2005 American Control Conference
June 8-10, 2005. Portland, OR, USA

Robust $\mathcal{H}_\infty$ Load-Frequency Control for Interconnected Power Systems with $\mathcal{D}$-Stability Constraints via LMI Approach

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***Abstract*— The aim of this paper is to investigate the problem of robust $\mathcal{H}_\infty$ load-frequency control for interconnected power systems with circular pole constraints. The problem we address is to design both state feedback and output feedback controllers such that, for all admissible parameter uncertainties, the closed-loop system satisfies not only the prespecified $\mathcal{H}_\infty$ norm constraint on the transfer function from the disturbance input to the system output, but also the prespecified circular pole constraint or $\mathcal{D}$-stability constraint on the closed-loop system matrix. The necessary and sufficient conditions for the existence of desired controllers are derived in terms of a linear maitrix inequality (LMI). The performance of the proposed robust controller is not only validated through simulation with the two-area power systems model, but also compared with the recent works of Wang *et al.* [5], Ray *et al.* [13], and the traditional LQR control method. The simulation results show that the system reponses with the proposed state feedback controller are the best transient reponses, compared with the three methods mentioned. Besides, it is also shown that the transient responses of the proposed output feedback controller with circular pole constraints is improved.**

I. INTRODUCTION

Load-Frequency Control (LFC) is one of the important issues in electric power system design and operation, because the loading in a power system inevitably changes. The change of load has resulted in the frequency change in power sytems. In addition, it is well-known that the operation objectives of the LFC are to maintain reasonably uniform frequency for dividing the load between generators and controlling the tie-line interchange schedules. As a result, it is necessary to design load frequency controller to provide adequate and reliable electric power. For LFC problem, there has been continuing interest in designing a suitable linear state feedback controller, e.g., an optimal control theory in [1] proposed an optimal control theory to get better performance since the early 1970s. These controller design methods considered are based on the centralized design with only nominal plant parameter and thus it is unable to apply directly to the uncertain interconnected power systems. Therefore, an important consideration in the LFC controller design of power system should inevitably include the class of parameter perturbation to obtain the exact models. Moreover, most physical plants to be controlled are often modelled approximately from linearization method around any operation points and may be subjected to plant parameter variation. In fact, it is irrefutable that LFC design, based on the nominal system parameter, is unable to guarantee both the stability and the desired performance of the power system with parameter uncertainties. Therefore, until now, many researchers have investigated and proposed various approaches to robust load frequency control problem such as variable structure control [2], adaptive control [3], robust control [4]-[6], and so on.

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In power system control, it is recently desirable to design controller achieving robust stability due to parametric uncertainties, especially in the study of robust $\mathcal{H}_\infty$ control problem whose the objective is to design controllers such that the closed-loop system is stable and the $\mathcal{H}_\infty$ norm of a specified closed-loop transfer function is minimized. Although the $\mathcal{H}_\infty$ control problem can be regarded as robustness against exogenous signal uncertainty, in the case when parameter uncertainty appears in the plant modelling, robust behavior on $\mathcal{H}_\infty$ performance as well as stability cannot be guaranteed by standard $\mathcal{H}_\infty$ control. This lead to the study of robust $\mathcal{H}_\infty$ control problem. However, both the standard $\mathcal{H}_\infty$ control and robust $\mathcal{H}_\infty$ control are little concerned with the transient behavior of the closed-loop system such as [7]-[9]. In order to improve the transient behavior, it is well-known that the pole location is directly related to the dynamical characteristics of linear system such as damping ratios, natural and damped natural frequencies. Therefore, it is also desired to contruct control systems to achieve better transient performance as well as robust stability simultaneously. A more practical way is to place the closed-loop poles in a suitable region of the complex plane, especially in circular region. For placing the closed-loop poles in a suitable disk, we are able to guarantee an upper bound on the damping ratio, the natural frequency, and the damped natural frequency. Hence, it may be concluded that the closed-loop poles in a specified region guarantees both stabiltiy (all closed-loop poles forced in the circular region of Left Half Plane (LHP)) and the transient performance such as settling time, maximum overshoot, and rise time. For the closed-loop pole placement in a specified region, there has been continuing interest in designing controller in both nominal and uncertain systems. Many researchers have investigated in this problem such as [10] and [11] for systems without uncertainties and [12] for systems with uncertainties.

However, there have recently been no papers that combine robust $\mathcal{H}_\infty$ load-frequency control for interconnected power systems with circular pole constraints not only to assure in robust stability, but also to improve better transient performance. Moreover, we extend the results of the study in robust $\mathcal{H}_\infty$ control with circular pole constraints using ARE approach from the state feedback control, [8] and [9], to the dynamic output feedback control via LMI approach.

This paper is organized as follows. In section 2, a statement of problem is provided. In Section 3, the necessary and sufficient conditions conditions for the existence of robust $\mathcal{H}_\infty$ controller design with circular pole constraints are derived in terms of a linear matrix inequality (LMI) in order to contruct a desired controller. In Section 4, an illustrative example of a two-area interconnected power system is considered to the feasibility of this proposed method and also compared with the recent works of Wang *et al.* [5], Ray *et al.* [13], and the traditional LQR control method for state feedback case. Also, it is shown that the transient reponses of the proposed output controller is improved. Finally, we conclude in Section 5.

Throughout this paper, $\Re^n$ and $\Re^{n\times m}$, respectively, denote the set of n-dimensional real vectors and the set of $n\times m$-dimensional real matrices. M' is the transpose of matrix M and the notion $X > Y$ where X and Y are symmetric matrices, means that $X-Y$ is positive definite. I_n stands for the $n\times n$ identity matrix.

