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Design of Cascade Fuzzy Sliding-Mode Controller

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Abstract: **In this paper, a new cascade fuzzy sliding-mode controller (CFSMC) for a class of uncertain underactuated systems is presented. Firstly, two states are chosen to construct the first-layer sliding-mode surface which can be considered as a general state variable. Then the first-level sliding surface and one of the left state variables are used to construct the second-layer sliding surface. This process continues till the last-level sliding surface is obtained. Considering there exists uncertainty in the system's model, fuzzy logic is used as an approximator to identify the uncertainty. Using Lyapunov law, we derive the sliding-mode control law and the related parameters adaptive law of fuzzy logic. The CFSMC can guarantee the stability of all the sliding surfaces, and the simulation results show the validity of this method.**

Keywords: **cascade, sliding-mode control, fuzzy logic, underactuated system**

I. INTRODUCTION

Underactuated systems are characterized by the fact that they have fewer actuators than the degrees of freedom to be controlled. That is to say, if the system has n degrees-of-freedom and m actuators ($m < n$), there are $n-m$ state-dependent equality constraints on the feasible acceleration of the system, which are sometimes referred to as second-order nonholonomic constraints. Underactuated systems have very important applications such as free-flying space robots, underwater robots, manipulators with structural flexibility, overhead crane, etc. It is obvious that underactuated systems have many advantages, which include decreasing the actuators' number, lightening the system, reducing the cost, etc.

Many papers about the control of underactuated mechanical systems models were published in the last few years [1][2]. However, Control of nonlinear underactuated systems has proved to be a challenging problem because the techniques developed for fully actuated systems cannot be directly used.

The sliding mode control (SMC) [3][4] which belongs to a kind of variable structure control system is a nonlinear feedback control whose structure is intentionally changed to achieve the desired performance. Usually, control laws include two parts which are switching control law and equivalent control law in a SMC system. The switching control law is used to drive the system states toward a specific sliding surface, and the equivalent control law guarantees the system states to stay on the sliding surface and converge to zero along the sliding surface. The equivalent control law is related to the system's model. Therefore, it is difficult to design the equivalent control law if the system model is unknown in advance.

Fuzzy controller based on conditional linguistic statements and approximation reasoning has been utilized in many fields where the controlled systems are complex, uncertain or model-free. The success of fuzzy controllers in those fields is due to the fact that it uses the expert control rules of conditional linguistic statements on the relationships of system variables and has the merit of emulating the behavior of a skilled operator. Recently, combinations of fuzzy control and SMC have achieved superior performance, and the fuzzy sliding-mode controller attracts more and more interests of many experts because of its special merits. There exist mainly three kinds of combinations of fuzzy control and SMC: (1) using fuzzy logic to adjust the sign function or saturation function of the SMC [5]; (2) using fuzzy logic to compensate the control of the SMC [6] or regulating the parameters of the SMC using the fuzzy logic [7]; (3) using fuzzy logic to identify the unknown part of the system model [8]. Using fuzzy logic to solve the uncertain term of the nonlinear system is an available idea for nonlinear uncertain system.

It is worth to mention that controller structure is also very important for controlling the complex nonlinear systems. Many scholars have done a great deal of work on it. Wang [9] presents a hierarchical fuzzy system. In the design part, he derived a

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gradient decent algorithm for tuning the parameters of the hierarchical fuzzy system to match the input-output pairs and the simulation results showed that the algorithm was effective and the hierarchical structure gave good approximation accuracy. Yi [10] presented a new fuzzy controller for anti-swing and position control of an overhead traveling crane based on the Single Input Rule Modules (SIRMs) dynamically connected fuzzy inference model. Therefore, using proper structure of the controller will predigest the design process and the complex degree of the controller.

