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Position Control of a PMSM Using Conditional Integrators

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Abstract— **We study the application of the "conditional integrator" technique to position control of a permanent magnet stepper motor. This is a recent approach to the output regulation of minimum-phase nonlinear systems, that results in better transient performance over conventional integral control. Global regulation results are provided for state-feedback control and semi-global results under output-feedback. Simulation results show that good tracking performance is achieved, in spite of partial knowledge of the machine parameters.**

I. Introduction

Permanent magnet stepper motors (PMSM) have become a popular alternative to the traditionally used brushed DC motors (BDCM) for many high performance motion control applications for several reasons : better reliability due to the elimination of mechanical brushes, better heat dissipation as there are no rotor windings, higher torque-to-inertia ratio due to a lighter rotor, lower price, and easy interfacing with digital systems [3]. They are now widely used in numerous motion control applications such as robotics, printers, process control systems etc. Some of the drawbacks of PM machines when operated in open loop are the occurrence of large overshoots and settling times, especially when the load inertia is high, and the fact that microstepping is not possible in the open loop mode of operation. As a result, over the years, many control algorithms that can improve the performance of PMSMs in a closed loop operation have been examined.

Zribi and Chiasson [12] used the technique of exact feedback linearization using full state feedback, with extensions to the partial state feedback case in [1], [2], and experimental validation of the controller in [1]. Adaptive solutions to the problem, under varying assumptions on the measurable states and on what parameters in the system are partially or wholly known, have appeared, for example, in the works of Dawson and co-workers [3], Khorrami and co-workers [6] and others [8]. A sliding mode controller along with implementation results was reported in Zribi *et al.* [13]. We present a new approach to position control based on a recently proposed technique for the output regulation of minimum-phase nonlinear systems [9], [11], which guarantees global regulation under state-feedback, and semi-global under output-feedback, under slightly differing assumptions. The rest of this paper is organized as follows. The system model is presented in Section 2, and state and output feedback designs in Sections 3 and 4 respectively. An extension to the case of sinusoidal reference signals is presented in Section 5, and our conclusions in Section 6.

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II. System Model

A schematic of a PMSM that has a slotted stator with two phases, and a PM rotor is shown in Fig 1.

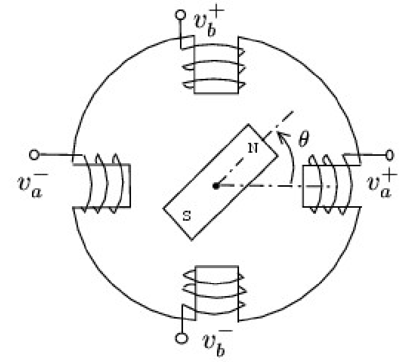

Fig. 1. Schematic of a two phase PMSM.

The mathematical model of the PMSM is given below [1], [12], [13]

$$\begin{aligned}
\frac{di_a}{dt} &= \frac{1}{L}(v_a - Ri_a + K_m\omega\,\sin(N_r\phi)) \\
\frac{di_b}{dt} &= \frac{1}{L}(v_b - Ri_b + K_m\omega\,\cos(N_r\phi)) \\
\frac{d\omega}{dt} &= \frac{1}{J}(K_m i_b\,\cos(N_r\phi) - K_m i_a\,\sin(N_r\phi) - B\omega - \tau_L) \\
\frac{d\phi}{dt} &= \omega
\end{aligned} \tag{1}$$

where i_a is the current in winding A, i_b is the current in winding B, ϕ is the angular displacement of the shaft of the motor, ω is the angular velocity of the shaft of the motor, v_a is the voltage across winding A, v_b is the voltage across winding B, N_r is the number of rotor teeth, J is the rotor and load inertia, B is the viscous friction coefficient, L and R are the inductance and resistance respectively of the phase windings, K_m is the motor torque (back-emf) constant, and τ_L is the load torque. The model neglects the slight magnetic coupling between the phases, the small change in inductance as a function of the rotor position, the *detent* torque [6], and the variation in inductance due to magnetic saturation. The DQ transformation [12] from the fixed axes variables (x_a, x_b) to the dq axes variables (x_d, x_q), defined by

$$\begin{bmatrix} x_d \\ x_q \end{bmatrix} \stackrel{\text{def}}{=} \begin{bmatrix} \cos(N_r\phi) & \sin(N_r\phi) \\ -\sin(N_r\phi) & \cos(N_r\phi) \end{bmatrix} \begin{bmatrix} x_a \\ x_b \end{bmatrix}$$

