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Modifications and Design of a Frequency Estimation Algorithm for Disturbance Rejection

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Abstract— **Modifications and design of an algorithm for identifying the frequency of periodic signal or disturbance are studied in this paper. The approach utilizes a feedback control system and can achieve true frequency estimation without error in the steady state. High frequency ripple has been observed in the frequency estimate of the original algorithm. This problem is analyzed on a theoretical basis in the paper. An alternative solution is presented to eliminate this ripple other than the previous adaptive notch filter approach. The design issue of the algorithm is also discussed in this paper, especially when multiple frequency components are present in the input signal. LQR optimal control technique is employed in determining the feedback gains in the estimation system. Simulations are conducted and results are presented.**

I. Introduction

In many engineering applications, it is necessary to identify the frequency of a noisy periodic signal with time-varying characteristics. Knowledge of the magnitudes and phases of the harmonic components are also important in some practical problems. An adaptive frequency identification algorithm based on the *internal model principle(IMP) of control theory* was recently presented by Brown and Zhang[1][2] for periodic noise rejection. The convergence and stability of this algorithm is given in [3]. The algorithm was simple in design, computationally light, and efficient in estimating and tracking the frequency of the periodic signal as well as the magnitude and phase.

The frequency estimated by the algorithm converges to the actual frequency of the periodic signal without error in the steady state. However, ripples may exist in the frequency estimate when the input signal's frequency varies rapidly. High frequency noises, especially harmonics, often exist in nonlinear system when periodic signals are involved. This is due to the system nonlinearity combining with the trigonometric characteristics. For example, ripples occur in the frequency estimates of a power system when the orthogonal filter's gains are amended adaptively by doing feedback [4]. Second harmonics also corrupt the behavior of a traditional phase locked loop (PLL) in communication systems. Low pass filters are required in PLLs to ensure convergence.

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A notch filter approach for ripple cancellation in the algorithm[1][2] was presented in [5]. In this paper, the adaptation of the original algorithm[1][2] is modified to improve the frequency tracking performance. Ripples can be eliminated by adding a new term, which compensates this oscillation with the application of the trigonometric relationship: $\sin^2\theta + \cos^2\theta = 1$. This method can be seen as an alternative to the notch filter approach. This is a progress report and more work on the comparison of these two approaches is expected to be done in near future.

Also, a design method for the internal-model(IM) control based frequency estimation system is presented in this paper. In [5], the design of the feedback loop is conducted via pole placement method. There is still significant room for optimization. Moreover, recent interest has focused on problems where signals contain multiple sinusoidal components such as harmonics. The design of a multi-feedback control system with multiple IM in parallel for cancelling the harmonic components, is difficult using the pole placement method. The strength of the estimation algorithm lies in the employment of feedback control and its parallel structure which allows the rejection of multi-frequency components. Thus, many modern control techniques can be applied to realize a good design of the linear feedback system. Among them, Linear Quadratic Regulator (LQR) for optimal control is found to be a good method to handle this task. In LQR design, the state feedback gains are calculated by minimizing a cost function of the control inputs and the feedback signals.

The outline of this paper is given as follows. In section II, the original algorithm is reformatted and reviewed. The ripple phenomenon of the algorithm is analyzed. A previously proposed solution which is based on a notch filter in frequency estimation loop is also introduced. An alternative solution based on a modification to the adaptation law is presented in section III. A new term that eliminates the oscillations is added to the frequency estimate. Section IV gives a design of the frequency estimation system for signals with harmonics based on LQR optimal control technique. Some simulation results are shown in section V. Finally, section VI concludes this paper.

II. Ripples in the Frequency Estimation Algorithm

The frequency estimation algorithm for disturbance rejection [1] is reviewed first in this section. The reason that the ripples are present in frequency estimates is then identified.