The paper proposes a new cascade fuzzy sliding mode controller (CFSMC) for a class of second-order under-actuated systems. The characteristic of the CFSMC is that its last-layer sliding surface is asymptotic stable and the fuzzy logic is used to approximate the uncertainty in the system model. Moreover, total control includes every layer's equivalent control, which guarantees that every subsystem can follow its own sliding surface to move to zero.

II. STRUCTURAL DESIGN OF THE CFSMC

Consider a single-input and multi-output nonlinear coupled system expressed in the following form:

$$\begin{aligned} \dot{x}_1 &= x_2 \\ \dot{x}_2 &= f_1(X) + b_1(X)u + d_1(t) \\ \dot{x}_3 &= x_4 \\ \dot{x}_4 &= f_2(X) + b_2(X)u + d_2(t) \\ y(t) &= [x_1, x_3]^T \end{aligned} \tag{1}$$

where $X = (x_1, x_2, x_3, x_4)^T$ is state variables vector; $f_1(X)$, $f_2(X)$, $b_1(X)$ and $b_2(X)$ are the nominal uncertain nonlinear functions; $d_1(t)$ and $d_2(t)$ are the bounded lumped disturbances which include the parameter variations and external disturbances; and u is the control input. The $f_1(X)$, $f_2(X)$, $b_1(X)$, $b_2(X)$, $d_1(t)$ and $d_2(t)$ are abbreviated as f_1, f_2, b_1, b_2, d_1, d_2 in the following description. The object of control is to design a single input u to simultaneously control the states (x_1, x_2, x_3, x_4) to achieve desired performance. This form can be treated as a norm expression of a class of under-actuated systems (such as Pendubot, Acrobot, overhead crane, Pendulum etc.).

For the state variables (x_1, x_2), we can construct a suitable pair of sliding surface as the first-layer:

$$s_1 = c_1 x_1 + x_2 \tag{2}$$

where c_1 is a real positive constant. Then, the first-layer surface s_1 can be considered as a general state variable. We can use it and one of the left system state variables to construct the second-layer surface s_2 formed as follows:

$$s_2 = c_2 x_3 + s_1 \tag{3}$$

where c_2 is a real positive constant. Similarly, the second-layer surface s_2 can also be thought as a general variable to construct the third-layer surface s_3 with the last system state variable which can be written as:

$$s_3 = c_3 x_4 + s_2 \tag{4}$$

where c_3 is a real positive constant. Using the equivalent control method, the equivalent control laws of the subsystems can be obtained:

$$u_{eq1} = -\frac{f_1 + c_1 x_2}{b_1} \tag{5}$$

$$u_{eq2} = -\frac{c_2 x_4 + c_1 x_2 + f_1}{b_1} \tag{6}$$

$$u_{eq3} = -\frac{c_3 f_2 |sgn(s_2)| + c_2 x_4 + c_1 x_2 + f_1}{c_3 b_2 \cdot |sgn(s_2)| + b_1} \tag{7}$$

Comment: When the third-layer sliding surface converges to zero, the u_{eq3} will guarantee the system to stay on the sliding surface until all the states of the third-layer sliding surface converge to zero (i.e. $x_4 = s_2 = 0$). Then, the u_{eq3} degenerates to u_{eq2} which describes the motion trajectory on the second-layer sliding surface. If the system states derivate from their sliding surfaces, the controller will become the third-layer sliding-mode controller again and continue to control the system states to converge to zero. Therefore, it is important to guarantee the third-layer surface is stable. That is to say if a sliding mode control law can guarantee the last-layer surface is stable, it will guarantee all the sliding surfaces and system states to converge to zero.

But, when the model of the system includes uncertainties, i.e. f_1, f_2 are unknown, the equivalent control law of the system is impossible to obtain. Therefore, we will approximate the uncertainties using fuzzy logic in the following section.

III. DESIGN OF FUZZY APPROXIMATORS

The fuzzy systems are mostly used as the parameterized identification models of unknown nonlinear functions. By adjusting the values of certain parameters, the output response of fuzzy model is modified to satisfy some prescribed system specifications.

