Post-Filtering Output Feedback Variable Structure Control

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***Abstract*— A full state constructed sliding mode is popularly used in a variable structure control system. To encounter with the insufficient measurement problem of the system state, a post-filtering approach to variable structure output feedback control is proposed. The proposed post-filter is used to convert the linear time invariant SISO system output into a pseudo-state sliding mode dynamics which is suitable for conventional VSS control law. It is a dynamic variable structure output feedback approach without a state observer. Stability of the digital post-filter is studied judiciously. The noncausal problem, due to the system with relative degree higher than one, discretization for inverse transfer function of the post-filter is recommended and concluded with the stable zeros conditions proposed by Åström *et. al.*.**

I. Introduction

The state feedback control concept is widely used in various control theory. In the variable structure control (VSC), a sliding mode hyperplane is set up deliberately such that perfect rejection of the matching disturbance is amazingly achieved [1]. However, the requirement of full state accessibility constrained the design engineers to realize the variable structure system (VSS). The extra sensor modules need be arranged in order to avoid the unexpected signal interference. This complicates the design problem and also brings up the overall system cost. The state estimator (or the state observer) first proposed by Luenberger is the most direct solution [2]-[3]. The unavailable system state is substituted with the approximated one. After that the various observer-based VSC controllers have still been developed. Since the system output can be more easily and economically obtained than the system state in most physical systems, the system output-based VSC approaches also attracted lots of interest in the past few decades.

If the system state is neither measurable nor accessible, the observer-type VSS approach provides the most general scheme to solve this problem [4]-[5]. Linear or sliding mode observers are popularly used in conjunction with conventional VSC to serve the control purposes. Zhu *et al.* developed the combined algorithm for robot control. The interaction of the controller and the observer is investigated [6]. Rundell *et al.* presented the combined approach for stabilization of the rotational motion of a vertical shaft magnetic bearing. The sliding mode observer is used to estimate the system state and the disturbance [7]. Korondi *et al.* introduced an observer-based discrete-time sliding mode controller for the two-mass system coupled by a flexible shaft [8]. Ahmed-Ali *et al.* combined the sliding observer with a sliding mode via the backstepping procedure. The output tracking for a class of uncertain triangular nonlinear systems is achieved [9]. Xiong *et al.* proposed the sliding mode observer for nonlinear uncertain systems with much less conservative conditions. They rendered the nonlinear system into two subsystems and then used the equivalent control concept based on this coordinate transformation [10].

Since the overall dynamic system becomes more complex while the auxiliary state observer is involved, the system output-based VSC approaches sprang up. Some of them sought to construct a static sliding surface in the range of the output subspace [11]-[13]. The remarkable advantage is to keep off the state observer in the control loop. This concise control framework is very contributive to realization. Nevertheless, this leads to an output feedback eigenstructure assignment problem with reduced order dynamics. To carry out the design procedure, the dynamic system must satisfy the well-known 'Kimura-Davidson' conditions [12]. The dynamic compensator of appropriate size is then developed without the above restriction [14]-[15]. A dynamic output feedback control generally has better performance than that of static feedback but the design procedure is more complicated. In [14], both the compensator type and the observer type of controllers are proposed by Diong *et al.* for stabilizing the multivariable linear systems. The major difference is that the observer type introduces the control as an input to the controller, while the compensator type does not. Furthermore, the compensator type controller requires a minimum phase condition, while the observer type does not [14]. Edwards *et al.* presented the dynamic compensator based output feedback sliding mode controller that need not

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satisfy the 'Kimura-Davidson' conditions [15]. Saaj *et al.* combined the sliding controller with the fast output sampling feedback technique for single-input-single-output (SISO) system. By proper design in the reaching law and the fast output sampling feedback technique, this methodology is more practical and easy for realization [16]. It can be considered as an improved version for the static output feedback approach. Ha *et al.* presented the dynamic output feedback sliding mode controller based on the pole placement technique [17]. A linear functional observer is also developed to implement the sliding function and the equivalent control. In this paper, a novel variable structure output feedback approach with a post-filter is proposed. Figure 1 shows the closed-loop system functional block diagram which is similar to the idea summarized by Young *et al.* [18], but the broad meaning of the called 'filtering' function we used here can be whatever type of control process.