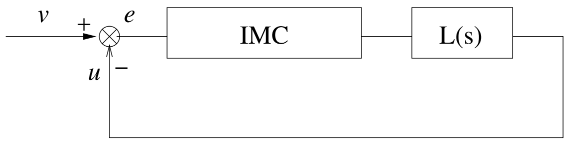

Fig. 1. The block diagram of the feedback disturbance rejection system

A. Review of the Algorithm

According to the fundamental *internal model principle of control theory* [6], a model of the dynamic structure of the disturbance should be included in the feedback loop for perfect disturbance rejection. The model for any periodic signal can be expressed as $\sum_{i=1}^{n} \frac{K_{1,i}s+K_{2,i}}{s^2+\omega_i^2}$. In a simple case where n=1, it can be written in state space form as:

$$\begin{aligned} \begin{bmatrix} \dot{x_1} \\ \dot{x_2} \end{bmatrix} &= \begin{bmatrix} 0 & \omega \\ -\omega & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} 0 \\ 1 \end{bmatrix} e \\ y &= \begin{bmatrix} K_1 & K_2 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} \end{aligned} \quad (1)$$

Note that the states $[x_1 \quad x_2]^T$ here denotes $[\frac{\omega x_1'}{K_f} \quad \frac{x_2'}{K_f}]^T$ where $[x_1' \quad x_2']^T$ is the state originally proposed in [2][3] with K_f the gain of the IM. This manipulation of states provides a simpler expression of our adaptation law and results in a normal form for the state space realization. Normal forms for time varying realizations of systems are shown to have better numerical properties [7].

A block diagram of this control system is shown in Fig 1 where $L(s)$ is a design function and IMC is the internal model controller in (1). With $L(s)$ chosen such that the closed loop system is stable, we let the periodic disturbance be $v = a\sin(\omega_c t + \phi)$, where ω_c is the frequency of this signal and ϕ the phase. In the steady state we have:

$$e(t) = A_e \sin(\omega_c t + \varphi) \quad (2)$$

$$x_1(t) = A\omega \sin(\omega_c t + \varphi - \theta) \quad (3)$$

$$x_2(t) = A\omega_c \cos(\omega_c t + \varphi - \theta) \quad (4)$$

where $A = \frac{A_e}{\omega^2-\omega_c^2}$,$\theta = \tan^{-1}(\frac{K_1\omega}{K_2\omega_c})$ and φ denotes the phase of e.
A phase diagram of $\frac{x_1}{\omega}$, $j\frac{x_2}{\omega_c}$ would show

$$\begin{aligned} \left|\frac{x_1(t)}{\omega} + j\frac{x_2(t)}{\omega_c}\right| &= \sqrt{\frac{x_1^2(t)}{\omega^2} + \frac{x_2^2(t)}{\omega_c^2}} \\ &= |A| \end{aligned} \quad (5)$$

$$\begin{aligned} \theta &= \angle\left(\frac{x_1(t)}{\omega} + j\frac{x_2(t)}{\omega_c}\right) \\ &= \omega_c t + \varphi \end{aligned} \quad (6)$$

where $\angle(\bullet)$ is defined to map to the set of real numbers rather than $[0, 2\pi)$. Then the frequency ω_c of the sinusoidal disturbance v can be identified by differentiating (6):

$$\frac{d\theta}{dt} = \omega_c \quad (7)$$

The derivative operation in (7) can be avoided by using the following method. From (6), (7) and $\frac{d}{dt}(\tan^{-1}(u)) = \frac{1}{1+u^2}\frac{du}{dt}$, we get

$$\begin{aligned} \omega_c &= \frac{d\theta}{dt} \\ &= \frac{d}{dt}\left(\tan^{-1}\frac{\omega_c x_1}{\omega x_2}\right) \\ &= \frac{1}{1+(\frac{\omega_c x_1}{\omega x_2})^2} \frac{\omega_c \dot{x_1} x_2 - \omega_c x_1 \dot{x_2}}{\omega x_2^2} \\ &= \frac{\omega\omega_c(\dot{x_1}x_2 - x_1\dot{x_2})}{(\omega_c x_1)^2 + (\omega x_2)^2} \end{aligned} \quad (8)$$