A fuzzy system includes four components: fuzzifier, fuzzy rule base, fuzzy inference engine, and defuzzifier. The fuzzy rule base consisting of fuzzy IF-THEN rules which describe linguistically the relationship between the input and output variables is the most important part of a fuzzy system. Wang [11] proved that if the singleton fuzzifier, product inference engine and center average defuzzifier were used, the fuzzy system could approximate any function to arbitrary accuracy. Wang [12] also proved that these fuzzy systems with Gaussian membership functions were universal approximators. Because of this distinct property, it is reasonable to use them as identification models for nonlinear systems.

Let the unknown nonlinear functions f_1, f_2 in (1) be modeled, respectively, by fuzzy systems on an arbitrarily chosen compact set $V \subset R^n$. Let $\hat{f}_1$ and $\hat{f}_2$ be the corresponding fuzzy models for f_1 and f_2, respectively. The knowledge base for the fuzzy logic system comprises a collection of fuzzy IF-THEN rules. In this paper, multi-input single-output (MISO) rules will be used in the formulation of the control law. The MISO IF-THEN rules have the following form [13]:

$$R^{(j)}: \text{IF } x_1 \text{ is } A_1^{r_1} \text{ and } \dots \text{ and } x_4 \text{ is } A_4^{r_4}, \text{ THEN } \hat{f}_1 \text{ is } \theta_1^{r_1 r_2 \cdots r_4} \tag{8}$$

where the system states $X = (x_1, \cdots x_n)^T \in V \subset R^n$ and $\hat{f}_1 \in W \subset R$ denote the linguistic variables associated with the input and output of the fuzzy logic system. $A_i^{r_i}$ and $\theta_1^{r_1 r_2 \cdots r_4}$ are labels of the fuzzy sets in V and W, respectively, and i denotes the number of input state of fuzzy logic system, i.e., $i = 1,2,3,4$ and j denotes the number of rules of the fuzzy logic system, i.e., $j = 1,2,\cdots M$. Fuzzy rule (8) can be implemented using fuzzy implication, which gives,

$$A_1^j \times \cdots \times A_4^j \to \theta_1^{r_1 r_2 \cdots r_4} \tag{9}$$

which is a fuzzy set defined in the product space $V \times W$. Based on generalizations of implications in multivalue logic, many fuzzy implication rules have been proposed in the fuzzy logic literature. In this paper, we define the implication rule used t-norm operator, which gives,

$$\mu_{A_1^j \times \cdots \times A_4^j} \to \theta_1^{r_1 r_2 \cdots r_4} = \mu_{A_1^j}(x_1) * \cdots * \mu_{A_4^j}(x_4) * \mu_{\theta_1^j}(\hat{f}_1) \tag{10}$$

where $*$ denotes t-norm, which corresponds to the conjunction "min" or "product" in general.

Similarly, for f_2, its fuzzy system can be built by defining the following rules:

$$R^{(j)}: \text{IF } x_1 \text{ is } A_1^{l_1} \text{ and } \dots \text{ and } x_4 \text{ is } A_4^{l_4}, \text{ THEN } \hat{f}_2 \text{ is } \theta_2^{l_1 l_2 \cdots l_4} \tag{11}$$

Further, we define the fuzzy basis functions $\xi_1^{r_1 r_2 \cdots r_4}(x)$, $\xi_2^{l_1 l_2 \cdots l_4}(x)$ to simplify the approximators for f_1, f_2, which can be expressed as:

$$\xi_1^{r_1 r_2 \cdots r_4}(x) = \frac{\prod_{i=1}^{4} \mu_{A_i^{r_i}}(x_i)}{\sum_{r_1=1}^{p_1} \cdots \sum_{r_4=1}^{p_4} (\prod_{i=1}^{4} \mu_{A_i^{r_i}}(x_i))} \tag{12}$$

$$\xi_2^{l_1 l_2 \cdots l_4}(x) = \frac{\prod_{i=1}^{4} \mu_{A_i^{l_i}}(x_i)}{\sum_{l_1=1}^{q_1} \cdots \sum_{l_4=1}^{q_4} (\prod_{i=1}^{4} \mu_{A_i^{l_i}}(x_i))} \tag{13}$$

