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Uniform Synchronization in Multi-Axis Motion Control

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Abstract— **This paper proposes a uniform motion synchronization strategy to treat multi-axis control problems. By using this strategy, the asymptotic convergence of both tracking and synchronization errors are achieved. Furthermore, transient motion performance may be improved by selecting proper position synchronization errors, all under the uniform framework. Simulations of a four-axis numerical example illustrate the effectiveness of the proposed approach.**

Keywords: adaptive control; position synchronization; uniform synchronization error; asymptotic convergence; transient performance.

I. Introduction

Motion synchronization in multiple number of axes or motors has drawn much attention, especially in modern manufacturing development. The cross-coupling concept to solve the synchronization problem has been proposed in early 1980s [1]. Since then, efforts of using this concept to improve synchronization performance of two-axis motions include [2], [3]. Other approaches include fuzzy logic coupling [4] and neuro-controller [5]. Synchronization, coordination, and cooperation are linked subjects in the study of multicomposed systems' behaviour. Much of recent work can be found in the field of coordinated robots, and multiple UAVs in formation flying, such as [6], [7], [8], [9].

The authors of this paper developed an adaptive coupling control algorithm for position synchronization of multiple axes [10], [11]. The cross coupling technology is incorporated into adaptive control design through feedback of position errors and synchronization errors. The proposed algorithm guarantees asymptotic convergence to zero of both position errors and synchronization errors. In [11], the synchronization errors are defined to be differential position errors of every adjacent pair of axes in sequence. In this paper, we propose a generic, uniform position synchronization error. Under this uniform strategy, both the position and synchronization errors are guaranteed of their asymptotic convergence. In addition, proper selection of position synchronization error will improve the transient synchronization performance.

II. Uniform Position Synchronization Error

Consider the dynamics of a motion system with n axes in the matrix format:

$$\underline{H}\ddot{\underline{x}} + \underline{C}\dot{\underline{x}} + \underline{F}(\underline{x}, \dot{\underline{x}}) = \underline{Y}\,\underline{\theta} = \underline{\tau} \tag{1}$$

where for each axis-i, H_i and C_i represent the inertia and centrifugal and Coriolis matrix respectively, and $\dot{H}_i - 2C_i$ is skew-symmetric; F_i represents the external disturbance. In this paper we deal with the system of a given structure, but suffering from parameter uncertainties. Without loss of generality, the dynamics equations of each axis can be reformulated as $Y_i\theta_i = \tau_i$, where Y_i is the regression matrix; θ_i represents the unknown parameters; and τ_i represents the torque that are to be designed for control.

Define the *position tracking error* of the ith axis as

$$e_i = x_i^d - x_i \tag{2}$$

The *position synchronization error* seeks the differences among the tracking errors of the multiple axes. For example, the synchronization errors defined in [11] are position error differences of every pair of adjacent axes. We denote it by Type I synchronization error as shown in the following equation:

$$\begin{array}{lcl} \epsilon_1 & = & e_1 - e_2 \\ \epsilon_2 & = & e_2 - e_3 \\ & \vdots & \\ \epsilon_i & = & e_i - e_{i+1} \\ & \vdots & \\ \epsilon_{n-1} & = & e_{n-1} - e_n \\ \epsilon_n & = & e_n - e_1 \end{array} \tag{3}$$

and the purpose of the control design is to achieve $e_i \to 0$ and $\epsilon_i \to 0$.

Generally speaking, there are many alternatives in selecting proper synchronization errors. For example, for each axis, one can compare its relative position with every one of other axes. Such synchronization information is more accurate and exhaustive, but may suffer from computational complexity. Even for the Type I synchronization of pair of adjacent axes as shown before, the designer may add some "weight" factors on each ϵ_i to indicate different priority levels of synchronization. Among technically infinite number of choices of synchronization errors, The question to the designer then, becomes how to select an *optimal* one to achieve best performance possible. In multi-axis motion control, it implies that the synchronization strategy and its corresponding control technique will 1) achieve the asymptotic convergence of tracking and synchronization errors simultaneously, 2) under acceptable control efforts, and 3) reach the best transient performance possible. In this paper, we extend our work in adaptive cross-coupling control to include a generic, uniform synchronization error, such that requirement 1) is guaranteed no matter which specific synchronization choice is; and by picking different choices of synchronization, one can evaluate their transient performance, as per requirement 3).

