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Fuzzy Model-Based Robust Controller Design for Hydrofoil Catamaran

Junsheng Ren, Yansheng Yang, Yunfeng Zheng and Tieshan Li

Abstract— **This paper addresses the robust fuzzy controller design for a class of nonlinear system with norm-bounded parametric uncertainties, and its application to attitude controller design for hydrofoil catamaran. First, a robust fuzzy controller is proposed, which can guarantee that the fuzzy system is asymptotically stable. The sufficient conditions are formulated in the form of linear matrix inequalities (LMI). The comparison between the new result and previous method shows that the stability conditions herein possess much less conservativeness. Second, The robust fuzzy controller is applied to attitude controller design for hydrofoil catamaran. Based on such a boat, "HC200B-A1", simulation researches demonstrate the drawbacks of conventional control strategies are overcome successfully.**

I. INTRODUCTION

Hydrofoil catamaran is a kind of high-speed boat, and composed of twin V-hulls and two or more rectangle hydrofoils at the bottom of the V-hulls. Both simulation study and tank experimental results [1], [2] manifest that waves will lead to roll, pitch, heave motion of large amplitude, which will discomfort passengers and seamen, and even destroy cargoes. Therefore, the special attitude stabilizer is indispensable for hydrofoil catamaran. Various studies on control systems for hydrofoil catamaran were studied [3], [4]. How ever, those traditional control strategies have several drawbacks. First, they are no longer efficient and feasible once out of the small neighborhood of the operating point, i.e. design speed. Second, they haven't taken account of the influences of environmental disturbances. Third, its zero left angles may subject to the fluctuations with time, because there exists the interferences between fore and aft hydrofoils.

Gain scheduling is an effective way of controlling systems whose dynamics change with the operating conditions. One drawback of conventional gain scheduling controller is that the parameter change may be rather abrupt across the region boundaries, which may result in unsatisfactory or unstable performance across the transition. Fuzzy gain scheduling has been proposed that utilizes fuzzy technique to determine the controller parameters. In fuzzy gain scheduling strategy, fuzzy inference mechanism is used to interpolate the controller parameters in the transition regions. As a result, it obviates the need to use exact mathematical descriptions in the interpolation of controller parameters [5], [6]. Therefore, fuzzy inference system makes it probable to have the feedback control covering the whole operating envelope [5] for hydrofoil catamaran. There exists voluminous literature on the subject of making use of various control techniques to attitude controller design for hydrofoil catamaran [3], [4]. However, very few papers are found that report Takagi-Sugeno (T-S) fuzzy system and the parallel distributed compensation (PDC) control structure, which can be used to control uncertain nonlinear systems, to satisfy the special requirements of attitude controller of hydrofoil catamaran.

Generally speaking, there are two kinds of fuzzy logic controller. One is model-free fuzzy controller, and the other is model-based fuzzy controller. Controller design based on T-S fuzzy model has been discussed in [5], [7]–[9]. Fuzzy model-based controller can combine the merits of both fuzzy controller and conventional linear theory, and furthermore guarantee stability in the sense of Lyapunov and control performance theoretically. Moreover, linear matrix inequality (LMI) techniques also make model-based fuzzy controller design more convenient. Since uncertainties will cause instability and degraded performance, robust fuzzy controller based on T-S fuzzy model had also discussed [8]. However, the stability conditions therein are of much conservativeness.

The main contributions of this paper are divided into two parts. First, a novel fuzzy model-based robust controller is proposed for a class of nonlinear system with norm-bounded parametric uncertainties. Because the controller design takes accounts of much closer relationship between the subsystems of the whole dynamic system, the conservativeness of the controller design is reduced efficiently. Second, by using the proposed controller scheme, attitude controller for hydrofoil catamaran are presented. Simulation researches manifest that the drawbacks of conventional control strategies have been overcome successfully.

This paper is organized as follows. T-S fuzzy system is constructed in Section II. The less conservative controller is proposed in Section III. In Section IV, the proposed scheme is applied to the attitude controller design for hydrofoil catamaran. Some conclusions are collected in Section V.

II. CONSTRUCTION OF T-S FUZZY MODELS

The development of T-S fuzzy model and its applications have been increasingly accelerated over the last decade. The T-S fuzzy system can be used to approximate the nonlinear

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system with norm-bounded parametric uncertainties [9], which is constructed as follows.