Therefore, by using the singleton fuzzifier, product inference engine, and average defuzzifier, the uncertain f_1, f_2 can be expressed as regression models:

$$\hat{f}_1 = \hat{f}_1(x \mid \theta_1) = \theta_1^T \xi_1(x) \tag{14}$$

$$\hat{f}_2 = \hat{f}_2(x \mid \theta_2) = \theta_2^T \xi_2(x) \tag{15}$$

where

$$\theta_1 = \left[\theta_1^{11\cdots1}, \dots, \theta_1^{p_1 p_2 \cdots p_4}\right]^T, \quad \xi_1 = \left[\xi_1^{11\cdots1}, \dots, \xi_1^{p_1 p_2 \cdots p_4}\right]^T,$$

$$\theta_2 = \left[\theta_2^{11\cdots1}, \dots, \theta_2^{q_1 q_2 \cdots q_4}\right]^T, \xi_2 = \left[\xi_2^{11\cdots1}, \dots, \xi_2^{q_1 q_2 \cdots q_4}\right]^T.$$

Then, the equivalent of the cascade sliding-mode controller can be written as:

$$\hat{u}_{eq} = -\frac{c_3 \hat{f}_2 \cdot |sgn(s_2)| + c_2 x_4 + c_1 x_2 + \hat{f}_1}{c_3 b_2 \cdot |sgn(s_2)| + b_1} \tag{16}$$

IV. DESIGN OF CONTROL LAW OF THE CFSMC

The total control law of the CFSMC can be assumed as the following form:

$$u = \hat{u}_{eq} + u_{sw} \tag{17}$$

where u_{sw} is the switching control of the CFSMC.

According to the above section, we define the optimal parameters of the approximators as:

$$\theta_1^* = \arg \inf_{\theta_1 \in R^p} \left[\max_{x \in R^n} \left| f_1(x) - \hat{f}_1(x \mid \theta_1) \right| \right] \tag{18}$$

$$\theta_2^* = \arg \inf_{\theta_2 \in R^q} \left[\max_{x \in R^n} \left| f_2(x) - \hat{f}_2(x \mid \theta_2) \right| \right] \tag{19}$$

Therefore, we can further define the estimation error of the optimal parameters vectors:

$$\varepsilon_1 = \theta_1 - \theta_1^* \tag{20}$$

$$\varepsilon_2 = \theta_2 - \theta_2^* \tag{21}$$

Then, we can obtain the switching control u_{sw} and the adaptive laws of the parameters θ_1, θ_2 using Lyapunov method.

The Lyapunov function can be defined as:

$$V = \frac{1}{2}s_3^2 + \frac{1}{2\gamma_1}\varepsilon_1^T\varepsilon_1 + \frac{1}{2\gamma_2}\varepsilon_2^T\varepsilon_2 \tag{22}$$

where γ_1, γ_2 are positive constants.

The Lyapunov stability condition can be derived as follows:

$$\begin{aligned}\dot{V} &= s_3\dot{s}_3 + \frac{1}{\gamma_1}\varepsilon_1^T\dot{\theta}_1 + \frac{1}{\gamma_2}\varepsilon_2^T\dot{\theta}_2 \\ &= s_3\left[c_3(f_2 + b_2u + d_2) + c_2x_4 + c_1x_2 + f_1 + b_1u + d_1\right] \\ &\quad + \frac{1}{\gamma_1}\varepsilon_1^T\dot{\theta}_1 + \frac{1}{\gamma_2}\varepsilon_2^T\dot{\theta}_2 \\ &= s_3\left[(c_3f_2 + f_1) + (c_3d_2 + d_1) + (c_3b_2 + b_1)(\hat{u}_{eq} + u_{sw})\right] \\ &\quad + \frac{1}{\gamma_1}\varepsilon_1^T\dot{\theta}_1 + \frac{1}{\gamma_2}\varepsilon_2^T\dot{\theta}_2\end{aligned} \tag{23}$$