changes the frame of reference from the fixed phase axes to axes that are moving with the rotor. The direct current i_d corresponds to the component of the stator magnetic field along the axis of the rotor magnetic field, while the quadrature current i_q corresponds to the orthogonal component. Defining the states, inputs, and outputs as $x_1 = i_d$, $x_2 = i_q$, $x_3 = \omega$, $x_4 = \phi$, $u_1 = v_d$, $u_2 = v_q$, $y_1 = i_d$, and $y_2 = \phi$, it can be verified that (1) can be rewritten in the form of the following state model for the PMSM

$$\left.\begin{array}{rcl} \dot{x}_1 &=& -k_1 x_1 + k_5 x_2 x_3 + k_6 u_1 \\ \dot{x}_2 &=& -k_1 x_2 - k_5 x_1 x_3 - k_2 x_3 + k_6 u_2 \\ \dot{x}_3 &=& k_3 x_2 - k_4 x_3 - d_0 \\ \dot{x}_4 &=& x_3 \\ y_1 &=& x_1 \\ y_2 &=& x_4 \end{array}\right\} \quad (2)$$

where the constants k_1 to k_6 and d_0 are related to N_r, J, B, L, R, K_m and τ_L by $k_1 = \frac{R}{L}$, $k_2 = \frac{K_m}{L}$, $k_3 = \frac{K_m}{J}$, $k_4 = \frac{B}{J}$, $k_5 = N_r$, $k_6 = \frac{1}{L}$, and $d_0 = \frac{\tau_L}{J}$. It is desired that the rotor angular position and direct-axis current track given references $\phi_d(t)$ and $I_{dd}(t)$ that tends to constant values as $t \to \infty$ [13].

III. State Feedback Control

It can be verified that the system (2) has full vector relative degree $\rho = \{1, 3\}$, globally in R^4. In ideal sliding mode control, a choice of sliding surface functions would have been

$$\left.\begin{array}{rcl} s_1 &=& e_1,\ e_1 \stackrel{\text{def}}{=} y_1 - I_{dd}, \\ s_2 &=& k_1^2 e_2 + k_2^2 \dot{e}_2 + \ddot{e}_2, \quad e_2 \stackrel{\text{def}}{=} y_2 - \phi_d \end{array}\right\}$$

with k_1^2 and k_2^2 chosen such that the polynomial $x^2 + k_2^2 x + k_1^2$ is Hurwitz, and the control designed to force the trajectories to reach the surfaces $s_i = 0$ in finite time and remain on them thereafter. However, as is well-known, this method suffers from the drawback of *chattering*, and can excite unmodelled high-frequency dynamics and degrade system performance. Replacing the discontinuous (ideal) control by a continuous approximation in a boundary layer of the sliding surface reduces chattering but at the expense of a finite steady-state error. Asymptotic regulation can be recovered in the continuous sliding mode control (CSMC) by introducing a conventional integrator $\dot{\sigma}_i = e_i$ as part of the sliding surface but (i) requires a redesign of the sliding surface parameters and (ii) degrades transient performance in comparison to ideal SMC, in part due to the increase in system order as a result of the integrator, and in part because of the interaction of the integrator with control saturation, which leads to the well-known problem of *windup*.

In order to recover the asymptotic regulation of ideal SMC and also retain its transient performance, we introduce integrators in the sliding surface design as follows

$$\left.\begin{array}{rcl} s_1 &=& k_0^1 \sigma_1 + e_1 \\ s_2 &=& k_0^2 \sigma_2 + k_1^2 e_2 + k_2^2 \dot{e}_2 + \ddot{e}_2 \end{array}\right\} \quad (3)$$

where σ_1 and σ_2 are the outputs of the conditional integrators

$$\left.\begin{array}{rcl} \dot{\sigma_1} &=& -k_0^1 \sigma_1 + \mu_1 \text{sat}(s_1/\mu_1) \\ \dot{\sigma_2} &=& -k_0^2 \sigma_2 + \mu_2 \text{sat}(s_2/\mu_2) \end{array}\right\} \quad (4)$$

$k_0^1, k_0^2 > 0$, the values of k_1^2 and k_2^2 are retained from the ideal SMC design, and μ_1, μ_2 are "sufficiently small" positive constants representing the widths of the boundary layers for s_1 and s_2 respectively. To see the relation of (4) to integral control, observe that inside the boundary layer $\{|s_i| \le \mu_i\}$, (4) reduces to

$$\left.\begin{array}{rcl} \dot{\sigma_1} &=& e_1 \\ \dot{\sigma_2} &=& k_1^2 e_2 + k_2^2 \dot{e}_2 + \ddot{e}_2 \end{array}\right\}$$

which implies that $e_i = 0$ at equilibrium. Thus, (4) represents a "conditional integrator" that provides integral action only inside the boundary layer.