II. PROBLEM STATEMENT

We are interested in an uncertain system which composes N-interconnected subsystems. In addition, each uncertain subsystem is described by the following equations (for $i = 1, 2, \cdots, N$).

$$\begin{aligned}
\dot{x}_i(t) &= (A_{ii} + \Delta A_{ii}(t))x_i(t) + \sum_{j=1, j\neq i}^{N} (A_{ij} + \Delta A_{ij}(t)) \\
&\quad \cdot x_j(t) + \sum_{j=1}^{N} (B_{ij} + \Delta B_{ij}(t))u_j(t) \\
&\quad + \sum_{j=1}^{N} (F_{ij} + \Delta F_{ij}(t))w_j(t) \qquad (1) \\
y_i(t) &= C_{ii}x_i(t) + \sum_{j=1, j\neq i}^{N} C_{ij}x_j(t) \qquad (2)
\end{aligned}$$

where $x_i(t) \in \Re^{n_i \times 1}$ is the state-vector of the ith subsystem, $u_i(t) \in \Re^{m_i \times 1}$ is the control-vector of the ith subsystem, $y_i(t) \in \Re^{p_i \times 1}$ is the output-vector, and $w_i(t) \in \Re^{m_i \times 1}$ is the disturbance vector of the ith subsystem. The matrices $A_{ii}, A_{ij}, B_{ii}, B_{ij}, F_{ii}, F_{ij}, C_{ii}$, and C_{ij} are of appropriate dimensions that each subsystem is assumed to be completely controllable and observable.

From (1) and (2), we can rewrite in the compact forms as:

$$\begin{aligned}
\dot{x}(t) &= (A + \Delta A)x(t) + (B_u + \Delta B_u)u(t) \\
&\quad + (F_w + \Delta F_w)w(t) \\
&= \tilde{A}x(t) + \tilde{B}_u u(t) + \tilde{F}_w w(t) \qquad (3) \\
y(t) &= Cx(t) \qquad (4)
\end{aligned}$$

where the system matrices $\tilde{A}$, $\tilde{B}_u$, $\tilde{F}_w$ and C have their proper dimensions. The matrices ΔA, ΔB_u, and ΔF_w represent the parametric perturbation in the system state matrix, control input matrix, and disturbance input matrix, respectively, which are assumed to be the following norm-bounded uncertainty form:

$$\begin{pmatrix} \Delta A & \Delta B_u & \Delta F_w \end{pmatrix} = H_1 \Delta(t) \begin{pmatrix} E & E_u & E_w \end{pmatrix} \qquad (5)$$

where $\Delta \in \Re^{k \times l}$ is an uncertain matrix bound by

$$\Delta'(t)\Delta(t) \leq I \qquad (6)$$

and H_1, E, E_u, and E_w are the constant matrices of appropriate dimensions which specify how the elements of the nominal matrices A, B_w and F_w are affected by the Δ. Also, ΔA ΔB_u, and ΔF_w are said to be admissible, if both (5) and (6) hold.

The problem of interest can be formulated in determining a linear feedback control $u(t) = Kx(t)$(or $u(t) = Ky(t)$ for output feedback case) such that the following performance criteria are simultaneously achieved.

1) All closed-loop poles are assigned in a stable disk region $\mathcal{D}(\alpha, r)$ in the complex plane with the center at $-\alpha + j0$ $(\alpha > 0)$ and the radius r $(r < \alpha)$.
2) The $\mathcal{H}_\infty$ norm of the disturbance transfer function $H(s)$ from $w(t)$ to $y(t)$ meets the constraint $\|H(s)\|_\infty \leq \gamma$ where γ is a given constant and $H(s) = C[sI - (\tilde{A} + \tilde{B}_u K)]^{-1}\tilde{F}_w$ for state feedback case or $H(s) = C[sI - (\tilde{A} + \tilde{B}_u KC)]^{-1}\tilde{F}_w$ for output feedback case.

In the next section, we will provide a procedure to develop a linear robust feedback controller which not only generates an actuating signal to control the regulated output of an interconnected power system in the presence of parameter variations, but also satisfies in the requirements 1) and 2).

III. MAIN RESULTS

In this section, we first provide some important lemmas which will be useful in the derivation of our main results.

Lemma 1 (Garcia and Bernussou [12]): Let $A \in \Re^{n \times n}$ be a given matrix. Then all the poles of the closed-loop system are located with a given circular region $\mathcal{D}(\alpha, r)$, i.e., $\lambda(A) \subset \mathcal{D}(\alpha, r)$, if and only if there exists $P > 0$ such that

$$A_r' Q A_r - Q < 0,$$

where $A_r = (A + \alpha I)/r$.

Lemma 2: Given a constant $\gamma > 0$ and a disk $\mathcal{D}(\alpha, r)$. Then both requirements (1) and (2) are satisfied if the following matrix inequality has a positive matrix $Q > 0$

$$\begin{pmatrix} \mathcal{Q} & A_\alpha' \\ A_\alpha & -Q^{-1} \end{pmatrix} < 0 \qquad (7)$$

where $\mathcal{Q} = -r^2 Q + \alpha(C'C + \gamma^{-2} Q F_w F_w' Q)$ and $A_\alpha = A + \alpha I$. In addition, from a Schur complement, (7) can be rewritten as:

$$\begin{pmatrix} -r^2 Q & A_\alpha' & \sqrt{\alpha}C' & \sqrt{\alpha}QF_w \\ A_\alpha & -Q^{-1} & 0 & 0 \\ \sqrt{\alpha}C & 0 & -I & 0 \\ \sqrt{\alpha}F_w'Q & 0 & 0 & -\gamma^2 I \end{pmatrix} < 0 \qquad (8)$$

Proof: By using a Schur complement, we can get the quadratic matrix inequality from (7) as follows.