In order to illustrate the impact of synchronization choices to the transient performance in multi-axis motion control, we pick a second synchronization error for comparison purpose (Type II):

$$
\begin{aligned}
\epsilon_1 &= 2e_1 - (e_2 + e_n) \\
\epsilon_2 &= 2e_2 - (e_3 + e_1) \\
&\vdots \\
\epsilon_i &= 2e_i - (e_{i+1} + e_{i-1}) \\
&\vdots \\
\epsilon_{n-1} &= 2e_{n-1} - (e_n + e_{n-2}) \\
\epsilon_n &= 2e_n - (e_1 + e_{n-1})
\end{aligned} \tag{4}
$$

In this paper, we formulate a generic, uniform position synchronization error:

$$
\begin{bmatrix} \epsilon_1 \\ \vdots \\ \epsilon_n \end{bmatrix} = \begin{bmatrix} \Lambda_{11} & \dots & \Lambda_{1n} \\ \vdots & \ddots & \vdots \\ \Lambda_{n1} & \dots & \Lambda_{nn} \end{bmatrix} \begin{bmatrix} e_1 \\ \vdots \\ e_n \end{bmatrix}, \text{or}, \underline{\epsilon} = \underline{T}\,\underline{e} \tag{5}
$$

where Λ_{ij} is a diagonal matrix. We call the matrix $\underline{T}$ synchronization transformation matrix.

To demonstrate the generality of our definition, the previous examples of synchronization error definitions (3) and (4) can be re-written in the similar format as follows:

$$
\begin{bmatrix} \epsilon_1 \\ \epsilon_2 \\ \vdots \\ \epsilon_{n-1} \\ \epsilon_n \end{bmatrix} = \begin{bmatrix} I & -I & & & \\ & I & -I & & \\ & & \ddots & \ddots & \\ & & & I & -I \\ -I & & & & I \end{bmatrix} \begin{bmatrix} e_1 \\ e_2 \\ \vdots \\ e_{n-1} \\ e_n \end{bmatrix}
$$

$$
\begin{bmatrix} \epsilon_1 \\ \epsilon_2 \\ \vdots \\ \epsilon_{n-1} \\ \epsilon_n \end{bmatrix} = \begin{bmatrix} 2I & -I & & & -I \\ -I & 2I & -I & & \\ & \ddots & \ddots & \ddots & \\ & & -I & 2I & -I \\ -I & & & -I & 2I \end{bmatrix} \begin{bmatrix} e_1 \\ e_2 \\ \vdots \\ e_{n-1} \\ e_n \end{bmatrix}
$$

We have the following conclusion of the generalized position synchronization error matrix.

Lemma 2.1: For diagonal matrix $\underline{E}$ with the same value in its diagonal elements, and of the same order as the synchronization transformation matrix $\underline{T}$ as defined in (5)

$$
\underline{E} = \begin{bmatrix} E & & \\ & \ddots & \\ & & E \end{bmatrix}
$$

$\underline{T}$ and $\underline{E}$ commute, i.e.,

$$
\underline{T}\,\underline{E} = \underline{E}\,\underline{T}, \underline{T}^T\underline{E} = \underline{E}\,\underline{T}^T \tag{6}
$$

The proof is obvious, and omitted from this paper.

Once the generalized position synchronization error is defined, our focus is to design the controller to guarantee the asymptotic convergence of both position tracking error $\underline{e}$ and position synchronization error $\underline{\epsilon}$. Such a control strategy is obtained, through a cross-coupling adaptive control technique, following the same approach as in [11].

III. Control Design Strategy

Consider the dynamics matrix equation of a motion system with n axes again in (1). Define the total feedback *generalized velocity and position error* as:

$$
\underline{r} = \dot{\underline{e}} + \alpha\underline{e} + \beta(\underline{\varepsilon} + \alpha\underline{\xi}) \tag{7}
$$

where $\alpha, \beta > 0$,

$$
\underline{\varepsilon} = \underline{T}^T\underline{\epsilon} \tag{8}
$$

and

$$
\underline{\xi} = \int_0^t \underline{\varepsilon}(w)dw \quad \text{or} \quad \dot{\underline{\xi}} = \underline{\varepsilon} \tag{9}
$$

