field in the United States during the last 10 years and is expected to be an even faster growing field in the next 25 years, as seen in the Department of Defence UAV Roadmap 2002-2027 [2]. The use of groups of cooperating UAVs in order to perform various missions is currently studied throughout the world and is considered a main research goal by the United States Air Force Research Laboratory (AFRL) [3].

Using a group of *agents* in order to achieve a certain goal cooperatively requires that each agent assumes a certain task at a given time. The global result of the group of agents acting together is the execution of a certain mission. Assigning the various agents to perform these tasks according to their capabilities is a challenge that requires the development of advanced algorithms. The algorithms may be classified into two main types: optimal algorithms and heuristic algorithms. While optimal algorithms (e.g. [4], [5], [6]) yield better results in task assignment, they are usually more sensitive to the system properties, the target behavior, environment changes and the possibility to estimate both. Heuristic algorithms (e.g. [7], [8], [9], [10], [11], [12]), on the other hand, are usually sub-optimal but more robust.

A related issue is formation flying (also referred to as *flocking*). This issue has been studied extensively in the last 20 years [13], [14], [15], [16], [17], [18], [19], [20], [21], following the seminal work of Reynolds [22]. Reynolds was the first to simulate flocking behavior based on the basic flocking rules of collision Avoidance, velocity Matching, flock Centering ,obstacle avoidance and migration. In this work we adopt Reynolds' flocking behavioral model in order to avoid collision and improve mission execution.

While most researchers assume a given system, propose a new control algorithm, and examine the algorithm's performance compared to other known algorithms, we take a novel approach and examine the effect of changes in the properties of a system (using the same algorithm) on mission execution performance. Using the taxonomy offered by Dudek et. al [1] we concentrate on the four main parameters: System size, communication range, communication Bandwidth and system composition.

We consider a search and destroy mission, where a group of autonomous armed UAVs is dispatched to an environment in order to detect and destroy time-critical targets. The group performance is analyzed using three figures of merit: Effectiveness, efficiency and complexity.

II. MODEL DEFINITION AND ANALYSIS

A. Scenario

The scenario is a "battle-field", comprised of "urban" and "open" terrains. In these settings a ground enemy is moving, usually with little to no distinguished signature. The enemy, however, in order to achieve his goals has to stop moving and create a distinguished signature (e.g. launch a missile). This signature is referred to as an *event* and can be detected by the UAVs' detectors. This may represent typical UAV missions [2].

In order to detect, identify and attack this enemy, a group of UAVs is dispatched to the area. These UAVs are autonomous, use communication and are equipped with various payloads:

- Detectors and Indicators - designed to detect a suspected target according to the unique signature of the event (e.g. SAM radar emission, launch,or movement). However, these sensors can not identify the target.

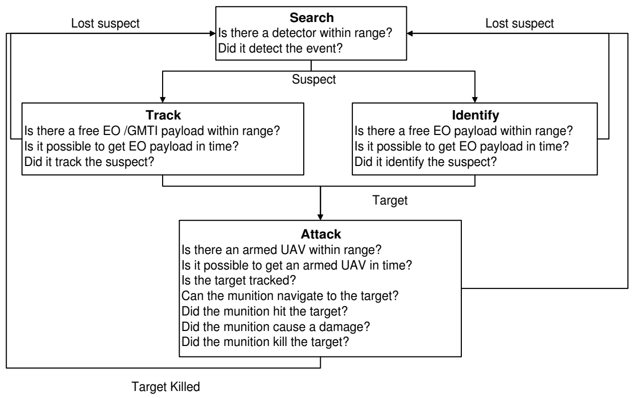

Fig. 1. Process Flowchart

- Electro-optic (EO) payload - used for target identification, tracking and munition guidance.
- Munitions - used for attacking identified targets.

The scenario takes place under conservative rules of engagement (i.e. no fire until absolute certainty of the target), typical to search and destroy missions in urban or civilian areas [23].

