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# On Dropping Noisy Packets in Kalman Filtering over a Wireless Fading Channel

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*Abstract*— **We consider the wireless estimation problem in fading environments which is inherent to many mobile sensor networks. Previous works on wireless estimation in sensor networks have extended the same design principles of data networks by carrying out their analysis under the assumption that the erroneous packets are simply dropped. In this paper, we show that proper use of the information contained in all the packets, including the erroneous ones, can improve the performance considerably. We derive an analytical expression to evaluate the performance improvement, for one class of communication noise characteristics, when the information of all the packets is used in the receiver. We also prove that keeping all the packets can prevent the potential instability that is introduced if erroneous packets are dropped. This suggests that applying the same design principles of data networks to sensor networks can lead to performance degradation as sensor network applications are typically delay sensitive and therefore may require different design strategies. The work also highlights the importance of sharing information between the physical and application layers since extracting the information contained in all the packets would necessitate a cross layer design approach.**

## I. INTRODUCTION

There has recently been considerable interest in sensor networks [1], [2]. Such networks have a wide range of applications such as environmental monitoring, surveillance, security, smart homes and factories, target tracking and military. Communication plays a key role in the overall performance of sensor networks as both sensor measurements and control commands are transmitted over wireless links.
Considering the impact of communication channels on wireless estimation/control is an emerging area of research. At this point, it is not yet clear what the right transmitter/receiver design strategies are for such applications. Estimation/control of a rapidly-changing dynamic system is a delay-sensitive application. Therefore, the communication protocols and designs suitable for other already-existing applications like data networks may not be entirely applicable to sensor networks. Data networks are not as sensitive to delays since the application is not real time. The receiver, therefore, can afford to drop erroneous packets and wait for retransmission. Control applications, on the other hand, are typically delay sensitive as we are racing against the dynamics of the system. Therefore, applying the same design strategies of data networks may not be appropriate. Authors in [3] have looked at the impact of packet loss on wireless Kalman filtering. Authors in [4] extended the work of [3] to multiple sensors. Both of these works assumed that the receiver is dropping the erroneous packets.
The main question this paper addresses is whether discarding erroneous packets[1] is an appropriate design strategy for delay-sensitive wireless estimation/control applications. In other words, we are interested in knowing if, and how much, we can improve the performance by using all the received packets in the estimation process. Results from [5] have shown that the useful rate (useful rate only considers the bits that are received correctly) of a system should be above a given threshold to ensure stability of the control process. Dropping the erroneous packets can result in a drop of the overall useful rate, increasing the chance of instability. Authors in [3] also confirmed this by finding a maximum tolerable packet loss probability beyond which the Kalman filtering process would go unstable.
In this paper we consider a mobile sensing/estimation process using Kalman filtering. Instead of dropping erroneous packets, we use the information contained in all the received packets. This changes the traditional form of Kalman filtering as channel dynamics are now introduced in the estimation process. We consider one class of communication noise characteristics. We derive an expression for the average error variance for the case of Kalman filtering over fading channels analytically. We also show the stability of the estimation process when all the packets are used in the receiver. This allows us to compare the performance with the scenario when the erroneous packets are dropped.
When keeping all the packets in the receiver, the physical layer would need to share information on the quality of the communication link with the application layer for estimation. Therefore, the paper also emphasizes the importance of cross-layer designs in sensor networks. The analysis can be a starting point for establishing design strategies in such networks.

## II. SYSTEM MODEL

Consider a mobile sensor observing a system with the following linear dynamics:

$$\begin{aligned} x_{k+1} &= Ax_k + w_k \\ y_k &= Cx_k + v_k, \end{aligned} \tag{1}$$

[1]Erroneous packets refer to those that are not received %100 accurately in the receiver. However, an erroneous packet may still carry some information about the transmitted sensor measurement.