Plant Rule i :

IF $z_1(t)$ is M_{i1} and, $\ldots$, and z_p is M_{ip},

THEN $\dot{x}(t) = (A_i + \Delta A_i)\, x(t) + (B_i + \Delta B_i)\, u(t)$,

$$(i = 1,\ 2,\ \cdots,\ r.) \tag{1}$$

where $z(t) = \{z_1(t), z_2(t), \cdots, z_q(t)\}$ denote the variables of premise part, $A_i \in \Re^{n\times n}$, $B_i \in \Re^{m\times n}$, $x(t)$ denotes state vector, $u(t)$ denotes control input vector, and M_{il} denotes fuzzy sets, r denotes the number of IF - THEN rules, and $\Delta A_i \in \Re^{n\times n}$, $\Delta B_i \in \Re^{m\times n}$ are the system's uncertainty matrices and satisfy Assumption 1.

Assumption 1: Uncertainty matrices ΔA_i and ΔB_i are norm-bounded, and have the following structures:

$$[\Delta A_i\ \Delta B_i] = D_i F_i(t)\, [E_{i1}\ E_{i2}] \tag{2}$$

where D_i, E_{i1} and E_{i2} are constant real matrices of appropriate dimensions, and $F_i(t) \in \Re^{i\times j}$ is unknown matrix-valued functions with Lebesgue-measurable elements and satisfies

$$F_i^T(t)\, F_i(t) \le I \tag{3}$$

By using the fuzzy inference method with a singleton fuzzifier, product inference, and center average defuzzifiers, the final output of T-S fuzzy model is obtained as

$$\dot{x}(t) = \sum_{i=1}^{r} h_i(z(t))[(A_i + \Delta A_i)x(t) + (B_i + \Delta B_i)u(t)] \tag{4}$$

where

$$h_i(z(t)) = \omega_i(z(t)) \Big/ \sum_{i=1}^{r} \omega_i(z(t)),$$
$$w_i(z(t)) = \prod_{j=1}^{p} M_{ij}(z(t)),$$

and $M_{ij}(z(t))$ denotes the degree of membership of $z(t)$ on M_{ij}. The degree of membership satisfies

$$\sum_{i=1}^{r} \omega_i(z(t)) > 0, \quad \omega_i(z(t)) \ge 0,\ i = 1,\ 2,\ \cdots,\ r.$$

Note that for all t, there exists

$$\sum_{i=1}^{r} h_i(z(t)) = 1,\ h_i(z(t)) \ge 0,\ i = 1,\ 2,\ \cdots,\ r.$$

For PDC scheme, fuzzy controller and fuzzy model (4) possess the same premises. Then, supposing that all the states are observable, the i-th controller rule can be expressed by

Controller Rule i :

IF $z_1(t)$ is M_{i1} and, $\ldots$, and z_p is M_{ip},

THEN $u(t) = -K_i x(t)$, $(i = 1,\ 2,\ \cdots,\ r.)$ (5)

At the consequent part, fuzzy control rules have linear state feedback gain. It has been proved that the controller using the PDC scheme is an approximator for any nonlinear state feedback controller [9]. The overall fuzzy controller can be represented as follows

$$u(t) = -\sum_{i=1}^{r} h_i(z(t)) K_i x(t) \tag{6}$$

Therefore, the design of fuzzy controller is to design local feedback gain K_is. Then, the combination of (4) and (6) results in the overall closed-loop fuzzy system

$$\begin{aligned}\dot{x}(t) = \sum_{i=1}^{r}\sum_{j=1}^{r} & h_i(z(t))h_j(z(t))\, [A_i - B_i K_j \\ & + D_i F_i\, (E_{i1} - E_{i2}K_j)]\, x(t)\end{aligned} \tag{7}$$

III. LMI-Based Robust Fuzzy Controller Design

Before proceeding with the research on stability conditions for the closed-loop fuzzy system (7), some useful lemmas are introduced first.

Lemma 1: [10] For any constants $\varepsilon > 0$, and any matrices X and Y with appropriate dimensions, we have

$$X^T Y + Y^T X \le \varepsilon X^T X + \varepsilon^{-1} Y^T Y \tag{8}$$

Lemma 2: For matrices X, Y with appropriate dimensions, there exists an arbitrary scalar $\varepsilon > 0$, such that

$$XY + (XY)^T \le \varepsilon X X^T + \varepsilon^{-1} Y^T Y \tag{9}$$

Proof: As for (8), let $X = A^T$, then $X^T = A$. From (8), we have

$$AY + Y^T A^T \le \varepsilon A A^T + \varepsilon^{-1} Y^T Y$$

Let $A \to X$, then we have (9).

