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Optimal actuator placement and model reduction for a class of parabolic partial differential equations using spatial $\mathcal{H}_2$ norm

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Index Terms— **parabolic partial differential equations, spatial norms, computational scheme, actuator placement, spillover effects.**

Abstract— **The present work focuses on the optimal, with respect to certain criteria, placement of control actuators for transport-reaction processes, mathematically modelled by linear parabolic partial differential equations. Using model decomposition to discretize the spatial coordinate, and the notions of spatial and modal controllability, the semi-infinite optimization problem is formulated as a nonlinear optimization problem in appropriate L_2 spaces. The formulated problem is subsequently solved using standard search algorithms. The proposed method is successfully applied to a representative process, modelled by a one-dimensional parabolic PDE, where the optimal location of a single point actuator is computed.**

I. Introduction

A large number of industrially important chemical processes exhibit variation of the process variables in space and can be categorized as transport-reaction processes. Examples include chemical vapor deposition reactors and plasma etching processes [1]. Process models can be derived from dynamic conservation equations and usually involve parabolic partial differential equation (PDE) systems.

The issue of control and optimization of transport-reaction processes has received a lot of attention in the last two decades [2], [3], [4]. One research direction focuses on the development of reduced-order models based on the property of parabolic PDEs that the eigenspectrum of the spatial operator can be partitioned into a finite size set of eigenvalues that are close to the imaginary axis and an infinite size set of eigenvalues that are far in the left half plane, implying that the dominant behavior of the system can be accurately captured by a finite number of eigenmodes [5]. Controllers are subsequently designed based on the reduced-order models [2], [6], [7]. An important issue for the controller design methodology, is the actuator placement such that the closed-loop system is controllable.

The conventional approach to actuator placement is to select the locations based on open-loop considerations to ensure that the necessary controllability, reachability or power factor requirements are satisfied [8], [9], [10], [11], [12]. For further work on different aspects of actuator and sensor placement based on controllability/observability, the reader is directed to the survey papers of van de Wal and de Jager in [13] and of Kubrusly and Malebranche in [14].

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The issue of integrating feedback control and optimal actuator placement has been examined for linear [15] and quasi-linear [16], [17] parabolic PDEs, and in actively controlled structures [18], [19], [20], [21], [22]. Using quadratic performance measures, parameterized by the actuator and sensor locations, the optimal position was found as the one yielding the minimum of the optimal control measures. For the linear parabolic PDEs this amounts to parameterizing the solution to the associated Algebraic Riccati Equation by the candidate actuator locations and minimizing the optimal value of the trace of the location-parameterized Riccati solution. For an in-depth exposure please see [23]. However, when using such performance measures, one might obtain an actuator location that in a sense is "averaged" over all frequencies; consider for example the specific case of the one-dimensional heat equation with Dirichlet boundary conditions and with constant diffusivity parameter. When the solution to the ARE is parameterized by the actuator locations, which are assumed to have a spatial distribution given by the spatial Dirac delta function (point actuator), one obtains the midpoint of the spatial domain as the optimal actuator location with the smallest performance index. This location is also the zero of the second eigenfunction, which means that the actuator will have no authority to handle exogenous signals containing the second eigenfrequency. Results have also been reported on the identification of optimal strategies for the activation of actuators (from a pool of available ones) according to predetermined criteria for distributed processes [24], [25], and the integration of these strategies with controller design [26], [27].

The present work deals with the optimal placement of control actuators for transport-reaction processes, mathematically modelled by linear parabolic partial differential equations. Using model decomposition to discretize the spatial coordinate, and based on the definitions of spatial and modal controllability [28], [29], the semi-infinite optimization problem is formulated as a nonlinear optimization problem in appropriate L_2 spaces. Standard optimal search algorithms are subsequently used to obtain the optimal location. The proposed method is successfully applied to a representative diffusion process, modelled by a one-dimensional parabolic PDE, where the location of a single point actuator that maximizes the spatial controllability of the system subject to constraints on modal controllability and spillover effects is computed. The approach considered here for actuator placement has a different flavor than earlier methods, in that it caters to specific low frequencies/modes of the system while at the same time takes into consideration

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medium-range frequencies often neglected in traditional model reduction techniques. The measure for actuator placement considers the enhancement of the actuator location over the dominant modes, while at the same time offers robustness with respect to the intermediate range of modes which certainly ameliorate spillover effects.