From (1) we have

$$\begin{aligned} \dot{x_1} &= \omega x_2 \\ \dot{x_2} &= -\omega x_1 + e \end{aligned}$$

Thus, equation (8) becomes

$$\omega_c = \frac{\omega\omega_c(\omega x_2^2 + \omega x_1^2 - e x_1)}{(\omega_c x_1)^2 + (\omega x_2)^2} \quad (9)$$

The certain equivalence principle is applied here by using ω instead of ω_c in the right hand side of (9). Thus, we have

$$\begin{aligned} \hat{\omega}_c &= \frac{\omega^2(\omega x_2^2 + \omega x_1^2 - e x_1)}{(\omega x_1)^2 + (\omega x_2)^2} \\ &= \omega - \frac{e x_1}{x_1^2 + x_2^2} \end{aligned} \quad (10)$$

Equation (10) is used to estimate the frequency ω_c of a sinusoidal disturbance.

The error ε between $\hat{\omega}_c$ and ω can be expressed as follows:

$$\begin{aligned} \varepsilon &= \hat{\omega}_c - \omega \\ &= -\frac{e x_1}{x_1^2 + x_2^2} \end{aligned} \quad (11)$$

An integral controller is used [1][2]to eliminate this error in the steady state:

$$\begin{aligned} \frac{d\omega}{dt} &= K_e \varepsilon \\ &= -K_e \frac{e x_1}{x_1^2 + x_2^2} \end{aligned} \quad (12)$$

where K_e is a constant gain.
From (12) we have

$$\begin{aligned} \omega &= \omega_0 + K_e \int_{t_0}^{t} \varepsilon d\tau \\ &= \omega_0 - K_e \int_{t_0}^{t} \frac{e x_1}{x_1^2 + x_2^2} d\tau \end{aligned} \quad (13)$$

where ω_0 is the initial value of ω of the internal model in the control system.

The formula (13) is the adaptation law for the disturbance cancellation system and is also the frequency identification mechanism. The convergence of this algorithm and stability of the feedback disturbance cancellation system above have been verified by singular perturbation theory and averaging theory in [3].

B. Reason for the Ripples in Frequency Updates

When tracking signals with rapidly varying frequencies, the original algorithm results in a ripple in the frequency estimates. This can be understood by examining the steady state behavior of the system. Ideally, the phase plot of $\left(\frac{x_1(t)}{\omega}, \frac{x_2(t)}{\omega_c}\right)$ should form a circle when $\omega = \omega_c$. If $\omega \neq \omega_c$, in the steady state, the phase plot will from an ellipse with major & minor axis of ω & ω_c and the derivative of the angle of the phase plot will no longer be a constant ω_c. The deviation from ideal can be expressed by substituting (2)(3)(4) to (11) as:

$$\varepsilon = -\omega \frac{(\omega^2 - \omega_c^2)\sin^2(\omega_c t + \varphi)}{\omega^2 \sin^2(\omega_c t + \varphi) + \omega_c^2 \cos^2(\omega_c t + \varphi)} \tag{14}$$

When ω is close to ω_c, the denominator of (14) can be approximated as follows

$$\omega^2 \sin^2(\omega_c t + \varphi) + \omega_c^2 \cos^2(\omega_c kT + \varphi) \approx \omega\omega_c \tag{15}$$

So (14) becomes

$$\begin{aligned} \varepsilon &\approx -\frac{\omega(\omega^2 - \omega_c^2)}{\omega\omega_c} \sin^2(\omega_c t + \varphi) \\ &= -\frac{\omega(\omega - \omega_c)(\omega + \omega_c)}{\omega\omega_c} \sin^2(\omega_c t + \varphi) \\ &\approx -2(\omega - \omega_c)\sin^2(\omega_c t + \varphi) \\ &\approx (\omega_c - \omega)(1 - \cos(2\omega_c t + 2\varphi)) \end{aligned} \tag{16}$$

The term $\cos(2\omega_c kT + 2\varphi)$ gives a multiplicative deviation from ideal. This explains the ripple or periodic noise in our frequency estimate. The frequency of this periodic noise is two times the fundamental of the input periodic signal. Thus, this ripple arises because of the application of certain equivalence principle to $tan^{-1}(\frac{\omega_c x_1}{\omega x_2})$ in (9).