■

Theorem 1: The closed-loop T-S fuzzy system (7) is asymptotically stable, if there exist feedback gains K_is and symmetric positive definite matrix P such that

$$\Xi_{ii}^T P + P\Xi_{ii} - X_{ii} < 0, (i = 1, \cdots, r) \tag{10}$$
$$\Xi_{ij}^T P + P\Xi_{ij} - X_{ij} < 0, (1 \le i < j \le r) \tag{11}$$
$$(X_{ij})_{r\times r} - S < 0, (1 \le i < j \le r) \tag{12}$$

where ε is an arbitrary positive number, S is a negative semi-definite matrix chosen by designer, $(X_{ij})_{r\times r}$ denotes matrix with element X_{ij}, and

$$\begin{aligned}\Xi_{ii} = & (A_i - B_i K_i)^T P + P\,(A_i - B_i K_i) + \varepsilon P D_i D_i^T P \\ & + \varepsilon^{-1} (E_{i1} - E_{i2}K_i)^T (E_{i1} - E_{i2}K_i),\end{aligned}$$

$$\Xi_{ij}=\Big[(A_i-B_iK_j)^TP+P(A_i-B_iK_j) \\ +(A_j-B_jK_i)^TP+P(A_j-B_jK_i) \\ +\varepsilon PD_iD_i^TP+\varepsilon PD_jD_j^TP \\ +\varepsilon^{-1}(E_{i1}-E_{i2}K_j)^T(E_{i1}-E_{i2}K_j) \\ +\varepsilon^{-1}(E_{j1}-E_{j2}K_i)^T(E_{j1}-E_{j2}K_i)\Big]\Big/2.$$

Proof: Choose the Lyapunov candidate as

$$V(t)=x^T(t)Px(t)$$

where P is symmetric positive definite matrix.

Then, the time derivative of the Lyapunov candidate $V(t)$ along the trajectory of (7) is given by

$$\begin{aligned}\dot{V}(t)&=\dot{x}^T(t)Px(t)+x^T(t)P\dot{x}(t)\\&=\sum_{i=1}^{r}\sum_{i=1}^{r}h_i(t)h_j(t)x^T(t)\Big[(A_i-B_iK_j)^TP\\&+(E_{i1}-E_{i2}K_j)^TF_i^TD_i^TP+P(A_i-B_iK_j)\\&+PD_iF_i(E_{i1}-E_{i2}K_j)]x(t)\end{aligned}$$

After some manipulations, we have

$$\begin{aligned}\dot{V}(t)&=\sum_{i=1}^{r}h_i^2(z(t))x^T(t)\Big[(A_i-B_iK_i)^TP\\&+P(A_i-B_iK_i)+(E_{i1}-E_{i2}K_i)^TF_i^TD_i^TP\\&+PD_iF_i(E_{i1}-E_{i2}K_i)]x(t)\\&+\sum_{i<j}^{r}h_i(z(t))h_j(z(t))x^T(t)\Big\{\Big[(A_i-B_iK_j)^TP\\&+P(A_i-B_iK_j)+(E_{i1}-E_{i2}K_j)^TF_i^TD_i^TP\\&+PD_iF_i(E_{i1}-E_{i2}K_j)]/2\\&+\Big[(A_j-B_jK_i)^TP+P(A_j-B_jK_i)\\&+(E_{j1}-E_{j2}K_i)^TF_j^TD_j^TP\\&+PD_jF_j(E_{j1}-E_{j2}K_i)]/2\}x(t)\end{aligned}$$

From Lemma 2 and Assumption 1, we have

$$\begin{aligned}\dot{V}(t)&\le\sum_{i=1}^{r}h_i^2(z(t))x^T(t)\Big[(A_i-B_iK_i)^TP\\&+\varepsilon PD_iD_i^TP+P(A_i-B_iK_i)\\&+\varepsilon^{-1}(E_{i1}-E_{i2}K_i)^T(E_{i1}-E_{i2}K_i)\Big]x(t)\\&+\tfrac{1}{2}\sum_{i<j}^{r}h_i(z(t))h_j(z(t))x^T(t)\Big\{\Big[(A_i-B_iK_j)^TP\\&+P(A_i-B_iK_j)+\varepsilon PD_iD_i^TP\\&+\varepsilon^{-1}(E_{i1}-E_{i2}K_j)^T(E_{i1}-E_{i2}K_j)\Big]\\&+\Big[(A_j-B_jK_i)^TP+P(A_j-B_jK_i)\\&+\varepsilon PD_jD_j^TP\\&+\varepsilon^{-1}(E_{j1}-E_{j2}K_i)^T(E_{j1}-E_{j2}K_i)\Big]\Big\}x(t)\end{aligned}$$