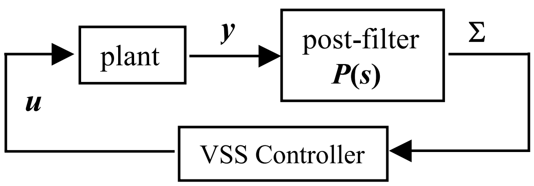

Figure 1. The functional block diagram of proposed closed-loop system

In our approach, the sliding vector is designed first as if the system state is fully accessible. The full state-sliding vector is a baseline and can be easily constructed according to the VSC theory. The proposed post-filter $P(s)$ is used to convert the linear time-invariant SISO system output into the pseudo state sliding mode dynamics. The synthesized sliding mode dynamics is then introduced into the conventional variable structure control law instead of the full state one.

The other parts of this paper are organized as following. Section II represents the design concept to synthesis the sliding mode dynamics briefly. Section III discusses both the digital realization problem and the stability issue. The numerical simulation examples are given in section IV. Final section is the conclusion.

II. The Post-Filter

Consider a linear time invariant (LTI) SISO system with relative degree one

$$\dot{x}_i(t) = x_{i+1}(t), \quad i = 1, \dots, n-1.$$

$$\dot{x}_n(t) = \sum_{j=1}^{n} (-\delta_j) x_j(t) + u(t) + v(t), \qquad (1a)$$

$$y(t) = \sum_{j=1}^{n} c_{j-1} x_j(t), \qquad (1b)$$

where $x_1(t)$, …, $x_n(t)$ are the state variables, $-\delta_1$, …, $-\delta_n$ are the relative coefficients, $u(t)$ is the control input, $y(t)$ is the system output, c_0, …, c_{n-1} are the relative output coefficients, and the unknown disturbance $v(t)$ is a smooth function of time with known bound

$$|\, v(t) \,| < v_{\max}. \qquad (1c)$$

Let $s=d/dt$ denote the differential operator. Using the Laplace transformation, the transfer function of the above LTI SISO system is shown in Figure 2.

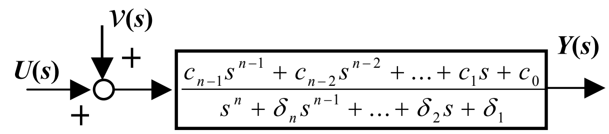

Figure 2. The transfer function of the LTI SISO system

If the system state is neither measurable nor accessible, a post-filter is introduced to construct a pseudo-state sliding surface from the system output. The synthesized one, instead of using the originally designed full-state sliding surface, is still complied with the conventional VSC theory as if the system state is fully accessible. At the beginning, the full state-sliding vector is designed deliberately in order to acquire desired performance and becomes a baseline for the constructed pseudo-state one. According to the VSC theory, the prescribed full-state sliding surface $\Sigma(x)$ is

$$\Sigma(x) = \Lambda x = [\lambda_0 \;\; \lambda_1 \;\; \dots \;\; \lambda_{n-2} \;\; 1]x = 0, \qquad (2)$$

where $x \in R^n$ is the state vector, and $\Lambda \in R^{1 \times n}$ is chosen properly. The relationship of the sliding surface and the system output can be found as

$$c_{n-1}\Sigma^{(n-1)}(x) + c_{n-2}\Sigma^{(n-2)}(x) + \cdots + c_1\Sigma^{(1)}(x) + c_0\Sigma(x) =$$
$$y^{(n-1)}(x) + \lambda_{n-2}y^{(n-2)}(x) + \cdots + \lambda_1 y^{(1)}(x) + \lambda_0 y(x). \qquad (3)$$

***proof*:**

The dynamic equation (1) can be written as following

$$x_1^{(0)} = x_1,$$
$$x_1^{(1)} = x_2,$$
$$x_1^{(2)} = x_2^{(1)} = x_3,$$
$$\dots,$$
$$x_1^{(n-1)} = x_{n-1}^{(1)} = x_n,$$
$$x_1^{(n)} = x_n^{(1)} = -\delta_n x_n - \delta_{n-1}x_{n-1} - \cdots - \delta_1 x_1 + u + v,$$
$$y = c_{n-1}x_n + c_{n-2}x_{n-1} + \cdots + c_1 x_2 + c_0 x_1. \qquad (4)$$

