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A New Discrete Variable Structure Control Algorithm Based on Sliding Mode Prediction

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Abstract—**In this paper, a new discrete variable structure control design approach based on sliding mode prediction algorithm for a class of discrete-time systems is presented. By employing a special model to predict future sliding mode value and an ideal reaching law as reference sliding mode trajectory, and at the same time, combining feedback correction and rolling optimization method which are extensively applied on predictive control strategy, a discrete-time variable structure control law is constructed. Simulation results illustrate that the closed-loop system has desired performance, such as strong robustness, fast convergence and chattering elimination.**

I. Introduction

Variable structure control (VSC) as a general design approach for robust control systems is well established. The long history of its development and main results have been reported since 1950s. Due to the widespread use of digital controllers, discrete-time techniques of a similar nature were developed [1-4]. In [4], Furuta proposed a variable-structure algorithm which drives the system state to an appropriately determined sector in state space. In [5], Gao *et al*. specified desired properties of the controlled system and used the so-called reaching law approach to design the corresponding control law. In [6], A. Bartoszewicz proposed a different reaching law, and the reaching law was based on the concept of time-varying switching surfaces introduced in [7].The reaching law in [6] has faster system error convergence speed, however, possess greater steady-state error as well, comparing with that of in [5]. In recent years, a number of papers have been done based on [5], see e.g., [8-13].

In this paper, for a class of discrete-time systems, in order to obtain desired closed-loop performance, such as wonderful chattering elimination, strong robustness and fast convergence, we combine predictive control method into the design of discrete quasi-sliding-mode variable structure control law. On one hand, the switching in the constructed control law enables the closed-loop system to possess robustness, which is a primary characteristic of sliding mode control. On the other hand, because of rolling optimization, control signal is able to optimize continuously and on-line. Therefore, compared with traditional sliding mode control strategy, sliding mode predictive control algorithm has great advantages. Firstly, new predictive control strategy can obtain minimal control signal and perfect system response performance at the same time. Secondly, the boundaries of uncertainties are not required in the control strategy, namely, the control strategy does not consider the worst case of the systems, and thus conservation is decreased, chattering problem be solved nicely. Furthermore, it is well known that discrete control algorithm is easy to be implemented on computer or single-chip computer, so the control law can be realized easily in real application.

The remainder of this paper is organized as follows. Section II presents notations and some definitions which are used throughout the paper. The main results, i.e., the sliding mode predictive control law that eliminates chattering, guarantees the robustness of system and provides optimal control signal, are derived in Section III. Robustness analysis is presented in Section IV. In Section V, performance of the discrete-time variable structure control law is verified by a numerical simulation example. Finally, Section VI gives conclusions of the paper.

II. Preliminary

Consider the following discrete-time system,

$$\boldsymbol{x}(k+1) = \boldsymbol{A}\boldsymbol{x}(k) + \Delta\boldsymbol{A}\boldsymbol{x}(k) + \boldsymbol{b}u(k) + \boldsymbol{w}(k) \tag{1}$$

where $\boldsymbol{x}(k) \in \Re^{n}$ is the state vector, $u(k) \in \Re^{1}$ is the control input, $\boldsymbol{A}$ and $\boldsymbol{b}$ are of appropriate dimensions, $\Delta\boldsymbol{A} \in \Re^{n\times n}$ represents parameter uncertainties and $\boldsymbol{w}(k) \in \Re^{n}$ denotes external disturbances. Assume the pair $(\boldsymbol{A}, \boldsymbol{b})$ is controllable and the matching conditions

$$\Delta\boldsymbol{A} = \boldsymbol{b}\overline{\boldsymbol{A}} \tag{2a}$$

$$\boldsymbol{w}(k) = \boldsymbol{b}\overline{w}(k) \tag{2b}$$

are satisfied, where $\overline{\boldsymbol{A}} \in \Re^{1\times n}$ is a row vector, $\overline{w} \in \Re^{1}$ is a scalar. For simplicity, we denote

$$\boldsymbol{f}(k) = \Delta\boldsymbol{A}\boldsymbol{x}(k) + \boldsymbol{w}(k) \tag{3}$$

In addition, we define sliding mode function

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$$s(k) = \boldsymbol{cx}(k) \tag{4}$$

where $\boldsymbol{c} \in \Re^{1\times \boldsymbol{n}}$ such that $\boldsymbol{cb} \neq 0$ and the resulting quasi-sliding mode motion is stable.