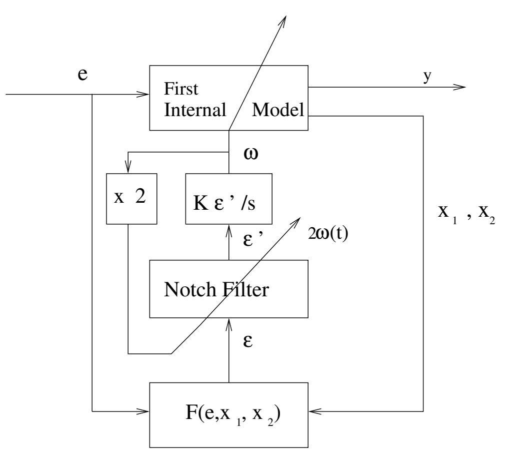

Fig. 2. The block diagram of the modified adaptation loop with a notch filter

In [5], it was suggested that a second order adaptive IIR notch filter $\frac{s^2+4\omega^2}{s^2+rs+4\omega^2}$ could be added in series with the integrator $\frac{K_e}{s}$ in (13) to minimize the ripple. ω is the estimated frequency of the periodic disturbance, and r the bandwidth. Let us call the filtered error signal ε' to distinguish it from the original ε. Figure 2 shows a block diagram of the improved adaptation loop. The sinusoidal term of $2\omega_c$ in (16) can be completely eliminated if the estimated frequency ω is close enough to the true frequency ω_c. Thus, the accuracy and the consistency of the frequency estimation for rapidly varying signals can be greatly improved.

III. Numerator Compensation Approach to the Ripple Problem

The $\sin^2(\omega_c t + \varphi)$ term in (16) can be expressed by $\frac{x_1^2}{x_1^2+x_2^2}$ in the steady state. Thus, this sinusoidal term can be removed by dividing (11) by $\frac{x_1^2}{x_1^2+x_2^2}$. From (3)(4), in the steady state, equation (11) becomes:

$$\varepsilon = -\frac{e}{2x_1} \tag{17}$$

Now, e and x_1 become zero periodically during the estimating process, ideally at the same time. But, dividing a small number by an approximately equal value results in extreme amplification of the noise of the system. Thus, equation (17) is not feasible for practical implementation.

Rather than eliminating the $\sin^2(\omega_c t + \varphi)$ term by dividing in (17) we can compensate it by applying the trigonometric formula, $\sin^2\theta + \cos^2\theta = 1$. Therefore, the goal is to find a mapping of the states of IM and feedback error to the term $(\omega_c - \omega)\cdot\cos^2(\omega_c t + \varphi)$. A new term $e_q x_2$, where $e_q = A_e\cos(\omega_c t + \varphi)$ is constructed to represent the quadrature of the feedback error e, is added to the original numerator to achieve this goal. The compensated frequency error ε is now:

$$\varepsilon = -\frac{1}{2}\frac{ex_1 + e_q x_2}{x_1^2 + x_2^2} \tag{18}$$

From (2)(3)(4) and the definition of e_q, (18) now becomes:

$$\begin{aligned} \varepsilon &= \frac{1}{2}\frac{\omega(\omega^2 - \omega_c^2)\sin^2(\omega_c t + \varphi) + \omega_c(\omega^2 - \omega_c^2)\cos^2(\omega_c t + \varphi)}{\omega^2\sin^2(\omega_c t + \varphi) + \omega_c^2\cos^2(\omega_c t + \varphi)} \\ &\approx -(\omega - \omega_c)(\sin^2(\omega_c t + \varphi) + \cos^2(\omega_c t + \varphi)) \\ &\approx -(\omega - \omega_c) \end{aligned}$$