The full-state sliding surface (2) becomes

$$\Sigma(x) = x_n + \lambda_{n-2}x_{n-1} + \cdots + \lambda_1 x_2 + \lambda_0 x_1$$
$$= x_{n-1}^{(1)} + \lambda_{n-2}x_{n-2}^{(1)} + \cdots + \lambda_1 x_1^{(1)} + \lambda_0 x_1$$
$$= x_1^{(n-1)} + \lambda_{n-2}x_1^{(n-2)} + \cdots + \lambda_1 x_1^{(1)} + \lambda_0 x_1. \qquad (5)$$

Taking differentiation of (5) with respect to time in sequence, we obtain

$$\begin{aligned}\Sigma^{(1)}(x) &= x_n^{(1)} + \lambda_{n-2}x_{n-1}^{(1)} + \cdots + \lambda_1 x_2^{(1)} + \lambda_0 x_1^{(1)} \\ &= x_{n-1}^{(2)} + \lambda_{n-2}x_{n-2}^{(2)} + \cdots + \lambda_1 x_1^{(2)} + \lambda_0 x_1^{(1)} \\ &= x_2^{(n-1)} + \lambda_{n-2}x_2^{(n-2)} + \cdots + \lambda_1 x_2^{(1)} + \lambda_0 x_2 , \qquad (6)\end{aligned}$$

……..

$$\begin{aligned}\Sigma^{(j-1)}(x) &= x_n^{(j-1)} + \lambda_{n-2}x_{n-1}^{(j-1)} + \cdots + \lambda_1 x_2^{(j-1)} + \lambda_0 x_1^{(j-1)} \\ &= x_{n-1}^{(j)} + \lambda_{n-2}x_{n-2}^{(j)} + \cdots + \lambda_1 x_1^{(j)} + \lambda_0 x_1^{(j-1)} \\ &= x_j^{(n-1)} + \lambda_{n-2}x_j^{(n-2)} + \cdots + \lambda_1 x_j^{(1)} + \lambda_0 x_j , \qquad (7)\end{aligned}$$

……..

$$\begin{aligned}\Sigma^{(n-1)}(x) &= x_n^{(n-1)} + \lambda_{n-2}x_{n-1}^{(n-1)} + \cdots + \lambda_1 x_2^{(n-1)} + \lambda_0 x_1^{(n-1)} \\ &= x_{n-1}^{(n)} + \lambda_{n-2}x_{n-2}^{(n)} + \cdots + \lambda_1 x_1^{(n)} + \lambda_0 x_1^{(n-1)} \\ &= x_n^{(n-1)} + \lambda_{n-2}x_n^{(n-2)} + \cdots + \lambda_1 x_n^{(1)} + \lambda_0 x_n , \qquad (8)\end{aligned}$$

The above differential equations can be summarized as

$$\begin{aligned}\Sigma^{(j)}(x) &= x_{j+1}^{(n-1)} + \lambda_{n-2}x_{j+1}^{(n-2)} + \lambda_{n-3}x_{j+1}^{(n-3)} + \cdots \\ &\cdots + \lambda_1 x_{j+1}^{(1)} + \lambda_0 x_{j+1}^{(0)}, \text{ for } j = 0,\ 1,\ \ldots,\ (n-1). \qquad (9)\end{aligned}$$

To find out the relationship between the sliding surface and the system output, we define the auxiliary equation as

$$\eta(x) = \begin{bmatrix} c_0 & c_1 & \ldots & c_{n-2} & c_{n-1} \end{bmatrix} \begin{bmatrix} \Sigma^{(0)}(x) \\ \Sigma^{(1)}(x) \\ \vdots \\ \Sigma^{(n-2)}(x) \\ \Sigma^{(n-1)}(x) \end{bmatrix} = \sum_{j=0}^{n-1} c_j \Sigma^{(j)}(x). \qquad (10)$$