At time $k+1$, there exists

$$\begin{aligned} s(k+1) &= \boldsymbol{cx}(k+1) \\ &= \boldsymbol{c}[\boldsymbol{Ax}(k) + \boldsymbol{b}u(k) + \boldsymbol{f}(k)] \\ &= \boldsymbol{c}[\boldsymbol{Ax}(k) + \boldsymbol{b}u(k)] + \boldsymbol{cf}(k) \end{aligned}$$

For the sake of clarity, we define following vectors,

$$\boldsymbol{S}(k+1) = [s(k+1), s(k+2), \cdots s(k+N)]^T \tag{5a}$$

$$\boldsymbol{U}(k) = [u(k), u(k+1), ..., u(k+M-1)]^T \tag{5b}$$

$$\boldsymbol{U}(k-1) = [u(k-1), u(k-2), \cdots, u(k-N)]^T \tag{5c}$$

$$\boldsymbol{X}(k-1) = [\boldsymbol{x}(k-1), \boldsymbol{x}(k-2), \cdots, \boldsymbol{x}(k-N)]^T \tag{5d}$$

$$\boldsymbol{F}(k) = [\boldsymbol{f}(k), \boldsymbol{f}(k+1), \cdots, \boldsymbol{f}(k+N-1)]^T \tag{5e}$$

$$\boldsymbol{V}(k) = [\boldsymbol{f}(k-1), \boldsymbol{f}(k-2), \cdots, \boldsymbol{f}(k-N)]^T \tag{5f}$$

where N and M are given scalars which will be determined in section III.

III. CONTROL LAW DESING

When there are no parameter uncertainties and disturbances, according to sliding mode function (3), sliding mode value at time $k+p$ can be described as follows,

$$s(k+p) = \boldsymbol{cA}^p \boldsymbol{x}(k) + \sum_{i=1}^{p} \boldsymbol{cA}^{i-1}\boldsymbol{b}u(k+p-i) \tag{6}$$

where $k \in \mathrm{Z}$ and $p \in \mathrm{Z}$ are sampling times.

It is clear that (6) may not converge always. As a result, we propose following sliding mode prediction model,

$$\begin{aligned} s_p(k+p) = \boldsymbol{cA}^p \boldsymbol{x}(k) + \sum_{i=1}^{p} \boldsymbol{cA}^{i-1}\boldsymbol{b}u(k+p-i) \\ +\zeta_p \operatorname{sgn}(s_p(k+p-1)) \end{aligned} \tag{7}$$

where $\zeta_p > 0$ is correct coefficient.

In this wise, predictive sliding mode value of time k on time $k-p$ can be deduced from (7),

$$\begin{aligned} s_{pp}(k|k-p) = \boldsymbol{cA}^p \boldsymbol{x}(k-p) + \sum_{i=1}^{p} \boldsymbol{cA}^{i-1}\boldsymbol{b}u(k-i) \\ +\zeta_p \operatorname{sgn}(s_p(k-1)) \end{aligned} \tag{8}$$

Obviously, if at time $k+p-1$, there exists $s(k+p-1)=0$, then (7) equals to (6).

Describe (7) in vector form,

$$\boldsymbol{S_p}(k+1) = \boldsymbol{\phi x}(k) + \boldsymbol{GU}(k) + \boldsymbol{L}\operatorname{sgn}(\boldsymbol{S}_p(k)) \tag{9}$$

where

$$\boldsymbol{S_p}(k+1) = [s_p(k+1), s_p(k+2), \cdots, s_p(k+N)]^T,$$
$$\boldsymbol{S_p}(k) = [s_p(k), s_p(k+1), \cdots, s_p(k+N-1)]^T,$$
$$\boldsymbol{L} = diag[\zeta_1, \zeta_2, ..., \zeta_N],$$
$$\operatorname{sgn}(\boldsymbol{S}_p(k)) = [\operatorname{sgn}(s_p(k)), \operatorname{sgn}(s_p(k+1)), ..., \operatorname{sgn}(s_p(k+N))]^T,$$
$$\boldsymbol{\phi} = \begin{bmatrix} \boldsymbol{cA} & \boldsymbol{cA}^2 & \cdots & \boldsymbol{cA}^N \end{bmatrix}^T,$$

$$\boldsymbol{G} = \begin{bmatrix} \boldsymbol{cb} & \boldsymbol{0} & \cdots & \cdots & \boldsymbol{0} \\ \boldsymbol{cAb} & \boldsymbol{cb} & \cdots & \cdots & \boldsymbol{0} \\ \vdots & \vdots & \cdots & \cdots & \vdots \\ \vdots & \vdots & \cdots & \cdots & \boldsymbol{cb} \\ \vdots & \vdots & \cdots & \cdots & \vdots \\ \boldsymbol{cA}^{N-2}\boldsymbol{b} & \boldsymbol{cA}^{N-3}\boldsymbol{b} & \cdots & \boldsymbol{cA}^{N-M}\boldsymbol{b} & \sum_{i=0}^{N-M-1} \boldsymbol{cA}^i\boldsymbol{b} \\ \boldsymbol{cA}^{N-1}\boldsymbol{b} & \boldsymbol{cA}^{N-2}\boldsymbol{b} & \cdots & \boldsymbol{cA}^{N-M+1}\boldsymbol{b} & \sum_{i=0}^{N-M} \boldsymbol{cA}^i\boldsymbol{b} \end{bmatrix}.$$

with N is prediction horizon and M is control horizon.