II. Mathematical Modelling

We consider the problem of computationally identifying the optimal locations of actuators for processes that can be mathematically described by parabolic partial differential equation (PDE) systems of the form:

$$\frac{\partial}{\partial t}x(t,\xi) = \mathcal{A}(\xi)x(t,\xi) + b(\xi;\xi_a)u(t), \tag{1}$$

where $x \in \mathbb{R}^n$ is the state, t is the time, $\xi \in \Omega$ is the spatial coordinate and Ω is a bounded smooth domain in $\mathbb{R}^n$, and $u(t) \in \mathbb{R}$ is the manipulated variable. Even though in the current manuscript we consider a single actuator, the results can be generalized to multiple actuators placement in a straightforward way. $\mathcal{A}$ is a second order (strongly) elliptic operator [30] of the form:

$$\mathcal{A}\phi = \sum_{j,k=1}^{n} \frac{\partial}{\partial \xi_j}\left(\alpha_{jk}(\xi)\frac{\partial \phi(\xi)}{\partial \xi_k}\right) + \sum_{j=1}^{n} \alpha_j(\xi)\frac{\partial \phi(\xi)}{\partial \xi_j} + \alpha_0(\xi)\phi(\xi),$$

for $\xi \in \Omega$, and $b(\xi;\xi_a)$ denotes the spatial distribution of the actuating device (e.g., boundary, distributed and/or pointwise) placed at location $\xi_a \in \Omega_\alpha$, where $\Omega_\alpha \subseteq \Omega$ denotes the domain of permissible actuator locations. All three cases of boundary conditions for the above PDE system may be considered: mixed (Robin), Neumann or Dirichlet [31], where the boundary, denoted by $\partial\Omega$, can be decomposed to $\partial\Omega = \Gamma_a \cup \Gamma_b$ with Γ_a denoting the part of the boundary where the actuator(s) *may* be placed and Γ_b $(= \partial\Omega \setminus \Gamma_a)$ the remainder of the boundary where actuators are not desired or allowed to be placed. Defining appropriate Hilbert spaces $\mathcal{H}_2 \subset L_2(\Omega)$ with inner product

$$\int_\Omega \psi_1(\xi)\psi_2(\xi)\,d\xi = \langle \psi_1, \psi_2 \rangle_{L_2}, \tag{2}$$

and norm $\|\psi\|_2 = \sqrt{\langle \psi_1, \psi_2 \rangle_{L_2}}$, the PDE system of (1) can be equivalently written in the following abstract form [30]:

$$\dot{x}(t) = \mathcal{A}x(t) + Bu(t), \tag{3}$$

where $x(t,\cdot) = x(t) \in \mathcal{H}$ is the state and B denotes the input operator either on the part of the boundary Γ_a where actuation is desired or permissible, or on the permissible part of the interior of the domain Ω_α.

The PDE of (1) can be solved independently for each mode by using the orthogonality properties of the eigenfunctions of spatial operator $\mathcal{A}$. In this case

$$\int_\Omega \mathcal{A}\phi_i(\xi)\phi_j(\xi)\,d\xi = \lambda_i\delta_{ij}, \quad \int_\Omega \phi_i(\xi)\phi_j(\xi)\,d\xi = \delta_{ij}, \tag{4}$$

where λ_i denotes the i^{th} eigenvalue, $\phi_i(\xi)$ denotes the i^{th} normalized eigenfunction and δ_{ij} denotes the Kronecker delta. We assume that the eigenvalues are ordered such that $\lambda_{i+1} \le \lambda_i, \forall i = 1,\ldots,\infty$. Employing the computed eigenfunctions as a basis function set for $\mathcal{H}$, $x(t,\xi)$ can be equivalently expressed as the expansion

$$x(t,\xi) = \sum_{i=1}^{\infty} x_i(t)\phi_i(\xi). \tag{5}$$

Using the properties of the Laplace transforms with $\mathcal{L}[x_i(t)] = X_i(s)$, one obtains

$$X_i(s;\xi_a) = \frac{\int_\Omega b(\xi;\xi_a)\phi_i(\xi)\,d\xi}{s - \lambda_i}\,U(s). \tag{6}$$