B. *Mission Analysis*

1) Process Overview: As shown earlier, search and destroy mission can be divided to search, identify, track and attack tasks. Fig. 1 shows an end-to-end process and the conditions that must be met in order to fulfill the mission. Notably, each task is divided to a number of conditions, which may be determined by one or more of the following properties:

- UAV capabilities (e.g. Velocity, communication).
- UAV location.
- UAV current mode (search or engagement).
- Payload capabilities.
- Environment conditions (e.g. urban vs. open).
- Target behavior.

2) Quantitative Analysis of the Process: To complete the mission, it is necessary that all the tasks are performed in the right order, meaning that all the conditions are fully met. Thus, the probability of a successful process is:

$$P_{des} = P_{det} \cdot P_{id|det} \cdot P_{t|det} \cdot P_{dam|id} \cdot P_{des|dam} \tag{1}$$

Note the difference between a fully cooperative group, where each task can be performed by any UAV, and a non-cooperative group, where **the same** UAV has to complete all the tasks. The following paragraphs discuss the various tasks.

a) Detection: A suspected target is detected if at least one sensor detects the target's event. The probability is given by:

$$P_{det} = P_{inplace} \cdot P_{det|inplace} \tag{2}$$

The first term in 2 is the probability that a sensor is located in vicinity to the event. The second term is the sensor's capability to detect the event, given its position.

Since no a-priori knowledge is assumed about the targets' locations, a sensor's probability to be located correctly is:

$$P_{inplace} = \frac{S_{det}}{S_{env}} = \frac{\int\int U(x,y)_i^D dxdy}{\int\int dxdy} \tag{3}$$
$$U(x,y)^D = \{0,\ \forall i\ \|(x,y)_{event} - (x,y)_i^D\| > R_{i,max}^D$$
$$U(x,y)^D = \{1,\ \exists i\ \|(x,y)_{event} - (x,y)_i^D\| \le R_{i,max}^D$$

Eq. 3 shows that while covering areas with two or more sensors may be useful, it may cause lack of coverage.

For all payloads, a target is sensed if:

$$D_i(x,y) \ge C(x,y) \tag{4}$$
$$D_i(x,y) = random_{unif}(0, P(x,y)_{i,max})$$
$$P(x,y)_{i,max} = P_{i,max}^D \cdot \frac{R_{i,max}^D - R}{R_{i,max}^D}$$
$$R = \|(x,y) - (x_i,y_i)\|$$

$R_{i,max}^D$-i-th sensor's maximum range

$P_{i,max}^D$-i-th sensor's max detection probability (@ R=0)

$C(x,y)$ - environment's complexity

Since an event takes place for a period of time and need to be detected only once during that period, the detection probability is:

$$P_{det|inplace} = 1 - [1 - P(D_i(x,y) \ge C(x,y))]^{t_D \cdot N_D} \tag{5}$$

where:

t_D - period of tame that detection is possible.

N_D - Number of detectors within range.

b) Identification: According to the model, identification can be made by means of electro-optic (EO) payloads only. Furthermore, an EO payload's field-of-view (FOV) is limited to one suspected area at a time. Therefore, the identification probability is:

$$P_{id|det} = P_{free\ EO\ in\ place} \cdot P_{id|in\ place} \tag{6}$$

The first term in 6 is the probability that the suspected target is within range of an EO payload that is not already engaged. Identification can be made until the suspect is declared "outdated", as it is necessary to declare suspects as "outdated" in order to deal with new targets when failing to engage previous suspects or targets.

$$P_{free\ EO\ in\ place} = P_{EO\ in\ place} \cdot P_{free\ EO} \tag{7}$$

The first term in 7 is similar to the case of detection. The second term depends on the following parameters:

- Rate of events.
- Number of EO payloads - allows for "parallel processing" of targets.
- Communication - allows for balancing the load between EO payloads.

- Outdating time - allows to free an EO payload from previous engagement.