If s_3 is not equal to zero, we have,

$$\begin{aligned}\dot{V} &= s_3\left[c_3(f_2 - \hat{f}_2) + (f_1 - \hat{f}_1) + (c_3d_2 + d_1) + (c_3b_2 + b_1)u_{sw}\right] \\ &\quad + \frac{1}{\gamma_1}\varepsilon_1^T\dot{\theta}_1 + \frac{1}{\gamma_2}\varepsilon_2^T\dot{\theta}_2 \\ &= s_3c_3\varepsilon_2^T\xi_2(x) + s_3\varepsilon_1^T\xi_1(x) + s_3(c_3d_2 + d_1) \\ &\quad + s_3(c_3b_2 + b_1)u_{sw} + \frac{1}{\gamma_1}\varepsilon_1^T\dot{\theta}_1 + \frac{1}{\gamma_2}\varepsilon_2^T\dot{\theta}_2 \\ &= s_3(c_3d_2 + d_1) + s_3(c_3b_2 + b_1)u_{sw} \\ &\quad + \varepsilon_1^T\left(s_3\xi_1(x) + \frac{1}{\gamma_1}\dot{\theta}_1\right) + \varepsilon_1^T\left(c_3s_3\xi_2(x) + \frac{1}{\gamma_2}\dot{\theta}_2\right)\end{aligned} \tag{24}$$

Choose

$$\dot{\theta}_1 = -\gamma_1 s_3\xi_1(x) \tag{25}$$

$$\dot{\theta}_2 = -\gamma_2 c_3 s_3\xi_2(x) \tag{26}$$

and

$$u_{sw} = -(c_3b_2 + b_1)^{-1}[\eta \cdot sgn(s_3) + ks_3] \tag{27}$$

where k and η are positive constants.

Define $d_m = \sup_{t\geq 0}\left|c_3d_2(t) + d_1(t)\right|$ and we can find that if the parameter η satisfies the condition $\eta > d_m$, we can obtain

$$\begin{aligned}\dot{V} &= s_3(c_3b_2 + b_1)u_{sw} + s_3(c_3d_2 + d_1) \\ &\leq -\eta s_3\, sgn(s_3) + |s_3|\cdot|c_3d_2 + d_1| - ks_3^2 \\ &= -\eta|s_3| + |s_3|\cdot|c_3d_2 + d_1| - ks_3^2 \\ &\leq -|s_3|(\eta - d_m) - ks_3^2 \\ &< 0\end{aligned} \tag{28}$$

At last, we can conclude that the total control law u of the CFSMC can be expressed as:

$$\begin{aligned}u &= \hat{u}_{eq} + u_{sw} \\ &= -\frac{c_3\hat{f}_2\cdot|sgn(s_2)| + c_2x_4 + c_1x_2 + \hat{f}_1}{c_3b_2\cdot|sgn(s_2)| + b_1} - \frac{k}{c_3b_2 + b_1}sgn(s_3)\end{aligned} \tag{29}$$

The control law of the CFSMC can guarantee the system is stable.