The command signal is defined as

$$
\underline{u} = \underline{r} + \dot{\underline{x}} = \dot{\underline{x}}^d + \alpha\underline{e} + \beta(\underline{\varepsilon} + \alpha\underline{\xi}) \tag{10}
$$

and choose the torque input as

$$
\underline{\tau} = \hat{\underline{H}}\dot{\underline{u}} + \hat{\underline{C}}\underline{u} + \hat{\underline{F}} + \underline{R}\,\underline{r} + \underline{E}\,\underline{\varepsilon} = \underline{Y}\hat{\underline{\theta}} + \underline{R}\,\underline{r} + \underline{E}\,\underline{\varepsilon} \tag{11}
$$

where $\hat{\underline{H}}, \hat{\underline{C}}, \hat{\underline{F}}$ are estimates of $\underline{H}, \underline{C}, \underline{F}$ respectively,

$$
\underline{R} = \begin{bmatrix} K_{R_1} & & \\ & \ddots & \\ & & K_{R_n} \end{bmatrix} \tag{12}
$$

$$
\underline{E} = \begin{bmatrix} K_\varepsilon & & \\ & \ddots & \\ & & K_\varepsilon \end{bmatrix} \tag{13}
$$

K_{R_i}, K_ε are positive definite diagonal gain matrices.

The estimated parameter vector is subject to the adaptation law,

$$
\dot{\hat{\underline{\theta}}} = \underline{\Gamma}\,\underline{Y}^T\underline{r} \tag{14}
$$

where

$$\underline{\Gamma} = \begin{bmatrix} \Gamma_1 & & \\ & \ddots & \\ & & \Gamma_n \end{bmatrix} \tag{15}$$

and Γ_i is a positive diagonal matrix.

Consider the model error $\underline{\tilde{\theta}} = \underline{\theta} - \underline{\hat{\theta}}$, it leads to

$$\dot{\underline{\tilde{\theta}}} = -\underline{\Gamma}\underline{Y}^T\underline{r} \tag{16}$$

Therefore, combining dynamics equation (1) and controller torque (11) we obtain the final closed-loop system matrix:

$$\underline{\underline{H}}\dot{\underline{r}} + \underline{\underline{C}}\underline{r} + \underline{\underline{R}}\underline{r} + \underline{\underline{E}}\underline{\varepsilon} = \underline{\underline{Y}}\underline{\tilde{\theta}} \tag{17}$$

Multiplied by $\underline{r}^T$ at left, one gets

$$\underline{r}^T\underline{\underline{H}}\dot{\underline{r}} + \underline{r}^T\underline{\underline{C}}\underline{r} + \underline{r}^T\underline{\underline{R}}\underline{r} + \underline{r}^T\underline{\underline{E}}\underline{\varepsilon} = \underline{r}^T\underline{\underline{Y}}\underline{\tilde{\theta}} = \underline{\tilde{\theta}}^T\underline{Y}^T\underline{r} \tag{18}$$

further,

$$\begin{aligned} \underline{r}^T\underline{\underline{E}}\underline{\varepsilon} &= (\dot{\underline{e}} + \alpha\underline{e} + \beta\underline{\varepsilon} + \alpha\beta\underline{\xi})^T\underline{\underline{E}}\underline{\varepsilon} \\ &= \dot{\underline{e}}^T\underline{\underline{E}}\underline{\varepsilon} + \alpha\underline{e}^T\underline{\underline{E}}\underline{\varepsilon} + \beta\underline{\varepsilon}^T\underline{\underline{E}}\underline{\varepsilon} \\ &\quad + \alpha\beta\underline{\xi}^T\underline{\underline{E}}\underline{\varepsilon} \end{aligned} \tag{19}$$

The main conclusion of asymptotic convergence of both position tracking error and position synchronization error is presented in the following theorem and its proof.

Theorem 3.1: The proposed control strategy (11,10,7,14) guarantee asymptotic convergence to zero of both position tracking error $\underline{e}$ and position synchronization error $\underline{\epsilon}$.