In practice, sliding mode prediction model will inevitably have some errors because of time-vary, nonlinear, or several of disturbances and so forth, therefore, the model output will not the same as real output. In predictive control method, a common way to solve the above problem is output error feedback correction approach, namely, closed-loop prediction. That is, firstly, at time k, calculate the error between real value and model output value; secondly, plus the error to model output value at time $k+1$; then take the sum as closed-loop output predictive value at time $k+1$.

Here, the error between present sliding mode value $s(k)$ and predictive sliding mode value $s_{pp}(k|k-p)$ is used to make feedback correction for future predictive sliding mode value $s(k+p)$, and the output of sliding mode prediction $\hat{s}_p(k+p)$ is given as follows,

$$\begin{aligned} \hat{s}_p(k+p) &= s_p(k+p) + h_p[s(k) - s_{pp}(k|k-p)] \\ &= cA^p \boldsymbol{x}(k) + \sum_{i=1}^{p} \boldsymbol{cA}^{i-1}\boldsymbol{b}u(k+p-i) \\ &\quad +\zeta_p \operatorname{sgn}(s_p(k+p-1)) \\ &\quad +h_p[s(k) - s_{pp}(k|k-p)] \end{aligned} \tag{10}$$

where $h_p \in \Re^1$ is correct coefficient. From the viewpoint of practice, usually, $h_1 = 1$,. $h_p < 1 (p \neq 1)$.

Rewrite (10) in vector form,

$$\hat{\boldsymbol{S}}_p(k+1) = \boldsymbol{S}_p + \boldsymbol{HE}(k) \tag{11}$$

where

$$\hat{\boldsymbol{S}}_{\boldsymbol{p}}(k+1) = [\hat{s}_p(k+1), \hat{s}_p(k+2), \cdots, \hat{s}_p(k+N)]^T,$$
$$\boldsymbol{S_{pp}}(k) = [s_{pp}(k|k-1), s_{pp}(k|k-2), \cdots, s_{pp}(k|k-N)]^T,$$
$$\boldsymbol{S}(k) = \begin{bmatrix} s(k) & s(k) & \cdots & s(k) \end{bmatrix}_{1\times N},$$
$$\boldsymbol{H} = diag[h_1, h_2, ..., h_N],$$
$$\boldsymbol{E}(k) = \boldsymbol{S}(k) - \boldsymbol{S}_{pp}(k).$$

In this paper, the purpose is to design a suitable control law by which the closed-loop system can obtain desired performance, including strong robustness, fast convergence and chattering elimination.

There are two parts for us to realize the objective. The first step is to drive $s(k)$ to follow a desired sliding mode trajectory such that $s(k)$ can always satisfy $s(k)=0$

after certain time. The second step is to obtain control law according to predictive control method.

In [5], Gao et al. presented an algorithm which drives the system state to the vicinity of a switching hyperplane in state space. They specified desired properties of the controlled system and used the so-called reaching law approach to design the corresponding control law. So far, this method has been used widely. However, it has some drawbacks, and one of them is the system can not converge to origin but to a vicinity of origin by employing the designed method.

Here we introduce the following reaching law,

$$
\begin{aligned}
&s(k+1) = (1-qT)s(k)\Phi(s(k)) - \varepsilon T\Psi(s(k))\operatorname{sgn}(s(k)) \\
&\Phi(s(k)) = \begin{cases} 1 & |s(k)| > \eta \\ 0 & |s(k)| \le \eta \end{cases}, \\
&\Psi(s(k)) = \begin{cases} 1 & |s(k)| > \eta \\ |s(k)|^2/\eta & |s(k)| \le \eta \end{cases}, \\
&0 < 1-qT < 1, 0 < \varepsilon T < 1, \eta = \varepsilon T/(1-qT)
\end{aligned}
\tag{12}
$$

where $T > 0$ is the sampling period.

Theorem 1: For arbitrary initial value of sliding mode function, the sliding mode trajectory which is determined by (12), will move into quasi-sliding-mode band $\{x \| s(x)| < \eta\}$ and stay on sliding mode surface ultimately.

Proof: If $|s(k)| > \eta$, then

$$s(k+1) = (1-qT)s(k) - \varepsilon T\operatorname{sgn}(s(k)) \tag{13}$$

Equation (13) is the reaching law in [5] where the sliding mode trajectory will move within the band $\{x \| s(x)| < \eta\}$ in finite time was verified.

Else, $s(k+1) = -(1-qT)|s(k)|^2\operatorname{sgn}(s(k))$

When $s(k) = 0$, there exists $s(k+1) = 0$, i.e., sliding mode trajectory will stay on sliding mode surface.

