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Delaying Instability and Voltage Collapse in Power Systems using SVCs with Washout Filter-Aided Feedback

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***Abstract*— This paper introduces washout filter-aided feedback in the design of Static Var Compensator (SVC) control to increase the range of stable operation of a power system susceptible to voltage collapse. The use of such a control automatically maintains the open-loop steady state operating conditions. In contrast to static state feedback designs, washout-filter based designs are more efficient because no control energy is spent on needlessly maintaining an unintentionally shifted system equilibrium. The control design is illustrated in a sample power system model that undergoes a Hopf bifurcation and voltage collapse as the reactive power demand is increased to a critical value. It is seen that the control meets the goals of delaying (or eliminating) the Hopf bifurcation while at the same time maintaining the open-loop operating condition.**

I. INTRODUCTION

Voltage collapse in electric power systems is recognized to be an important mechanism for loss of stability (see [5] and references therein). Significant effort has been devoted to studying mechanisms of voltage collapse. More recently, several authors have considered control techniques to mitigate voltage collapse.

In this paper, we introduce washout filter-aided feedback in the design of SVC control to increase the range of stable operation of a power system susceptible to voltage collapse. The use of such a control automatically maintains the open-loop steady state operating conditions. In contrast to static state feedback designs, washout-filter based designs are more efficient because no control energy is spent on needlessly maintaining an unintentionally shifted system equilibrium. We illustrate the control design in a sample power system model that undergoes a Hopf bifurcation and voltage collapse as the reactive power demand is increased to a critical value. We show that the control meets the goals of delaying (or eliminating) the Hopf bifurcation while at the same time maintaining the open-loop operating condition.

Similar to [13], [12], we take the setting of control through a Static Var Compensator (SVC). The effect of the control on the system is studied using bifurcation analysis. We show that washout filter-aided feedback performs better than static state feedback in the sense of improved control efficiency. Static state feedback changes the operating condition of the open-loop system. This results in ineffective control action and may also result in degrading system performance. Also, in static state feedback control design, an estimate of the operating point to be stabilized has to be provided to the controller. Washout filter-aided feedback, on the other hand, does not change the value of the operating points of the open-loop system since the control vanishes by nature at steady state. Moreover, washout filter-aided feedback provides automatic tracking of the operating point to be stabilized even in the presence of model uncertainty. Thus, washout filter-aided feedback can be used to stabilize the system over a wide range of operating conditions without the need to supply the controller with the open-loop values of these operating conditions.

The designed SVC control with washout filter-aided feedback is tested on a single generator infinite bus system with induction motor load. The power system loses stability through a supercritical Hopf bifurcation. Bifurcation analysis and time simulations are used to demonstrate the effectiveness of the controller in eliminating the Hopf bifurcation and stabilizing the branch of unstable equilibrium points of the open-loop system.

In [13], [12], [14], an SVC control was used to improve the voltage instability. Direct feedback linearization was utilized to simplify the control design. However, for this type of control to be effective, the value of operating point of the line voltage of the open loop system for each reactive power demand has to be provided to the controller. In [10], [8], the effect of SVC with static state feedback on voltage collapse in a model power system was studied. Numerical simulations were used to demonstrate the effect of the control on the power system.

The paper proceeds as follows. In Sec. II, the power system model used in this paper is presented. In Sec. III, the SVC controller with static state feedback is presented and the design of the SVC controller with washout filter-aided feedback control is carried out. In Sec. IV, time simulation results that demonstrate the effectiveness of the proposed controller are presented and compared with those of the controller with static state feedback.

II. Power system model

Consider the single generator power system model with induction motor load [11] depicted in Fig. 1. This model may be viewed as an equivalent circuit for a local area of interest connected to a large (external) network. The external network is modeled as an infinite bus, i.e., a voltage source providing constant voltage magnitude and phase regardless of power flow. In Fig. 1, the infinite bus has terminal voltage $E_0\angle 0$ (a phasor). Here, E_0 is the voltage magnitude, and 0 is taken as the voltage phase angle. The generator terminal voltage is $E_m\angle\delta_m$ (see Fig. 1). Figure 1 also depicts the complex admittances of the transmission lines connected to the generator and the infinite bus terminals, a load and a capacitor. Note that the load voltage is $V_L\angle\delta$. The equations describing the dynamics of this model are