Proof: Define a positive definite function

$$V = \frac{1}{2}\underline{r}^T\underline{\underline{H}}\underline{r} + \frac{1}{2}\underline{\tilde{\theta}}^T\underline{\Gamma}^{-1}\underline{\tilde{\theta}} + \frac{1}{2}\underline{\epsilon}^T\underline{\underline{E}}\underline{\epsilon} + \frac{1}{2}\alpha\beta\underline{\xi}^T\underline{\underline{E}}\underline{\xi} > 0 \tag{20}$$

Differentiating V with respect time yields $\dot{V} =$

$$\begin{aligned} &\underline{r}^T\underline{\underline{H}}\dot{\underline{r}} + \tfrac{1}{2}\underline{r}^T\dot{\underline{\underline{H}}}\underline{r} + \underline{\tilde{\theta}}^T\underline{\Gamma}^{-1}\dot{\underline{\tilde{\theta}}} + \dot{\underline{\epsilon}}^T\underline{\underline{E}}\underline{\epsilon} + \alpha\beta\dot{\underline{\xi}}^T\underline{\underline{E}}\underline{\xi} \\ &= \underline{r}^T\underline{\underline{H}}\dot{\underline{r}} + \tfrac{1}{2}\underline{r}^T\dot{\underline{\underline{H}}}\underline{r} - \underline{r}^T\underline{\underline{Y}}\underline{\tilde{\theta}} + \dot{\underline{\epsilon}}^T\underline{\underline{E}}\underline{\epsilon} \\ &+\alpha\beta\underline{\varepsilon}^T\underline{\underline{E}}\underline{\xi} \quad \text{from (16) and (9)} \\ &= \underline{r}^T\underline{\underline{H}}\dot{\underline{r}} + \tfrac{1}{2}\underline{r}^T\dot{\underline{\underline{H}}}\underline{r} - \underline{r}^T\underline{\underline{H}}\dot{\underline{r}} - \underline{r}^T\underline{\underline{C}}\underline{r} - \underline{r}^T\underline{\underline{R}}\underline{r} \\ &-\underline{r}^T\underline{\underline{E}}\underline{\varepsilon} + \dot{\underline{\epsilon}}^T\underline{\underline{E}}\underline{\epsilon} + \alpha\beta\underline{\varepsilon}^T\underline{\underline{E}}\underline{\xi} \quad \text{from (18)} \\ &= -\underline{r}^T\underline{\underline{R}}\underline{r} - \dot{\underline{e}}^T\underline{\underline{E}}\underline{\varepsilon} - \alpha\underline{e}^T\underline{\underline{E}}\underline{\varepsilon} - \beta\underline{\varepsilon}^T\underline{\underline{E}}\underline{\varepsilon} \\ &-\alpha\beta\underline{\xi}^T\underline{\underline{E}}\underline{\varepsilon} + \dot{\underline{\epsilon}}^T\underline{\underline{E}}\underline{\epsilon} + \alpha\beta\underline{\varepsilon}^T\underline{\underline{E}}\underline{\xi} \quad \text{from (19)} \end{aligned}$$

Consider the synchronization transformation matrix and Lemma 2.1,

$$\dot{\underline{\epsilon}}^T\underline{\underline{E}}\underline{\epsilon} = \dot{\underline{e}}^T\underline{T}^T\underline{\underline{E}}\underline{\epsilon} = \dot{\underline{e}}^T\underline{\underline{E}}\underline{T}^T\underline{\epsilon} = \dot{\underline{e}}^T\underline{\underline{E}}\underline{\varepsilon}$$

$$\underline{\epsilon}^T\underline{\underline{E}}\underline{\epsilon} = \underline{e}^T\underline{T}^T\underline{\underline{E}}\underline{\epsilon} = \underline{e}^T\underline{\underline{E}}\underline{T}^T\underline{\epsilon} = \underline{e}^T\underline{\underline{E}}\underline{\varepsilon}$$

The above differential function becomes

$$\dot{V} = -\underline{r}^T\underline{\underline{R}}\underline{r} - \alpha\underline{\epsilon}^T\underline{\underline{E}}\underline{\epsilon} - \beta\underline{\varepsilon}^T\underline{\underline{E}}\underline{\varepsilon} \le 0 \tag{21}$$