Otherwise, $s(k) \ne 0$, let $v(k) = s^2(k)$,

$$
\begin{aligned}
\Delta v(k) &= v(k+1) - v(k) \\
&= [-(1-qT)|s(k)|^2\operatorname{sgn}(s(k))]^2 - s^2(k) \\
&= (1-qT)^2|s(k)|^4 - s^2(k) \\
&= s^2(k)[(1-qT)^2 s^2(k) - 1] \\
&\le s^2(k)[\varepsilon^2 T^2 - 1] \\
&< 0
\end{aligned}
$$

Consequently, sliding mode trajectory will definitely move into quasi-sliding-mode band and stay on sliding mode surface ultimately. □

Apparently, the reaching law (12) can be considered as a good reference sliding mode trajectory for the design.

Thus, taking the reaching law (12) as reference sliding mode trajectory, the following equations are obtained,

$$
\begin{cases}
s_r(k+p) = (1-qT)s_r(k+p-1)\Phi(s_r(k+p-1)) \\
\qquad -\varepsilon T\Psi(s_r(k+p-1))\operatorname{sgn}(s_r(k+p-1)) \\
s_r(k) = s(k)
\end{cases}
$$

$$
\begin{aligned}
&\Phi(s_r(k+p-1)) = \begin{cases} 1 & |s_r(k+p-1)| > \eta \\ 0 & |s_r(k+p-1)| \le \eta \end{cases}, \\
&\Psi(s_r(k+p-1)) = \begin{cases} 1 & |s_r(k+p-1)| > \eta \\ |s_r(k+p-1)|^2/\eta & |s_r(k+p-1)| \le \eta \end{cases}.
\end{aligned}
\tag{14}
$$

Now, corresponding optimization cost function is defined as

$$J_p = \sum_{i=1}^{N} q_i[\hat{s}_p(k+i) - s_r(k+i)]^2 + \sum_{j=1}^{M} r_j[u(k+j-1)]^2 \tag{15}$$

where $s_r(k+i)$ is the sliding mode reference trajectory, q_i and r_j are weight coefficients.

Describe (15) in vector form

$$
\begin{aligned}
J_p &= [\hat{\boldsymbol{S}}_p(k+1) - \boldsymbol{S}_r(k+1)]^T \boldsymbol{Q}[\hat{\boldsymbol{S}}_p(k+1) - \boldsymbol{S}_r(k+1)] \\
&\quad + \boldsymbol{U}(k)^T \boldsymbol{R}\boldsymbol{U}(k) \\
&= [\boldsymbol{\phi x}(k) + \boldsymbol{GU}(k) + \boldsymbol{L}\operatorname{sgn}(\boldsymbol{S}_p(k)) + \\
&\quad \boldsymbol{E}(k) - \boldsymbol{S}_r(k+1)]^T \boldsymbol{Q}[\boldsymbol{\phi x}(k) + \boldsymbol{GU}(k) \\
&\quad + \boldsymbol{L}\operatorname{sgn}(\boldsymbol{S}_p(k)) + \boldsymbol{HE}(k) - \boldsymbol{S}_r(k+1)] \\
&\quad + \boldsymbol{U}(k)^T \boldsymbol{RU}(k)
\end{aligned}
\tag{16}
$$

where

$$\boldsymbol{S}_r(k+1) = [s_r(k+1), s_r(k+2), ..., s_r(k+N)]^T,$$

$$\boldsymbol{Q} = diag[q_1, q_2, ..., q_N], \boldsymbol{R} = diag[r_1, r_2, ..., r_M].$$

Optimize (16), i.e., $\frac{\partial J_p}{\partial \boldsymbol{U}(k)} = 0$, thus predictive sliding mode control law can be obtained,

$$
\begin{aligned}
\boldsymbol{U}(k) = -(\boldsymbol{G}^T\boldsymbol{QG} + \boldsymbol{R})^{-1}\boldsymbol{G}^T\boldsymbol{Q}[\boldsymbol{\phi x}(k) + \\
\boldsymbol{L}\operatorname{sgn}(\boldsymbol{S}_p(k)) + \boldsymbol{HE}(k) - \boldsymbol{S}_r(k+1)]
\end{aligned}
\tag{17}
$$

The control law (17) can calculate all control input signals from time k to time $k+M-1$ at one time. However, only present control input signal is implemented, the next time control input signal $u(k+1)$ will be calculated recursively by the control law (17). As a result, the control law (17) can be written equally as follows,

$$
\begin{aligned}
u(k) = -[1, 0, \cdots, 0](\boldsymbol{G}^T\boldsymbol{QG} + \boldsymbol{R})^{-1}\boldsymbol{G}^T\boldsymbol{Q}[\boldsymbol{\phi x}(k) \\
+ \boldsymbol{L}\operatorname{sgn}(\boldsymbol{S}_p(k)) + \boldsymbol{HE}(k) - \boldsymbol{S}_r(k+1)]
\end{aligned}
\tag{18}
$$