$$\begin{aligned}
A'QA + (\alpha^2 - r^2)Q + \alpha(A'Q + QA \\
+ \gamma^{-2}QF_wF_w'Q + C'C) < 0 \qquad (9)
\end{aligned}$$

or

$$A_\alpha' Q A_\alpha - r^2 Q + \alpha(\gamma^{-2} Q F_w F_w' Q + C'C) < 0 \qquad (10)$$

It is easy to show that the circular pole requirement (1) will be met by using Lemma 1 as follows:

$$Q - A_r' Q A_r > \frac{\alpha}{r^2}(\gamma^{-2} Q F_w F_w' Q + C'C) > 0 \qquad (11)$$

where F_w is of full row rank. Next, we can rearrange (9) as follows:

$$A'Q + QA + \gamma^{-2} Q F_w F_w' Q + C'C + \Sigma < 0 \qquad (12)$$

where $\Sigma = \alpha^{-1}[A'QA + (\alpha^2 - r^2)Q] > 0$. To show that the requirement (2) is also met, this proof of $\|H(s)\|_\infty \leq \gamma$ is completely similar to that of Theorem 1 of Wang [8]. ∎

Lemma 3 (Xie [15]): Given matrices G, H and E of appropriate dimensions and $G = G'$, then

$$G + H\Delta E + E'\Delta' H' < 0$$

holds for any admissible uncertain matrix Δ satisfying $\Delta'\Delta \leq I$, if and only if there exists a scalar $\epsilon > 0$ such that

$$G + \epsilon H H' + \epsilon^{-1} E' E < 0$$

Definition 1: The unforced uncertain system in (3) (setting $u(t) = 0$) is satisfied with the above performance criteria, if there exists $Q > 0$ such that

$$\begin{pmatrix} \tilde{\mathcal{Q}} & \tilde{A}_\alpha' \\ \tilde{A}_\alpha & -Q^{-1} \end{pmatrix} < 0 \qquad (13)$$

where $\tilde{\mathcal{Q}} = -r^2 Q + \alpha(C'C + \gamma^{-2} Q \tilde{F}_w \tilde{F}_w' Q)$.
If these uncertainties can be represented in (5) according to Definition 1, a necessary and sufficient LMI condition is stated as follows:

Theorem 1: As for the unforced uncertain linear system, the desired circular pole region $\mathcal{D}(\alpha, r)$ and the $\mathcal{H}_\infty$ norm bound constraint $\gamma > 0$ on attenuation of disturbance be given. The system (3) is satisfied with requirement 1) and 2) if and only if there exists a scalar $\epsilon > 0$ and a symmetrical positive definite matrix $P \in \Re^{n\times n}$ such that

$$\begin{pmatrix} -r^2P & PA'_\alpha & \sqrt{\alpha}PC' & \sqrt{\alpha}F_w & 0 & PE' & \epsilon H_1 & 0 \\ A_\alpha P & -P & 0 & 0 & 0 & 0 & 0 & \epsilon H_1 \\ \sqrt{\alpha}CP & 0 & -I & 0 & 0 & 0 & 0 & 0 \\ \sqrt{\alpha}F'_w & 0 & 0 & -\gamma^2 I & \sqrt{\alpha}E'_w & 0 & 0 & 0 \\ 0 & 0 & 0 & \sqrt{\alpha}E_w & -\epsilon I & 0 & 0 & 0 \\ EP & 0 & 0 & 0 & 0 & -\epsilon I & 0 & 0 \\ \epsilon H'_1 & 0 & 0 & 0 & 0 & 0 & -\epsilon I & 0 \\ 0 & \epsilon H'_1 & 0 & 0 & 0 & 0 & 0 & -\epsilon I \end{pmatrix} < 0 \quad (14)$$

Proof: We begin with pre- and post-multiplying (13) by the matrix $\mathcal{U}$ = diag$\{P, I\}$ to yield:

$$\begin{pmatrix} \tilde{\mathcal{P}} & P\tilde{A}'_\alpha \\ \tilde{A}_\alpha P & -P \end{pmatrix} = \mathcal{U}\begin{pmatrix} \tilde{\mathcal{Q}} & \tilde{A}'_\alpha \\ \tilde{A}_\alpha & -Q^{-1} \end{pmatrix}\mathcal{U} < 0 \qquad (15)$$

where $\tilde{\mathcal{P}} = -r^2P + \alpha(PC'CP + \gamma^{-2}\tilde{F}_w\tilde{F}'_w)$ and $P = Q^{-1}$.

From Definition 1, we need to show that (14) is equivalent to (15). Based on (6), (15) can be rewritten as:

$$\begin{pmatrix} -r^2P & PA'_\alpha & \sqrt{\alpha}PC' & \sqrt{\alpha}F_w \\ A_\alpha P & -P & 0 & 0 \\ \sqrt{\alpha}CP & 0 & -I & 0 \\ \sqrt{\alpha}F'_w & 0 & 0 & -\gamma^2 I \end{pmatrix} + \mathcal{H}\tilde{\Delta}\mathcal{E} + (\mathcal{H}\tilde{\Delta}\mathcal{E})' < 0. \quad (16)$$

where

$$\mathcal{H} = \begin{pmatrix} H_1 & 0 \\ 0 & H_1 \\ 0 & 0 \\ 0 & 0 \end{pmatrix}, \tilde{\Delta} = \begin{pmatrix} \Delta & 0 \\ 0 & \Delta \end{pmatrix}$$

$$\mathcal{E} = \begin{pmatrix} 0 & 0 & 0 & \sqrt{\alpha}E_w \\ EP & 0 & 0 & 0 \end{pmatrix}$$

