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Impact of Correlation Errors on Optimum Kalman Filter Matrices Gains Identification in Multicoordinate Systems

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***Abstract*— This paper investigates the impact that errors in the innovation correlation calculations has upon the steady-state Kalman filter gain identification. This issue arises in all real time applications, where the correlations must be calculated from experimental data. The algorithm proposed by [1] is considered and equations describing the impact are established. Simulation results are presented and discussed. Finally, experimental results for the algorithm in [1], applied to estimate the states of a servo system, are presented.**

I. INTRODUCTION

The Kalman filter has been widely used in many engineering applications. This is mainly due to its ability to deal with linear systems corrupted by uncertainties and provide an optimum state estimate, according to the minimum mean square error optimality criteria, in a recursive way. See [2], [3] and [4] for details. To accomplish this, the process noise covariance matrix Q and the measurement noise covariance matrix R must be known. In many practical situations, Q and R are unknown or known only approximately, thereby providing a suboptimum filter. Several authors have presented schemes for the identification of the unknown covariances or the steady-state gain of the optimum filter. Among them, we have [5] and [6] that use the innovation correlation functions as the necessary information for the identification of the steady-state optimum filter gains.

In [1] it is proposed a generalization of the identification algorithm presented by [6], with sensors in different coordinate systems and considering uncertainties in the mappings among the local coordinate systems and the central coordinate systems. When only one sensor is employed at the central coordinate system, and there are no uncertainties in the mappings, this algorithm provides the same equations proposed by [6].

This paper is an extension of the results presented in [7]. Here, it is presented an analysis of the impact that errors on evaluation of the innovation correlation functions have on the identification of the steady-state matrices gains of the optimum filter, when using the approach proposed by [1]. As mentioned before, [1] is a generalization of [6]. Therefore, the equations and the method developed in this paper can be applied to the scenario proposed by [6].

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This paper is organized as follows: in section II the problem is presented together with a summary of the algorithm proposed by [1]. In section III, equations related to the impact that errors on the evaluation of the innovations correlation functions have on the identification of the steady-state matrices gain of the optimum filters are developed. Finally, simulation results are presented in section IV and an analysis of a practical implementation applied to a position servo system is presented in section V.

II. PROBLEM FORMULATION

Consider a time-invariant multivariable stochastic linear discrete system with N sensors

$$x_{k+1} = Ax_k + Gw_k \tag{1}$$

$$y_k^i = H^i x_k^i + v_k^i \qquad i = 1, \ldots, N \tag{2}$$

$$E\{w_k\} = 0, \quad E\{w_k w_j^T\} = Q\delta(k-j) \tag{3}$$

$$E\{v_k^i\} = 0, \quad E\{v_k^i v_j^{i^T}\} = R^i\delta(k-j) \tag{4}$$

$$E\{x_k w_r^T\} = 0, E\{x_k v_r^{i^T}\} = 0, E\{w_k v_r^{i^T}\} = 0, \forall k, r, i \tag{5}$$

$$E\{v_k^i v_r^{j^T}\} = 0, \ \forall k, r, i \neq j \tag{6}$$

where $E\{\cdot\}$ denotes expectation and $\delta(k-j)$ denotes the Kronecker delta function. The vector $x_k \in \Re^n$ is the state vector to be estimated at the k^{th} moment and $w_k \in \Re^q$ are the uncertainties in the state characterized by the covariance matrix Q. There are N sensors in the system (2), each of which corresponds to a local measurement referred to their local coordinate system. The symbol $y_k^i \in \Re^r$ describes the measurement made by sensor i, with uncertainty $v_k^i \in \Re^r$, specified by the covariance matrix R^i at the k^{th} moment. In (1), the state vector x_k is represented in the central coordinate system at the k^{th} moment while the state vectors x_k^i, in (2) are represented in their local coordinate system at the k^{th} moment. Q and R^i are bounded positive definite matrices, that is, $Q > 0$ and $R^i > 0$. Initial state x_0 is normally distributed with zero mean and covariance P_0. It is assumed that the system (1)-(2) is stochastically observable and controllable.