IV. Robustness Analysis

Under the control law (17), the vector form of practical quasi-sliding-mode motion for the closed-loop system is given,

$$
\begin{aligned}
\boldsymbol{S}(k+1) &= \boldsymbol{\phi x}(k) + \boldsymbol{GU}(k) + \boldsymbol{DF}(k) \\
&= \boldsymbol{\phi x}(k) + \boldsymbol{G}\{-(\boldsymbol{G}^T\boldsymbol{QG} + \boldsymbol{R})^{-1} \\
&\quad \boldsymbol{G}^T\boldsymbol{Q}[\boldsymbol{\phi x}(k) + \boldsymbol{L}\operatorname{sgn}(\boldsymbol{S}_p(k)) \\
&\quad + \boldsymbol{HE}(k) - \boldsymbol{S}_r(k+1)]\} + \boldsymbol{DF}(k)
\end{aligned}
\tag{19}
$$

where

$$\boldsymbol{D} = \begin{bmatrix} \boldsymbol{c} & \boldsymbol{0} & \cdots & \boldsymbol{0} \\ \boldsymbol{cA} & \boldsymbol{c} & \cdots & \boldsymbol{0} \\ \vdots & \vdots & \vdots & \vdots \\ \boldsymbol{cA}^{N-1} & \boldsymbol{cA}^{N-2} & \cdots & \boldsymbol{c} \end{bmatrix},$$

$$V(k) = [\boldsymbol{f}(k-1), \boldsymbol{f}(k-2), \cdots, \boldsymbol{f}(k-N)]^T$$

with $\mathbf{0}$ denotes n dimension zero vector.

When time $k-p$ is counted as start point, the practical value of sliding mode function at time k is given by

$$s(k) = \boldsymbol{c}\boldsymbol{A}^p \boldsymbol{x}(k-p) + \sum_{i=1}^{p} \boldsymbol{c}\boldsymbol{A}^{i-1}\boldsymbol{b}u(k-i) + \sum_{i=1}^{p} \boldsymbol{c}\boldsymbol{A}^{i-1}\boldsymbol{f}(k-i) \tag{20}$$

Moreover, the predictive value of sliding mode at time k which is based on time $k-p$ can be calculated by (8). Thus $\boldsymbol{E}(k)$ in (19) can be described as follows,

$$\boldsymbol{E}(k) = \tilde{\boldsymbol{D}}\boldsymbol{V}(k) - \boldsymbol{L}\,\mathrm{sgn}(\boldsymbol{S}_p(k)) \tag{21}$$

where

$$\tilde{\boldsymbol{D}} = \begin{bmatrix} \boldsymbol{c} & \mathbf{0} & \cdots & \mathbf{0} \\ \boldsymbol{c} & \boldsymbol{cA} & \cdots & \mathbf{0} \\ \vdots & \vdots & \vdots & \vdots \\ \boldsymbol{c} & \boldsymbol{cA} & \cdots & \boldsymbol{cA}^{N-1} \end{bmatrix}$$

According to the cost function (16), the action of weight coefficient matrix $\boldsymbol{R}$ is to limit the control input $\boldsymbol{U}$. Thus, for the sake of clarity, it is sound to suppose that $\boldsymbol{R}=0$, i.e., there is not limitation for control input $\boldsymbol{U}$.

Now, (19) can be written as

$$\boldsymbol{S}(k+1) = \boldsymbol{S}_r(k+1) - \boldsymbol{H}\tilde{\boldsymbol{D}}\boldsymbol{V}(k) + \boldsymbol{D}\boldsymbol{F}(k) + (\boldsymbol{H}-\boldsymbol{I})\boldsymbol{L}\,\mathrm{sgn}(\boldsymbol{S}_p(k)) \tag{22}$$

Because of rolling optimization, only present control input signal is implemented. The practical sliding mode motion of the closed-loop system can be described as follows

$$s(k+1) = [1, 0, \cdots, 0][\boldsymbol{S}_r(k+1) - \boldsymbol{H}\tilde{\boldsymbol{D}}\boldsymbol{V}(k) + \boldsymbol{D}\boldsymbol{F}(k) + (\boldsymbol{H}-\boldsymbol{I})\boldsymbol{L}\,\mathrm{sgn}(\boldsymbol{S}_p(k))] \tag{23}$$

On account of $h_1 = 1$, (23) simplifies to

$$s(k+1) = s_r(k+1) - \boldsymbol{cf}(k-1) + \boldsymbol{cf}(k)$$

Here, we define $d(k) = \boldsymbol{cf}(k)$,

then

$$s(k+1) = s_r(k+1) + d(k) - d(k-1) \tag{24}$$

Theorem 2: If the change rate of disturbances is bounded, namely, the following inequality holds,

$$\left|\boldsymbol{cf}(k) - \boldsymbol{cf}(k-1)\right| = \left|d(k) - d(k-1)\right| \le \lambda \tag{25}$$

where λ is a positive constant, then the closed-loop control system, which is constructed by (1), (4) and (18), is robustly stable.