By using Lemma 3, the matrix inequality (16) holds for all Δ satisfying with (6) if and only if there exists a matrix $P > 0$ and a scalar $\epsilon > 0$ such that

$$\begin{pmatrix} -r^2P & PA'_\alpha & \sqrt{\alpha}PC' & \sqrt{\alpha}F_w \\ A_\alpha P & -P & 0 & 0 \\ \sqrt{\alpha}CP & 0 & -I & 0 \\ \sqrt{\alpha}F'_w & 0 & 0 & -\gamma^2 I \end{pmatrix} + \epsilon\mathcal{H}\mathcal{H}' + \epsilon^{-1}\mathcal{E}'\mathcal{E} < 0 \quad (17)$$

or

$$\begin{pmatrix} -r^2P & PA'_\alpha & \sqrt{\alpha}PC' & \sqrt{\alpha}F_w \\ A_\alpha P & -P & 0 & 0 \\ \sqrt{\alpha}CP & 0 & -I & 0 \\ \sqrt{\alpha}F'_w & 0 & 0 & -\gamma^2 I \end{pmatrix} - \mathcal{K}'\mathcal{L}^{-1}\mathcal{K} < 0 \qquad (18)$$

where

$$\mathcal{K} = \begin{pmatrix} 0 & 0 & 0 & \sqrt{\alpha}E_w \\ EP & 0 & 0 & 0 \\ \epsilon H'_1 & 0 & 0 & 0 \\ 0 & \epsilon H'_1 & 0 & 0 \end{pmatrix}, \mathcal{L} = \begin{pmatrix} -\epsilon I & 0 & 0 & 0 \\ 0 & -\epsilon I & 0 & 0 \\ 0 & 0 & -\epsilon I & 0 \\ 0 & 0 & 0 & -\epsilon I \end{pmatrix}$$

Then using the Schur complement, it is straightforward to verify that this inequality (18) is equivalent to (14). ∎

A. Robust $\mathcal{H}_\infty$ Controller Design with Circular Pole Constraints

For the design problem, we consider the following norm-bounded uncertain systems (3). The uncertainties are described by

$$\begin{pmatrix} \Delta A & \Delta B_u & \Delta F_w \\ \Delta C & * & * \end{pmatrix} = \begin{pmatrix} H_1 \\ H_2 \end{pmatrix}\Delta(t)\begin{pmatrix} E & E_u & E_w \end{pmatrix} \qquad (19)$$

where ΔC is assumed to be used in practice and $*$ are neglected. Next, two approaches for robust $\mathcal{H}_\infty$ controller with circular pole constraints are proposed by using Theorem 1.

1) State Feedback Controller Design: We consider the system in (3) and desire to acheive not only placing all closed-loop poles of uncertain systems in $\mathcal{D}(\alpha, r)$ region, but also satisfying the prespecified $\mathcal{H}_\infty$ norm constraint simultaneously via a state feedback $u(t) = Kx(t)$. Therefore, the closed-loop system can be represented by:

$$\dot{x}(t) = A_{cl}(\Delta)x(t) + B_{cl}(\Delta)w(t) \qquad (20)$$
$$y(t) = C_{cl}x(t) \qquad (21)$$

where

$$\begin{pmatrix} A_{cl}(\Delta) & B_{cl}(\Delta) \end{pmatrix} = \begin{pmatrix} A_{cl} & B_{cl} \end{pmatrix} + H_1\Delta\begin{pmatrix} \tilde{E} & E_w \end{pmatrix},$$
$$A_{cl} = A + B_uK, B_{cl} = F_w, C_{cl} = C = I_n, \tilde{E} = E + E_uK.$$

Theorem 2: As for the uncertain linear systems (3), the desired circular pole region $\mathcal{D}(\alpha, r)$ and the $\mathcal{H}_\infty$ norm bound constraint $\gamma > 0$ on the disturbance rejection are given. The closed-loop system can acheive the expected performance requirement 1) and 2) if and only if there exist $P > 0$, Y, and a scalar $\epsilon > 0$ such that

$$\begin{pmatrix} -r^2P & \mathcal{A}'_{cl} & \sqrt{\alpha}PC' & \sqrt{\alpha}F_w & 0 & \mathcal{E}'_{cl} & \epsilon H_1 & 0 \\ \mathcal{A}_{cl} & -P & 0 & 0 & 0 & 0 & 0 & \epsilon H_1 \\ \sqrt{\alpha}CP & 0 & -I & 0 & 0 & 0 & 0 & 0 \\ \sqrt{\alpha}F'_w & 0 & 0 & -\gamma^2 I & \sqrt{\alpha}E'_w & 0 & 0 & 0 \\ 0 & 0 & 0 & \sqrt{\alpha}E_w & -\epsilon I & 0 & 0 & 0 \\ \mathcal{E}_{cl} & 0 & 0 & 0 & 0 & -\epsilon I & 0 & 0 \\ \epsilon H'_1 & 0 & 0 & 0 & 0 & 0 & -\epsilon I & 0 \\ 0 & \epsilon H'_1 & 0 & 0 & 0 & 0 & 0 & -\epsilon I \end{pmatrix} < 0 \qquad (22)$$

where $\mathcal{A}_{cl} = (A_\alpha P + B_uY)$ and $\mathcal{E}_{cl} = EP + E_uY$.