 

where $x_k$ and $y_k$ represent the state and observation respectively. $w_k$ and $v_k$ represent zero-mean process and observation noises with variances of $Q$ and $R$ respectively. We assume scalar quantities in this paper to facilitate mathematical derivations. We are interested in estimating unstable dynamics and therefore we take $|A| > 1$. $y_k$ is then quantized, transformed into a packet of bits and transmitted over a mobile fading channel:

$$\hat{y}_k = y_k + n_k, \tag{2}$$

where $\hat{y}_k$ is the receiver version of the observation and $n_k$ represents zero-mean communication noise. Let $G_k$ represent the variance of $n_k$. $G_k$ is a function of $SNR_k$, the instantaneous received Signal to Noise Ratio at $k^{th}$ transmission. $SNR_k$ is a stochastic process and its distribution is a function of the environment and level of the mobility of the sensor:

$$G_k = f(SNR_k). \tag{3}$$

Function $f$ is a decreasing function that depends on the transmitter/receiver design principles as well as the transmission environment. We consider a receiver that discards the packets at $SNR = 0$, which means $f(0) = \infty$. Furthermore we take $f(\infty) = 0$, which assumes that quantization noise is negligible. Depending on the environment, $SNR_k$ would have different distributions. In a narrowband fading environment, we will have $SNR_k = \frac{|h_k|^2 \sigma_x^2}{\sigma_n^2}$ where $\sigma_x^2$ and $\sigma_n^2$ represent the transmitted signal power and receiver noise power respectively. $h_k$ is the value of the channel at $k^{th}$ transmission. In an environment with no LOS path, it is reasonable to assume that $|h_k|$ has Rayleigh fading distribution which results in an exponential distribution for $SNR_k$ [6]. We investigate the impact of such environments on the estimation process. Furthermore, we take $h_k$ and therefore $SNR_k$ to be independent from one transmission to the next. This will be the case as long as the time interval between consecutive transmissions is bigger than channel coherence time [6].

## III. Estimation

The distant node estimates the state based on the received observation using a Kalman filter [7]. Let $\hat{x}_k$ represent the estimate of $x_k$ at the receiver:

$$\hat{x}_k = E\left[x_k | \hat{y}_0, \hat{y}_1, \ldots, \hat{y}_{k-1}\right]. \tag{4}$$

Let $P_k$ represent variance of the estimation error of the $k^{th}$ transmission:

$$P_k = E\left[(x_k - \hat{x}_k)^2\right]. \tag{5}$$

In the presence of communication noise, we will have the following equation for updating estimation error variance:

$$P_{k+1} = A^2 P_k - \frac{A^2 P_k^2 C^2}{R + C^2 P_k + f(SNR_k)} + Q. \tag{6}$$

This is different from typical Kalman filtering as $P_k$ is a function of $h_k$ through $f(SNR_k)$ and therefore is a random process. To focus on the impact of the communication channel, in this paper we assume, $R = 0$, $Q = 0$ and $C = 1$ .

## IV. Scenario 1: Dropping Erroneous Packets

In this case, if the receiver gets a packet that contains an error, it drops it and will not use it in the estimation process. This translates into the following $f$ function:

$$f_{drop,ideal}(SNR_k) = \begin{cases} 0 & SNR_k \geq SNR_{Thresh.} \\ \infty & else \end{cases} \tag{7}$$

$SNR_{Thresh.}$ is the minimum $SNR_k$ that would result in packet loss probability of zero in theory[2]. In practice, however, a small fraction of those packets that are perceived to contain no error, may be erroneous. We take this into account in Section VI.

$f_{drop,ideal}$ can model the strategy used in [3] and [4] for dropping packets. There they showed that there is a critical average packet loss probability above which the estimation error variance would get unbounded. Here we briefly relate that to $SNR$. Eq. 6 will be as follows in this case:

$$\begin{gathered} P_{k+1} = A^2 P_k - A^2 \eta_k P_k \\ \overline{P_{k+1}} = A^2 \overline{P_k}(1 - \overline{\eta_k}) \\ \eta_k = \begin{cases} 1 & SNR_k \geq SNR_{Thresh.} \\ 0 & else \end{cases} \end{gathered} \tag{8}$$

where $\overline{\eta_k} = Prob\left\{SNR_k \geq SNR_{Thresh.}\right\}$, which will be as follows for an exponentially distributed $SNR_k$:

$$\overline{\eta_k} = e^{-\lambda SNR_{Thresh.}}, \tag{9}$$

where [3] $\lambda = \frac{1}{\overline{SNR_k}}$. Then,

$$\overline{P_{k+1}} \;=\; P_0\left[A^2(1 - e^{-\lambda SNR_{Thresh.}})\right]^{k+1}. \tag{10}$$