Since $\dot{V} \le 0$ in Eq.(21), the V is either decreasing or constant. Due to the fact that V is non-negative, we conclude that $V \in \mathcal{L}_\infty$; hence $\underline{r} \in \mathcal{L}_\infty$ and $\underline{\tilde{\theta}} \in \mathcal{L}_\infty$. With $\underline{r} \in \mathcal{L}_\infty$, we can conclude from Eq.(7) that $(\dot{\underline{e}} + \alpha\underline{e}) \in \mathcal{L}_\infty$ and $(\underline{\varepsilon} + \alpha\underline{\xi}) \in \mathcal{L}_\infty$, and $\underline{e}(t)$, $\dot{\underline{e}}(t)$, $\underline{\xi}(t)$, $\dot{\underline{\xi}}(t) \in \mathcal{L}_\infty$ based on their definitions. Because of the boundedness of $\underline{x}^d(t)$ and $\dot{\underline{x}}^d(t)$, we conclude that $\underline{x}(t) \in \mathcal{L}_\infty$ and $\dot{\underline{x}}(t) \in \mathcal{L}_\infty$ from Eq.(2). Taking $\underline{\tilde{\theta}} \in \mathcal{L}_\infty$ and $\underline{\theta}$ a constant vector, $\underline{\hat{\theta}} \in \mathcal{L}_\infty$ is obtained. With the previous boundedness statements and the fact that $\ddot{\underline{x}}^d(t)$ is also bounded, the variable $\underline{u} \in \mathcal{L}_\infty$ and $\underline{Y}(\cdot) \in \mathcal{L}_\infty$ can be concluded from their definitions in Eq.(10) and Eq.(1). Hence, the torque input $\underline{\tau}(t) \in \mathcal{L}_\infty$ is also determined from Eq.(11). The preceding information can also be used to get $\ddot{\underline{x}}(t)$, $\dot{\underline{r}}(t) \in \mathcal{L}_\infty$. Now we have explicitly illustrated that all signals in the adaptive synchronization controller and system remain bounded during the closed-loop operation.

From Eq.(21), we show that $\underline{r}(t) \in \mathcal{L}_2$, $\underline{T}^T\underline{\xi}(t) \in \mathcal{L}_2$ and $\underline{\xi}(t) \in \mathcal{L}_2$. Hence, $(\underline{e}+\alpha\dot{\underline{e}})(t) \in \mathcal{L}_2$ and $(\underline{\varepsilon}+\alpha\underline{\xi}) \in \mathcal{L}_2$ can be concluded from Eq.(7). We further conclude that $\dot{\underline{e}}(t) \in \mathcal{L}_2$ since $\underline{T}^T\boldsymbol{\Xi}(t) \in \mathcal{L}_2$ and $(\underline{e} + \alpha\dot{\underline{e}})(t) \in \mathcal{L}_2$. Moreover, from Eq.(5) we have $\underline{e}(t) = \underline{T}^{-1}\underline{\xi}(t)$, hence, $\underline{e}(t) \in \mathcal{L}_2$ because of $\underline{\xi}(t) \in \mathcal{L}_2$. When $\ddot{\underline{x}}(t) \in \mathcal{L}_\infty$ and $\ddot{\underline{x}}^d(t)$ is bounded, $\ddot{\underline{e}}(t) \in \mathcal{L}_\infty$ is concluded. By differentiating Eq.(5) twice and considering $\dot{\underline{e}}(t) \in \mathcal{L}_2$ and $\ddot{\underline{e}}(t) \in \mathcal{L}_\infty$, we conclude that $\dot{\underline{\xi}}(t) \in \mathcal{L}_2$ and $\ddot{\underline{\xi}}(t) \in \mathcal{L}_\infty$. Thus, Barbalat's Lemma [12], [13], [14] can be applied to conclude that

$$\lim_{t\to\infty}\underline{\xi}(t) = 0 \quad \text{and} \quad \lim_{t\to\infty}\dot{\underline{\xi}}(t) = 0$$
$$\lim_{t\to\infty}\underline{e}(t) = 0 \quad \text{and} \quad \lim_{t\to\infty}\dot{\underline{e}}(t) = 0$$

■

A block diagram of the overall control system is shown in Figure 1.