Substitute (10) and (11) into the above inequality, and we obtain

$$\dot{V}(t)<\begin{bmatrix}h_1(z(t))x(t)\\h_2(z(t))x(t)\\\vdots\\h_r(z(t))x(t)\end{bmatrix}^T\times\begin{bmatrix}X_{11}&X_{12}\dots X_{1r}\\X_{21}&X_{22}\dots X_{2r}\\\vdots&\vdots\ddots\vdots\\X_{r1}&X_{r2}\dots X_{rr}\end{bmatrix}\times\begin{bmatrix}h_1(z(t))x(t)\\h_2(z(t))x(t)\\\vdots\\h_r(z(t))x(t)\end{bmatrix}$$

Next, substituting (12) into the above inequality yields

$$\begin{aligned}\dot{V}(t)&<\begin{bmatrix}h_1(z(t))x(t)\\h_2(z(t))x(t)\\\vdots\\h_r(z(t))x(t)\end{bmatrix}^TS\begin{bmatrix}h_1(z(t))x(t)\\h_2(z(t))x(t)\\\vdots\\h_r(z(t))x(t)\end{bmatrix}\\&=x^T(t)Sx(t)\\&\le-|\lambda_{\max}(S)|x^T(t)x(t)\end{aligned}$$

where $\lambda_{\max}(S)$ denotes the maximal eigenvalue of negative semi-definite S. Therefore, the closed-loop fuzzy system (7) is asymptotically stable.

■

The search for the common matrix P and K_is nowadays can resort to some efficient numerical methods in terms of LMIs. How ever, the conditions are not jointly convex in K_is and P in Theorem 1. Hence, Theorem 2 is proposed, in which the LMIs are tractable.

Theorem 2: The closed-loop T-S fuzzy system (7) is asymptotically stable, if there exist matrices M_is and symmetric positive definite matrix N such that

$$\begin{bmatrix}\Omega_{ii}&*\\E_{i1}N-E_{i2}M_i&-\varepsilon I\end{bmatrix}<0 \tag{13}$$

$$\begin{bmatrix}\Omega_{ij}&*&*\\E_{i1}N-E_{i2}M_j&-\varepsilon I&*\\E_{j1}N-E_{j2}M_i&0&-\varepsilon I\end{bmatrix}<0 \tag{14}$$

$$(Y_{ij})_{r\times r}-Q<0 \tag{15}$$

where

$$\begin{aligned}\Omega_{ii}&=A_iN-B_iM_i+N^TA_i-M_i^TB_i^T+\varepsilon D_iD_i^T-Y_{ii},\\\Omega_{ij}&=A_iN+A_jN-B_iM_j-B_jM_i+N^TA_i+N^TA_j\\&\quad-M_j^TB_i^T-M_i^TB_j^T+\varepsilon D_iD_i^T+\varepsilon D_jD_j^T-Y_{ij}-Y_{ij}^T,\end{aligned}$$

Q is negative semi-definite matrix chosen by the controller designer, $*$ denotes the transposed element in the symmetric position, $1\le i<j\le r$ and I is identity matrix. Furthermore, feedback gain K_is and symmetric positive definite matrix P are obtained by

$$P=N^{-1},\ K_i=M_iN^{-1} \tag{16}$$

Proof: Multiply (13)-(15) with P^{-1} both left and right side. Let $N=P^{-1}$, $M_i=K_iP^{-1}$, $Y_{ij}=NX_{ij}N$ and

$Q = (N)_{r\times r} S (N)_{r\times r}$, by use of Schur complements, we obtain (13) - (15).

■

Similar results were presented in [8] as follows.