$$\dot{\delta}_m = \omega \quad (1)$$

$$M\dot{\omega} = -d_m\omega + P_m - E_m V_L Y_m \sin(\delta_m - \delta) \quad (2)$$

$$K_{qw}\dot{\delta} = -K_{qv2}{V_L}^2 - K_{qv}V_L + Q(\delta_m,\delta,V_L) - Q_0 - Q_1 \quad (3)$$

$$TK_{q\omega}K_{pv}\dot{V}_L = K_{p\omega}K_{qv2}{V_L}^2 + (K_{p\omega}K_{pv} - K_{q\omega}K_{pv})V_L + K_{q\omega}(P(\delta_m,\delta,V_L) - P_0 - P_1) - K_{p\omega}(Q(\delta_m,\delta,V_L) - Q_0 - Q_1) \quad (4)$$

The state variables are δ_m (the generator phase angle, closely related to the mechanical angle of the generator rotor), ω (the rotor speed), δ (the load voltage phase angle) and V_L (the magnitude of the load voltage). The load includes a constant PQ load (denoted by P_1, Q_1) in parallel with an induction motor with constant real and reactive powers P_0, Q_0, respectively. The real and reactive powers supplied to the load by the network are, respectively,

$$P(\delta_m,\delta,V_L) = -E_0^{'}V_L Y_0^{'}\sin(\delta) + E_m V_L Y_m \sin(\delta_m - \delta) \quad (5)$$

$$Q(\delta_m,\delta,V_L) = E_0^{'}V_L Y_0^{'}\cos(\delta) + E_m V_L Y_m \cos(\delta_m - \delta) - (Y_0^{'} + Y_m){V_L}^2 \quad (6)$$

The load bus includes a capacitor as part of its constant impedance representation in order to maintain the voltage magnitude at a nominal and reasonable value. The adjusted values of the Thevenin equivalent circuit with the capacitor are

$$E_0^{'} = \frac{E_0}{\sqrt{1 + C^2Y_0^{-2} - 2CY_0^{-1}\cos\theta_0}}$$

$$Y_0^{'} = Y_0\sqrt{1 + C^2Y_0^{-2} - 2CY_0^{-1}\cos\theta_0}$$

$$\theta_0^{'} = \theta_0 + \tan^{-1}\left\{\frac{CY_0^{-1}\sin\theta_0}{1 - CY_0^{-1}\cos\theta_0}\right\}$$

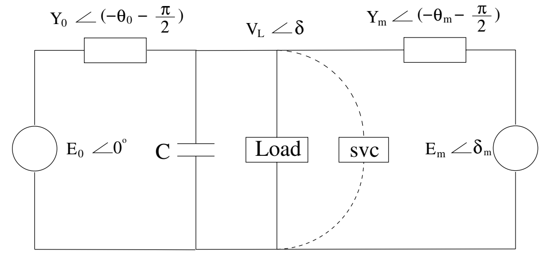

Fig. 1. Single generator infinite bus power system model.

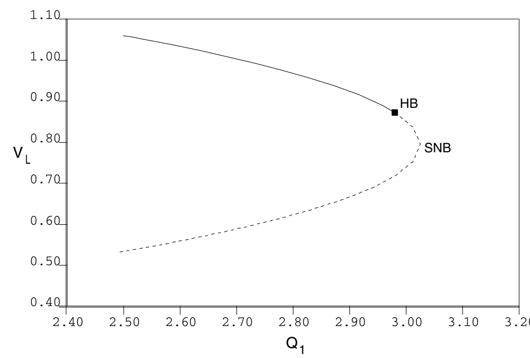

Fig. 2. Bifurcation diagram for the power system (open-loop).

The system parameters appearing in (1)-(6) are:
$M = 0.01464$, $C = 3.5$, $E_m = 1.05$, $Y_0 = 3.33$, $\theta_0 = 0$, $\theta_m = 0$, $K_{p\omega} = 0.4$, $K_{pv} = 0.3$, $K_{q\omega} = -0.03$, $K_{qv} = -2.8$, $K_{qv2} = 2.1$, $T = 8.5$, $P_1 = 0.6$, $P_0 = 0.0$, $Q_0 = 1.3$, $E_0 = 1.0$, $Y_m = 5.0$, $P_m = 1.0$, $d_m = 0.05$. All values are in per unit except for angles, which are in degrees.