The information about the detected suspect must be available to the EO payload for identification. Therefore, the probability to identify given that the EO payload is in place:

$$P_{id|in\ place} = P_{info|detect} \cdot P_{id|info} \tag{8}$$

This information depends mainly on communication between the UAVs:

- If no communication is used, the suspect's information can arrive only from the UAV itself. Thus the probability is 1 for a UAV that detected the suspect and 0 otherwise.
- If communication is used, the suspect's information arrives from any detection, meaning that the probability is 1.

The second term of 8 is given by the following:

$$P_{id|info} = P_{id|in\ FOV} \cdot P_{in\ FOV|suspect\ info} \tag{9}$$

and:

$$P_{id|in\ FOV} = 1 - [1 - P(D_i^{EO} \geq C)]^{t_{outdate}} \tag{10}$$

D_i^{EO} - the i'th EO detection probability at (x,y)

C - the terrain complexity at (x,y)

The probability that the suspect is in FOV given the last known location of it, is roughly given by:

$$P_{in\ FOV|info} = \frac{\Delta x_{suspect}}{FOV} = \frac{\bar{v}_{suspect} \cdot \Delta t}{FOV} \tag{11}$$

Eq. 11 shows that as time passes since the last suspect location update, the chances that the target will be in the payload's FOV are diminishing. This supports the "outdating" of targets and suspects.

In summary, the probability to identify is dependant on many factors, but it is evident that:

- It may be beneficial to have more EO payloads than events.
- Using communication improves the chances to balance the load between EO payloads.
- It is important to "tune" the time it takes until a suspect/target is declared as "outdated".

c) Tracking: Tracking can be achieved by means of an EO payload or a Ground Moving Target Indicator (GMTI) radar. Therefore, the probability to track a suspect/target is:

$$P_{track|detect} = P^{EO}_{track|detect} + P^{G}_{track} \tag{12}$$

Note the two main differences between EO payloads' and GMTI payloads' tracking capabilities:

- EO payloads need prior knowledge on the enemy location, whereas GMTI payloads can detect new suspects according to their movement.
- EO payloads are limited to tracking within their FOV, whereas GMTI payloads may cover the entire footprint in the payload's range.

In order to track a target, an EO payload functions exactly as in identifying a target. Therefore, the same probability is valid for both cases. In the case of a GMTI payload the tracking probability is the same as the detection probability in the general case.

Tracking conclusions:

- Since EO payloads can engage a single target at a time, they are considered as "expensive" in terms of resources assignment and only one EO payload will be assigned to a known enemy at a time. This limits the probability of a successful EO tracking.
- GMTI payloads, however, are able to track as many targets as there are in their potential footprint, and so using more GMTI payloads is of great benefit to a successful tracking probability.

d) Causing Damage to a Target: A munition is launched given that the target is identified and tracked, and the following conditions are met:

- There is a UAV in munition range from the target.
- The UAV has information about the target's identification and location.

Once the munition is airborne the conditions that it will damage the target are:

- Navigate - the target's location is updated at least once every few time-steps (the autonomous flight time of the munition).
- Hit - once the munition is in proximity to the target it identifies it and homes itself on it.

The probability of a successful launch at a target is therefore defined as:

$$P_{dam|id} = P_{in\ place} \cdot P_{info|id} \cdot P_{nav|launch} \cdot P_{dam|nav} \tag{13}$$

The first term is similar to other payloads in place cases. Note that as the scenario continues, more munitions are used, making it less probable that a munition is located in range of a target. In this case it is possible to track the target and wait for an armed UAV, if such a UAV is within communication range and not engaged. The probability to have target information was discussed earlier.

The munition is assumed to successfully navigate if at least one tracking device will track the target at least once before the munition considers it to be lost and aborts itself (a common safety mechanism to minimize collateral damage). Therefore:

$$P_{nav|launch} = 1 - (1 - P_{t|det})^{t_{autonomous}} \tag{14}$$

The probability that the munition damages the target depends mostly on the munition's capabilities, which are not in the scope of this work.

e) Target Destruction: The last probability in eq. 1 depends on the munition's ability to destroy a target when hitting it. Sometimes the term hit-to-kill ratio, or how many hits are necessary to destroy a target (in average), is used.