By setting

$$B_i(\xi_a) \triangleq \int_\Omega b(\xi;\xi_a)\phi_i(\xi)\,d\xi,$$

one then obtains the Laplace transform of the spatial distributed state

$$X(s,\xi;\xi_a) = \sum_{i=1}^{\infty} \frac{B_i(\xi_a)\phi_i(\xi)}{s - \lambda_i}\,U(s). \tag{7}$$

The resulting single input-infinite output transfer function is given by

$$G(s,\xi;\xi_a) = \sum_{i=1}^{\infty} \frac{B_i(\xi_a)\phi_i(\xi)}{s - \lambda_i}. \tag{8}$$

III. Actuator Location Optimization

In this section we formulate the optimization problem for the identification of optimal actuator locations, according to certain objectives. We initially introduce the necessary mathematical background leading to the definition of the objective function.

A. Spatial norms

The spatial $\mathcal{H}_2$ norm [29] of the transfer function $G(s,\xi)$ is defined as

$$\|G\|^2_{\mathcal{H}_2} \triangleq \frac{1}{2\pi}\int_{-\infty}^{\infty}\int_\Omega tr\{G^*(j\omega,\xi)G(j\omega,\xi)\}\,d\xi\,d\omega. \tag{9}$$

Using the orthogonality property of the eigenfunctions, the above norm simplifies to

$$\|G\|^2_{\mathcal{H}_2} = \sum_{i=1}^{\infty} \|G_i(s,\xi_a)\|_2^2 \tag{10}$$

where

$$G_i(s,\xi_a) \triangleq \frac{B_i(\xi_a)}{s - \lambda_i} \tag{11}$$

B. Spatial and Modal controllability measures

We now utilize the above notion of spatial $\mathcal{H}_2$ norm to propose various measures for actuator placement. We assume that an actuator is located at position ξ_a having a spatial distribution function $b(\xi;\xi_a)$. Then

$$B_i(\xi_a) = \int_\Omega b(\xi;\xi_a)\phi_i(\xi)\,d\xi. \tag{12}$$

We now define

$$f_i(\xi_a) \triangleq \|G_i(s,\xi_a)\|_2 = \left\| \frac{B_i(\xi_a)}{s-\lambda_i} \right\|_2. \tag{13}$$

Following [28], we define the i^{th} *modal controllability* at location ξ_a as

$$\mathcal{M}_i(\xi_a) = \frac{f_i(\xi_a)}{\max_{\xi\in\Omega_\alpha} f_i(\xi)} \times 100\%, \quad i=1,2,\ldots, \tag{14}$$

and which describes controller authority of location ξ_a over the i^{th} mode. If the modal controllability at a specific location of a given mode is zero, it means that the controller has no authority over that mode. This also coincides with the notion of approximate controllability for the class of PDEs with Riesz-spectral operators [32]. The requirement for approximate controllability is

$$\langle b, \phi_i \rangle_{L_2} \neq 0 \quad \forall i.$$

Indeed, when the i^{th} mode has a zero modal controllability at location ξ_a, it means that $f_i(\xi_a) \equiv 0$ and hence $B_i(\xi_a) = 0$, or that

$$\int_\Omega b(\xi;\xi_a)\phi_i(\xi)\, d\xi = 0.$$

Continuing, we also consider the *spatial controllability* [29] defined for the first N system modes via

$$\begin{aligned} \mathcal{S}^N(\xi_a) \quad &\triangleq \quad \frac{\sqrt{\sum_{i=1}^{N} f_i^2(\xi_a)}}{\max_{\xi\in\Omega_\alpha} \sqrt{\sum_{i=1}^{N} f_i^2(\xi)}} \times 100\% \\ &= \quad \frac{\sqrt{\sum_{i=1}^{N} \|G_i(s,\xi_a)\|_2^2}}{\max_{\xi\in\Omega_\alpha} \sqrt{\sum_{i=1}^{N} \|G_i(s,\xi)\|_2^2}} \times 100\%. \end{aligned} \tag{15}$$