The reactive power demand of the load is denoted by Q_1 and will be taken as the bifurcation parameter. This power system model undergoes a supercritical Hopf bifurcation as Q_1 is increased beyond a critical value ($Q_1^* = 2.98021$). The bifurcation diagram of the open-loop system is depicted in Fig. 2. All bifurcation diagrams in this paper were produced using the software package AUTO [4].

III. SVC control

The considered SVC configuration consists of a fixed capacitor connected in parallel with a thyristor controlled reactor [7]. The effective reactance of the controlled reactor is a function of the firing angle of the thyristor. The SVC function, therefore, is equivalent to providing a continuously controlled shunt susceptance to the system [3], [8]. The SVC used in this paper is modeled as a first order linear differential equation

$$\dot{B} = \frac{1}{T_{SVC}}(K_{SVC}u - B), \qquad B_{min} \le B \le B_{max} \quad (7)$$

where B is the susceptance of the SVC, K_{SVC} is the SVC gain, T_{SVC} is the SVC time constant and u is the control input to the SVC (see Fig. 3). SVCs are usually connected at nodes whose voltages need to be controlled. This could be at the middle of a transmission line or at a load bus. In this

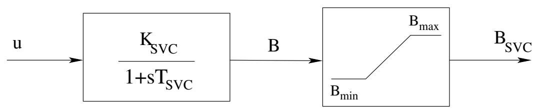

Fig. 3. Schematic diagram of the SVC model.

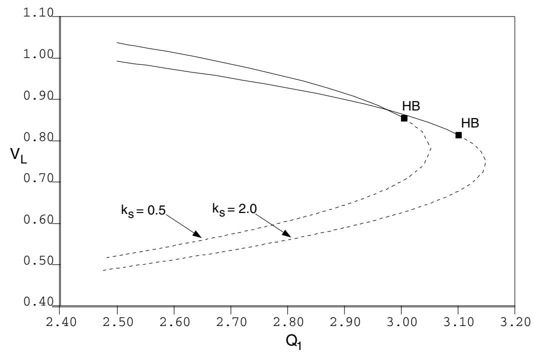

Fig. 4. Bifurcation diagram for the power system with SVC using static state feedback for two values of the controller gain; $k_s = 0.5$, $k_s = 2.0$.

paper, the SVC is connected across the load as depicted in Fig. 1. The net effect of the variable susceptance introduced to the system by the SVC is modeled as an additional reactive power Q_{added} that is added to the load reactive power [3]

$$Q_{added} = BV_L^2. \tag{8}$$

A. Static state feedback

In static state feedback, the SVC input is given by

$$u = k_s(V_L - V_o) \tag{9}$$

where V_o is the reference voltage, a targeted operating voltage. Using SVC with static state feedback results in

$$\dot{B} = \frac{1}{T_{SVC}}(K_{SVC}k_s(V_L - V_o) - B). \tag{10}$$

With SVC control, the reactive power supplied by the SVC to the power system is given by (8) which is added to Q_1 in Eqs. (3),(4). The bifurcation diagram of the power system with SVC using static state feedback for two values of the controller gain ($k_s = 0.5$, $k_s = 2.0$) is depicted in Fig. 4. The SVC constants take the following values: $T_{SVC} = 0.01$ s and $K_{SVC} = 1$. Comparing Fig. 4 with the open-loop bifurcation diagram shown in Fig. 2, it is clear that static state feedback alters the open-loop operating points. This may result in degrading the system performance since the system was not designed to operate at the new operating points. Also, a more serious consequence is that the voltage at these new operating points is lower than the nominal operating condition.