IV. Impact of Position Synchronization

The advantage of the proposed adaptive coupling controller over traditional controllers without synchronization lies in the ability to control coordinates of axes while regulating the relative positions/velocities amongst them. The control strategy addresses not only the convergence of position error $\underline{e}$, but also how these errors converge to zero, represented by the position synchronization error $\underline{\epsilon}$. Although those independent controls without synchronization

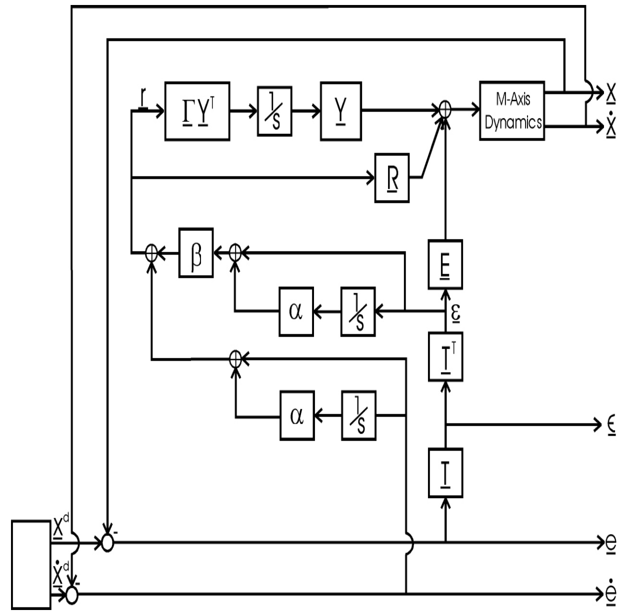

Fig. 1. Overall System Block Diagram

ensure that $\underline{e} \to 0$ and eventually $\underline{\epsilon} \to 0$, they cannot guarantee satisfactory transient performance of synchronization. In other words, the proposed control algorithm explicitly guarantees that position error and position synchronization error converge to zero simultaneously.

Further, as shown in (5), the generic, uniform position synchronization allows for different choices of error definitions, all represented by the selection of the synchronization transformation matrix $\underline{T}$. Different choices of $\underline{T}$ will affect the synchronization performance. This is further indicated by $\dot{V}$ of (21), the derivative of a Lyapunov function along the trajectory.

If we define a *mixed position and integral error* as $\underline{f} = \underline{e} + \alpha \int_0^t \underline{e}$, then the generalized velocity and position error $\underline{r}$ in (7) can be rewritten as

$$\underline{r} = \dot{\underline{e}} + \alpha \underline{e} + \beta(\underline{\varepsilon} + \alpha \underline{\xi}) = \dot{\underline{f}} + \beta \underline{T}^T \underline{T} \underline{f} \tag{22}$$

and the derivative $\dot{V}$ becomes:

$$\begin{aligned} \dot{V} &= -\underline{r}^T \underline{R} \underline{r} - \alpha \underline{\epsilon}^T \underline{E} \underline{\epsilon} - \beta \underline{\varepsilon}^T \underline{E} \underline{\varepsilon} \\ &= -\left(\dot{\underline{f}} + \beta \underline{T}^T \underline{T} \underline{f}\right)^T \underline{R} \left(\dot{\underline{f}} + \beta \underline{T}^T \underline{T} \underline{f}\right)^T \\ &\quad -\alpha \underline{e}^T \underline{T}^T \underline{E} \underline{T} \underline{e} - \beta \underline{T}^T \underline{T} \underline{E} \underline{T}^T \underline{T} \underline{e} \end{aligned}$$

Clearly it indicates that the attraction of $\dot{V}$ depends on the $\underline{T}$. In this paper, we use the following 2-norm formula to evaluate the position tracking and synchronization tracking performance for motion on each axis i:

$$J_{\text{position}} = \|e_i(t)\|_2 \tag{23}$$

$$J_{\text{synchronization}} = \|\epsilon_i(t)\|_2 \tag{24}$$

V. Numerical Example

To demonstrate the generalized position synchronization error and its impact on transient performance, simulations were performed on a four(4)-axis system ([10] with modification) in which motion synchronization is required. The desired tracking of each axis is specified by:

$$x_i^d = 1 - e^{-t} \qquad i = 1, 2, 3, 4 \tag{25}$$

The parameter matrix of the 4-axis dynamic model is given by:

$$\begin{aligned} \underline{H} &= \begin{bmatrix} 10t & & & \\ & 9t & & \\ & & 8t & \\ & & & 7t \end{bmatrix} \\ \underline{C} &= \begin{bmatrix} 10/2 & & & \\ & 9/2 & & \\ & & 8/2 & \\ & & & 7/2 \end{bmatrix} \\ \underline{F} &= \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} \end{aligned} \tag{26}$$

where no external disturbance during the motion is assumed.