Please note that the term

$$\sqrt{\sum_{i=1}^{N} \|G_i(s,\xi_a)\|_2^2}$$

corresponds to the truncated $\mathcal{H}_2$ spatial norm since

$$\begin{aligned} \|G(s,\xi;\xi_a)\|_{\mathcal{H}_2}^2 &= \sum_{i=1}^{\infty} \|G_i(s,\xi_a)\|_2^2 \\ &= \sum_{i=1}^{N} \|G_i(s,\xi_a)\|_2^2 + \sum_{i=N+1}^{\infty} \|G_i(s,\xi_a)\|_2^2 \\ &= \|G^N(s,\xi_a)\|_{\mathcal{H}_2}^2 + \mathbf{S}(\xi_a), \end{aligned}$$

where $\mathbf{S}(\xi_a) \triangleq \sum_{i=N+1}^{\infty} \|G_i(s,\xi_a)\|_2^2$ denotes the higher mode contribution. Therefore, the spatial controllability is also given in terms of the truncated spatial $\mathcal{H}_2$ norm via

$$\mathcal{S}^N(\xi_a) = \frac{\|G^N(s,\xi_a)\|_{\mathcal{H}_2}}{\max_{\xi\in\Omega_\alpha} \|G^N(s,\xi)\|_{\mathcal{H}_2}}.$$

Following [28], the spatial controllability at location ξ_a is a measure of controller authority placed at location ξ_a over the entire spatial domain in an average sense. Using the above notions of controllability, we can now state the location optimization problem.

Actuator placement optimization: It is desired to find the actuator location ξ_a that provides a maximum of the spatial controllability while at the same time maintains a reasonable level of controller authority over each mode. The constrained optimization problem is thus formulated as:

$$\begin{aligned} &\max_{\xi\in\Omega_\alpha} \mathcal{S}^N(\xi) \\ &\text{s.t.} \\ &\mathcal{M}_i(\xi) \geq \beta_i, \ \ i=1,2,\ldots,N. \end{aligned} \tag{Pc-I}$$

Remark 1: The above can also be alternately formulated as an unconstrained optimization problem in which one considers the set of admissible locations Ω_a given by

$$\Omega_a = \left\{ \xi \in \Omega_\alpha : \left| \int_\Omega b(\xi;\xi)\phi_i(\xi)\, d\xi \right| \geq \gamma_i, \, i=1,\ldots,N \right\},$$

where the controllability bounds γ_i are related to the level of modal controllabilities β_i. One then restricts the search over the space Ω_a and hence the constrained optimization problem is reformulated as

$$\max_{\xi\in\Omega_a} \mathcal{S}^N(\xi). \tag{Pc-II}$$

Remark 2: The formulated optimization problem Pc-I can be solved using standard search algorithms such as Newton-based, interior-point or direct-search methods [33], [34]. Due to the nonlinear nature of the objective function and the inequality constraints, global optimization methods are preferred [35]. For the unconstrained formulation of Pc-II, in the event the set Ω_a is disjoint, branch and bound algorithms [34] or genetic algorithms [18] can also be used to obtain the optimal actuator locations.

C. Spillover effects

In order to address implicitly the issue of actuator spillover inadvertently affecting the closed-loop process response, we reformulate the optimization problem to include an additional constraint that guarantees a maximum level of spatial controllability for higher-frequency modes. Capitalizing on the property of parabolic systems that the higher modes become progressively more stable ($\lambda_{i+1} \leq \lambda_i$ and $\lambda_i \to \infty$ as $i \to \infty$), we assume that the spillover effect on a finite number of higher modes $\phi_i(\xi)$, $i = N+1,\ldots,M$ need be considered. The spatial controllability for these modes is similarly defined as:

$$\mathcal{S}_M^N(\xi_a) \triangleq \frac{\sqrt{\sum_{i=N+1}^{M} f_i^2(\xi_a)}}{\max_{\xi\in\Omega_\alpha} \sqrt{\sum_{i=N+1}^{M} f_i^2(\xi)}} \times 100\%. \tag{16}$$

In this case, the optimization problem of Pc-I is now formulated as

$$\begin{aligned} &\max_{\xi\in\Omega_\alpha} \mathcal{S}^N(\xi) \\ &\text{s.t.} \\ &\mathcal{M}_i(\xi) \geq \beta_i,\ \ i = 1,2,\ldots,N, \\ &\mathcal{S}_M^N(\xi) \leq c. \end{aligned} \tag{Pc-III}$$

where c is the allowable level for spatial controllability for spillover reduction.