Since $\ddot{e_2} = k_3 x_2 - k_4 x_3 - d_0 - \phi_d^{(2)}$ is required to construct s_2, the parameters k_3, k_4 and d_0 will need to be known. We assume that k_1, k_2 and k_6 are unknown, corresponding to uncertainties in the resistance R and inductance L of the phase windings, while k_5, being the number of rotor teeth, is precisely known. Let $\theta = [k_1, k_2, k_6]^T$ denote the vector of unknown parameters. It can be verified that the expressions for $\dot{s}_i$ take the form

$$\left.\begin{array}{rcl} \dot{s}_1 &=& k_0^1(-k_0^1 \sigma_1 + \mu_1 \text{sat}(s_1/\mu_1)) \\ && +F_1(x, e_1, I_{dd}^{(1)}, \theta) + a_{11}(x, \theta) u_1 \\ \dot{s}_2 &=& k_0^2(-k_0^2 \sigma_2 + \mu_2 \text{sat}(s_2/\mu_2)) \\ && +F_2(x, e_2, \dot{e}_2, \ddot{e}_2, \phi_d^{(3)}, \theta) + a_{22}(x, \theta) u_2 \end{array}\right\}$$

where $F_1(\cdot) = -\theta_1 x_1 + k_5 x_2 x_3 - I_{dd}^{(1)}$, $F_2(\cdot) = k_1^2 \dot{e}_2 + k_2^2 \ddot{e}_2 - \phi_d^{(3)} - k_4(k_3 x_2 - k_4 x_3 - d_0) - k_3 \theta_1 x_2 - k_3 k_5 x_1 x_3 - k_3 \theta_2 x_3$, $a_{11}(\cdot) = \theta_3$, and $a_{22}(\cdot) = k_3 \theta_3$. The control u is taken as

$$u_i = \frac{-\hat{F}_i(x, e, \varpi) - \beta_i(x, e, \varpi)\ \text{sat}(s_i/\mu_i)}{\hat{a}_{ii}},\ i = 1, 2 \quad (5)$$

where $e^T = [e_1, e_2, \dot{e}_2, \ddot{e}_2]$, $\varpi^T = [I_{dd}^{(1)},\ \phi_d^{(3)}]$, $\hat{a}_{11}(\cdot) = \hat{\theta}_3$, $\hat{a}_{22}(\cdot) = k_3 \hat{\theta}_3$, $\hat{\theta}_3 > 0$ being a nominal value of θ_3. We will choose the nominal control component $\hat{F}_i(\cdot)$ to cancel all known/nominal terms in $F_i(\cdot)$, i.e.,

$$\begin{array}{rcl} \hat{F}_1(\cdot) &=& -\hat{\theta}_1 x_1 + k_5 x_2 x_3 - I_{dd}^{(1)} \\ \hat{F}_2(\cdot) &=& k_1^2 \dot{e}_2 + k_2^2 \ddot{e}_2 - \phi_d^{(3)} - k_4(k_3 x_2 - k_4 x_3 - d_0) \\ && -k_3 \hat{\theta}_1 x_2 - k_3 k_5 x_1 x_3 - k_3 \hat{\theta}_2 x_3 \end{array}$$

where $\hat{\theta}_1 > 0$ and $\hat{\theta}_2 > 0$ are nominal values of θ_1 and θ_2 respectively. To make the choice of $\beta_i(\cdot)$ precise, suppose that $\theta_i \in [\theta_i^m, \theta_i^M]$, where $0 < \theta_i^m < \theta_i^M$ and θ_i^m and θ_i^M are known. Let $\Delta_i(\cdot) = F_i(\cdot) - (\theta_3/\hat{\theta}_3)\hat{F}_i(\cdot)$ and $\rho_i(x, e, \varpi)$ be such that $\sup\left|\frac{\hat{\theta}_3 \Delta_i(\cdot)}{\theta_3}\right| \le \varrho_i(x, e, \varpi)$, where the supremum is taken over all $x \in R^4$, $e \in R^4$, $\theta_i \in [\theta_i^m, \theta_i^M]$ and $\varpi \in R^2$. The functions β_i are chosen as $\beta_i(\cdot) = \varrho_i(\cdot) + q_i$, where $q_i > 0$. The results of [9, Theorem 2] allow us to conclude that the controller (3) - (5) achieves global regulation, provided μ_1 and μ_2

are sufficiently small. In addition, the above CSMC with conditional integrators "recovers the performance" (see [9, Theorem 3] for a precise statement of this result) of the corresponding ideal SMC

$$\left.\begin{array}{rcl} s_1 & = & e_1 \\ s_2 & = & k_1^2 e_2 + k_2^2 \dot{e}_2 + \ddot{e}_2 \\ u_i & = & \frac{-\hat{F}_i(x,e,\varpi)-\beta_i(x,e,\varpi)\ \mathrm{sgn}(s_i)}{\hat{a}_{ii}},\ i=1,2 \end{array}\right\}$$