B. Washout filter-aided feedback

A washout filter (also sometimes called a washout circuit) is a high pass filter that washes out, i.e., rejects steady state inputs, while passing transient inputs [2]. The main benefit of using washout filters is that all the equilibrium points of the open-loop system are preserved (i.e., their location isn't changed). In addition, washout filters facilitate automatic tracking of the targeted operating point, which results in vanishing control action once stabilization has been achieved and steady state condition is reached. Washout filters have been used in feedback control in many applications (e.g., [9], [1], [6]). In continuous-time setting, the transfer function of a typical washout filter is

$$\begin{aligned} G(s) &= \frac{y(s)}{x(s)} \\ &= \frac{s}{s+d} = 1 - \frac{d}{s+d} \end{aligned}$$

Here, d is the reciprocal of the filter time constant which is positive for a stable filter and negative for an unstable filter. With the notation

$$z(s) := \frac{1}{s+d}\, x(s) \tag{11}$$

the dynamics of the state z of the filter can be written as

$$\dot{z} = x - dz, \tag{12}$$

along with the output equation

$$y = x - dz. \tag{13}$$

The control input u is taken as a function of the output of the washout filter y. Note that the output of the washout filter vanishes in steady state and hence the equilibrium points of the open-loop system are not moved by feedback through washout filters.

The dynamics of the SVC control with washout filter-aided feedback is given by

$$\dot{B} = \frac{1}{T_{SVC}}(K_{SVC}k_w(V_L - dz) - B) \tag{14}$$

$$\dot{z} = V_L - dz \tag{15}$$

where z is the state of the washout filter, d is the washout filter constant and k_w is the controller gain to be chosen. With SVC control, the reactive power supplied by the SVC to the power system is given by (8) which is added to Q_1 in Eqs. (3),(4). The SVC constants take the following values: $T_{SVC} = 0.01$ s and $K_{SVC} = 1$.

Suppose that we are interested in designing an SVC with washout filter-aided controller to eliminate the Hopf bifurcation and stabilize the branch of unstable operating points of the open-loop system. We consider the Jacobian matrix of the closed-loop system (the system with SVC and washout filter-aided feedback) evaluated at $Q_{1_s} = 3.02578122$ (the value of Q_1 where the saddle node bifurcation occurs). The controller parameters (k_w, d) are then chosen such that all the eigenvalues of the closed-loop system Jacobian are stable except for an eigenvalue at the origin which corresponds to the saddle node bifurcation. The Jacobian

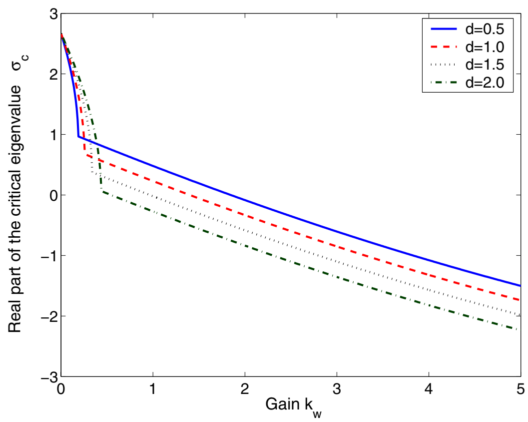

Fig. 5. Variation of the real part of the complex conjugate pair of eigenvalues undergoing Hopf bifurcation as a function of the control gain k_w for four values of the washout filter constant d (calculated for $Q_1 = 3.02578122$: Hopf bifurcation elimination).

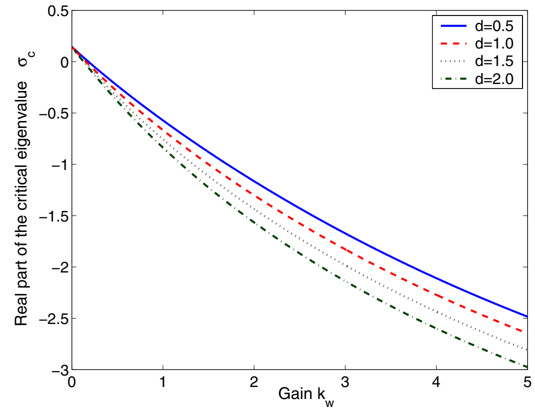

Fig. 6. Variation of the real part of the complex conjugate pair of eigenvalues undergoing Hopf bifurcation as a function of the control gain k_w for four values of the washout filter constant d (calculated for $Q_1 = 2.99$: Hopf bifurcation delay).