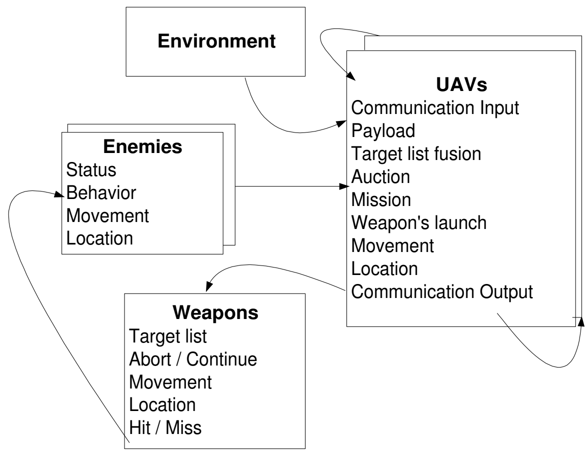

Fig. 2. Simulated Entities - Block Diagram

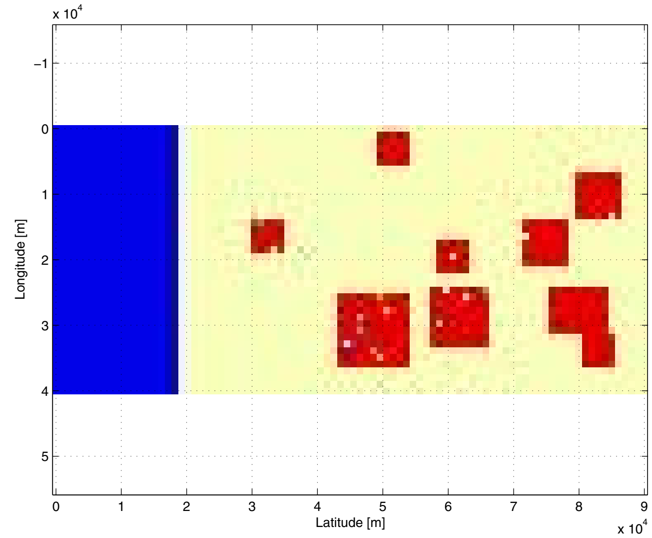

Fig. 3. Environment Map. The dark spots represent the higher probabilities to lose a target, or "towns". The black region to the left represents a "sea" where no enemies would move

III. Simulated Environment

A. Simulation Overview

The simulation is written in MATLAB. Fig. 2 shows the simulated entities and their relationships in a block diagram. The main entities in the simulation are:

- Environment - a simulated area where the enemy forces move.
- Enemies - representing a number of enemy vehicles, and updated every time-step.
- UAVs - representing a number of agents, and updated every time-step.
- Munitions - representing a number of munitions, launched by the UAVs at a target. A munition is initialized at launch, updated every time-step until it is terminated by either aborting or hitting the target.

1) The Environment: The environment is a grid of probability values, representing the probability to lose a target. The whole environment is assigned a minimum probability, representing "open" terrain, and then a number of randomly sized "urban" terrains are added, by assigning higher probability values to lose a target. This can be visualized as seen in fig. 3. Dark spots represent the higher probabilities to lose a target, or "towns".

2) Enemy: The enemy is simulated as multiple non-cooperative agents. Non-cooperative behavior is assumed to be valid for para-military groups with little to no organization. Each enemy agent's location and velocity is initialized to a random number, and then for every time-step, all the enemy agents are updated in the following process:

- Status - "Alive" until "killed" by the UAVs munitions.
- Behavior - the enemy agent's default behavior is to move. A random decision causes it to create an event and remain static for a random period of time. Then, the agent shall continue to move.
- Movement - the enemy is assumed to take a "fire and run" policy, where after an event it moves in a varying velocity.
- Location - the enemy's location is integrated from the previous time-step, using the enemy's velocity.