The adaptation will converge to the right frequency without ripples only if the variable e_q is estimated accurately. The problem now becomes a quadrature estimation problem. A simple method would be to set $e_q = -\omega\int edt$. This did not perform well in simulations. Instead, another internal model in a feedback system can be used to estimate the quadrature of e. The structure of this observer is shown in Fig. 3. Where IMC is the same sinusoid internal model controller as in (1), $C(s)$ is a stabilizing transfer function, ω the frequency estimated. In this feedback system, the signal e from Fig. 1 is taken as input. In the steady state, one state of the IMC, x_2^q, cancels the input signal e. In other words, x_2^q is of the same magnitude and in phase of signal e. The other state x_1^q is the quadrature of x_2^q. Therefore, x_1^q can be seen approximately as the quadrature of e. By using

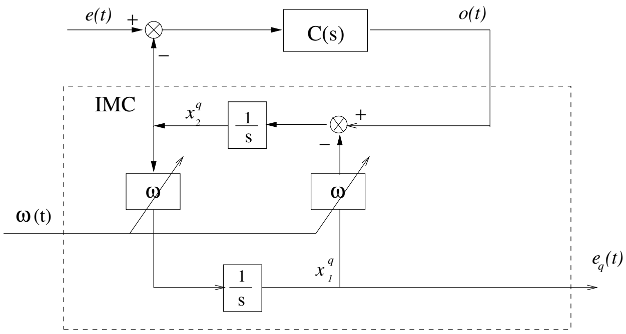

Fig. 3. The block diagram of the quadrature estimation system for numerator compensation

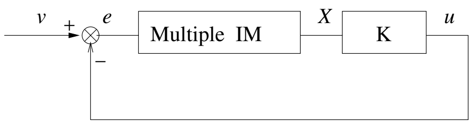

Fig. 4. Frequency estimation system with multiple IMs

the method shown above, the ripple effect in the frequency estimates can be avoided.

IV. LQR Design for the Frequency Estimation System

The design of the frequency estimation system can be easily formatted as a state-feedback design problem. LQR [8] optimal control approach is found to be a good method for designing a frequency estimation system proposed in [1][2] with multiple IMs for harmonic cancellation. Figure 4 shows a block diagram of this frequency estimation system where IM is of form given in (1). Output of the multiple IMs is the full state vector X. $L(s)$ in Fig. 1 can be set equal to a vector K. v is the input periodic signal, e the input signal of IM. The stability and convergence of the algorithm with multiple IMs has been studied in [1]. The state-feedback gain vector K can be determined by using optimal LQR technique for optimal control.

Considering the model $\dot{x} = Ax + Bu$ in Fig. 4 for multiple IMs in parallel, it can be rewritten as:

$$\begin{bmatrix} \dot{x_1} \\ \dot{x_2} \\ . \\ . \\ . \\ . \\ \dot{x_{2N}} \end{bmatrix} = \begin{bmatrix} A_1 & & & \\ & . & & \\ & & . & \\ & & & . \\ & & & & A_N \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ . \\ . \\ . \\ . \\ x_{2N} \end{bmatrix} + \begin{bmatrix} 0 \\ 1 \\ . \\ . \\ . \\ 0 \\ 1 \end{bmatrix} e$$

where $A_i = \begin{bmatrix} 0 & \omega_i \\ -\omega_i & 0 \end{bmatrix}$, N denotes the number of IM in the system, $\omega_i = i \times \omega$ the frequency of ith harmonics.