The control gains in (11,10,7,14) are chosen to be: $\alpha = 4, \beta = 50$, and

$$\underline{R} = \begin{bmatrix} k_r & & & \\ & k_r & & \\ & & k_r & \\ & & & k_r \end{bmatrix}, \underline{E} = \begin{bmatrix} k_\epsilon & & & \\ & k_\epsilon & & \\ & & k_\epsilon & \\ & & & k_\epsilon \end{bmatrix}, \tag{27}$$

where $k_r = 500$ and $k_\epsilon = 10$.

Three test cases are designed for comparison purpose:

- **Case I**: We assume independent adaptive control without synchronization by choosing $\beta = 0$ and $k_\epsilon = 0$
- **Case II**: We select the position synchronization error as in (3), i.e.,

$$\underline{\epsilon}^a = \underline{T}_a \underline{e}, \underline{T}_a = \begin{bmatrix} 1 & -1 & 0 & 0 \\ 0 & 1 & -1 & 0 \\ 0 & 0 & 1 & -1 \\ -1 & 0 & 0 & 1 \end{bmatrix} \tag{28}$$

- **Case III**: We select a different position synchronization error as in (4), i.e.,

$$\underline{\epsilon}^b = \underline{T}_b \underline{e}, \underline{T}_b = \begin{bmatrix} 2 & -1 & 0 & -1 \\ -1 & 2 & -1 & 0 \\ 0 & -1 & 2 & -1 \\ -1 & 0 & -1 & 2 \end{bmatrix} \tag{29}$$

The results of each case on every axis are shown in Figures 2, 3, 4. The transient position and synchronization performance are evaluated quantitatively using the norm format of the position error and the synchronization error respectively, on each axis. The results are listed in Table I.

TABLE I
Position and Synchronization Performance Evaluation

Error	Case I	Case II	Case III
$\|\|e_1\|\|$	0.1160	0.1027	0.1025
$\|\|e_2\|\|$	0.1069	0.1026	0.1025
$\|\|e_3\|\|$	0.0979	0.1022	0.1023
$\|\|e_4\|\|$	0.0888	0.1021	0.1023
$\|\|\epsilon_1^a\|\|$	9.3603e-003	8.3748e-004	2.5181e-004
$\|\|\epsilon_2^a\|\|$	9.3528e-003	2.4002e-003	1.6654e-003
$\|\|\epsilon_3^a\|\|$	9.3453e-003	9.7379e-004	3.1386e-004
$\|\|\epsilon_4^a\|\|$	2.8043e-002	3.7688e-003	1.9153e-003
$\|\|\epsilon_1^b\|\|$	3.7397e-002	4.5809e-003	2.1036e-003
$\|\|\epsilon_2^b\|\|$	3.7406e-004	1.7983e-003	1.5409e-003
$\|\|\epsilon_3^b\|\|$	3.9633e-004	2.1428e-003	1.6450e-003
$\|\|\epsilon_4^b\|\|$	3.7382e-002	4.5403e-003	2.0577e-003

Several observations are obtained: 1) In all three cases, the position tracking error $\underline{e}$ is asymptotically stable and the tracking performance is relatively consistent; 2) however, the synchronization tracking performance is different when comparing control algorithms with and without cross coupling strategy. Generally speaking, the adaptive control without position synchronization shows poorer synchronization tracking response; 3) further, the synchronization tracking performance is different with different structures of position synchronization error choices. In this numerical example, the position synchronization error $\underline{\epsilon}^b$ in Case III shows better tracking performance than $\underline{\epsilon}^a$ in Case II.

VI. Conclusions

The purpose of multi-axis motion control is to ensure proper synchronization, coordination, or cooperation among multicompoesed systems. It requires that the synchronization stragety and its corresponding control technique will 1) achieve the asymptotic convergence of tracking and synchronization errors simultaneously, 2) under acceptable control efforts, and 3) reach the best transient performance possible, among technically infinite number of choices of synchronization errors. In this paper, we extend our work in adaptive cross-coupling control to include a generic, uniform synchronization error, such that requirement 1) is guaranteed no matter which specific synchronization choice is, and by picking different choices of synchronization, one can evaluate their transient performance, as per requirement 3). Simulation results of the numerical example illustrate the effective impact of synchronization strategies to the transient performance. We haven't solved the optimization problem of synchronization errors, and tradeoff between synchronization choices and control capabilities. These issues are topics under our on-going investigation.