B. Group Capabilities

The UAVs are simulated as multiple cooperative agents. Each UAV's location is initialized so the UAVs are equally distributed in the environment. Then, each UAV of the flock performs the following two algorithms:

1) Task Assignment Algorithm: Each UAV receives information from communication and gathers information on the environment using its payloads. Then, the UAV makes a list of targets and tasks need to be performed and calculates its ability to perform the various tasks (*bid*). Finally, each UAV calculates the result of the *auction*, i.e. checks if its bid is the best (given the information received by the UAV from communication). If so, it assigns itself to the task it is best fitted to do.

Bid calculation of a task is based on two main criteria:

- The UAV current assignment. If it is assigned, the UAV will not bid for more tasks until finishing the current assignment.
- The UAV's ability to perform a task according to the task and its payloads' capabilities.

The UAVs transmit to the other UAVs their list of targets and for each target their list of tasks' bids. If there exists communication between the UAVs, each one can check if it is the best bidder and therefore the winner of the auction to perform a task. If no communication exists, the agent assumes that it is the best bidder and assign itself to the task, making it possible for more than one agent to perform a single task and the whole group may perform less efficiently.

The algorithm assumes that it is enough for one UAV to deal with a certain task. This may not be the case in a complicated task (e.g. following a vehicle in a city). The auction mechanism proposed earlier can be generalized to choosing a number of agents that are best bidders.

Another note is that once a UAV has assigned itself to a task, the UAV will perform this task until the target is killed or lost. This may exclude the UAV from a future auction and may inflict suboptimal behavior.

2) Flocking Algorithm: The flocking algorithm in use is the implementation of Reynolds' behavioral algorithm [22]. Each agent calculates its desired velocity as follows:

$$\vec{v}_d^k = \sum_{i=1}^{4} w_i^k \vec{v}_i^k \tag{15}$$

where w is a weight function, k is UAV's index and i is algorithm law index, given by the following list:

1) Alignment - causing the UAV's velocity to match the mean velocity vector of the group.
2) Cohesion - causing the UAV to converge with the other UAVs.
3) Separation - causing the UAV to disperse from the other UAVs.
4) Collision Avoidance - keeping the UAV from colliding with the nearest neighbors.

C. UAV Kinematics

A UAV has two modes of movement:

- Flocking - a UAV will flock with the other UAVs until it commits itself to deal with a target.
- Target following - when a UAV is committed to a target it will fly at full speed to the last known target's position. This mode is used until the target is killed or deemed as "lost".

The desired velocity is translated into acceleration, velocity and position using the following equations:

$$\vec{a^k}(t) = \frac{(\vec{v^k} \times \vec{v_d^k}) \times \vec{v^k})}{\| (\vec{v^k}) \|^2 \| \vec{v_d^k} \|} g\sqrt{(n_{max}^k)^2 - 1} \tag{16}$$

$$\vec{v^k}(t) = \vec{v^k}(t - \Delta t) + \vec{a^k}(t)\Delta t \tag{17}$$

$$\vec{x^k}(t) = \vec{x^k}(t - \Delta t) + \vec{v^k}(t)\Delta t \tag{18}$$

IV. Results and Discussion

The performance of the group is measured using 3 parameters:

- Effectiveness - measured by the following ratio:

$$effectiveness = \frac{K}{E} \tag{19}$$

where K is the number of targets killed and E is number of enemy events.

- Efficiency - measured by the following ratio:

$$efficiency = \frac{K}{LM} \tag{20}$$

where K is the same as earlier and LM is the number of launched munitions at the targets. Since the flock's number of munitions is finite, this measurement is important.

- Complexity - the complexity of the algorithm is dependent on various properties. The more complex the case is, the harder it becomes to implement it in a real-time UAV mission computer. We measure the complexity according to the mean length of the UAV's target list, which is the main input to the task assignment algorithm.