of the closed-loop system evaluated at Q_{1_s} is

$$A = \begin{bmatrix} 0 & 1 & 0 & 0 & 0 & 0 \\ -276.86 & -3.42 & 276.86 & -85.9 & 0 & 0 \\ 33.33 & 0 & -27.95 & 11.37 & 21.08 & 0 \\ -3.64 & 0 & 1.76 & -1.49 & -3.31 & 0 \\ 0 & 0 & 0 & 100k_w & -100 & -100k_w d \\ 0 & 0 & 0 & 1 & 0 & -d \end{bmatrix}$$

Denote the critical pair of eigenvalues (the eigenvalues corresponding to the Hopf bifurcation in the open-loop system) by $\sigma_c \pm j\omega_c$. Figure 5 shows the effect of the controller gain k_w on the real part of the critical eigenvalues σ_c for four values of the washout filter constant d when the reactive power load is $Q_1 = 3.02578122$. Note that for each d, there is a minimum controller gain that is needed to eliminate the Hopf bifurcation. Note also that one can design the controller to delay the Hopf bifurcation to any value between the old Hopf bifurcation point and the saddle node bifurcation point. For example, one can calculate the minimum control gain required to delay the Hopf bifurcation to an intermediate point between Q_{1_s} and Q_1^*. Figure 6 shows the variation of the real part of the critical eigenvalues as a function of the control gain at the operating point $Q_1 = 2.99$. It is clear from Figs. 5-8 that a higher controller gain is required to eliminate the Hopf bifurcation than to delay the Hopf bifurcation.

The eigenvalues of the closed-loop system change smoothly as a function of the controller parameters and Q_1. Since we have shown that there exist controllers that either eliminate the Hopf bifurcation or delay it, there exists a controller that stabilizes any operating point between the open-loop Hopf bifurcation point and the saddle-node bifurcation point.

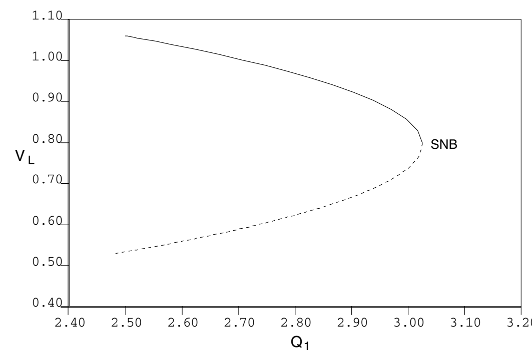

Fig. 7. Bifurcation diagram of the closed-loop system with SVC and washout filter-aided feedback with $k_w = 1.834$ and $d = 1$. The Hopf bifurcation is eliminated by this feedback control.

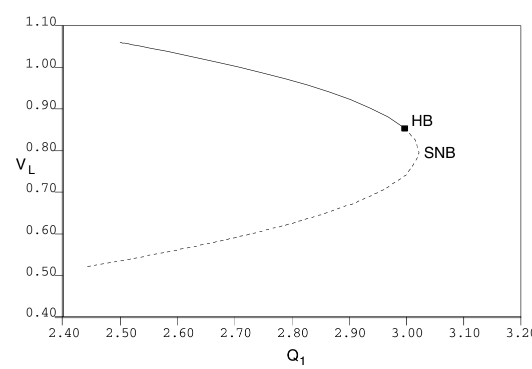

Fig. 8. Bifurcation diagram for the power system with SVC using washout filter-aided feedback with gain $k_w = 0.8$ and $d = 1$. The Hopf bifurcation is delayed to $Q_1 = 2.99$.

IV. Time Simulation Results

The designed control is tested on the power system model of Sec. II. As mentioned above, this power system model undergoes a supercritical Hopf bifurcation as the reactive power load Q_1 is increased through the critical value $Q_1^* = 2.98021$ [11]. This model has also been used in several studies on voltage collapse. Figure 2 shows the bifurcation diagram of the open-loop system.

To demonstrate the effectiveness of the controller, a simulation of the system undergoing a step change in the active power demand is carried out. A step change in the active power demand was chosen instead of a step change in the load reactive power demand since an instantaneous change in the reactive power demand is not realistic.