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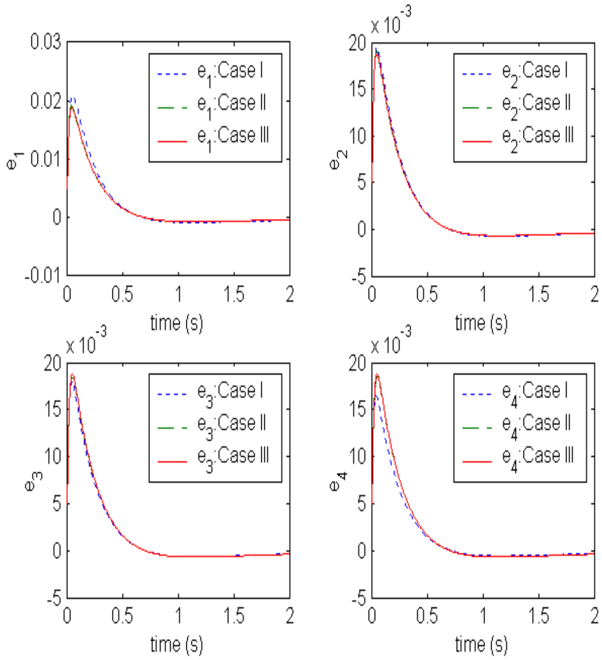

Fig. 2. Position tracking error $\underline{e}$ on each axis of each case

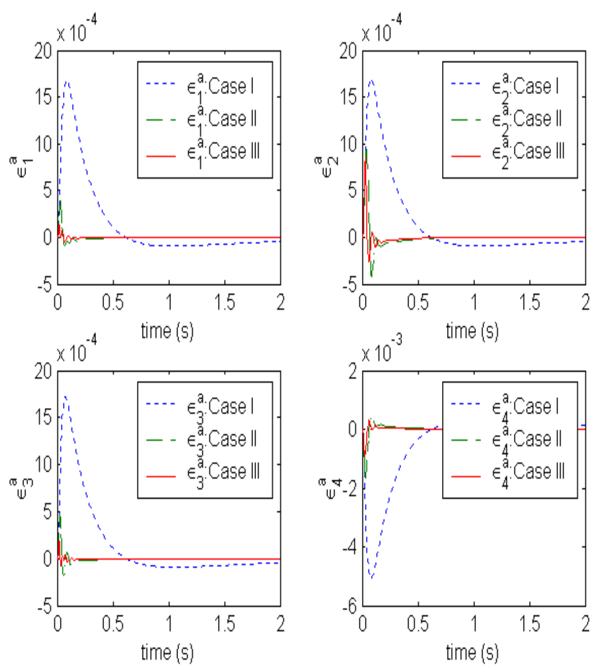

Fig. 3. Position synchronization error $\underline{\epsilon}^a$ on each axis of each case

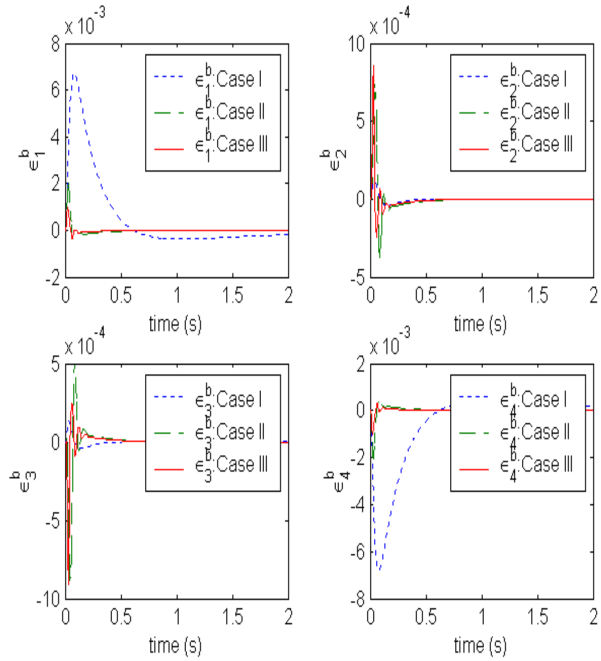

Fig. 4. Position synchronization error $\underline{\epsilon}^b$ on each axis of each case