Remark: The trajectory of the system states are shown in figure 1. From the above derivation process, we can find that the total control law shown as (31) of the CFSMC can guarantee the last layer surface is stable. That is to say, the total control law can guarantee the system states to converge to zero surface (state 1) from any initial condition (state 0). Then, the equivalent control law u_{eq3} will assure that the system states stay on the sliding surface until they converge to the origin (state 2). At the origin, there is $s_2 = 0$ (i.e. the state 2 is equal to the state 3). Therefore, the equivalent control law of the third-layer surface will degenerate to the second-layer equivalent u_{eq2} which can guarantee the system states trajectory to stay on the second-layer sliding mode surface. Then, the system states can converge to state 4 along the sliding surface $s_2 = 0$. Then, the primal

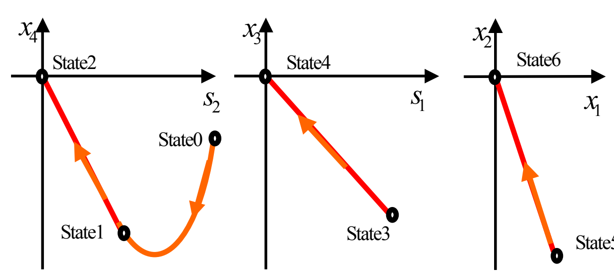

Fig.1 sketch map of convergent curves

third-layer surface degenerates to the first-layer sliding surface (state 5) and converges to the zero point (from state 5 to state 6) at last. During this process, if the system states derivate from the sliding surface $s_2 = 0$, the controller will become the third-layer controller which has been proven to be stable.

Similarly, this idea can be extended to even higher order nonlinear systems by constructing the cascade sliding surfaces until the last state variable.

V. SIMULATION RESULTS

Overhead crane system (shown as Fig.2) is a typical under-actuated system. The control object of the over crane is to move the trolley to its destination and complement anti-swing of the load at the same time.

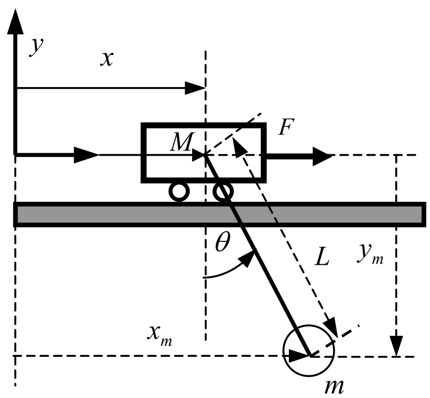

Fig.2 Overhead Crane System

For simplicity, in the paper, the following assumptions are made:

(a) The trolley and the load can be regarded as point masses;

(b) Friction force which may exists in the trolley and elongation of the rope due to tension force can be neglected;

(c) The trolley moves along the rail and the load moves in the x-y plane.

Using Lagrange's method, we can obtain the model of the overhead crane system as follow:

$$x \quad : \quad (m+M)\ddot{x} + mL\left(\ddot{\theta}\cos\theta - \dot{\theta}^2 \sin\theta\right) = F \quad (30)$$

$$\theta \quad : \quad \ddot{x}\cos\theta + L\ddot{\theta} + g\sin\theta = 0 \quad (31)$$

where M and m are the masses of the trolley and the load, respectively. θ is the sway angle of load and L is the length of suspension rope.

Where $x_1 = e = x^d - x; x_2 = \dot{x}^d - \dot{x}, x_3 = \theta, x_4 = \dot{\theta}$ are the displacement error of the trolley in the horizontal direction, the velocity error of the trolley in the horizontal direction, the sway angle of the load and the sway angle velocity of the load.

The parameters of the overhead crane can be chosen as: $M = 1kg$, $m = 0.8kg$, $L = 0.305m$, and the parameters of the CFSMC are chosen as $c_1 = 1.4$, $c_2 = 2.4$, $c_3 = 0.1$, $k = 0.7$.

The initial conditions of the overhead crane system are $\left(x_0, \dot{x}_0, \theta_0, \dot{\theta}_0\right) = (0,0,0,0)$ and the expectations are $\left(x^d, \dot{x}^d, \theta^d, \dot{\theta}^d\right) = (2m,0,0,0)$.