The mappings among the local and the central coordinate systems are described by

$$x_k^i = J^i x_k + T^i \qquad i = 1, \ldots, N \tag{7}$$

where $J^i \in \Re^{nxn}$ represents a rotation and $T^i \in \Re^n$ represents a translation of the local coordinate systems referred to the central coordinate systems, which have uncertainties characterized by

$$J^i = J_0^i + \Delta J^i \tag{8}$$

$$T^i = T_0^i + \Delta T^i \tag{9}$$

$$E\{\Delta \overline{J}^i\} = 0,\ E\{\Delta \overline{J}^i \Delta \overline{J}^{i^T}\} = Q_J^i \tag{10}$$

$$E\{\Delta T^i\} = 0,\ E\{\Delta T^i \Delta T^{i^T}\} = Q_T^i \tag{11}$$

$$E\{\Delta \overline{J}^i \Delta T^{j^T}\} = 0, E\{\Delta \overline{J}^i x_k^T\} = 0, \forall k, i, j \tag{12}$$

$$E\{\Delta \overline{J}^i w_k^T\} = 0, E\{\Delta \overline{J}^i v_k^{j^T}\} = 0, \forall k, i, j \tag{13}$$

$$E\{\Delta \overline{J}^i \Delta \overline{J}^{j^T}\} = 0, \forall i \neq j \tag{14}$$

$$E\{\Delta T^i x_k^T\} = 0, E\{\Delta T^i w_k^T\} = 0, \forall k, i, j \tag{15}$$

$$E\{\Delta T^i v_k^{i^T}\} = 0, \forall k, i, j \tag{16}$$

$$E\{\Delta T^i \Delta T_k^{i^T}\} = 0, \forall i \neq j \tag{17}$$

where

$$\Delta \overline{J}^i = [\Delta J_{11}^i, \ldots, \Delta J_{1n}^i, \ldots, \ldots, \Delta J_{n1}^i, \ldots, \Delta J_{nn}^i]^T \tag{18}$$

The uncertainties w_k, v_k^i, $\Delta \overline{J}^i$, ΔT^i are assumed white Gaussian, i. e.,

$$w_k \sim N(0, Q), v_k^i \sim (0, R^i) \tag{19}$$

$$\Delta \overline{J}^i \sim N(0, Q_J^i), \Delta T^i \sim N(0, Q_T^i) \tag{20}$$

It is also assumed that

$$rank \left[\overset{\sim}{H^i}^T,\ A^T\ \overset{\sim}{H^i}^T,\ \cdots,\ (A^T)^{n-1}\ \overset{\sim}{H^i}^T \right]^T = n \tag{21}$$

$$rank\left[G,\ AG,\ \cdots,\ (A)^{n-1}G\right] = n \tag{22}$$

In (21) $\overset{\sim}{H^i} = H^i J_0^i$.

A. Multisensor Integration

When the multisensor system is completely known, that is, all the matrices A, G, H^i, J_0^i, Q, R^i, Q_J^i and Q_T^i and the vector T_0^i are precisely known, then, the multisensor integration algorithm proposed by [8], accordingly the author, is optimum and can be rewritten, in steady-state, according to [1], as

$$\hat{x}_{k+1|k} = A\hat{x}_{k|k-1} + \sum_{i=1}^{N} K^i v_k^i \tag{23}$$

$$v_k^i = y_k^i - H^i J_0^i \hat{x}_{k|k-1} - H^i T_0^i, \quad i = 1, \ldots, N \tag{24}$$

In (23), K^i are the steady-state optimum filter gains represented in the central coordinate system and are described in [1].

B. Identification Algorithm

In order to apply the summarized algorithm, it is necessary the knowledge of the covariance matrices Q, R^i, Q_J^i and Q_T^i to evaluate the matrices K^i. Based on the prior information, Q^*, R^{i^*}, $Q_J^{i^*}$ and $Q_T^{i^*}$, the filter, in steady-state, is the given by

$$\hat{x}^*_{k+1|k} = A\hat{x}^*_{k|k-1} + \sum_{i=1}^{N} K^{i^*} v_k^{i^*} \tag{25}$$

$$v_k^{i^*} = y_k^i - H^i J_0^i \hat{x}^*_{k|k-1} - H^i T_0^i, \quad i = 1, \ldots, N \tag{26}$$

Hereafter, the superscript $*$ indicates that the quantities are not the optimal ones.