For the purpose of simulation, we use the following values for the system parameters, obtained from [13] : $K_m = 0.1349\ Nm/A$, $J = 4.1295 \times 10^{-4}\ kgm^2$, $B = 0.0013\ Nm/rad/s$, $N_r = 50$ and $\tau_L = 0.2\ kg$. The resistance R and inductance L are assumed to be unknown, with nominal values of $\hat{R} = 20\ \Omega$ and $\hat{L} = 35\ mH$ respectively. Also, $R \in [R^m, R^M]$, and $L \in [L^m, L^M]$, with $L^m = 30\ mH$, $L^M = 40\ mH$, $R^m = 19\ \Omega$, and $R^M = 21\ \Omega$. The actual values of the resistance and inductance are taken as $R = 19\ \Omega$ and $L = 40\ mH$. The current reference is taken as $I_{dd}(t) = 0A$ [7]. The values of the controller parameters are taken as $k_0^1 = 20$, $k_0^2 = 100$, $k_1^2 = 7.5 \times 10^4$, $k_2^2 = 550$, $\mu_1 = 0.1$, and $\mu_2 = 50$. Initial values for all the states are taken as zero. The functions $\beta_i(\cdot)$ are taken as

$$\begin{array}{rcl} \beta_1(\cdot) & = & [(R^M - \hat{R})|x_1| + (L^M - \hat{L})N_r|x_2 x_3| + q_1]/\hat{L}, \\ q_1 & = & 2.1 k_0^1 \mu_1 L^M \\ \beta_2(\cdot) & = & [(R^M - \hat{R})k_3|x_2| + (L^M - \hat{L})|k_1^2 \dot{e}_2 + k_2^2 \ddot{e}_2 \\ & & -k_4(k_3 x_2 - k_4 x_3 - d_0) - k_3 k_5 x_1 x_3| + q_2]/\hat{L}, \\ q_2 & = & 2.1 k_0^2 \mu_2 L^M \end{array}$$

The desired angular position is taken as $\phi_d(t) = 0.03142\ [u(t) + u(t-0.5)]$, where $u(t)$ is the unit step function. The results of the simulation are shown in Fig 2. We focus our attention on the error e_2, since it corresponds to the variable ϕ of physical interest and also because it is the harder of the two outputs to control. However, our observations regarding e_2 also hold for e_1. From the figure, it is clear that good tracking performance with very little overshoot is achieved, independent of the magnitude of $\phi_d(t)$.

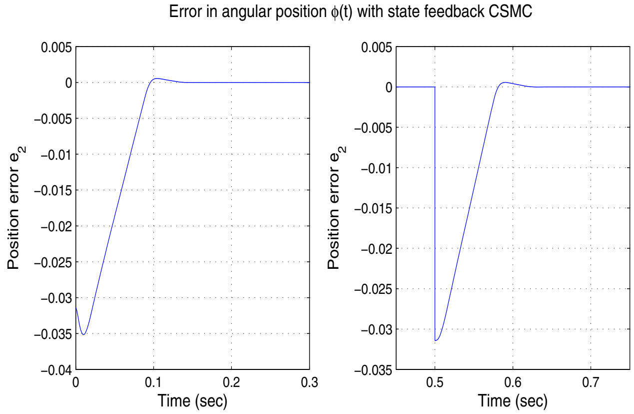

Fig. 2. Tracking error performance under state-feedback integral control.