The general form of the LQR problem is to find the control $u = Kx$, which minimizes a cost function [8]:

$$\jmath = \int_0^\infty [x^T Q x + e^T R e] dt \qquad (19)$$

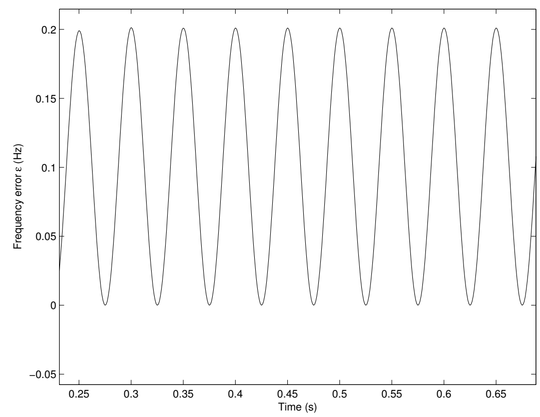

Fig. 5. Estimated frequency difference

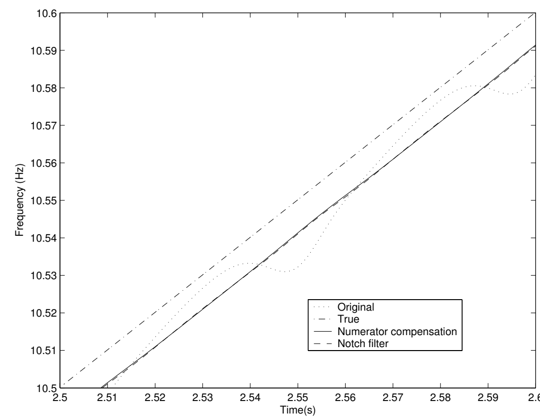

Fig. 6. Detailed frequency estimates around 2.5 seconds

Where x, e are the state and input vectors of the linear system and Q, R are weighting matrices of the designer's choice.

Because of the LTI assumption LQR optimal control is only suitable for the case when the frequency range of the input signal is constrained in a certain range known *a priori*. A design example is given in the next section.

V. Matlab Simulations

A. Simulations on Ripple Rejection

The continuous time frequency estimation system models with original algorithm and with modifications have been constructed in MATLAB/SIMULINK. The parameter settings for the systems are $L(s)$=1000, K_f=1, r=100, K_e=-200 and IMC=$\frac{s+236.05}{s^2+\omega^2}$.

A sinusoid $v(t) = \sin(2\pi 10 t)$ is first fed to the frequency estimation systems as the periodic disturbance. For demonstration purpose, the frequency of the IM is fixed at 9.9 Hz (or $K_e = 0$). The frequency difference ε estimated by (11) is shown in Fig. 5. It is seen that the estimated frequency difference oscillates around 0.1 Hz at a frequency of 20 Hz, which is consistent with our analysis in II-B.

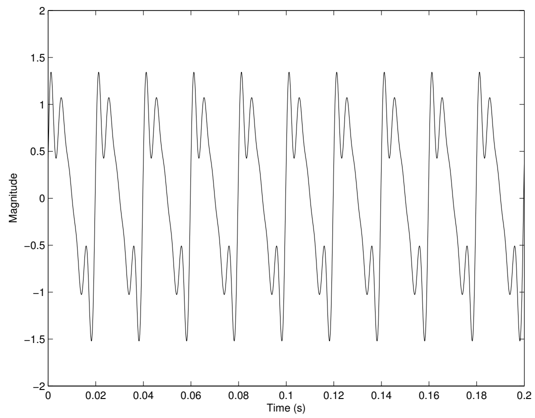

Fig. 7. Input signal:$x(t) = \sin(2\pi 50t) + 0.2\sin(2\pi 100t + 0.2) + 0.3\sin(2\pi 150t + 0.3) + 0.4\sin(2\pi 200t + 0.4) + 0.2\sin(2\pi 250t + 0.5)$