A simulation of the system undergoing a step change in the active power demand P_1 from 0.6 p.u. to 0.55 p.u. at time $t = 60$ s is carried out (see Fig. 9). Numerical simulations demonstrate that the load voltage collapses shortly after the step change in the active power demand as shown in Fig. 10. When SVC control is used, the voltage is stabilized and the Hopf bifurcation is delayed. Figure 11 (a) shows the closed-loop simulation of the load voltage when SVC with static state feedback is used with controller gain $k_s = 0.8$ and $V_o = V_L(0) = 0.8814$. As a result of the step change in the load power demand, the load voltage experienced a transient oscillatory behavior before settling to a new equilibrium value $V_L = 0.8799$ (see Fig. 11 (a)). Figure 11 (b) shows the control input signal to the SVC when static state feedback is used. Note that the control input does not vanish after the system settles in the new steady state. Figure 12 (a) shows the closed-loop simulation of the load voltage when SVC with washout filter-aided feedback is used with control parameters $k_w = 0.8$ and $d = 1$. As a result of the step change in the load active demand, the load voltage experienced a transient oscillatory behavior before settling to a new equilibrium value $V_L = 0.8793$. Note that the new steady state value for the voltage is different from the steady state value resulted when static state feedback was used. Figure 12 (b) shows the control input signal to the SVC when washout filter aided feedback is used. Note that the control input vanishes once stabilization is achieved and the new steady state is reached.

Next, another simulation is carried out where reactive power demand is ramped from 2.9 p.u. at time t=35 s to 3.0 p.u. at time t=85 s (see Fig. 13). This loading scenario will drive the open-loop power system to voltage collapse. Figures 14 and 15 show the voltage of the closed-loop system in response to the loading scenario of Fig. 13 when SVC with static state feedback and $V_o = V_L(0) = 0.9248$ and SVC with washout filter-aided feedback controllers were used, respectively.

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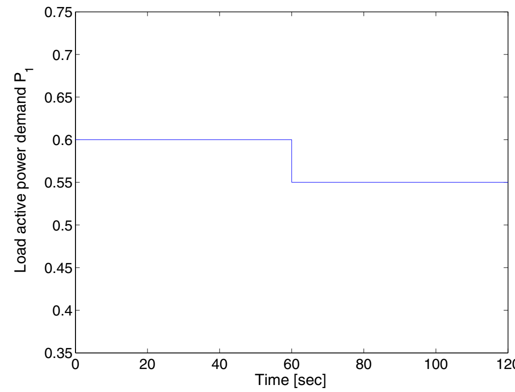

Fig. 9. A sudden decrease in active power demand P_1 from 0.6 p.u. to 0.55 p.u. at time t=60 seconds.

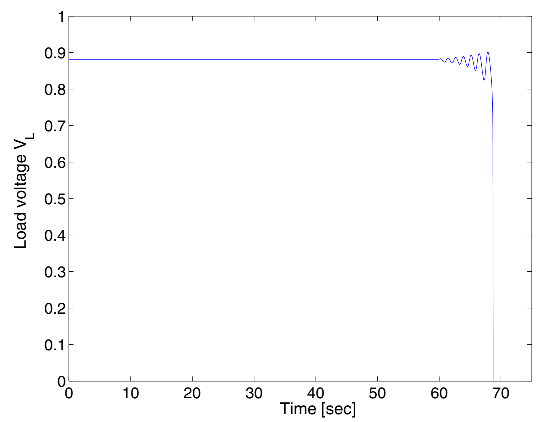

Fig. 10. Open-loop plot of load voltage versus time for a sudden decrease in active power demand P_1 from 0.6 p.u. to 0.55 p.u. at time t=60.

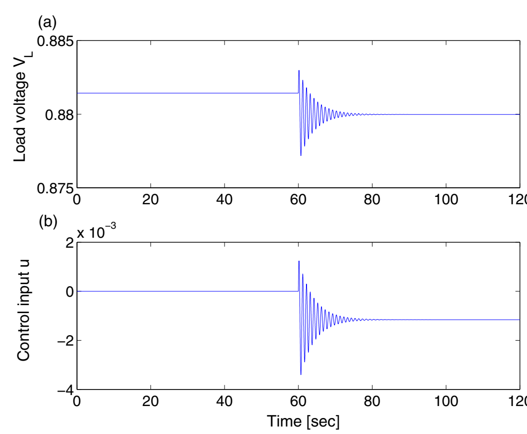

Fig. 11. Closed-loop simulation of a decrease in the load active power demand P_1 from 0.6 p.u. to 0.55 p.u. at time t=60 seconds for the power system with SVC and static state feedback ($B(0) = 0$). (a) load voltage, (b) controller input u.