Based on the innovations $v_k^{i^*}$ [1] proposed the following algorithm to identify the steady-state optimum gains K^i.

1) Evaluation of correlations C_0^i:

$$C_{0_m}^i = C_0^{i^*} - \tilde{H}^i \tilde{W}_m^i \tilde{H}^{i^T}, \quad i = 1, \ldots, N \tag{27}$$

2) Identification of the optimal gains K^i:

$$K_m^i = \left[\overline{B}^{i^\dagger} \left(\overline{A}^i + \overline{D}^i C_{0_m}^i \right) - A\tilde{W}_m^i \tilde{H}^{i^T} \right] \left(C_{0_m}^i \right)^{-1} \tag{28}$$

where $\overline{B}^{i^\dagger}$ is the pseudoinverse of $\overline{B}^i$ and $\overline{A}^i$ is given by 35.

3) Calculating W_{m+1}:

$$W_{m+1} = \left(A - \sum_{i=1}^{N} K^{i^*} \tilde{H}^i \right) W_m \left(A - \sum_{i=1}^{N} K^{i^*} \tilde{H}^i \right)^T + \sum_{i=1}^{N} \left(K_m^i - K^{i^*} \right) C_{0_m}^i \left(K_m^i - K^{i^*} \right) \tag{29}$$

$$\overline{B}^i = \begin{bmatrix} \tilde{H}^i \\ \tilde{H}^i A \\ \tilde{H}^i A^2 \\ \vdots \\ \tilde{H}^i A^{n-1} \end{bmatrix}, \quad \overline{D}^i = \begin{bmatrix} -\tilde{H}^i \tilde{M}^i \\ -\tilde{H}^i A \tilde{M}^i \\ -\tilde{H}^i A^2 \tilde{M}^i \\ \vdots \\ -\tilde{H}^i A^{n-1} \tilde{M}^i \end{bmatrix} \tag{30}$$

where

$$\tilde{H}^i = H^i J_0^i \tag{31}$$

$$M^i = \sum_{a=1, a\neq i}^{N} K^{a*} \tilde{H}^a, \quad \tilde{M}^i = M^i (\tilde{H}^i)^{-1} \tag{32}$$

4) Calculating $\tilde{W}^i$:

$$\tilde{W}_{m+1}^i = W_{m+1} \tag{33}$$

The subscript m is the iteration index and the matrix W_0 can be a null matrix. Denoting the identified steady-state gains by $\hat{K}^i$ the filtering is performed by

$$\hat{x}^*_{k+1|k} = A\hat{x}^*_{k|k-1} + \sum_{i=1}^{N} \hat{K}^{i^*} v_k^{i^*} \tag{34}$$

$$\overline{A}^i = \begin{bmatrix} C_1^{i^*} + \tilde{H}^i \left(K^{i^*} + \tilde{M}^i\right) C_0^{i^*} \\ C_2^{i^*} + \tilde{H}^i \left(K^{i^*} + \tilde{M}^i\right) C_1^{i^*} + \tilde{H}^i A \left(K^{i^*} + \tilde{M}^i\right) C_0^{i^*} \\ \vdots \\ C_n^{i^*} + \tilde{H}^i \left(K^{i^*} + \tilde{M}^i\right) C_{n-1}^{i\ *} + \tilde{H}^i A \left(K^{i^*} + \tilde{M}^i\right) C_{n-2}^{i\ *} + \ldots + \tilde{H}^i A^{n-1} \left(K^{i^*} + \tilde{M}^i\right) C_0^{i^*} \end{bmatrix} \tag{35}$$

The correlation functions $C_j^{i^*}$ can be estimated by

$$\begin{aligned} C_j^{i^*} &= \frac{1}{L} \sum_{a=1}^{L-j} \upsilon_{a-j}^{i^*} (\upsilon_a^{i\ *})^T \\ & \quad i = 1, \ldots, N,\ j = 0, 1, \ldots, n \end{aligned} \tag{36}$$

where L is the number of observations made by sensor i. The correlations obtained by means of (36) are asymptotically unbiased and consistent.