Therefore,

$$\begin{gathered} if \quad \overline{SNR} > \frac{SNR_{Thresh.}}{ln(\frac{A^2}{A^2-1})} \Rightarrow lim_{k\to\infty}\overline{P_k} \to 0 \\ if \quad \overline{SNR} < \frac{SNR_{Thresh.}}{ln(\frac{A^2}{A^2-1})} \Rightarrow lim_{k\to\infty}\overline{P_k} \to \infty \end{gathered} \tag{11}$$

We can see From Eq. 11 that depending on the average $SNR$ of the environment, the estimation process may get unstable in this scenario. This motivates the work presented in the next sections. For average $SNR$ above the minimum required, the estimation error may not converge to zero asymptotically (as is predicted by Eq. 11) in practice. We explore this in more details in Section VI.

[2]The receiver may not decide on dropping packets directly based on $SNR_k$. For instance it may use a checksum bit in the simplest scenario. Since any other used measure is a function of $SNR_k$, we find it useful to express $f_{drop,ideal}$ as a function of this fundamental quantity. Function $f_{drop,ideal}$ can have different forms as a function of $SNR_k$, depending on the packet drop mechanism. Eq. 7 assumes that the receiver drops the packets when $SNR_k < SNR_{Thresh.}$.

[3]this assumes that $SNR_k$ is a wide-sense stationary process.

## V. Scenario 2: Keeping all the packets

In this section we investigate the case where the receiver keeps all the packets. In general, function $f$ would have different forms depending on the environment and transmitter/receiver design. We pursue the analysis with the following $f$:

$$f_{keep}(SNR_k) = \frac{\beta}{SNR_k} \tag{12}$$

for $\beta \geq 0$.

### A. Performance Evaluation

In this section we will find an expression for $\overline{P_k}$. Using Eq. 6 and 12 we will have,

$$P_{k+1} = \frac{A^2 P_k \beta}{\beta + SNR_k P_k} = \frac{A^2 P_k}{1 + \gamma_k P_k}, \tag{13}$$

where $\gamma_k = \frac{SNR_k}{\beta}$ with $\lambda_n = \frac{1}{\overline{\gamma_k}} = \lambda\beta$. It is shown in Appendix A that averaging Eq. 13 over $\gamma_k$ will result in the following (insert $c = 1$, $d = P_k$ and $a = A^2 P_k$ in Eq. 30):

$$\overline{P_{k+1,0}} = A^2 \lambda_n S(\frac{\lambda_n}{P_k}), \tag{14}$$

where $S(\zeta) = e^{\zeta} Expint(\zeta)$, for an arbitrary $\zeta$, with $Expint$ representing exponential integral: $Expint(\zeta) = \int_{\zeta}^{\infty} \frac{e^{-t}}{t} dt$. $\overline{P_{k+1,i}}$ refers to the case that $P_{k+1}$ is averaged over $\gamma_k$, $\gamma_{k-1}$, ..., $\gamma_{k-i}$ and $\overline{P_{k+1}} = \overline{P_{k+1,k}}$. Inserting $P_k$ as a function of $P_{k-1}$, we will have,

$$\overline{P_{k+1,0}} = A^2 \lambda_n S(\frac{\lambda_n (1 + \gamma_{k-1} P_{k-1})}{A^2 P_{k-1}}). \tag{15}$$

We need to take an average over[4] $\gamma_{k-1}$. Appendix B shows that the following equality holds for a $q > 0$ and an exponentially distributed random variable $\gamma$ with $\overline{\gamma} = \frac{1}{\lambda_n}$:

$$\overline{S(\frac{\lambda_n(1+q\gamma)}{qA^{2i}})} = \frac{S(\frac{\lambda_n}{A^{2i}q})}{1 - \frac{1}{A^{2i}}} - \frac{S(\frac{\lambda_n}{q})}{1 - \frac{1}{A^{2i}}} \quad i \geq 1 \quad |A| > 1 \tag{16}$$