As a result, a state feedback controller gain can be chosen as:

$$K = YP^{-1} \qquad (23)$$

Proof: According to Theorem 1, the system (20) can achieve the expected performance requirement (1) and (2) if and only if there exist $P > 0$ and a scalar $\epsilon > 0$ such that

$$\begin{pmatrix} -r^2P & \mathcal{A}'_{cl} & \sqrt{\alpha}PC' & \sqrt{\alpha}F_w & 0 & \mathcal{E}'_{cl} & \epsilon H_1 & 0 \\ \mathcal{A}_{cl} & -P & 0 & 0 & 0 & 0 & 0 & \epsilon H_1 \\ \sqrt{\alpha}CP & 0 & -I & 0 & 0 & 0 & 0 & 0 \\ \sqrt{\alpha}F'_w & 0 & 0 & -\gamma^2 I & \sqrt{\alpha}E'_w & 0 & 0 & 0 \\ 0 & 0 & 0 & \sqrt{\alpha}E_w & -\epsilon I & 0 & 0 & 0 \\ \mathcal{E}_{cl} & 0 & 0 & 0 & 0 & -\epsilon I & 0 & 0 \\ \epsilon H'_1 & 0 & 0 & 0 & 0 & 0 & -\epsilon I & 0 \\ 0 & \epsilon H'_1 & 0 & 0 & 0 & 0 & 0 & -\epsilon I \end{pmatrix} < 0 \qquad (24)$$

where $\mathcal{A}_{cl} = (A_\alpha + B_uK)P$, and $\mathcal{E}_{cl} = (E + E_uK)P$. By the change of variables $KP = Y$, the result is apparent. ∎

Remark 1: By Theorem 2, a robust state feedback controller with circular pole constraints that minimize a prespecified $\mathcal{H}_\infty$ norm constraint for all Δ with $\Delta'\Delta \leq I$ can be simultaneously obtained. Hence, the design problem is reformulated as the following optimization problem:

$$\begin{array}{ll} \text{minimize} & \gamma^2 \\ \text{subject to} & (22),\ P > 0, \text{ and } \epsilon > 0 \end{array} \qquad (25)$$

which can be efficiently solved by using convex optimization algorithm.

2) Dynamic Output Feedback Controller Design: We consider the system in (3) and desire to place all closed-loop poles of uncertain systems in $\mathcal{D}(\alpha, r)$ region and satisfy the prespecified $\mathcal{H}_\infty$ norm constraint simultaneously via a full order output feedback controller. Hence, the state space equations of the desired controller can be shown as follows:

$$\dot{x}_K(t) = A_Kx_K(t) + B_Ky(t) \qquad (26)$$
$$u(t) = C_Kx_K(t) \qquad (27)$$

where $x_K(t) \in \Re^{n_K \times n_K}$ is the state of the controller, and A_K, B_K, and C_K are matrices with the appropriate dimensions that can be determined. Hence, the overall closed-loop system is given by:

$$\begin{aligned} \dot{x}_{cl}(t) &= A_{cl}(\Delta)x_{cl}(t) + B_{cl}(\Delta)w(t) && (28) \\ y(t) &= C_{cl}(\Delta)x_{cl}(t) && (29) \end{aligned}$$

where

$$\begin{pmatrix} A_{cl}(\Delta) & B_{cl}(\Delta) \\ C_{cl}(\Delta) & * \end{pmatrix} = \begin{pmatrix} A_{cl} & B_{cl} \\ C_{cl} & * \end{pmatrix} + \bar{H}_1 \Delta \begin{pmatrix} \tilde{E} & E_w \end{pmatrix},$$

$$\begin{aligned} x_{cl}(t) &= \begin{pmatrix} x(t) \\ x_K(t) \end{pmatrix}, A_{cl} = \begin{pmatrix} A & B_u C_K \\ B_K C & A_K \end{pmatrix}, \\ \bar{H}_1 &= \begin{pmatrix} H_1 \\ B_K H_2 \end{pmatrix}, \tilde{E} = \begin{pmatrix} E & E_u C_K \end{pmatrix}, \\ B_{cl} &= \begin{pmatrix} F_w \\ 0 \end{pmatrix}, C_{cl} = \begin{pmatrix} C & 0 \end{pmatrix}. \end{aligned}$$

Theorem 3: As for the uncertain linear systems (3), the desired circular pole region $\mathcal{D}(\alpha, r)$ and the $\mathcal{H}_\infty$ norm bound constraint $\gamma > 0$ on the disturbance rejection are given. The closed-loop system can acheive the expected performance requirement 1) and 2) if and only if there exist X, Y, $\bar{A}, \bar{B}, \bar{C}$, and a scalar $\epsilon > 0$ such that

$$\begin{pmatrix} \Omega_{11} & \Omega_{21}' & \Omega_{31}' & \Omega_{41}' & 0 & \Omega_{51}' & \Omega_{61}' & 0 \\ \Omega_{21} & \Omega_{22} & 0 & 0 & 0 & 0 & 0 & \Omega_{61}' \\ \Omega_{31} & 0 & -I & 0 & 0 & 0 & 0 & 0 \\ \Omega_{41} & 0 & 0 & -\gamma^2 I & \sqrt{\alpha}E_w' & 0 & 0 & 0 \\ 0 & 0 & 0 & \sqrt{\alpha}E_w & -\epsilon I & 0 & 0 & 0 \\ \Omega_{51} & 0 & 0 & 0 & 0 & -\epsilon I & 0 & 0 \\ \Omega_{61} & 0 & 0 & 0 & 0 & 0 & -\epsilon I & 0 \\ 0 & \Omega_{61} & 0 & 0 & 0 & 0 & 0 & -\epsilon I \end{pmatrix} < 0 \qquad (30)$$

where

$$\begin{aligned} \Omega_{11} &= -\begin{pmatrix} r^2 X & r^2 I \\ r^2 I & r^2 Y \end{pmatrix} \\ \Omega_{21} &= \begin{pmatrix} AX + \bar{B}C + \alpha X & A + \alpha I \\ \bar{A} + \alpha I & YA + B_u \bar{C} + \alpha Y \end{pmatrix} \\ \Omega_{22} &= -\begin{pmatrix} X & I \\ I & Y \end{pmatrix} \\ \Omega_{31} &= \sqrt{\alpha} \begin{pmatrix} CX & C \end{pmatrix} \\ \Omega_{41} &= \sqrt{\alpha} \begin{pmatrix} F_w' & F_w' Y \end{pmatrix} \\ \Omega_{51} &= \begin{pmatrix} EX + E_u \bar{C} & E \end{pmatrix} \\ \Omega_{61} &= \epsilon \begin{pmatrix} H_1' & H_1' Y + H_2' \bar{B}' \end{pmatrix} \end{aligned}$$

As a result, a dynamic output feedback controller can be constructed as:

$$\begin{aligned} A_K &= (N')^{-1}(\bar{A} - YAX - N'B_K CX - YB_u C_K M)M^{-1} \\ B_K &= (N')^{-1}\bar{B} \\ C_K &= \bar{C}M^{-1} \end{aligned}$$

where X and Y are arbitrary nonsingular matrices satisfying $M'N = I - XY$.