We now come to the main point of this paper: suppose the very realistic case, bound to occur in any practical application, in which there are errors in the evaluation of the correlation function, given by (36), modelled here as

$$\begin{aligned} C_j^{i^*} &= C_{j_{nom}}^{i^*} + \Delta C_j^{i^*} \\ & \quad i = 1, \ldots, N,\ j = 0, 1, \ldots, n \end{aligned} \tag{37}$$

where $C_{j_{nom}}^{i^*}$, $j = 1, \ldots, N$, $j = 0, 1, \ldots, n$ are the theoretical values of the correlation functions and are presented in [1]. From a practical viewpoint, a very important issue is: What is the impact of the errors $\Delta C_j^{i^*}$ on the identification of the optimum filter gains K^i? Section III deals with this issue.

III. IMPACT OF CORRELATION ERRORS ON THE OPTIMUM FILTER GAINS IDENTIFICATION

Suppose $C_j^{i^*}$, given by (37), in (27) and (28). It is also assumed that all inverses indicated exist. Due to errors $\Delta C_j^{i^*}$, $i = 1, 2, \ldots, N$, $j = 0, 1, \ldots, n$, $C_{0_m}^i$, $\hat{K}_m^i$ and W_{m+1}, in (27), (28) and (29), respectively, do not converge anymore to the desired steady-state values C_0^i, K^i and W. Instead, they will converge to $\bar{C}_0^i$, $\bar{K}^i$ and $\bar{W}$. That is, errors $\Delta C_j^{i^*}$, $i = 1, 2, \ldots, N$, $j = 0, 1, \ldots, n$ will imply on errors in (27), (28) and (29), modelled as $\Delta C_{0_m}^i$, ΔK_m^i and ΔW_{m+1}, respectively. Hence, we have

$$\bar{C}_{0_m}^i = C_{0_m}^i + \Delta C_{0_m}^i \tag{38}$$

$$\bar{K}_m^i = K_m^i + \Delta K_m^i \tag{39}$$

$$\bar{W}_m = W_m + \Delta W_m \tag{40}$$

By using, (37), (27), (28), (29) and considering (38)-(40), in a similar way as presented in [7], we can develop the following equations for the evaluation of the impact of the correlation errors on the identification of the steady-state optimum filter gains:

1) Initial condition:

$$\Delta W_0 \neq 0 \tag{41}$$

2) Calculating ΔC_0^i:

$$\Delta C_{0_m}^i = \Delta C_0^{i^*} - \tilde{H}^i \Delta \tilde{W}_m^i \tilde{H}^{i^T} \tag{42}$$

3) Estimating ΔK^i:

$$\begin{aligned} \Delta K_m^i = & \left[\overline{B}^{i\dagger} \left(\Delta \overline{A}^i + \overline{D}^i \Delta C_{0_m}^i\right) - A \Delta \tilde{W}_m^i \tilde{H}^{i^T}\right] \\ & \times \left(C_0^i + \Delta C_{0_m}^i\right)^{-1} - K^i \left[\left(C_0^i\right)^{-1} + \left(\Delta C_{0_m}^i\right)^{-1}\right]^{-1} \\ & \times \left(C_0^i\right)^{-1} \end{aligned} \tag{43}$$