IV. Output Feedback

Suppose that the angular velocity ω of the motor shaft is unavailable for feedback. It is easy to verify that for the system to have a uniform vector relative degree and be transformable to normal form, none of the positive constants k_i and d_0 need to be exactly known. Accordingly, we will assume that in the present case, in addition to the resistance R and inductance L, the parameters K_m, B, J and the load torque τ_L are all unknown, and take the vector θ of unknown parameters as $\theta = [k_1, k_2, k_3, k_4, k_6, d_0]^T$. Since ω is unavailable for feedback, so is $\dot{e}_2 = \omega - \dot{\phi}_d$. Furthermore, even if ω were available for feedback, since $\ddot{e}_2 = k_3 i_q - k_4 \omega - d_0 - \phi_d^2$, and k_3, k_4 and d_0 are unknown, $\ddot{e}_2$ would be unavailable for feedback. Therefore, we estimate $\dot{e}_2$ and $\ddot{e}_2$ in (3) using the high-gain observer (HGO) [4]

$$\left.\begin{array}{rcl} \dot{z}_1 & = & z_2 + \alpha_1(e_2 - z_1)/\epsilon \\ \dot{z}_2 & = & z_3 + \alpha_2(e_2 - z_1)/\epsilon^2 \\ \dot{z}_3 & = & \alpha_3(e_2 - z_1)/\epsilon^3 \end{array}\right\} \quad (6)$$

where the positive constants α_1, α_2, and α_3 are chosen such that the polynomial $\lambda^3 + \alpha_1\lambda^2 + \alpha_2\lambda + \alpha_3$ is Hurwitz. We replace $\dot{e}_2$, $\ddot{e}_2$, and also s_2 in (4) by their estimates z_2, z_3, and

$$\hat{s}_2 = k_0^2\sigma_2 + k_1^2 e_2 + k_2^2 z_2 + z_3$$

respectively. Finally, motivated in part by the goal of simplifying the design, and in part by the need to work with saturated controls (this is required in order to prevent the peaking phenomenon associated with HGOs [4]), we (i) make use of the flexibility in choosing $\hat{F}_i$, and let $\hat{F}_i = 0$, and (ii) choose $\beta_i(\cdot)$ as a constant M_i (say), which is equal to the maximum physically allowable value for the control component $|u_i|$. With this choice, the final expression for the control then becomes

$$\begin{array}{rcl} u_1 & = & -M_1\ \mathrm{sat}(s_1/\mu_1) \\ u_2 & = & -M_2\ \mathrm{sat}(\hat{s}_2/\mu_2) \end{array} \quad (7)$$

The results of [11, Theorem 1] show that provided μ_1, μ_2, and ϵ are "sufficiently small", and M_1, M_2 can be chosen "sufficiently large", the controller (7) can achieve semi-global regulation. Moreover, when M_1 and M_2 are limited in magnitude, there is a tradeoff between the control magnitude and the region of attraction. A performance recovery result similar to the one in the previous section, which says the output feedback CSMC controller with integral action (7) recovers the performance of a corresponding state feedback ideal SMC can be found in [11, Theorem 2].

For the purpose of simulation, we let K_m, J, B, N_r, τ_L, k_0^1, k_0^2, k_1^2, k_2^2, μ_1, μ_2 and I_{dd} retain their values from the previous simulation. Also we take $R = 19.5\ \Omega$, $L = 30\ mH$, and $\phi_d = 0.03142\ rads$. The HGO gains are taken as $\alpha_1 = 17$, $\alpha_2 = 80$ and $\alpha_3 = 100$, and the saturation levels for the controls as $M_1 = M_2 = 50$. We compare the performance of the output feedback controller with the partial state-feedback design $u_i = -M_i\ \mathrm{sat}(s_i/\mu_i)$, which makes use of measurements of e_1, e_2, $\dot{e}_2$ and $\ddot{e}_2$. This could, for instance, be the case when the full state x is available for feedback and the parameters k_3, k_4 and d_0 are known. Fig 3(a) shows the results of the simulation for $\epsilon = 10^{-4}$, and we see that good tracking performance is achieved by the output-feedback controller, which uses minimal information about the system. Fig 3(b) shows the effect of ϵ on the recovery of the state-feedback performance, and it is clear from the figure that the error e_2 under output-feedback approaches the error e_2 under state-feedback as ϵ tends to zero.

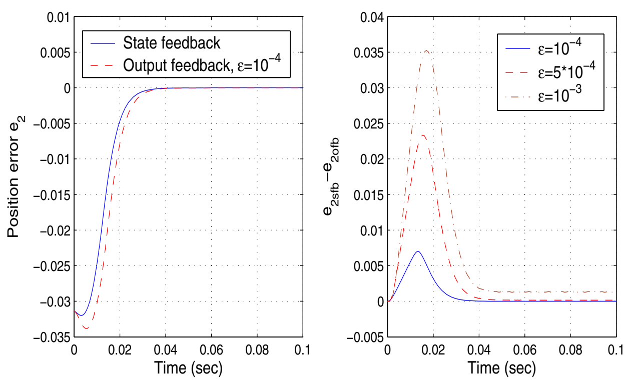

Fig 3(a) : State/output feedback integral control

Fig 3(b) : Performance recovery of sfb as ε tends to zero

Fig. 3. Tracking error performance under the output-feedback integral control.