To introduce (9) into (10), the auxiliary equation becomes

$$\begin{aligned}\eta(x) &= c_{n-1}\Sigma^{(n-1)}(x) + c_{n-2}\Sigma^{(n-2)}(x) + \cdots + c_1\Sigma^{(1)}(x) + c_0\Sigma(x) \\ &= c_{n-1}x_n^{(n-1)} + c_{n-1}\lambda_{n-2}x_n^{(n-2)} + \ldots + c_{n-1}\lambda_1 x_n^{(1)} + c_{n-1}\lambda_0 x_n^{(0)} \\ &\quad + c_{n-2}x_{n-1}^{(n-1)} + c_{n-2}\lambda_{n-2}x_{n-1}^{(n-2)} + \cdots + c_{n-2}\lambda_1 x_{n-1}^{(1)} + c_{n-2}\lambda_0 x_{n-1}^{(0)} \\ &\quad + \ldots.. \\ &\quad + c_1 x_2^{(n-1)} + c_1\lambda_{n-2}x_2^{(n-2)} + \cdots + c_1\lambda_1 x_2^{(1)} + c_1\lambda_0 x_2^{(0)} \\ &\quad + c_0 x_1^{(n-1)} + c_0\lambda_{n-2}x_1^{(n-2)} + \cdots + c_0\lambda_1 x_1^{(1)} + c_0\lambda_0 x_1^{(0)} \qquad (11)\end{aligned}$$

Therefore,

$$\eta(x) = \begin{bmatrix} c_0 & c_1 & \ldots & c_{n-2} & c_{n-1} \end{bmatrix} \begin{bmatrix} x_1^{(n-1)} \\ x_2^{(n-1)} \\ \vdots \\ x_{n-1}^{(n-1)} \\ x_n^{(n-1)} \end{bmatrix} +$$

$$\lambda_{n-2}\begin{bmatrix} c_0 & c_1 & \ldots & c_{n-2} & c_{n-1} \end{bmatrix} \begin{bmatrix} x_1^{(n-2)} \\ x_2^{(n-2)} \\ \vdots \\ x_{n-1}^{(n-2)} \\ x_n^{(n-2)} \end{bmatrix} + \ldots..$$

$$\ldots.. + \lambda_1\begin{bmatrix} c_0 & c_1 & \ldots & c_{n-2} & c_{n-1} \end{bmatrix} \begin{bmatrix} x_1^{(1)} \\ x_2^{(1)} \\ \vdots \\ x_{n-1}^{(1)} \\ x_n^{(1)} \end{bmatrix}$$

$$+\lambda_0\begin{bmatrix} c_0 & c_1 & \ldots & c_{n-2} & c_{n-1} \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ \vdots \\ x_{n-1} \\ x_n \end{bmatrix}$$

$$= y^{(n-1)}(x) + \lambda_{n-2}y^{(n-2)}(x) + \cdots + \lambda_1 y^{(1)}(x) + \lambda_0 y(x). \qquad (12)$$

If the system state is not fully accessible, the fictitious sliding surface can be obtained with the SISO system output information only. Let $Y(s)$ and $\Sigma(s)$ be the Laplace transformation of the system output and the sliding surface respectively. The post-filter, $P(s)=\Sigma(s)/Y(s)$, becomes

$$P(s) = \frac{\Sigma(s)}{Y(s)} = \frac{s^{n-1} + \lambda_{n-2}s^{n-2} + \cdots + \lambda_1 s + \lambda_0}{c_{n-1}s^{n-1} + c_{n-2}s^{n-2} + \cdots + c_1 s + c_0}. \qquad (13)$$

After the sliding mode dynamics is successfully synthesized, the conventional VSS control methods can be easily applied.

Lemma 1. For a LTI SISO system with relative degree one, the post-filter (13) is able to convert the system output information into the pseudo-state sliding mode dynamics (3).