4) Calculating ΔW:

$$\begin{aligned} \Delta W_{m+1} = & \left(A - \sum_{i=1}^{N} K^{i^*} \tilde{H}^i\right) \Delta W_m \\ & \times \left(A - \sum_{i=1}^{N} K^{i^*} \tilde{H}^i\right)^T + \sum_{i=1}^{N} \left[\left(\bar{\bar{K}}^i + \Delta K_m^i\right) \Delta C_{0_m}^i\right. \\ & \times \left(\bar{\bar{K}}^i + \Delta K_m^i\right)^T + \left(\bar{\bar{K}}^i + \Delta K_m^i\right) C_0^i \left(\bar{\bar{K}}^i + \Delta K_m^i\right)^T \\ & \left. - \left(\bar{\bar{K}}^i\right) C_0^i \left(\bar{\bar{K}}^i\right)^T\right] \end{aligned} \tag{44}$$

where

$$\Delta \tilde{W}_m^i = \Delta W_m \tag{45}$$

$$\bar{\bar{K}} = K^i - K^{i^*} \tag{46}$$

$\Delta \overline{A}^i$ is given by (49) and C_0^i are the theoretical values of the correlations.

Therefore, given $\Delta C_j^{i^*}$, $i = 1, 2, \ldots, N$, $j = 0, 1, \ldots, n$, by using (42), (43) and (44), in a recursive manner, we can evaluate the impact of correlation errors on the identification of the steady-state optimum filter gains, in a multicoordinated scheme.

IV. ILLUSTRATIVE EXAMPLE

In order to show the influence of the covariances errors on the identification of the steady-state filter gains, and how it can degrade the state estimates, let us consider a similar system as presented in [1] and described in section II with 5 sensors, that is $N = 5$ and

$$A = \begin{bmatrix} \cos(3^\circ) & -0.5 \sin(3^\circ) \\ 2 \sin(3^\circ) & \cos(3^\circ) \end{bmatrix} \tag{47}$$

$$G = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}, H^i = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \tag{48}$$

$$
\Delta \overline{A}^i = \begin{bmatrix} \Delta C_1^{i*} + \tilde{H}^i \left(K^{i*} + \tilde{M}^i\right) \Delta C_0^{i*} \\ \Delta C_2^{i*} + \tilde{H}^i \left(K^{i*} + \tilde{M}^i\right) \Delta C_1^{i*} + \tilde{H}^i A \left(K^{i*} + \tilde{M}^i\right) \Delta C_0^{i*} \\ \vdots \\ \Delta C_n^{i*} + \tilde{H}^i \left(K^{i*} + \tilde{M}^i\right) \Delta C_{n-1}^{i}{}^{*} + \tilde{H}^i A \left(K^{i*} + \tilde{M}^i\right) \Delta C_{n-2}^{i}{}^{*} + \ldots + \tilde{H}^i A^{n-1} \left(K^{i*} + \tilde{M}^i\right) \Delta C_0^{i*} \end{bmatrix} \tag{49}
$$

$$
Q = \begin{bmatrix} 0.5^2 & 0 \\ 0 & 0.5^2 \end{bmatrix} m^2, R^i = \begin{bmatrix} 2^2 & 0 \\ 0 & 2^2 \end{bmatrix} m^2 \tag{50}
$$

$$
Q^* = \begin{bmatrix} 1.5^2 & 0 \\ 0 & 1.5^2 \end{bmatrix} m^2, R^{i^*} = \begin{bmatrix} 0.1^2 & 0 \\ 0 & 0.1^2 \end{bmatrix} m^2 \tag{51}
$$

for $i = 1, \ldots, N$.

The nominal values for the rotation matrices are

$$
J_0^i = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}, i = 1, \ldots, N \tag{52}
$$

while the nominal translation vectors are

$$
T_0^1 = \begin{bmatrix} 10 \\ 15 \end{bmatrix} m,\ T_0^2 = \begin{bmatrix} 1 \\ 2 \end{bmatrix} m,\ T_0^3 = \begin{bmatrix} -12 \\ 6 \end{bmatrix} m,
$$

$$
T_0^4 = \begin{bmatrix} -5 \\ -11 \end{bmatrix} m,\ T_0^5 = \begin{bmatrix} 13 \\ -7 \end{bmatrix} m \tag{53}
$$