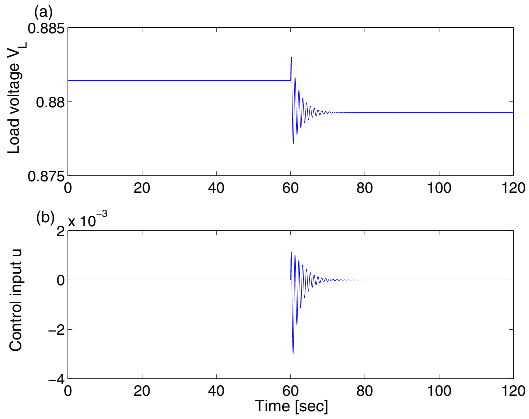

Fig. 12. Closed-loop simulation of a decrease in the load active power demand P_1 from 0.6 p.u. to 0.55 p.u. at time t=60 seconds for the power system with SVC with washout filter ($B(0) = 0$, $z(0) = \frac{V_L(0)}{d} = 0.8814$). (a) load voltage, (b) controller input u.

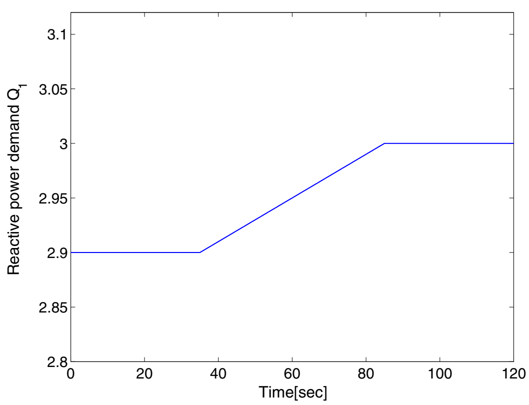

Fig. 13. The load reactive power demand Q_1 is ramped from 2.9 p.u. at time t=35 sec to 3.0 p.u. at time t=85 sec.

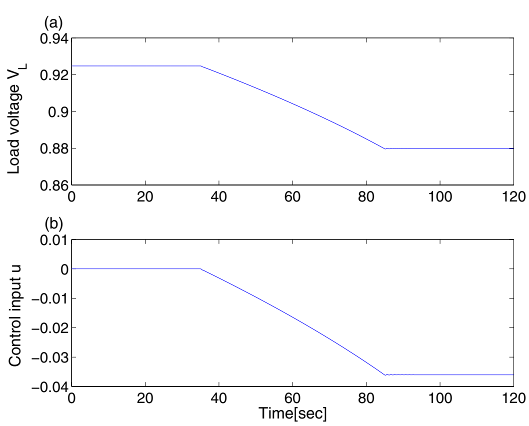

Fig. 14. Closed-loop simulation of a ramp increase in the load reactive power demand Q_1 from 2.9 p.u. at t=35 sec. to 3.0 p.u. at time t=85 sec. for the power system with SVC and static state feedback ($B(0) = 0$). (a) load voltage, (b) controller input u.

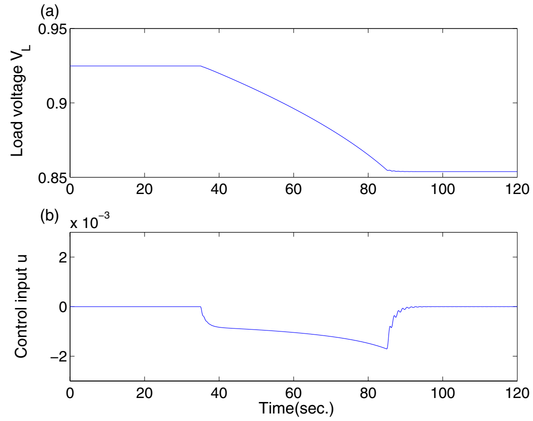

Fig. 15. Closed-loop simulation of a ramp increase in the load reactive power demand Q_1 from 2.9 p.u. at t=35 sec. to 3.0 p.u. at time t=85 sec. for the power system with SVC and washout filter-aided feedback ($B(0) = 0$, $z(0) = \frac{V_L(0)}{d} = 0.9248$). (a) load voltage, (b) controller input u.

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