III. Digital Realization of the Post-Filter $P(s)$

Due to the revolutionized diffusion of digital controllers, discrete time implementations of variable structure systems become one of the main topics in modern VSC theory. Properness and stability of the proposed post-filter are both of the major concerns in digital realization. If $P(s)$ is proper, it can be discretized directly for digital implementation. Otherwise, we encounter a realization problem with the relative degree higher than one. The realization approaches and the stability issue are presented and discussed in this section.

A. Discretization

(i) If the coefficient $c_{n-1}\neq 0$, $P(s)$ is obviously proper. To discretize (13) by zero-order-hold process yields the general form

$$P(z) = \frac{\Sigma(z)}{Y(z)} = \frac{b_0 z^{n-1} + b_1 z^{n-2} + \cdots + b_{n-2}z + b_{n-1}}{z^{n-1} + a_1 z^{n-2} + \cdots + a_{n-2}z + a_{n-1}}, \qquad (14)$$

where $b_0, \ldots, b_{n-1}, a_1, \ldots, a_{n-1}$ are suitable coefficients. The discretized post-filter becomes the $(n-1)^{th}$–order general form of the infinite-impulse-response (IIR) filter

$$P(z^{-1}) = \frac{\Sigma(z^{-1})}{Y(z^{-1})} = \frac{b_0 + b_1 z^{-1} + \cdots + b_{n-2}z^{-n+2} + b_{n-1}z^{-n+1}}{1 + a_1 z^{-1} + \cdots + a_{n-2}z^{-n+2} + a_{n-1}z^{-n+1}}$$

$$=(\sum_{i=0}^{n-1} b_i z^{-i}) \Big/ (\sum_{i=0}^{n-1} a_i z^{-i}) , \quad (15)$$

where a_0=1. Therefore, the discrete-time sliding mode Σ_k now becomes

$$\Sigma_k = b_0 y_k + b_1 y_{k-1} + \cdots + b_{n-2} y_{k-n+2} + b_{n-1} y_{k-n+1} - a_1 \Sigma_{k-1} - \cdots$$
$$\cdots - a_{n-2}\Sigma_{k-n+2} - a_{n-1}\Sigma_{k-n+1} = \sum_{i=0}^{n-1} b_i y_{k-i} - \sum_{i=1}^{n-1} a_i \Sigma_{k-i} . \quad (16)$$

(ii) If $c_{n-1} = 0$ and $c_{n-2} \neq 0$, the inverse of $P(s)$ is discretized instead of $P(s)$ and yields

$$\frac{1}{P(z)} = \frac{Y(z)}{\Sigma(z)} = \frac{\alpha_0 z^{n-2} + \alpha_1 z^{n-3} + \cdots + \alpha_{n-3} z + \alpha_{n-2}}{z^{n-1} + \beta_1 z^{n-2} + \cdots + \beta_{n-2} z + \beta_{n-1}} , (17)$$

where $\alpha_0, \alpha_1, \ldots, \alpha_{n-2}, \beta_1, \beta_2, \ldots, \beta_{n-1}$ are suitable coefficients. The difference equation becomes

$$y_{k+n-1} + \beta_1 y_{k+n-2} + \cdots + \beta_{n-1} y_k =$$
$$\alpha_0 \Sigma_{k+n-2} + \cdots + \alpha_{n-3}\Sigma_{k+1} + \alpha_{n-2}\Sigma_k . \quad (18)$$

If $\alpha_0 \neq 0$, the discrete-time sliding mode Σ_k can be expressed as

$$\Sigma_k = \frac{1}{\alpha_0}(y_{k+1} + \beta_1 y_k + \cdots + \beta_{n-1} y_{k-n+2} - \alpha_1 \Sigma_{k-1} - \cdots$$
$$\cdots - \alpha_{n-3}\Sigma_{k-n+3} - \alpha_{n-2}\Sigma_{k-n+2}). \quad (19)$$

However, the above equation is not causal since the future output value y_{k+1} is involved at the k^{th} sampling instant. If the system is designed to be BIBO stable, a similar idea used in [19] is introduced to approximate Σ_k with Σ_{k-1}. Hence,

$$\Sigma_{k-1} = \frac{1}{\alpha_0}(y_k + \beta_1 y_{k-1} + \cdots + \beta_{n-1} y_{k-n+1} - \alpha_1 \Sigma_{k-2} - \cdots$$
$$\cdots - \alpha_{n-3}\Sigma_{k-n+2} - \alpha_{n-2}\Sigma_{k-n+1})$$
$$= \frac{1}{\alpha_0}[\sum_{i=0}^{n-1} \beta_i y_{k-i} - \sum_{i=1}^{n-2} \alpha_i \Sigma_{k-(i+1)}] \cong \Sigma_k , \quad (20)$$

where β_0=1.