Proof: (Necessity) It is obvious that the matrix P and the controller parameters in the matrix A_{cl} in (31) are unknown and occur in nonlinear fashion.

$$\begin{pmatrix} -r^2 P & PA_{cl}' & \sqrt{\alpha}PC_{cl}' & \sqrt{\alpha}B_{cl} & 0 & P\tilde{E}' & \epsilon\bar{H}_1 & 0 \\ A_{cl}P & -P & 0 & 0 & 0 & 0 & 0 & \epsilon\bar{H}_1 \\ \sqrt{\alpha}C_{cl}P & 0 & -I & 0 & 0 & 0 & 0 & 0 \\ \sqrt{\alpha}B_{cl}' & 0 & 0 & -\gamma^2 I & \sqrt{\alpha}E_w' & 0 & 0 & 0 \\ 0 & 0 & 0 & \sqrt{\alpha}E_w & -\epsilon I & 0 & 0 & 0 \\ \tilde{E}P & 0 & 0 & 0 & 0 & -\epsilon I & 0 & 0 \\ \epsilon\bar{H}_1' & 0 & 0 & 0 & 0 & 0 & -\epsilon I & 0 \\ 0 & \epsilon\bar{H}_1' & 0 & 0 & 0 & 0 & 0 & -\epsilon I \end{pmatrix} < 0 \quad (31)$$

Consequently, we apply a method of changing variables from Scherer [14] so that the matrix inequality (31) can be reduced to an LMI in all variables.

First, we define the partitioning of the matrix P and P^{-1} as

$$P := \begin{pmatrix} X & M' \\ M & U \end{pmatrix}, P^{-1} := \begin{pmatrix} Y & N' \\ N & Q \end{pmatrix},$$

where the order of controller n_K is equal to the order of plant n. Then we pre-multiply and post-multiply (31) by $\text{diag}\{\Theta_2, \Theta_2, I, I, I, I, I, I\}$ and its transpose, respectively, thereby we can get:

$$\left(\begin{array}{c|c|c|c|c|c|c|c} \Theta_2'(-r^2 P)\Theta_2 & (\star)' & (\star)' & (\star)' & (\star)' & (\star)' & (\star)' & (\star)' \\ \hline \Theta_2' A_{cl} P \Theta_2 & \Theta_2'(-P)\Theta_2 & (\star)' & (\star)' & (\star)' & (\star)' & (\star)' & (\star)' \\ \hline \sqrt{\alpha}C_{cl}P\Theta_2 & 0 & -I & (\star)' & (\star)' & (\star)' & (\star)' & (\star)' \\ \hline \sqrt{\alpha}B_{cl}'\Theta_2 & 0 & 0 & -\gamma^2 I & (\star)' & (\star)' & (\star)' & (\star)' \\ \hline 0 & 0 & 0 & \sqrt{\alpha}E_w & -\epsilon I & (\star)' & (\star)' & (\star)' \\ \hline \tilde{E}P\Theta_2 & 0 & 0 & 0 & 0 & -\epsilon I & (\star)' & (\star)' \\ \hline \epsilon\bar{H}_1'\Theta_2 & 0 & 0 & 0 & 0 & 0 & -\epsilon I & (\star)' \\ \hline 0 & \epsilon\bar{H}_1'\Theta_2 & 0 & 0 & 0 & 0 & 0 & -\epsilon I \end{array}\right) < 0$$

where

$$\Theta_1 := \begin{pmatrix} X & I \\ M & 0 \end{pmatrix}, \Theta_2 := \begin{pmatrix} I & Y \\ 0 & N \end{pmatrix},$$

and we use $(\star)'$ to replace blocks that are readily inferred by symmetry.

Then it is also apparent that

$$P\Theta_2 = \Theta_1, \Theta_2' P \Theta_2 = \Theta_1' \Theta_2 = \begin{pmatrix} X & I \\ I & Y \end{pmatrix} > 0.$$

We substitute $A_{cl}, \bar{H}_1, \tilde{E}$, and P in (31), and need to change the controller variables to new ones as:

$$\begin{aligned} \bar{A} &:= YAX + N'B_K CX + YB_u C_K M + N'A_K M \\ \bar{B} &:= N'B_K \\ \bar{C} &:= C_K M. \end{aligned}$$

Additionally, we can easily check each term in (31) as follows:

$$\begin{aligned} \Theta_2'(-r^2 P)\Theta_2 &= -\begin{pmatrix} r^2 X & r^2 I \\ r^2 I & r^2 Y \end{pmatrix} \\ \Theta_2' A_{cl} P \Theta_2 &= \begin{pmatrix} AX + \bar{B}C + \alpha X & A + \alpha I \\ \bar{A} + \alpha I & YA + B_u \bar{C} + \alpha Y \end{pmatrix} \\ \Theta_2' P \Theta_2 &= \begin{pmatrix} X & I \\ I & Y \end{pmatrix} \\ \sqrt{\alpha}C_{cl}P\Theta_2 &= \sqrt{\alpha}\begin{pmatrix} CX & C \end{pmatrix} \\ \sqrt{\alpha}B_{cl}'\Theta_2 &= \sqrt{\alpha}\begin{pmatrix} F_w' & F_w' Y \end{pmatrix} \\ \tilde{E}P\Theta_2 &= \begin{pmatrix} EX + E_u\bar{C} & E \end{pmatrix} \\ \epsilon\bar{H}_1'\Theta_2 &= \epsilon\begin{pmatrix} H_1' & H_1'Y + H_2'\bar{B}' \end{pmatrix} \end{aligned}$$

which imply that (30) holds.