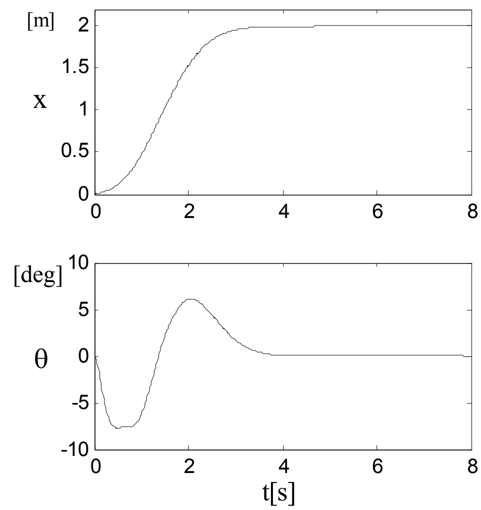

Fig.3 Output of trolley displace and load swing angle

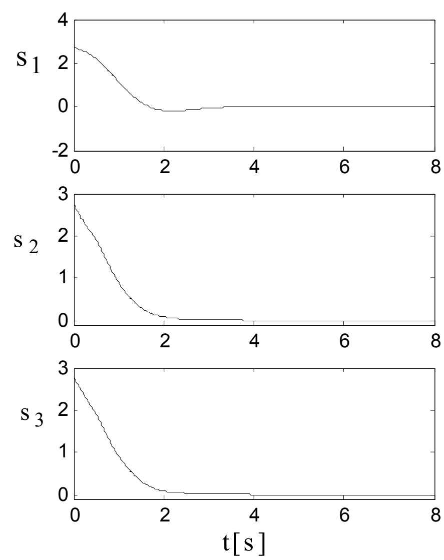

Fig.4 Convergent curves of all sliding surfaces

Fig.3 shows the displacement and velocity of the trolley, the swing angle and angle velocity of the load under short distance condition with the CFSMC. The simulation results show that the CFSMC can control the trolley to destination and implement anti-sway control at the same time. Fig.4 shows the convergent curve of all the sliding surfaces under short distance. From Fig.4, we can find that all the sliding surfaces can converge to zero.Fig.5 shows the control force of the controller. Fig.6 shows

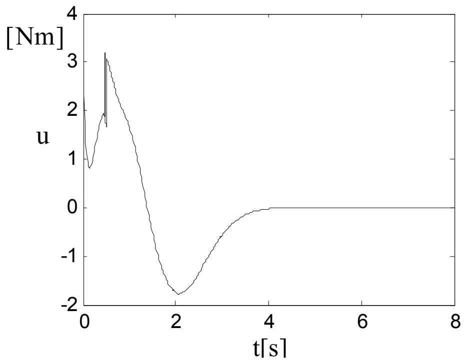

Fig.5 the output torque of the controller

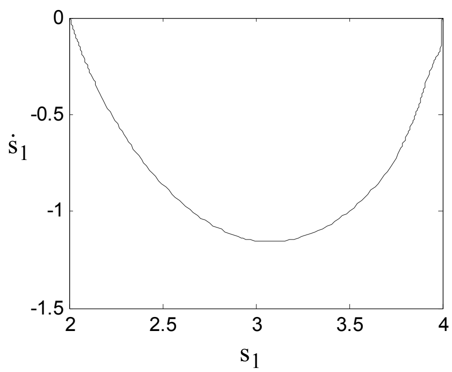

Fig.6 Phase plane curve of the sub-sliding mode plane s_1

the phase plane curve of the first-layer sliding surface s_1. The simulation results show the validity of the CFSMC.

VI. CONCLUSION

A new cascade fuzzy sliding-mode controller for a class of uncertain underactuated systems is presented in this paper. The method can control one-input and multi-output nonlinear systems efficiently. The paper has proved that the last-layer sliding surface is stable, which can guarantee all other sliding surfaces and system states to converge to zero. The simulation results also show the validity of the CFSMC. Similarly, this idea can be extended to even higher order systems by introducing the cascade sliding surfaces. Therefore, this method gives a viable solution for a class of underactuated mechanical systems.

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