Using Eq. 16 for $i = 1$, we will have,

$$\overline{P_{k+1,1}} = \frac{A^2 \lambda_n S(\frac{\lambda_n}{A^2 P_{k-1}})}{1 - \frac{1}{A^2}} - \frac{A^2 \lambda_n S(\frac{\lambda_n}{P_{k-1}})}{1 - \frac{1}{A^2}}. \tag{17}$$

It can be seen from Eq. 14 and 17 that we will have the following after $m+1$ steps of averaging:

$$\overline{P_{k+1,m}} = \sum_{i=0}^{m} B_{i,m} S(\frac{\lambda_n}{A^{2i} P_{k-m}}). \tag{18}$$

The goal is to find $B_{i,m}$ for $m = k$. Let $D_k(i,m) = S(\frac{\lambda_n}{A^{2i} P_{k-m}})$. Then,

$$\overline{P_{k+1,m}} = \sum_{i=0}^{m} B_{i,m} D_k(i,m) \tag{19}$$

with $B_{0,0} = A^2 \lambda_n$.
Substituting $P_{k-m}$ as a function of $P_{k-m-1}$ and averaging over $\gamma_{k-m-1}$ (using Eq. 16) will result in the following for $-1 \leq m \leq k-1$:

$$\begin{aligned} \overline{P_{k+1,m+1}} &= \textstyle\sum_{i=0}^{m} \frac{B_{i,m}}{\xi_{i+1}} D_k(i+1, m+1) \\ &- \textstyle\sum_{i=0}^{m} \frac{B_{i,m}}{\xi_{i+1}} D_k(0, m+1) \\ &= -\left[\textstyle\sum_{i=0}^{m} \frac{B_{i,m}}{\xi_{i+1}}\right] D_k(0, m+1) \\ &+ \textstyle\sum_{z=1}^{m+1} \frac{B_{z-1,m}}{\xi_z} D_k(z, m+1) \\ &= \textstyle\sum_{z=0}^{m+1} B_{z,m+1} D_k(z, m+1), \end{aligned} \tag{20}$$

where $\xi_i = 1 - \frac{1}{A^{2i}}$ and the last equality is written using Eq. 19. Therefore for $0 \leq z \leq m+1$,

$$B_{z,m+1} = \begin{cases} -\sum_{i=0}^{m} \frac{B_{i,m}}{\xi_{i+1}} & z = 0 \\ \frac{B_{z-1,m}}{\xi_z} & z \neq 0 \end{cases} \tag{21}$$

Then $\overline{P_{k+1,k}}$ will be as follows as a function of $P_0$:

$$\overline{P_{k+1}} = \overline{P_{k+1,k}} = \sum_{i=0}^{k} B_{i,k} e^{\frac{\lambda\beta}{A^{2i} P_0}} Expint(\frac{\lambda\beta}{A^{2i} P_0}), \tag{22}$$

where $B_{i,k}$ for $0 \leq i \leq k$ is calculated using $B_{i,k-1}$ for $0 \leq i \leq k-1$ (using Eq. 21). This means that $\overline{P_{k+1}}$ is calculated using the coefficients of $\overline{P_k}$.

### B. Stability of the Estimation Process

In this part we show that the average error variance always stays bounded as long as $\overline{SNR} \neq 0$. We have,

$$P_{k+1} = \frac{A^2 P_k \beta}{\beta + SNR_k P_k}. \tag{23}$$

Let $\overline{Z}_r$ represent average of $Z$ with respect to $r$ for an arbitrary random function $Z$ and a random variable $r$. $\overline{Z}$ represents average of $Z$ with respect to all its variables. First we take an average of $P_{k+1}$ with respect to $P_k$. It is easy to show that the function $Z(r) = \frac{r}{r+u}$ is concave for any $r \geq 0$ and $u \geq 0$. Therefore we will have the following by first applying the Jensen's inequality and then averaging over $SNR_k$:

$$\begin{aligned} \left[P_{k+1}\right]_{P_k} &= \overline{\left[\frac{A^2 P_k \beta}{\beta + SNR_k P_k}\right]_{P_k}} \\ &\leq \frac{A^2 \overline{P_k} \beta}{\beta + SNR_k \overline{P_k}} \Rightarrow \\ \overline{P_{k+1}} &= \overline{\left[\overline{\left[P_{k+1}\right]_{P_k}}\right]_{SNR_k}} \\ &= \overline{\left[\overline{\left[\frac{A^2 P_k \beta}{\beta + SNR_k P_k}\right]_{P_k}}\right]_{SNR_k}} \\ &\leq \overline{\left[\frac{A^2 \overline{P_k} \beta}{\beta + SNR_k \overline{P_k}}\right]_{SNR_k}} \\ &= A^2 \beta \lambda e^{\lambda\beta/\overline{P_k}} Expint(\lambda\beta/\overline{P_k}). \end{aligned} \tag{24}$$

The last equality of Eq. 24 is proved in Appendix A. Next we examine the conditions under which the upper bound of

[4] We can take averages over $\gamma_i$s for $0 \leq i \leq k$ one at a time since they are assumed independent (see Section II).

$\overline{P_{k+1}}$ is smaller than $\overline{P_k}$. We want,

$$A^2\beta\lambda e^{\lambda\beta/\overline{P_k}} Expint(\lambda\beta/\overline{P_k}) < \overline{P_k}. \qquad (25)$$

Call $\alpha_1 = \lambda\beta$, $\alpha_2 = \overline{P_k}$ and $\zeta = \frac{\alpha_1}{\alpha_2}$, then we are interested in finding those conditions under which

$$g(\zeta) = A^2\zeta e^{\zeta} Expint(\zeta) < 1 \qquad (26)$$

Function $g$ has the following characteristics:

1) $g(0) = 0$
2) $g(\infty) = A^2$
3) $g(\zeta)$ is an increasing function of $\zeta$ for $\zeta \geq 0$

where 1 and 2 can be confirmed easily and 3 is proved in Appendix C.

If $|A| < 1$ then Eq. 26 will always hold as expected. Here, however, we are interested in estimation of unstable processes, i.e. $|A| > 1$. Therefore for a $\zeta_0 > 0$, we will have $g(\zeta_0) = 1$ meaning that,

$$\zeta < \zeta_0 \quad \Rightarrow \quad g(\zeta) < 1 \qquad (27)$$

Therefore,

$$if \quad \overline{P_k} > \frac{\lambda\beta}{\zeta_0} \Rightarrow \overline{P_{k+1}} < \overline{P_k} \qquad (28)$$

It can be seen from Eq. 28 that as $\overline{P_k}$ is increasing, once it reaches $\frac{\lambda\beta}{\zeta_0}$, it will decrease. Therefore, unless $\lambda = \infty$, the average estimation error variance stays bounded. Case of $\lambda = \infty$ corresponds to the case of $\overline{SNR} = 0$. This is the only case where the estimation process will get unstable, as one would expect. Such a case results in zero Signal to Noise Ratio in every transmission, which makes it an open loop process. However, for any $\overline{SNR} > 0$, the process is stable.

## VI. Comparing the Two Scenarios

To confirm the mathematical derivations of the past sections, we simulate wireless Kalman filtering of a system with linear dynamics in a fading environment. The following parameters are chosen for this example: $A = 4$, $\beta = .1$ and $x_0 = 5$. First we look at scenario 2, the case in which information of all the packets is used in the receiver. Solid lines of Fig. 1 show the performance evaluated through analysis (Eq. 22) for scenario 2. To confirm mathematical derivations of scenario 2, we also simulate this system. We can see that the simulation results (stars of Fig. 1) confirm the mathematical derivations. Fig. 1 shows average error variance at different $\overline{SNR}$. As can be seen the error stays bounded even for $\overline{SNR}$ as low as -40dB if the information in all the packets is used properly in the receiver.