Remark 3: In a similar fashion as before, we can reformulate the constrained optimization problem as an unconstrained one by restricting the search in a subset of Ω_a that also ensures a certain lever of spatial controllability for spillover reduction. Thus we define

$$\Omega_{as} = \left\{\xi \in \Omega_a : \mathcal{S}_M^N(\xi) \leq c\right\}, \tag{17}$$

and therefore the constrained optimization problem is reformulated as

$$\max_{\xi\in\Omega_{as}} \mathcal{S}^N(\xi). \tag{Pc-IV}$$

IV. Numerical experiment

We consider a representative spatially distributed process, mathematically described by the following PDE

$$\frac{\partial}{\partial t}x(t,\xi) = 0.01\frac{\partial^2}{\partial \xi^2}x(t,\xi) + b(\xi;\xi_a)u(t),$$

subject to Neumann boundary conditions:

$$\frac{\partial x}{\partial \xi}\Big|_{\xi=0} = 0,\ \ \frac{\partial x}{\partial \xi}\Big|_{\xi=L} = 0,$$

where the spatial domain is defined as $\Omega = [0,L]$, with $L = 2$. The spatial distribution of the actuating device placed at location ξ_a has the form

$$b(\xi;\xi_a) = \begin{cases} \dfrac{1}{\varepsilon} & \text{if } \xi_a - \dfrac{\varepsilon}{2} \leq \xi \leq \xi_a + \dfrac{\varepsilon}{2} \\ 0 & \text{otherwise.} \end{cases}$$

For the specific system, the associated eigenvalues and eigenfunctions (normalized) can be computed analytically and are of the form

$$\lambda_i = -\left(\frac{i\pi}{L}\right)^2 \text{ and } \phi_i(\xi) = \sqrt{\frac{2}{L}}\cos\left(\frac{i\pi\xi}{L}\right),\ i = 1,\ldots,\infty.$$

The permissible actuator domain is defined as $\Omega_\alpha = [0+\varepsilon/2, L-\varepsilon/2]$ with the span of the actuating device $\varepsilon = L/500$. To compute the optimal actuator location, we assume that the first 5 modes are important for the calculation of the objective function $\mathcal{S}^N(\xi)$, (i.e. $N = 5$). For spillover reduction the next ten higher modes are considered and thus

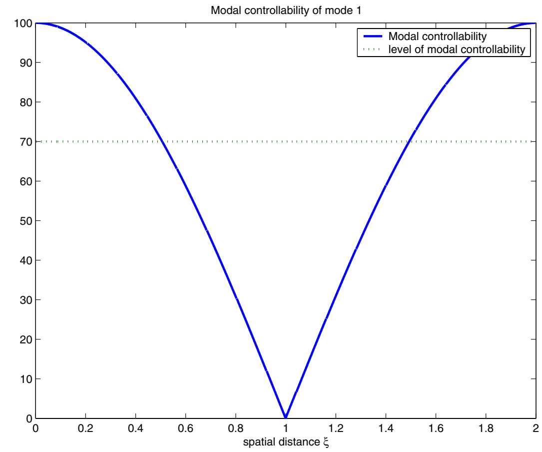

Fig. 1. Percentage modal controllability of mode 1.

$M = 15$. The associated constrained optimization problem of Pc-III takes the specific form

$$\begin{aligned} &\max_{\xi\in\Omega} \mathcal{S}^5(\xi) \\ &\text{s.t.} \\ &\mathcal{M}_i(\xi) \geq 70\%,\ \ i = 1,2,\ldots,5, \\ &\mathcal{S}_{15}^5(\xi) \leq 60\%. \end{aligned} \tag{Pc-V}$$

The minimum level of modal controllability for the first five modes was chosen as $b_i = 70\%$ and the contributions of the next ten modes to the spatial norm was limited to $c = 60\%$.