The covariance matrices associated to the uncertainties on the mappings are

$$
Q_J^i = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0.68 & -0.68 & 0 \\ 0 & -0.68 & 0.68 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} 10^{-6},
$$

$$
Q_J^{i^*} = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0.68 & -0.68 & 0 \\ 0 & -0.68 & 0.68 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} 10^{-7},
$$

$$
Q_T^i = \begin{bmatrix} 0.2^2 & 0 \\ 0 & 0.2^2 \end{bmatrix} m^2,
$$

$$
Q_T^{i^*} = \begin{bmatrix} 0.5^2 & 0 \\ 0 & 0.5^2 \end{bmatrix} m^2,\ i = 1, \ldots, N \tag{54}
$$

The initial conditions used were

$$
x_0 = \begin{bmatrix} 10 \\ 0 \end{bmatrix} m,\ \hat{x}_{0|-1} = \hat{x}_{0|-1}^* \begin{bmatrix} 0 \\ 0 \end{bmatrix} m \tag{55}
$$

$$
P_{0|-1} = P_{0|-1}^* = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} m^2 \tag{56}
$$

$$
W_0 = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix} m^2,\ \Delta W_0 = \begin{bmatrix} 0.1 & 0 \\ 0 & 0.1 \end{bmatrix} m^2 \tag{57}
$$

The algorithm was implemented in *Matlab 6.5* and the noise generator seeds were initialized with state zero. The correlations considerer were C_0^i, C_1^i and C_2^i.

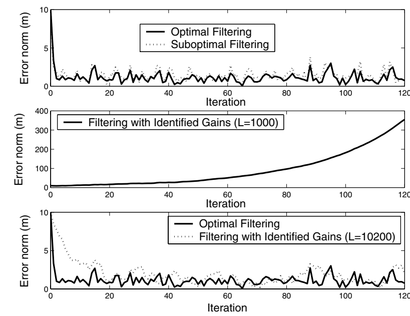

Fig. 1. Error Norms for Filtering with Optimal, Suboptimal and Identified Gains with Errors on the Correlation Matrices $(N = 5)$

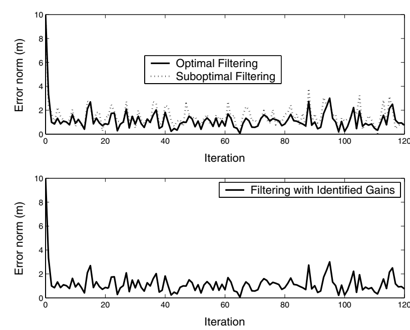

Fig. 2. Error Norms for Filering with Optimal, Suboptimal and Identified Gains with Theoretical Correlation Matrices $(N = 5)$

Applying the identification algorithm proposed by [1] we have Fig. 1. With $L = 1000$ the estimates obtained by using the identified gains are divergent. Its is due to the correlation matrices estimated by means of (36) be distinct of the theoretical values given in [1]. These errors arise from the limitation of samples and due the *Matlab* noise generator be pseudo-random. Better results are obtained with $L = 10200$. Besides the large number of samples, it was expected better results compared to the suboptimum filter. In Fig. 2 it is shown the results of the algorithm proposed by [1] when the correlations are given by their theoretical ones.

In order to analyze the impact that errors on the evaluation of each correlation matrix have on the identification of the optimal matrices gains, let´s consider a percentage error over the nominal values of the correlations indicated on the horizontal axis of the Figs. 3-5, while the other correlations are given by their nominal values, that is, with no errors.

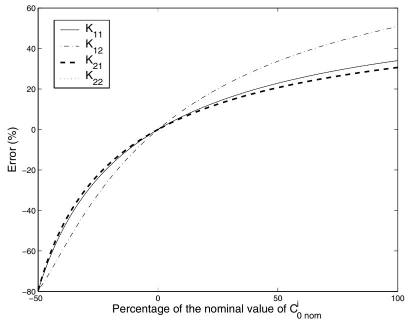

Fig. 3. Gain Identification Errors due to Error in $C_0^{i^*}$, $i = 1, \ldots, 5$

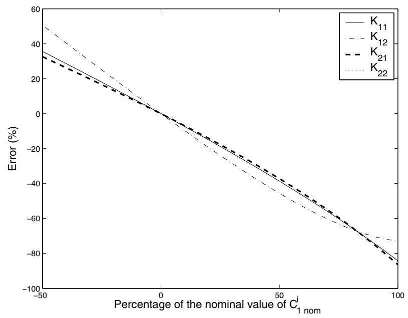

Fig. 4. Gain Identification Errors due to Error in $C_1^{i^*}$, $i = 1, \ldots, 5$

In Figs. 3-5 we have this impact for a system with $N = 5$ and it is presented the results for the sensor 1.