As noted in [11], the state-feedback CSMC with conditional integrators can be thought of as the following two-step modification to ideal SMC : (i) Take $s_i = s_i^* + k_0^i\ \sigma_i$, where $s_i^* = 0$ is the sliding surface under ideal SMC, and σ_i is the state of the conditional integrator $\dot{\sigma}_i = -k_0^i\sigma_i + \mu_i \mathrm{sat}(s_i/\mu_i)$, $\mu_i, k_0^i > 0$ and μ_i "sufficiently small", (ii) Replace $\mathrm{sgn}(s_i^*)$ in ideal SMC with $\mathrm{sat}(s_i/\mu_i)$. In the output-feedback design, there is the additional step of replacing s_i with $\hat{s}_i$. The performace of such a control is then "close" to that of ideal SMC. In light of the preceding remark, it becomes imperative to explain the advantage of the proposed technique over ideal SMC. Since we do not require the boundary layer widths μ_1 and μ_2 to be arbitrarily small in order to achieve asymptotic regulation, but only "small enough" to stabilize the equilibrium point (and also small enough for performance recovery of ideal SMC), we expect less sensitivity of the method to the problem of chattering. As a demonstration of this, consider a sampled-data implementation of the above controller, i.e., we assume that the inputs to the controller are sampled and held signals, with a zero-order hold, and likewise for the controller outputs. We redo the previous simulation (with the state feedback CSMC) and compare the results versus the ideal sliding mode control $u_i = -M_i\ \mathrm{sgn}(s_i)$. The sampling period is assumed to be $T = 0.1$ms. The results are shown in Fig 4, and we see that asymptotic regulation is lost with the ideal SMC, and there is considerable chattering in the control v_q. Asymptotic regulation is retained with the CSMC with conditional integrator, and there is no chattering in the control.

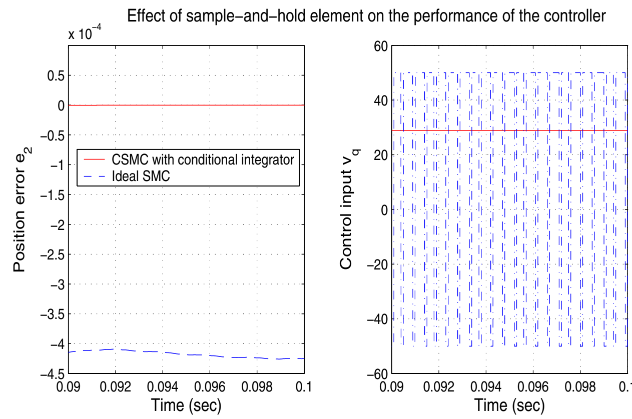

Fig. 4. Effect of sampled-time implementation on ideal SMC and CSMC with conditional integrator.

V. Extension to sinusoidal exogenous signals

The integral control designs of the previous sections were done with the goal of point-to-point motion of the PMSM. In many positioning applications, the desired trajectory for the position is a sinusoid [3]. Specifically, suppose that the desired trajectory asymptotically converges to $\phi_d(t) = r_0\ \sin(\omega_0 t)$ [1], which is generated by the neutrally stable exosystem

$$\dot{w} = \begin{bmatrix} 0 & \omega \\ -\omega & 0 \end{bmatrix} w \stackrel{\text{def}}{=} S_0 w,\ \ w^T(0) = [0,\ \ r_0],\ \ r(t) = w_1$$

We show that the conditional servocompensator design of [10], which is a natural extension of the conditional integrator design of [11] to the servocompensator design of [5], can be applied to this case. To that end, our first goal is to identify a suitable linear internal model that generates the steady-state values of the control inputs v_d and v_q. As before, the desired reference for the current i_d is a constant I_{dd}. It can be verified that with steady-state values $x_{1ss}(t) = I_{dd}$ and $x_{4ss}(t) = r_0\sin(\omega_0 t)$ respectively for x_1 and x_4, the steady-state values of x_2 and x_3 are given by $x_{2ss} = [d_0 + k_4 r_0\omega_0\ \cos(\omega_0 t) - r_0\omega_0^2\ \sin(\omega_0 t)]/k_3$ and $x_{3ss} = r_0\omega_0\ \cos(\omega_0 t)$ respectively, and the steady state values of the control inputs v_d and v_q are given by