TABLE I
Dudek's Taxonomy's properties used in the experiments

Axis	Subdivision	Value/Remarks
Collective size	SIZE-ALONE	1
	SIZE-PAIR	2
	SIZE-LIM	3-10
	SIZE-INF	N/A
Communication Range	COM-NONE	0
	COM-NEAR	10,000m
	COM-INF	1e10 m
Communication Topology	TOP-BROAD	Used always
	TOP-ADD	N/A
	TOP-GRAPH	N/A
	TOP-TREE	N/A
Communication Bandwidth	BAND-ZERO	same as COMM-NONE
	BAND-LOW	not used
	BAND-MOTION	self-created target list
	BAND-INF	entire target list
Collective Reconfigurability	ARR-STATIC	Dependent on UAV velocity
	ARR-COM	which is dependent on
	ARR-DYN	number of group size
Processing Ability	PROC-SUM	N/A
	PROC-FSA	N/A
	PROC-PDA	N/A
	PROC-TME	used always
Collective Composition	CMP-IDENT	used
	CMP-HOM	same as CMP-IDENT
	CMP-HET	used

Table I summarizes Dudek's taxonomy's properties and the values that are assigned to them in the experiments. The flock's total capabilities (payloads' footprints, number of munitions) are taken as equal for any number of UAVs. Meaning that the more UAVs in the flock results with a less capable UAV.

Figure 4 shows the results of a Monte-Carlo simulation. Notably, using more UAVs improves the ability to perform multiple missions in parallel, making the group more effective against multiple events, or enemy coordination. This result is true for any communication used, including no-communication. However, for large number of UAVs, the group is becoming less effective, since for every target more than one UAV is needed (e.g., consumption of all munitions).

Using communication improves, as expected, the group's ability to perform cooperatively, as can be seen in its efficiency. However, this is achieved at the cost of computational cost, as the UAVs create longer target lists. The length of the target list in the case of infinite communication is linear with the number of UAVs, since they echo the

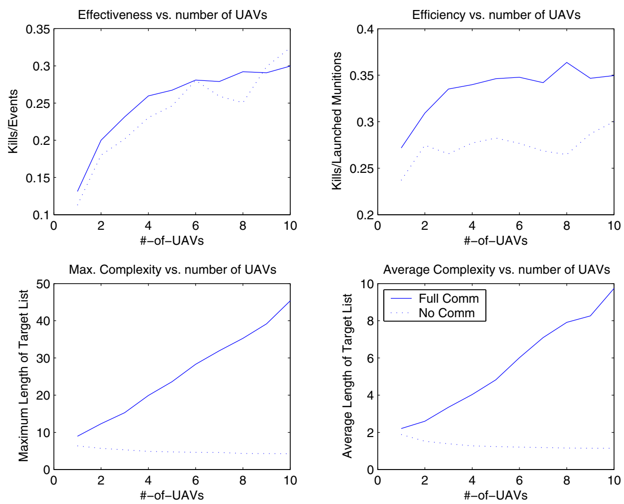

Fig. 4. Simulation Results

target's to one another. Reducing the size of the target list can be achieved by forcing the UAVs to communicate only "their" target, thus reducing bandwidth, or limiting the communications range. Notably, the flock's performance under limited communication does not fall from the case of infinite communication. Using no communication, however, causes the UAV's target list to consist of the targets each UAV detected. Since we decrease the detector's range in order to keep the total group's capabilities constant with the number of UAVs, the total target list length decreases.

V. Conclusions

We use the taxonomy proposed by Dudek et al. in order to investigate the performance of a group of autonomous UAVs cooperating in mission execution against a group of enemy agents acting in an unknown environment. We showed that increasing the number of UAVs in the group proves to be beneficial as it allows the group to react to more enemy events (e.g. SAM launches). We also show that using communication helps creating better cooperation between the flock members, however using infinite communication has a great cost of computational complexity, and it may be sufficient to use finite range or bandwidth communication.

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