An analysis of the Figs. 3-5 indicates that the identification errors can be considerable. Additionally, these errors are less significant as the index j of the correlation functions increases. This effect can be analyzed based in (49). In this equation, as the lag index j increases, the j-lag error in the correlation matrix associated, $j > 1$, appears $n - j + 1$ times, providing less influence on the identification errors. Besides, as the lag index j increases, the nominal value of the correlation matrices becomes smaller. Therefore, the errors considered in the simulation, given in percentage of the theoretical values of the correlation functions of the innovations, are also smaller.

Based on (44) another important conclusion can be drawn. In the multisensor scheme, in order to evaluate ΔW_{m+1}, it is necessary the information of $\Delta C_{0_m}^i$ and ΔK_m^i, $i = 1, 2, \ldots, N$. This leads to more erroneous information involved in the identification of the gains K^i, compared with the single sensor case [7]. Hence, if there are errors in the correlation functions, the filter gains identification will degrade as the number of sensors increases. Consequently, it could degrade the state estimates as shown in the illustrative example. Experimental results, considering only one sensor, are presented in section V.

V. EXPERIMENTAL RESULTS

In order to evaluate the performance, in real time applications, of the gain identification algorithm of [1], summarized in section II, it was applied to perform the tuning of a

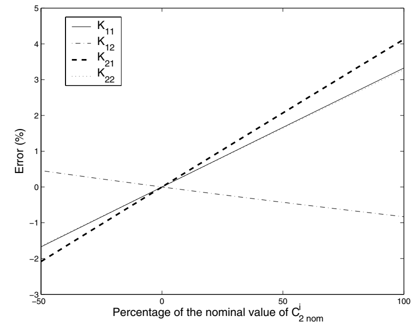

Fig. 5. Gain Identification Errors due to Error in $C_2^{i^*}$, $i = 1, \ldots, 5$

Kalman filter used in a position servo system. In [1] it is presented only a numerical example. The servo system consisted of a three-phase AC induction motor driven by a PWM (Pulse Width Modulation) inverter. Only position measurements are available. A microcomputer PC compatible platform with a proprietary data acquisition and PWM generator board was in charge of generation of the control law, by means of a C software. This control law is described in [9] and makes use of the angular speed estimate to adapt the controller parameters. The continuous mechanical model of the motor is

$$\begin{bmatrix} \dot{\omega}_R(t) \\ \dot{\theta}_R(t) \end{bmatrix} = \begin{bmatrix} -\frac{Bn}{Jn} & 0 \\ 1 & 0 \end{bmatrix} \begin{bmatrix} \omega_R(t) \\ \theta_R(t) \end{bmatrix} + \begin{bmatrix} \frac{1}{Jn} \\ 0 \end{bmatrix} T_e(t) \tag{58}$$

$$y(t) = \begin{bmatrix} 0 & 1 \end{bmatrix} \begin{bmatrix} \omega_R(t) \\ \theta_R(t) \end{bmatrix} \tag{59}$$

where $\omega_R(t)$, $\theta_R(t)$, $Bn = 0.001\ Kg \cdot m/A$, $Jn = 0.0133\ Kg{\cdot}m^2$ and T_e are the angular speed (rad/s), angular position (rad), damping coefficient, moment of inertia and electrical torque, respectively. The parameters Bn and Jn were obtained by means of an adaptive control scheme described in [10]. The sampled time mechanical model was obtained by using a 128.5 Hz sampling frequency. More details about the control law used, nominal parameter of the motor and the models involved are found in [9]. In order to account for modelling errors due to errors on Bn and Jn it is included a state uncertainty as described in [11]. It is also considered measurement noise. Hence, we have