(Sufficiency:) Due to limited spaces, the proof of a sufficient condition is omitted. ■

Remark 2: It is obvious that (30) is not an LMI due to the product of a scalar ϵ with variables Y and $\bar{B}$, respectively. As a result, the LMI software fails to solve (30). However, we are able to achieve difficulty by setting ϵ as a prior value and then apply the LMI software. We need to tune ϵ until the solver returns a feasible solution.

Remark 3: In LFC problem, when the $\mathcal{H}_\infty$ constraint is not considered ($\gamma \to \infty$), the problem reduces to robust LFC controller design with circular pole constraints considered in [16]. Hence, (14) is reduced to the following LMI:

$$\begin{pmatrix} -r^2P & PA'_\alpha & PE' & 0 \\ A_\alpha P & -P & 0 & \epsilon H_1 \\ EP & 0 & -\epsilon I & 0 \\ 0 & \epsilon H'_1 & 0 & -\epsilon I \end{pmatrix} = \begin{pmatrix} -r^2P & PA'_\alpha & PE' & 0 \\ A_\alpha P & -P & 0 & H_1 \\ EP & 0 & -I & 0 \\ 0 & H'_1 & 0 & -I \end{pmatrix} < 0$$

Similarly, when the circular pole constraint is absent, the problem is reduced to the standard robust $\mathcal{H}_\infty$ controller design. As a result, (14) is reduced to the following LMI:

$$\begin{pmatrix} A'P + PA & PC' & F_w & PE' & \epsilon H_1 \\ CP & -I & 0 & 0 & 0 \\ F'_w & 0 & -\gamma^2 I & E'_w & 0 \\ EP & 0 & E_w & -\epsilon I & 0 \\ \epsilon H'_1 & 0 & 0 & 0 & -\epsilon I \end{pmatrix} < 0 \qquad (32)$$

IV. Simulation Results

The performance of the proposed robust $\mathcal{H}_\infty$ controller with circular pole constraints for interconnected power systems based on LMI approach is investigated. The model of power system in [13] is consideration in this paper. This model is the two-area interconnected power system where two stream plants are interconnected via tie-line.

To demonstrate the efficiency of the proposed method, we first study the robustness of the proposed controller against the variation of system parameters. In this paper, a step load change of 0.1 p.u. in the load demand for Area 1 (ΔP_{d1} = 0.1) is applied to the system and we choose weighting matrices $Q = I_9$ and $R = I_2$ for LQR method in state feedback case. Then, both the proposed robust $\mathcal{H}_\infty$ controller and the standard one are compared in output feedback case. In studying LFC problem, we consider the two cases in designing robust controller as follows.

State feedback case: We would like to enforce all closed-loop poles in $\mathcal{D}(17, 15)$, assume $\epsilon = 0.01457$, and minimize $\|H(s)\|_\infty = 107.9 \leq 110$. Our results are compared with those of Wang *et. al.* [5], Ray *et. al.* [13] and the traditional LQR method. The responses of $\Delta f_1(t)$ and $\Delta f_2(t)$ are shown in Figure 1 and 2 where consist of the upper and lower bounds of parameter variations and nominal parameter. It is well-known that the difference in the shape of the transient responses from the nominal responses are an indication of the performance robustness. To show performance robustness of all method, we define the norms ($\mathcal{N}$) and absolute sums (Σ) as follows:

$$\mathcal{N}_{ubp} = \left[\sum_{j=0}^{k}[x_{np}(t_k) - x_{ubp}(t_k)]^2\right]^{1/2}$$

$$\mathcal{N}_{lbp} = \left[\sum_{j=0}^{k}[x_{np}(t_k) - x_{lbp}(t_k)]^2\right]^{1/2}$$

$$\Sigma_{ubp} = \sum_{j=0}^{k}|x_{np}(t_k) - x_{ubp}(t_k)|$$

$$\Sigma_{lbp} = \sum_{j=0}^{k}|x_{np}(t_k) - x_{lbp}(t_k)|$$

where $x_{np}(t_k)$, $x_{ubp}(t_k)$ and $x_{lbp}(t_k)$ represent the nominal, upper bound and lower bound value of any state $x(t_k)$ at time t_k, respectively. The numerical results of the performance robustness are present in Table I.

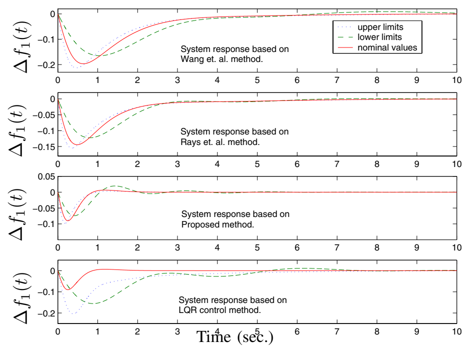


Fig. 1. The responses of $\Delta f_1(t)$ of four methods.