Proof: From (24), we can see that $s(k+1)$ is composed of two parts,

$$s_1(k+1) = s_r(k+1), s_1(0) = s(0) \tag{26a}$$

$$s_2(k+1) = d(k) - d(k-1), s_2(0) = 0 \tag{26b}$$

According to **Theorem 1**, there exists a special k_0 such that $\left|s_1(k+1)\right|$ as small as possible. Due to the hypothesis (25), (26b) can always satisfy $\left|s_2(k+1)\right| \le \lambda$. Thus, when $k > k_0$, the following inequality holds,

$$\left|s(k+1)\right| = \left|s_1(k+1) + s_2(k+1)\right| \le \lambda + \boldsymbol{o}(s) \tag{27}$$

where $\boldsymbol{o}(s)$ denotes infinitesimal of higher order. Consequently, the practical sliding mode motion of the closed-loop system will definitely converge to a λ vicinity of sliding mode surface and stay on it subsequently. Because the stability and dynamic performance of ideal quasi-sliding mode will be guaranteed by pole assignment method, the closed-loop control system which is constructed by (1), (4) and (18), is robustly stable. □

Remark 1: Usually, external disturbances and parameter uncertainties have bounded change rates, and sometimes they are constant indeed, i.e., generally speaking, (25) can be satisfied. Therefore, under the control law (18), quasi-sliding-mode band will only have relation to the change of external disturbances and parameter uncertainties in a control period. For slowly varying external disturbances and parameter uncertainties, or control period $T_c \to 0$, quasi-sliding-mode band will be very small. Especially, system states can enter into ideal quasi-sliding-mode motion when $d(k)$ is a constant.

Remark 2: It is well known that one of notorious effects for variable structure control is chattering phenomena which can excite the high frequency oscillation of controlled system. Thus under most of conditions, chattering is undesired in VSC systems. In this paper, we take uncertainties whose boundaries are not required into consideration. On the contrary, in traditional sliding mode control strategy, the boundaries of uncertainties are known is precondition in most works, that is, they considered the worst case. It is inevitable to result in overwhelming conservation and severe chattering. Clearly, our algorithm can deal with chattering very well. However, chattering-like phenomena may exist in the closed-loop system with high-sampling rates. If such phenomena are objectionable, recently reported chattering eliminating methods for VSC can be used to alleviate the problem [7], [14].

V. NUMERICAL EXAMPLE

In order to illustrate the properties of the proposed control law design method, a numerical example is studied here.

Consider a second order system,

$$\begin{aligned} x_1(k+1) &= 1.2x_1(k) + 0.1x_2(k) \\ x_2(k+1) &= 0.5x_1(k) + 0.6x_2(k) + u(k) \end{aligned}$$

According to pole placement method, $\boldsymbol{c}$ is designed as $\boldsymbol{c} = [15, 1]$, thus the sliding mode on the switching plane $s(x) = 0$ is stable. Based on predictive control strategy, it is suitable to select the predictive horizon $N = 10$, the control horizon $M = 5$, and the correct coefficient matrix $\boldsymbol{H} = diag[1\ 0.8\ 0.6\ 0.5\ 0.4\ 0.3\ 0.2\ 0.1\ 0.05\ 0.01]$.

Setting the initial values $\boldsymbol{x}(0) = [5\quad 1]^T$ and

$s(0)=120$, choosing $1-qT=0.7, \varepsilon T=0.15$ and. $\boldsymbol{Q}=8\times \boldsymbol{I}_{N\times N}, \boldsymbol{R}=2.15\times \boldsymbol{I}_{M\times M}, \boldsymbol{L}=0.8\times I_{N\times N}$ Here, the external disturbance vector is constant $\boldsymbol{w}(k)=[0,1]^T$, and parameter uncertainty matrix is $\Delta \boldsymbol{A}=\begin{bmatrix}-0.1 & 0\\ 1.5 & 0.1\end{bmatrix}$.

For the sake of comparison, firstly, we employ the algorithm proposed in this paper and the reaching law method in [5] to nominal system respectively; secondly, we employ the both to uncertain system respectively.