The main idea proposed in this paper is to create an auxiliary dynamic function such that the SISO system output can be converted into the sliding mode dynamics. This provides a solution while the system state is neither measurable nor accessible. Furthermore, the conventional VSC theory still can be applied as if the full system state is accessible. It is seen that the post-filter $P(s)$ may not be realizable if $c_{n-1} = 0$. A compromise approach is to choose the approximated sliding mode (20) if the steady state tolerance of the VSC system is accepted.

B. Stability of the post-filter

To discretize a continuous-time system via the zero-order-hold process, all stable poles are transformed into the unit circle. This is a common sense to check the stability of the proper $P(s)$. If $P(s)$ is improper, we suggested $1/P(s)$ to be discretized instead of $P(s)$. The zeros of $P(s)$, decided by the coefficients of the sliding vector, are equivalent to the poles of $1/P(s)$ in this case. However, the stable condition of zeros between continuous-time system and its discrete-time counterpart is not obvious. Åström *et. al.* first pointed out this issue and attracted lots of investigation later. Åström *et. al.* show the following conditions to guarantee the stable zeros of the SISO system [20]. If

$$\begin{aligned} &(i)\ \mathrm{Re}(\xi_i) < 0, \\ &(ii)\ 1/P(0) \neq 0, \\ &(iii)\ \pi > \arg P(j\omega) > 0, \quad \mathit{for}\ 0 < \omega < \infty, \end{aligned} \quad (21)$$

where ξ_i are the poles of $1/P(s)$, then all the zeros of $P(z)$ are stable. Since ξ_i are decided by the coefficients set $\Lambda = [\lambda_0\ \lambda_1\ \ldots\ \lambda_{n-2}\ 1]$ of the desired sliding mode dynamics. The above conditions (21) are valuable for us to check the stability problem of the improper post-filter.

Corollary 1. If the stability conditions (21) are assured. The improper post-filter can be digital realized to construct the discrete-time sliding mode dynamics in the variable structure system.

Remark 1. The improper post-filter is due to the physical system with relative degree two or higher. Hence, the extra derivative of the system output is unavoidable. Fridman pointed out that any differentiation would involve some parasitic dynamics that would inevitably result in loss of accuracy and chattering [21]. Although the discretized problem of the improper $P(s)$ can be solved by **Corollary 1**, the seriously degraded system performance should be evaluated carefully in practical applications.

IV. Numerical Example

Let $\Sigma(x) = \Lambda x = [\lambda_0\ \lambda_1\ \ 1]x = [1\ \ 1\ \ 1]x$. If a given dynamic system is full state accessible, the most conventional and extensively used discrete-time VSS control is

$$u_k = -K[sgn(\Sigma_k)], \quad (22)$$

where K is the proper gain value. This switching type VSS control (22) will be used as a baseline to evaluate the closed-loop performance with the proposed post-filter.

A. P(s) is proper

Given the linear time-invariant SISO system

$$\dot{x} = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & -3 & 3 \end{bmatrix} x + \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} u + \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} v, \quad (23a)$$

$$y = [\tfrac{1}{2}\ \ \tfrac{1}{9}\ \ 1]x , \quad (23b)$$

with the initial condition $x_1(0)$=3, $x_2(0)$=−1, $x_3(0)$=2, and the unknown external disturbance $v(t) = 0.5 + 5\sin(0.1t) - \cos t$. If the system state of (23) is not accessible, the post-filter $P(s)$ is constructed as

$$P(s)=\frac{\Sigma(s)}{Y(s)}=\frac{s^2+\lambda_1 s+\lambda_0}{c_2 s^2+c_1 s+c_0}=\frac{s^2+s+1}{s^2+\frac{1}{9}s+\frac{1}{2}}. \quad (24)$$