$$\begin{aligned} u_{1ss} &= \gamma_1 + \gamma_2\ \cos(\omega_0 t) \\ &\quad +\gamma_3\ \sin(2\omega_0 t) + \gamma_4\ \cos(2\omega_0 t) \\ u_{2ss} &= \gamma_5 + \gamma_6\ \sin(\omega_0 t) + \gamma_7\ \cos(\omega_0 t) \end{aligned} \qquad (8)$$

for some constants γ_1 to γ_7. The steady state value of the control u_{1ss} satisfies an identity of the form $L_s^q\chi = c_0\chi + c_1 L_s\chi + \cdots + c_{q-1}L_s^{q-1}\chi$, where $L_s\chi = (\frac{\partial\chi}{\partial w})S_0 w$ with $q = 5$, $c_0 = 0$, $c_1 = -4\omega_0^4$, $c_2 = 0$, $c_3 = -5\omega_0^2$, and $c_4 = 0$,

[1] More general reference trajectories can be considered, as long as they satisfy the conditions of [10, Assumption 3].

while u_{2ss} does so with $q = 3$, $c_0 = 0$, $c_1 = -\omega_0^2$ and $c_2 = 0$. An explanation of this identity can be found in the paragraph following [10, Assumption 5]. Roughly speaking, the identity guarantees that the existence of a linear internal model that generates the trajectories of the exosystem, along with a number of higher-order deformations that result from the plant nonlinearities. Let

$$S_1 = \begin{bmatrix} 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \\ 0 & -4\omega_0^4 & 0 & -5\omega_0^2 & 0 \end{bmatrix}, \; S_2 = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & -\omega_0^2 & 0 \end{bmatrix}$$

be the internal model matrices corresponding to u_{1ss} and u_{2ss}, and

$$J_1 = [\ 0 \quad 0 \quad 0 \quad 0 \quad 1\ ]^T, \; J_2 = [\ 0 \quad 0 \quad 1\ ]^T$$

We take σ_1 and σ_2 as outputs of the conditional servcompensators

$$\dot{\sigma}_i = (S_i - J_i K_0^i)\sigma_i + \mu_i J_i \mathrm{sat}(\hat{s}_i/\mu_i) \qquad (9)$$

where

$$\left.\begin{array}{rcl} s_1 & = & K_0^1 \sigma_1 + e_1 \\ s_2 & = & K_0^2 \sigma_2 + k_1^2 e_2 + k_2^2 \dot{e}_2 + \ddot{e}_2 \end{array}\right\} \qquad (10)$$

The matrices K_0^i are chosen such that $S_i - J_i K_0^i$ are Hurwitz, the scalars k_1^2 and k_2^2 are chosen such that the polynomial $x^2 + k_2^2 x + k_1^2$ is Hurwitz, $\hat{s}_1 = s_1$, $\hat{s}_2 = K_0^2\sigma_2 + k_1^2 e_2 + k_2^2 z_2 + z_3$, where z_2 and z_3 are estimates of $\dot{e}_2$ and $\ddot{e}_2$ respectively, provided by the high-gain observer (6). The control is taken as in (7), i.e.,

$$u_i = -M_i \ \mathrm{sat}(\hat{s}_i/\mu_i) \qquad (11)$$

This completes the design of the controller. Note from (9) that servocompensation is provided only "conditionally", inside the boundary layers $|s_i| \leq \mu_i$. When $|s_i| \geq \mu_i$, (9) represents the dynamics of an exponetially stable system, driven by an $O(\mu_i)$ term, and therefore $\|\sigma_i\| = O(\mu_i)$, provided $\sigma_i(0) = O(\mu_i)$. Analytical results for stability and performance of the controller (9) - (11), similar to the ones in the previous sections, can be found in [10, Theorem 1 & 2].