Figures 1-5 depict the modal controllabilities for the first five modes. Considering the inequality constraint of controllability level of 70% in the optimization program of Pc-V (dashed line in Figures 1-5), is can be observed that the set of admissible locations now becomes $\Omega_a = [0.002, 0.4412] \cup [1.5588, 1.998]$. Due to the symmetry of the process and the objective function with respect to the center of the spatial domain $\xi = 1$, we focus our attention to region $[1.5588, 1.998]$. If we do not consider spillover effects, the solution to the optimization problem of Pc-V is computed to be location $\xi^{opt} = 1.998$. We observe in Figure 7 that this value maximizes the objective function. The issue with the chosen location though is that it also maximizes the spill-over effect.

The spillover spatial controllability $\mathcal{S}_{15}^5(\xi)$ is shown in Figure 6. Constraining the acceptable level of spill-over to 60%, we observe that the set of spatial locations that ensure that $\mathcal{S}_{15}^5(\xi) \leq 60\%$ is given by $[0.0699, 0.1589] \cup [1.8411, 1.9301]$. Its intersection with Ω_a would then yield the admissible set $\Omega_{as} = [0.0699, 0.1588] \cup [1.8412, 1.9301]$.

Solving the optimization problem of Pc-IV, the optimal actuator location was computed to be $\xi = 1.9301$, shown in Figure 7 which depicts the spatial controllability $\mathcal{S}^5(\xi)$ as a function of the spatial distance ξ.

In addition, Table I summarizes the levels of controllabilities at the optimal location $\xi_{opt} = 1.9301$. This optimal location maintains a spatial controllability above 98% while

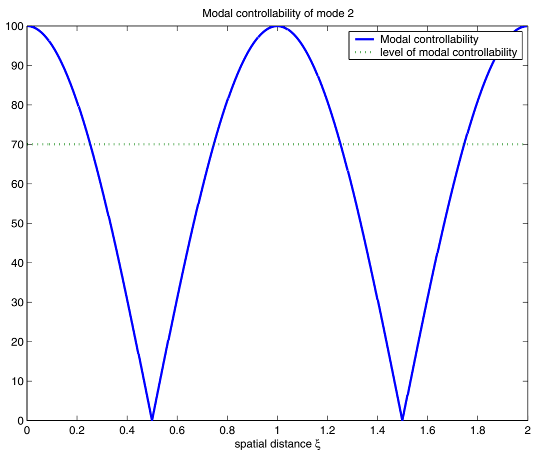

Fig. 2. Percentage modal controllability of mode 2.

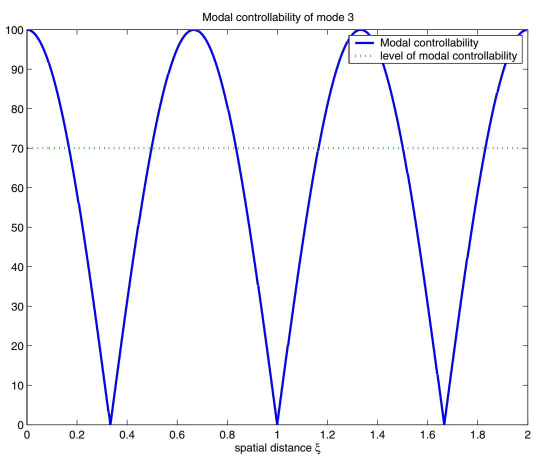

Fig. 3. Percentage modal controllability of mode 3.

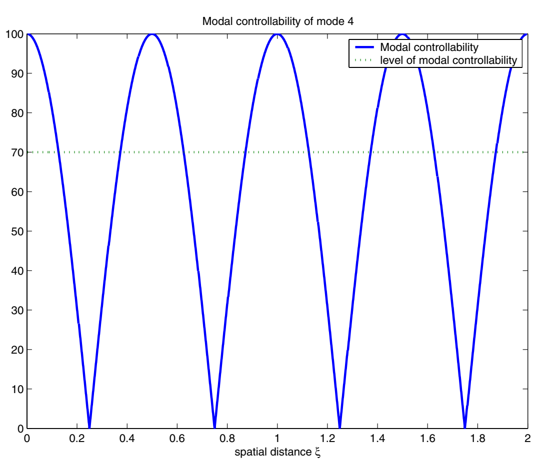

Fig. 4. Percentage modal controllability of mode 4.