$$x_{k+1} = Ax_k + Bu_k + G\gamma_k \tag{60}$$

$$y(t) = Cx_k + F\varphi_k \tag{61}$$

where

$$A = \begin{bmatrix} 0.9994 & 0 \\ 0.0078 & 1 \end{bmatrix},\ B = \begin{bmatrix} 0.5846 \\ 0.0023 \end{bmatrix},\ G = \begin{bmatrix} 1 \\ 0 \end{bmatrix} \tag{62}$$

$$C = \begin{bmatrix} 0 & 1 \end{bmatrix},\ F = \begin{bmatrix} 1 \end{bmatrix},\ u_k = T_{e_k} \tag{63}$$

The mapping associated to the communication network is

$$J_0 = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix},\ T_0 = \begin{bmatrix} 0 \\ 0 \end{bmatrix} \tag{64}$$

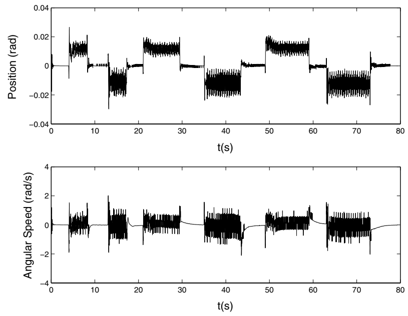

Fig. 6. Estimation error for a filter with suboptimum gains

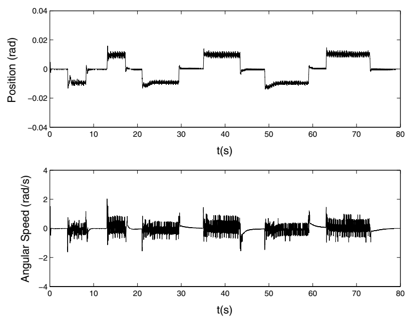

Fig. 7. Estimation error for a filter with identified gains

with no uncertainties. This mapping means that the local coordinate system or the reference of the sensor is the central coordinate system.

The prior information about the state and noise uncertainties is

$$Q^* = 0.001,\ R^* = 0.02 \tag{65}$$

that gives the suboptimum filter gain

$$K^* = \begin{bmatrix} 0.2128 \\ 0.0584 \end{bmatrix} \tag{66}$$

In the evaluation of the innovation correlation it was considered $L = 10200$. The identified gains are

$$K = \begin{bmatrix} 13.1774 \\ 1.431 \end{bmatrix} \tag{67}$$

The position and angular speed estimation errors are presented in figure 6, for the suboptimal case. The position and angular speed estimation errors are presented in figure 7, for the identified filter gain case. Since we have errors on the evaluation of the correlation functions, the identified gains are not the optimal ones but provide better state estimates compared with the estimates obtained with the gain based on the prior information.

VI. CONCLUSIONS

This paper investigated the impact that errors in the evaluation of the correlation of the innovations of a suboptimal filter have on the identification of the optimal matrices gains. The analysis was based on an identification scheme for multisensor scheme with uncertainties in the mappings among the sensors and the central processor that generalizes an previous work for only one sensor with no mappings. It was shown that those errors can lead to erroneous steady-state identified gains and consequently to state estimates degradation. Equations were developed to quantify the identification gains errors and to show the relationship of the variables involved. Based on the illustrative example it is clear that errors in the correlation functions are less impacting as the number of sensors are reduced and, in order to identify the steady-state gains, it is necessary that the evaluation of the correlation functions be extremely accurate, which can be a difficult requirement to be guaranteed in practice.

A practical implementation, that uses an adaptive control law, based on estimated states was also presented. In this application it was shown that the identification algorithm for the optimum gains can be a valuable tool for tuning Kalman filters, thereby, providing less errors on the estimates of the states. In order to improve the convergence of the algorithm, the correlation function of the innovations of a suboptimal filter must be evaluated as precise as possible.

VII. ACKNOWLEDGMENTS

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