TABLE I

Comparison of Performance Robustness of All Methods

State	Method	$\mathcal{N}_{upb}$	$\mathcal{N}_{lpb}$	Σ_{upb}	Σ_{lpb}
$\Delta f_1(t)$	Wang *et. al.* [5]	1.064	2.153	56.657	147.126
	Rays *et. al.* [13]	0.653	1.338	27.417	71.050
	Our method	0.393	0.860	11.939	39.868
	LQR control	2.709	3.802	174.428	227.336
$\Delta f_2(t)$	Wang *et. al.*	0.852	2.559	57.542	198.313
	Rays *et. al.*	0.360	1.142	21.650	78.434
	Our method	0.224	0.783	8.529	41.416
	LQR control	2.432	4.172	175.244	232.315

Furthermore, from these figures, it is obvious that the deviation from the nominal value of the perturbed system (upper and lower limits) is much less in case of the proposed method compared with that of three methods mentioned. This means that the performance robustness of the proposed controller is superior compared to these methods. Also, it is very clear that the transient performance is improved and can be achieved such as maximum overshoot, settling time, and rise time are both smaller and shorter, while the other methods cannot. These are related to pole locations in a specified region. In Figure 3, it it apparent that all closed-loop poles are placed in a desired region.

Output feedback case: Now, we wish to show the result of the output feedback controller design satisfying both requirements. The proposed robust $\mathcal{H}_\infty$ controller with circular pole constraint is compared with the standard robust $\mathcal{H}_\infty$ controller (without circular pole constraint) and the closed-loop pole locations of both controllers is also shown. We choose to put all closed-loop poles in $\mathcal{D}(200.05, 200)$ and $\mathcal{D}(300.16, 300)$, respectively, assume $\epsilon = 0.0001$, and set $\|H(s)\|_\infty = 5463.15$ according to remark 2. The responses of $\Delta f_1(t)$ and $\Delta f_2(t)$, and the pole placement of all closed-loop poles in the specified region of this case are shown in Figure 4, 5, and 6, respectively. From these figures, it is obvious that the transient responses in increment frequency deviation $\Delta f_1(t)$ and $\Delta f_2(t)$ do decay faster and exhibit smaller overshoot as well as shorter settling time, when the the circular pole constraint is included in our proposed controller design. Furthermore, the more all closed-loop poles is pushed toward LHP, the more the settling time and overshoot decrease quickly. Also, we are able to use this circular pole constraint in regulating the control input and avoiding the saturation in any systems.

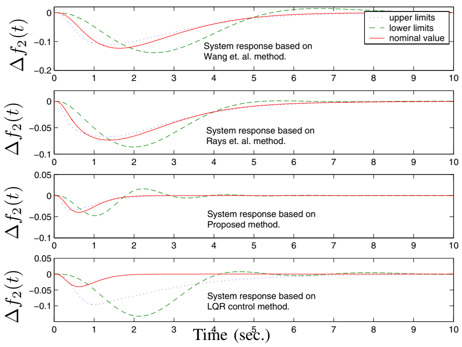


Fig. 2. The responses of $\Delta f_2(t)$ of four methods.

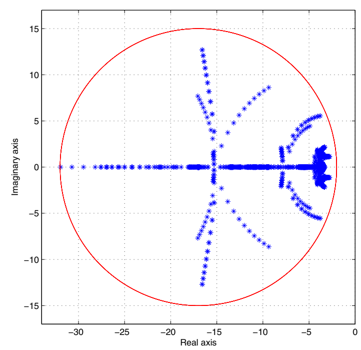


Fig. 3. Closed-loop pole locations in the state feedback case.

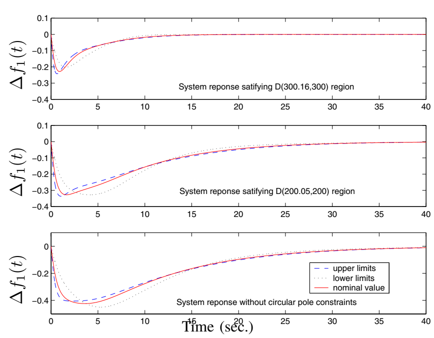


Fig. 4. The responses of $\Delta f_1(t)$ in the output feedback case.

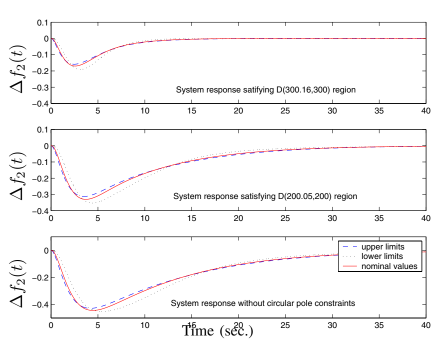
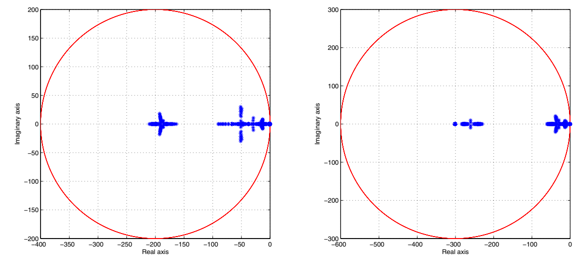


Fig. 5. The responses of $\Delta f_2(t)$ in the output feedback case.



(a) $\mathcal{D}(200.05, 200)$ (b) $\mathcal{D}(300.16, 300)$

Fig. 6. Closed-loop pole locations in the output feedback case.

V. Conclusions

In this paper, a controller design problem involving both circular pole constraint and the prespecified $\mathcal{H}_\infty$ norm constraint for interconnected power systems via LMI approach is investigated. The aim of this paper is to obtain robust stability and to improve the transient responses of the uncertain system. The necessary and sufficient conditions for the existence of a desired controller have proved and the feasible solution to LMI can be used to figure out a controller. In comparision with Wang *et al.*[5], Ray *et al.* [13], and the traditional LQR control method, the system responses with the proposed state feedback controller are the best transient responses with respect to settling time, rise time, and maximum overshoot in all cases of the system parameter variations. Besides, it is also shown that the transient responses of the proposed output feedback controller with circular pole constraints is improved.

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