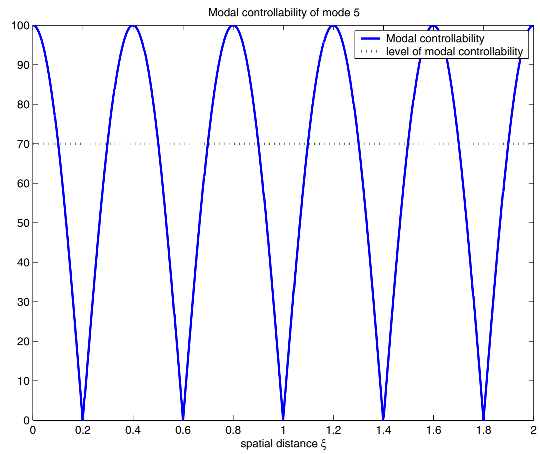

Fig. 5. Percentage modal controllability of mode 5.

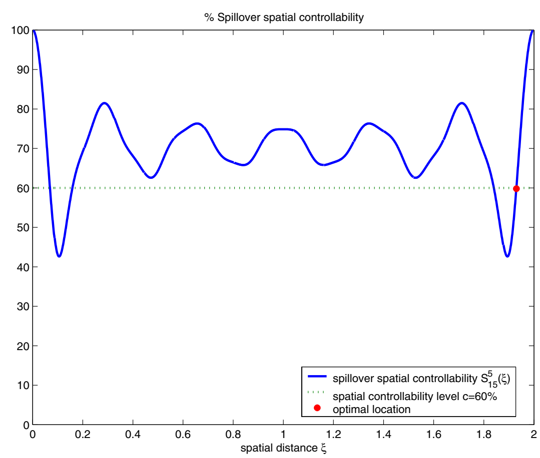

Fig. 6. Percentage spillover spatial controllability $\mathcal{S}^5_{15}(\xi)$.

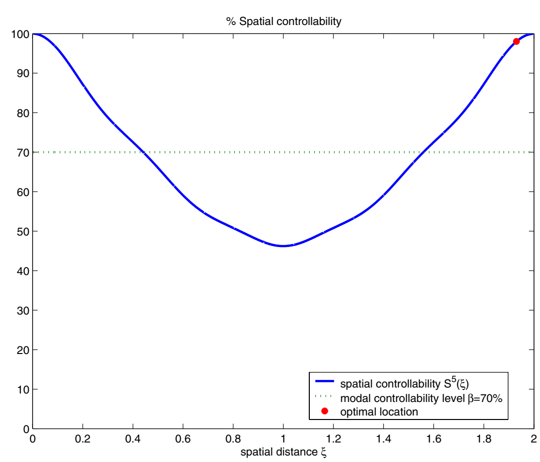

Fig. 7. Percentage spatial controllability, $\mathcal{S}^5(\xi)$.

TABLE I

CONTROLLABILITY VALUES AT OPTIMAL ACTUATOR LOCATION.

$\mathcal{S}^5(\xi^{opt})$	98.0123%
$\mathcal{M}^1(\xi^{opt})$	99.3987%
$\mathcal{M}^2(\xi^{opt})$	97.6001%
$\mathcal{M}^3(\xi^{opt})$	94.6272%
$\mathcal{M}^4(\xi^{opt})$	90.5154%
$\mathcal{M}^5(\xi^{opt})$	85.3142%
$\mathcal{S}^5_{15}(\xi^{opt})$	59.808%

maintaining a modal controllability of the first five modes above 85%. Additionally, the spatial controllability of the 6th to 15th modes $\mathcal{S}^5_{15}(\xi)$ is also maintained at a level below 60%. In fact, at the optimal actuator location, one has $\mathcal{S}^5_{15}(\xi^{opt}) = 59.7913\%$.

Remark 4: Note that due to symmetry of *both* process and objective function with respect to the center of the process $\xi = 1$, position $\xi = 0.0699$ is also an optimal location. The issue of two identical optimal locations due to symmetry is circumvented during the optimal search, by limiting the search in set $\xi \in [1, 1.998]$.

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