To discretize $P(s)$ with the sampling period T=0.01 second yields

$$P(z)=\frac{\Sigma(z)}{Y(z)}=\frac{z^2-1.9899z+0.99}{z^2-1.9988z+0.9989}. \quad (25)$$

From (25), the sliding mode dynamics becomes

$$\Sigma_k=y_k-1.9899y_{k-1}+0.99y_{k-2}+1.9988\Sigma_{k-1}-0.9989\Sigma_{k-2}. \quad (26)$$

Now (26) can be simply introduced to the conventional VSS control (22). Let the gain value K=12. The closed-loop system performance is compared and shown in Figure 3. The comparison of the state trajectory is shown in Figure 4.

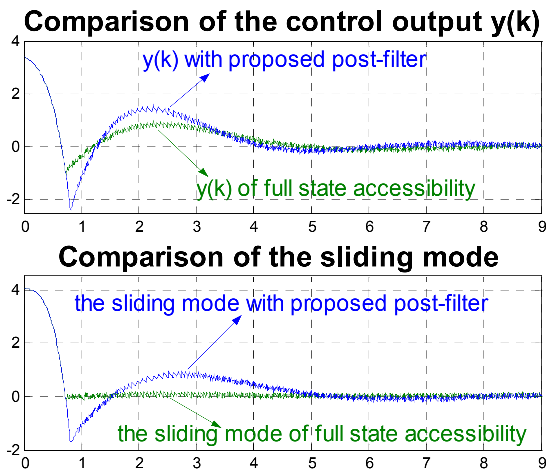

Figure 3. The comparison of the closed-loop system performance

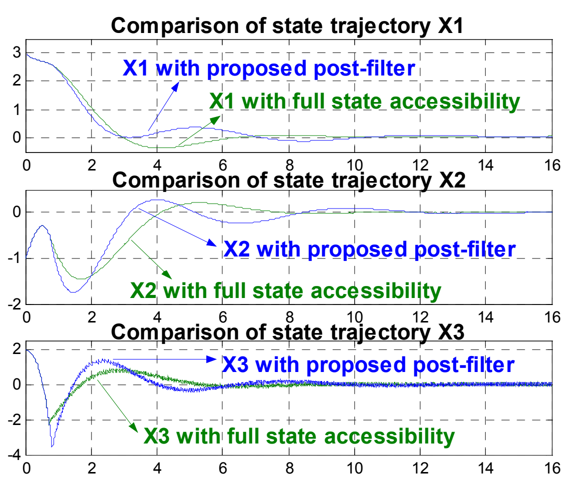

Figure 4. The comparison of the state trajectory

It is obviously that the proposed approach with the fictitious sliding variable, obtained from the system output via the post-filter, still has perfect rejection of the matching disturbance in the VSS.

B. A flexible system with improper P(s) [22]

Let us consider a similar system model used by Meckl *et. al.* as follows [22]

$$\frac{Y(s)}{U(s)}=\frac{bs+k}{(m_1+m_2)(s^4+bs^3+ks^2)}, \quad (27)$$

where $U(s)$ and $Y(s)$ represent the Laplace transform of the applied force u and the endpoint position y respectively. The nominal system parameters are given by m_1=4, m_2=5, k=8, and b=2. The dynamic equation becomes

$$\begin{aligned}\dot{x}_1&=x_2,\\ \dot{x}_2&=x_3,\\ \dot{x}_3&=x_4,\\ \dot{x}_4&=-8x_3-2x_4+u+v,\\ y&=\frac{8}{9}x_1+\frac{2}{9}x_2,\end{aligned} \quad (28)$$

with the initial condition $x_1(0)$=0.5, $x_2(0)$=−1, $x_3(0)$=2, $x_4(0)$=5, and the external disturbance $v(t)=1+5\sin(10t)-\cos t$. The relative degree of the above mechanical system is 3. Let the sliding surface be $\Sigma(x)=x_4+\lambda_2 x_3+\lambda_1 x_2+\lambda_0 x_1$.