To see how much we lose by dropping the erroneous packets, next we compare the performance of the two aforementioned scenarios. For scenario 1, case of dropping erroneous packets, function $f_{drop,ideal}$ of Section IV assumed that as long as $SNR$ is above $SNR_{Thresh.}$, the communication noise is negligible. As we want to compare the two scenarios, we have to use a more practical version of function $f$ for scenario 1. This will result in the following function:

$$f_{drop}(SNR_k) = \begin{cases} \frac{\beta}{SNR_k} & SNR_k \geq SNR_{Thresh.} \\ \infty & else \end{cases} \qquad (29)$$

Fig. 2 shows the performance of the two scenarios as a function of $\overline{SNR}$ and for different $SNR_{Thresh.}$s. We can see that dropping erroneous packets results in instability. Performance loss increases as $SNR_{Thresh.}$ increases. In this simulation, for scenario#1, we considered the cases where the information on the quality of the link is used by the estimator for those packets that are kept in the receiver. If such a cross-layer feedback is not available, then the performance of scenario#1 would be even worse. The graph confirms that keeping all the packets and sharing information of the physical layer (communication noise variance) with the application layer can improve the performance considerably.

## VII. Conclusion

In this paper, we considered the wireless sensing/estimation problem in fading environments which is key to many mobile sensor networks. We showed that applying the same design strategies of data networks to sensor networks may lead to performance degradation since sensor network applications are typically delay sensitive and therefore may require different design principles. In particular, we looked at the impact of dropping erroneous packets, which is done in data networks, on the estimation quality in sensor-network applications. We showed that proper use of the information contained in all the packets can improve the performance considerably compared to the case of dropping erroneous packets. We derived a mathematical expression to evaluate the performance when the receiver uses all the packets for one class of communication noise characteristics. We also proved that keeping all the packets can prevent the potential instability that is introduced if erroneous packets are dropped. Our mathematical results were also confirmed by simulations. Finally our analysis emphasized the importance of sharing information between the physical and application layers.

## VIII. Future Work

We are working on extending the work presented in this paper to different $f$ functions that would characterize communication noise of different environments and protocols. Furthermore, we are working on extending the work to more general cases where $R \neq 0$, $Q \neq 0$ and $C \neq 1$ and to the vector case.

## References

[1] C. Chong and S. Kumar, "Sensor networks: evolution, opportunities and challenges," Proceedings of the IEEE, vol. 91, issue 8, Aug. 2003, pages:1247-1256

[2] B. Sinopoli, C. Sharp, L. Schenato, S. Schaffert and S. Sastry, "Distributed control applications within sensor networks," Proceedings of the IEEE, vol. 91, issue 8, Aug. 2003, Pages:1235-1246

[3] B.Sinopoli, L. Schenato, M. Franceschetti, K. Poolla, M. Jordan, S. Sastry, "Kalman filtering with intermittent observations," Proceedings of the 42nd IEEE Conference on Decision and Control, Dec. 9-12, 2003, Volume: 1, Pages:701 - 708

[4] X. Liu and A. J. Goldsmith, "Kalman Filtering with Partial Observation Losses," 43rd IEEE Conference on Decision and Control, 2004

[5] N. C. Martins and M. A. Dahleh, "Fundamental limitations of feedback control with communication constraints," http://www.cds.caltech.edu/doyle-fest/presentations/pdfs/dahleh.pdf

[6] William Jakes, *Microwave Mobile Communications*. IEEE Press, 1974

[7] T. Kailath, A. H. Sayed, B. Hassibi, *Linear Estimation*. Prentice Hall information and system sciences series

## IX. APPENDIX A

Let $\gamma$ be an exponentially distributed random variable with $\overline{\gamma} = \frac{1}{\lambda_n}$. Then we will have the following for arbitrary $a \geq 0$, $c \geq 0$ and $d \geq 0$:

$$\begin{aligned} \overline{\left[\frac{a}{c+d\gamma}\right]} &= a\int_0^\infty \lambda_n \frac{e^{-\lambda_n \gamma}}{c+d\gamma}d\gamma \\ &= a\frac{\lambda_n e^{\frac{\lambda_n c}{d}}}{d}\int_c^\infty \frac{e^{-\frac{\lambda_n z}{d}}}{z}dz \\ &= a\frac{\lambda_n e^{\frac{\lambda_n c}{d}}}{d}Expint(\frac{\lambda_n c}{d}) \end{aligned} \tag{30}$$