For the purposes of simulation, all values are retained from the one in the previous section, except for the reference $\phi_d(t)$ and the matrices K_0^i. The former is chosen as $\phi_d(t) = r_0 \cos(\omega_0 t)$, which is simply phase-shifted from $\phi_d(t) = r_0 \sin(\omega_0 t)$ by $\pi/2$, and chosen this way to have a non-zero initial error $e_2(0)$. Two sets of values of (r_0, ω_0) are used, $(r_0, \omega_0) = (\pi/2, 2)$ and $(r_0, \omega_0) = (\pi/10, 5)$. The matrices K_0^i are chosen to place the eigenvalues of $S_1 - J_1 K_0^1$ and $S_2 - J_2 K_0^2$ at $\{-1, -2 \pm j, -3 \pm j\}$ and $\{-1, -2 \pm j\}$ respectively. We compare the performance against a CSMC design that does not include a servocompensator, i.e., $\hat{s}_1 = e_1$, $\hat{s}_2 = k_1^2 e_2 + k_2^2 z_2 + z_3$, where z_2 and z_3 are as defined above, and $u_i = -M_i \ \mathrm{sat}(\hat{s}_i/\mu_i)$. The results of the simulation are shown in Fig 5. As before, we see that good tracking performance is achieved by the output feedback servocompensator design, which uses minimal information about the system. The transient performance of the CSMC without servocompensator is close to the one with a servocompensator (indistinguishable in the figure), but the steady-state error is non-zero.

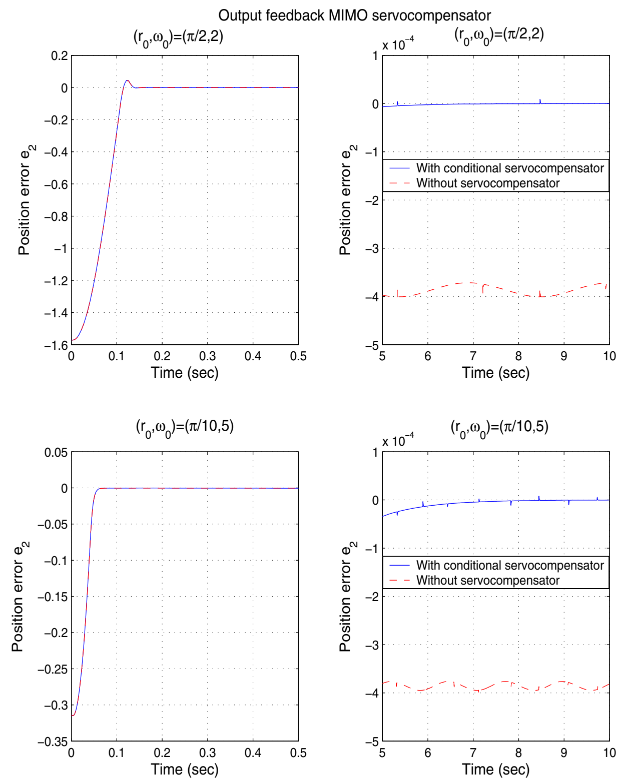

Fig. 5. Sinusoidal reference tracking using output-feedback servocompensators.

Before we present our conclusions, we make the following observations. The assumption that the load torque τ_L is constant can be relaxed : (i) for the constant exogenous signals case, since we only require that τ_L be asymptotically constant, and $\phi(t)$ and $\omega(t)$ approach a constant and zero respectively, we can allow $\tau_L = f_\tau(\phi(t), \omega(t))$, where $f_\tau(\cdot)$ is a suficiently smooth function of its arguments, and (ii) for the sinusoidal exogenous signals case, on account of the need for the identity following (8) that the steady-state control must satisfy, it can be verified that $f_\tau(\cdot)$ will have to be a polynomial function of its inputs, and that its form must be known, i.e., $f_\tau(\phi, \omega) = \sum_{i \in I, j \in J} \alpha_{ij} \phi^i \omega^j$, where $I, J \subset Z_{\geq 0}$ known, and α_{ij} possibly unknown. Furthermore, when the polynomial condition is violated, for example, when the load torque is of the form $\tau_L = N \sin(\phi)$ say, then, as shown in [5, Theorem 2], polynomial approximations may be used to achieve practical regulation of the error.

VI. Conclusions

In this paper, we presented an approach for position control of a permanent magnet stepper motor. In the constant references case, we looked at both a state feedback design for global regulation, and an output feedback design for regional/semi-global regulation. An extension of the design to the case of sinusoidal references was provided. Analytical results for stability and performance of the proposed method are provided. Simulation results show that good tracking performance is achieved in all cases, in spite of partial knowledge of the machine parameters. The state feedback design for constant exogenous signals was also presented in an earlier work [9], but the current paper extends those results to the case of output feedback and sinusoidal exogenous signals. While we restricted our presentation to unknown plant parameters θ, since our approach can be thought of as a modification to ideal SMC, our problem formulation (see [10], [11]) allows for matched disturbances δ that could possibly depend on θ and x, with the analytical results about regulation and performance remaining unchanged. Lastly, while we specifically considered PMSMs in this paper, our results can be extended to other types of motors, such as permanent magnet synchronous and dc motors.

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