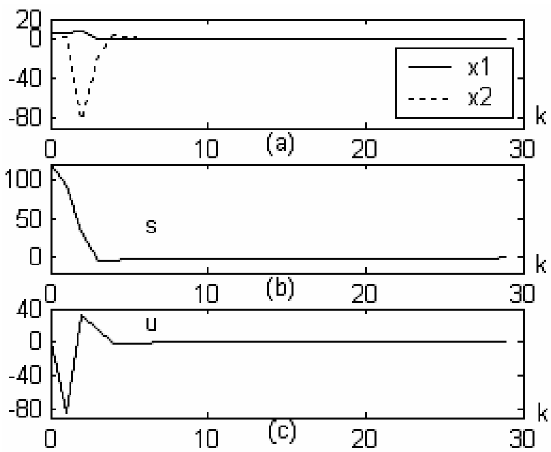

Fig. 1. Responses of nominal system (Proposed algorithm in this paper)

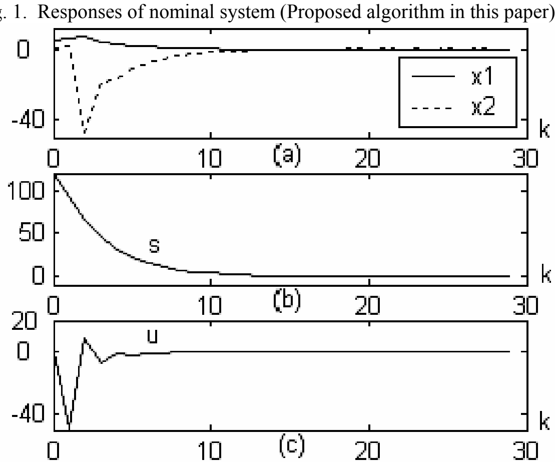

Fig. 2. Responses of nominal system (Reaching law method in [5])

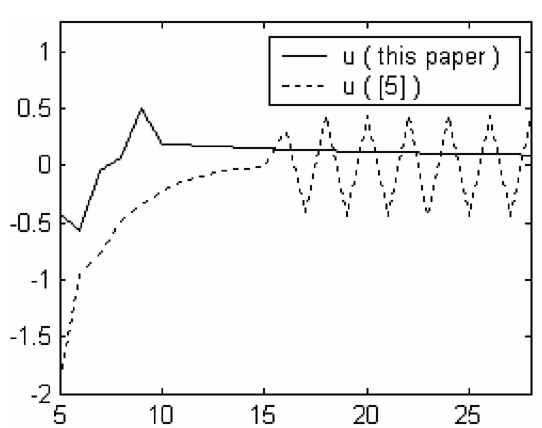

Fig. 3. Compare of the two control signals (for nominal system)

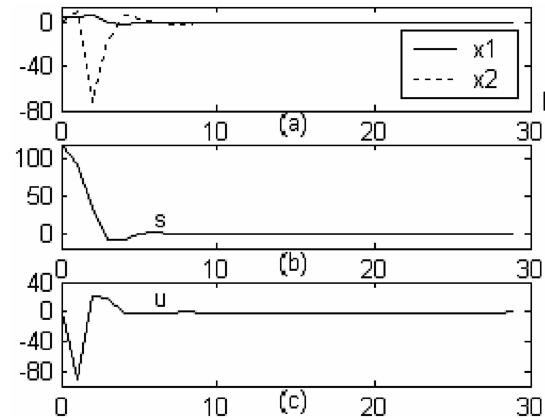

Fig. 4. Responses of uncertain system (Proposed algorithm in this paper)

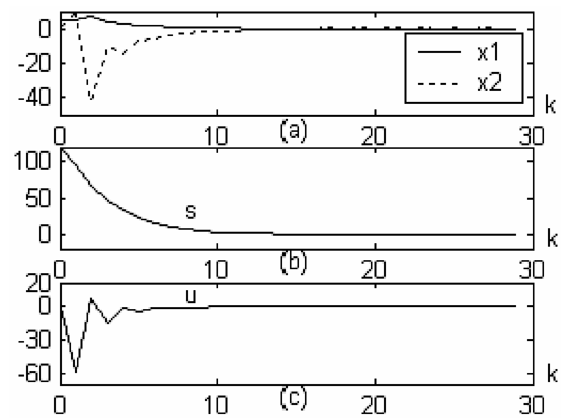

Fig. 5. Responses of uncertain system (Reaching law method in [5])

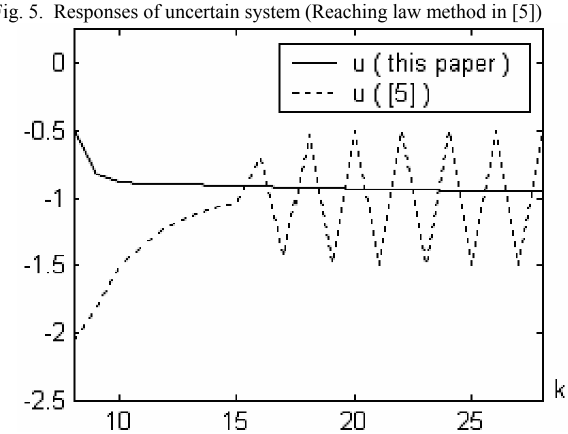

Fig. 6. Compare of the two control signals (for uncertain system)