## X. APPENDIX B

Let $\gamma$ be an exponentially distributed random variable with $\overline{\gamma} = \frac{1}{\lambda_n}$. Let $d$ and $c$ represent positive scalars where $d < \lambda_n$. Then we will have,

$$\begin{aligned} \overline{S(d\gamma+c)} &= \overline{e^{d\gamma+c}Expint(d\gamma+c)} \\ &= \int_0^\infty \lambda_n e^{(d-\lambda_n)\gamma+c}Expint(d\gamma+c)d\gamma \\ &= \frac{\lambda_n}{d-\lambda_n}e^{(d-\lambda_n)\gamma+c}Expint(d\gamma+c)|_{\gamma=0}^{\gamma=\infty} \\ &+ \frac{d\lambda_n}{d-\lambda_n}\int_0^\infty \frac{e^{-\gamma\lambda_n}}{d\gamma+c}d\gamma \\ &= -\frac{\lambda_n}{d-\lambda_n}e^c Expint(c) \\ &+ \frac{\lambda_n}{d-\lambda_n}e^{\frac{\lambda_n c}{d}}Expint(\frac{\lambda_n c}{d}) \\ &= \frac{\lambda_n}{\lambda_n-d}S(c) - \frac{\lambda_n}{\lambda_n-d}S(\frac{\lambda_n c}{d}) \end{aligned} \tag{31}$$

Inserting $c = \frac{\lambda_n}{qA^{2i}}$ and $d = \frac{\lambda_n}{A^{2i}}$ will result in Eq. 16.

## XI. APPENDIX C

Let $g(\zeta) = A^2\zeta e^{\zeta}Expint(\zeta)$ for $\zeta \geq 0$. Then we will have,

$$\frac{dg(\zeta)}{d\zeta} = A^2\left[e^{\zeta}(1+\zeta)Expint(\zeta) - 1\right]. \tag{32}$$

Since $\frac{e^{-c}}{c} > \frac{e^{-c}(c+2)}{(c+1)^2}$ for an arbitrary $c \geq 0$, we will have,

$$\int_\zeta^\infty \frac{e^{-c}}{c}dc > \int_\zeta^\infty \frac{e^{-c}(c+2)}{(c+1)^2}dc \quad for \quad \zeta \geq 0, \tag{33}$$

resulting in

$$\begin{gathered} Expint(\zeta) > \frac{e^{-\zeta}}{1+\zeta} \quad for \quad \zeta \geq 0 \Rightarrow \\ \frac{dg(\zeta)}{d\zeta} = A^2\left[e^{\zeta}(1+\zeta)Expint(\zeta) - 1\right] > 0 \quad \zeta \geq 0. \end{gathered} \tag{34}$$

Hence[5] $g(\zeta)$ is an increasing function of $\zeta$ for $\zeta \geq 0$.

[5] $\frac{dg(\zeta)}{d\zeta}$ is only zero at $\zeta = \infty$.

Fig. 1 Performance of scenario 2 (keeping all the packets)

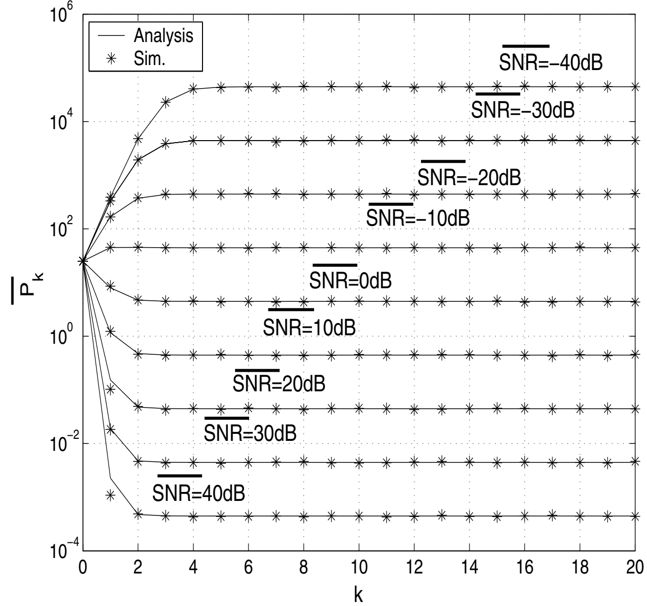


Fig. 2 Comparing the two scenarios