According to (13), the proposed post-filter $P(s)$ is constructed as

$$P(s)=\frac{\Sigma(s)}{Y(s)}=\frac{s^3+\lambda_2 s^2+\lambda_1 s+\lambda_0}{(\frac{2}{9})s+\frac{8}{9}}. \quad (29)$$

The coefficients λ_0, λ_1, and λ_2, which determine the zeros of $P(s)$, are chosen deliberately in order to obtain both of the desired sliding mode dynamics and the stability conditions (21). Since the pole of $P(s)$ is –4, a reasonable choice for the zeros in (29) are –0.5, –2, and –6, respectively. The sliding vector becomes $\Lambda=[\lambda_0\ \ \lambda_1\ \ \lambda_2\ \ 1]=[6\ \ 16\ \ 8.5\ \ 1]$.

To realize $P(s)$ with (17), we obtain

$$\frac{1}{P(z)}=\frac{Y(z)}{\Sigma(z)}$$

$$=\frac{0.1095\text{e-}004z^2+0.0027\text{e-}004z-0.1036\text{e-}004}{z^3-2.9170z^2+2.8355z-0.9185}. \quad (30)$$

The stable zeros of $1/P(z)$ are –0.9851, and 0.9608. The poles of $1/P(z)$ are 0.9950, 0.9802, and 0.9418 which are all in the unit circle. From (20), the approximated sliding mode is

$$\Sigma_{k-1}=91324y_k-266390y_{k-1}+258950y_{k-2}-83881y_{k-3}$$

$$-0.0247\Sigma_{k-2}+0.9461\Sigma_{k-3}\cong\Sigma_k. \quad (31)$$

Now (31) can be applied to the modified VSS control law

$$u_k=-K[sgn(\Sigma_{k-1})]. \quad (32)$$

where gain value K =50. Figure 5 shows the performance of both (32) with improper $P(s)$ and (22) of full state accessibility.

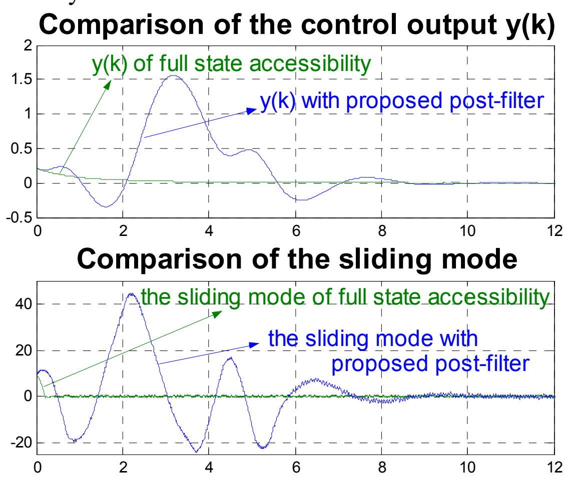

Figure 5. The comparison of closed-loop system performance

We observe that the performance of the full state accessible VSS method is much better than the proposed post-filtering approach. This is due to the system with relative degree two or higher as the analysis by Fridman [21]. Although the proposed approach provides an alternate and simple solution for a LTI SISO system with realtive degree one than ever, this limitation for real applications should be noticed.

V. Conclusion

In most physical systems, the system state is not always accessible. The full-state feedback VSS method has its constraint to deal such a case. In this paper, a post-filtering approach to LTI SISO system output feedback control is proposed. The major significance of the proposed post-filter is to synthesize a pseudo-state sliding surface instead of the full-state sliding surface in VSS control loop. The VSC design procedure can be easily applied with the system output as if the full system state is accessible. The post-filter requires only partial information of the system parameters, c_i's. The uncertainties of δ_i's do not influence the structure of the post-filter. In fact, the variations of δ_i's still remain in the closed-loop system. However, they can be considered as the matched uncertainties for SISO system. Furthermore, since the post-filter does not need the control signal u as input, all exogenous disturbances that enters through the control channels will not affect the constructed sliding mode dynamics. Robustness can be achieved via VSC. However, an improved version of the post-filtering algorithm for the system of relative degree higher than one is necessary for us to investigate in the future.

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