TABLE I
PERFORMANCE COMPARE BETWEEN ALGORITHM IN THIS PAPER AND REACHING LAW METHOD IN [5]

		Nominal system		Uncertain system	
		Algorithim in this paper	Reaching law method in [5]	Algorithim in this paper	Reaching law method in [5]
Convergence time(step)	x	6	12	8	13
	s	3	10	6	15
	u	10	15	10	17
Convergence speed		fast	slow	very fast	very slow
Chattering elimination		good	bad	good	bad
Robustness		/	/	good	bad

Obviously, Fig.1 shows that the closed-loop system states converge in finite time (about 6 steps) and have good steady-state performance. And Fig. 4 shows that states converge soon as well, when parameter uncertainty and external disturbance enter into system (about 8 steps). These results indicate the system has fast convergence and strong robustness. One can see from Fig.1 (1.b) and Fig. (4.b) that sliding surface is smooth and quasi-sliding-mode band is slight. Fig. 3 and Fig.6 indicate that chattering is eliminated.

By comparing Fig.1 with Fig.2, and from TABLE I, it is clear that although state and control signal in Fig.1 are larger than those in Fig.2 at the very beginning, so as by comparing Fig.4 with Fig.5, Fig.1 and Fig.4 illustrate that the algorithm in this paper make closed-loop system have faster convergence speed, stronger robustness and chattering elimination in the following time, especially in steady-state. As a result, the obtained algorithm in this paper is better on the whole, and the control strategy is efficient.

VI. CONCLUSIONS

In this paper, the approaches of model prediction, rolling optimization and feedback correction, which are widely used in predictive control strategy, are introduced into discrete quasi-sliding-mode variable structure control design. After constructing sliding mode predictive discrete variable structure control law, proof for the robust stability of the closed-loop system without control limitation is given. Simulation results verify the efficiency of the presented control law design method.

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REFERENCES

[1] V. Utkin, "Variable structure systems with sliding modes," *IEEE Trans. Automat. Contr.*, vol. AC-22, pp. 212-222, 1977.

[2] S. Sarpturk, S. Y. Istefanopulos, and O. Kaynak, "On the stability of discrete-time sliding mode contril systems," *IEEE Trans. Automat. Contr.*, vol. 32, pp. 930-932, 1987.

[3] U. Kotta, "Comments on 'On the stability of discrete-time sliding mode control systems'," *IEEE Trans. Automat. Contr.*, vol.34, pp. 1021-1022, 1989.

[4] K. Furuta, "Sliding mode control of a discrete system," *Syst. Contr. Lett.*, vol. 14, pp. 1456-152, 1990.

[5] W. Gao, Y. Wang, and Homaifa, "Discrete-time variable structure control systems," *IEEE Trans. Ind. Electron*., vol.42, pp. 117-122, 1995.

[6] A. Bartoszewicz, "Discrete-time quasi-sliding-mode control strategies," *IEEE Trans. Ind. Electron.*, vol. 45, pp. 633-637, 1998.

[7] S.B. Choi, D. W. Park, and S. Jayasuriya, "A time-varying sliding surface for fast and robust tracking control of second-order uncertain syatems," Automatica, vol. 30, no. 5, pp. .899-904, 1994.

[8] Chang. Zhai, Ming. Wu, "Variable structure control method for discrete time systems," *Jour. Shan. Jiao.*, vol. 34, no. 5, pp. 719-722, 2000.

[9] K. Mao, Z. Wang, and D. Zhang, "New method of variable structure control for discrete time systems," *Elec. Mach. Contr.*, vol. 5, no. 13, pp. 181-185, 2001.

[10] Li. Song, Le. Bu, and Qiong. Yao, "Controller for active power distribution based on discrete variable structure control theory," *Contr. Theo. Appl.*, vol. 18, no. 4, pp. 539-542, 2001.

[11] Qiong. Yao, Li. Song, and Hong Wen, "Proportional-constant-variable rate control for discrete-time variable structure systems," *Control and Decision*, vol. 15, no. 3, pp. 329-332, 2001.

[12] Shuang. He, Wen. Yi, and Pei. Bo, "Chattering-free discrete quasi-sliding mode controller," *Control and Decision*, vol. 16, no. 3, pp. 380-382, 2001.

[13] L. Song, H, Li, and S. Chen, "Sliding mode prediction-based discrete variable structure control and its application to ship autopilot," *Chin. Soc. for Elec. Eng.*, vol.23, pp. 160-163, 2003.

[14] J. C. Hung, "VSC-on-demand control systems," in Proc. First Chinese World Congress on Intell. Contr. and Intell. Automation. Beijing: Science Press, vol.3, pp.2186-2191, 1993.