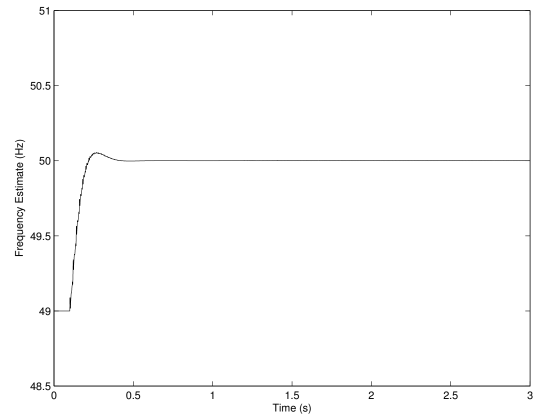

Fig. 8. Frequency estimate of a signal with high order harmonics

Another periodic signal is constructed in MATLAB as the disturbance that needs to identified. The frequency of this signal follows a ramp function as follows:

$$f(t) = \begin{cases} 10 & : & 0 < t < 2s \\ 10 + (t-2) & : & 2 \leq t < 3s \\ 11 & : & t \geq 3s \end{cases} \tag{20}$$

Frequency estimation results around 2.5 seconds with and without notch filter in estimation loop is shown in Fig. 6. It is shown that the ripple is eliminated from the frequency estimates by introducing a notch filter to estimation loop.

The modified algorithm with numerator compensation is then applied to this signal. The parameters for the quadrature estimation system are set as $C(s)$=1,K_2=50 and K_1=0. The frequency estimation results around 2.5 seconds is also shown in Fig. 6. Similar to the results obtained by the notch filer approach, the ripple effect in the frequency estimates is seen to be avoided to a great extent by utilizing the numerator compensation.

B. Simulations on LQR System Design

Next, a signal having up to fifth harmonic components is constructed. The definition of this signal is shown below:

$$\begin{aligned} x(t) &= \sin(2\pi 50t) + 0.2\sin(2\pi 100t + 0.2) \\ &\quad +0.3\sin(2\pi 150t + 0.3) + 0.4\sin(2\pi 200t + 0.4) \\ &\quad +0.2\sin(2\pi 250t + 0.5) \end{aligned} \tag{21}$$

The original signal profile is shown in Fig. 7. The frequency estimation system with 5 IMs is designed by using LQR optimal control technique introduced in IV with the LQR parameters selected as Q=I_{10}, R=0.0005 and the adaptation gain K_e=-50. The state-feedback gain vector K is calculated as [-3.9635 63.1212 1.6945 63.2229 5.6584 62.9919 10.0416 62.4433 17.1872 60.8654]. The pole placement approach would require a specification of 10 closed-loop poles and controller parameters selected for an appropriate guess at ω. Design would then need to be tested for sensitivity to variation in ω.

The initial guess of the frequency is set equal to 49 Hz and the adaptive algorithm is turned on at 0.1 seconds. Frequency estimation results are shown in Fig. 8. It is shown that the estimated frequency converge to the true value, 50 Hz, in about 0.2 seconds. In the steady state the frequency estimate is perfect or free of noise. We can see that the frequency estimates provided by LQR design are fast and accurate.

VI. CONCLUSIONS

A previously developed adaptive algorithm for identifying the frequency of periodic signal or disturbance is reformatted and studied in this paper. It is based on the feedback control system and can converge to the true frequency without error in the steady state. High frequency oscillation/ripple has been observed in the frequency estimates of the algorithm. This oscillation results in a longer convergence and inaccurate estimation. This phenomenon is analyzed on a theoretical basis in the paper. The frequency of the ripple is identified as twice of the fundamental of the periodic disturbance. An alternative solution is presented to eliminate this ripple in the original algorithm other than the previous notch filter approach. Moreover, the design issue of the algorithm is also discussed in the paper, especially when multiple frequency components are present in the input signal. LQR optimal control technique is employed in determining the multiple feedback gains in the frequency estimation system. Simulations results are presented and confirm the theoretical analysis.

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