Theorem 3: If there exist a symmetric and positive definite matrix P, some matrices K_is, and some ε_{ij}, $(i,\ j = 1,\ \ldots,\ r)$ such that the following LMIs are satisfied, then the continuous-time T-S fuzzy system (7) is asymptotically stable via the T-S fuzzy model-based state-feedback controller (6):

$$\begin{bmatrix} \Psi_{ii} & * & * \\ E_{1i}Q + E_{2i}M_i & -\varepsilon_{ii}I & * \\ D_i^T & 0 & -\varepsilon_{ii}^{-1}I \end{bmatrix} < 0 \qquad (17)$$

$$\begin{bmatrix} \Upsilon_{ij} & * & * & * & * \\ E_{1i}Q + E_{2i}M_j & -\varepsilon_{ij}I & * & * & * \\ E_{1j}Q + E_{2j}M_i & 0 & -\varepsilon_{ij}I & * & * \\ D_i^T & 0 & 0 & -\varepsilon_{ij}^{-1}I & * \\ D_j^T & 0 & 0 & 0 & -\varepsilon_{ij}^{-1}I \end{bmatrix} < 0 \qquad (18)$$

where

$$\begin{aligned} \Psi_{ii} =& QA_i^T + A_i^T Q + M_i^T B_i^T + B_i M_i, \\ \Upsilon_{ij} =& QA_i^T + A_i Q + QA_j^T + A_j Q + M_j^T B_i^T + B_i M_j \\ &+ M_i^T B_j^T + B_j M_i, \end{aligned}$$

and $Q = P^{-1}$, $M_i = K_i P^{-1}$, where $*$ denotes the transposed elements in the symmetric positions.

How ever, in comparison with Theorem 3, the results in Theorem 1 and 2 are much more relaxed. Comparisons are made as follows.

Example 1: Consider a T-S fuzzy model with two IF-THEN rules

Rule 1: IF $x_1(t)$ is M_{11},
THEN $\dot{x}(t) = (A_1 + \Delta A_1)\, x(t) + (B_1 + \Delta B_1)\, u(t)$;
Rule 2: IF $x_1(t)$ is M_{21},
THEN $\dot{x}(t) = (A_2 + \Delta A_2)\, x(t) + (B_2 + \Delta B_2)\, u(t)$,

where

$$A_1 = \begin{bmatrix} 2 & -10 \\ 1 & 0 \end{bmatrix},\ B_1 = \begin{bmatrix} 1 \\ 0 \end{bmatrix},$$

$$A_2 = \begin{bmatrix} a & -10 \\ 1 & 1 \end{bmatrix},\ B_2 = \begin{bmatrix} b \\ 0 \end{bmatrix},$$

and for the simplicity of the comparisons, the uncertainty matrices ΔA_i and ΔB_i are assumed to be zero matrices.

The local feedback gains for the two subsystems are both obtained by placing the poles at -1 and -15. Herein, parameter a is supposed to vary between 2 and 6. By using the method in 3, LMI problem of obtaining common P is feasible only when $b < 6.5$. The method in [7] is somewhat relaxed, however, when $b > 10$, the LMI problem is no longer feasible. In contrast, with Theorem 1 and 2, the LMI problem is always feasible even when $b < 10^7$. From the comparisons, it can be concluded that theorem 1 and 2 can relax the condition of asymptotical stability for T-S fuzzy system (7), and thus lower the conservativeness of the controller design.

TABLE I

PRINCIPAL PARTICULARS OF HYDROFOIL CATAMARAN, HC200B-A1

$L.B.P$(m)	breadth(m)	d_0(m)	Δ (ton)	speed(knot)
35.84	11.584	3.84	200	40
chord (m)	span (m)	x_{f1} (m)	x_{f2} (m)	$\alpha_{1,2}$ (°)
0.96	8.32	6.56	-15.84	2

IV. APPLICATION TO ATTITUDE CONTROLLER DESIGN FOR HYDROFOIL CATAMARAN

To design the attitude controller for hydrofoil catamaran, the mathematical model of hydrofoil catamaran is established for the boat's heave and pitch motion first [2].

$$\begin{cases} m(\ddot{\xi} + U\dot{\theta}) = \sum\limits_{i=1}^{2} F_{fi} + \sum\limits_{i=1}^{2} F_{fpi} + F_H + mg\cos\theta \\ I_{yy}\ddot{\theta} = -\sum\limits_{i=1}^{2} (x_{fi} - x_g)F_{fi} - \sum\limits_{i=1}^{2} (x_{fpi} - x_g)F_{fpi} \\ \qquad -(x_b - x_g)\nabla\cos\theta - 2(x_H - x_g)L_H \end{cases} \qquad (19)$$

where m is the mass of boat, U is the along-ship velocity, F_{fi} is the force arising from hydrofoil, F_{fpi} is the force from flap, g is gravitational acceleration, i.e. $9.8\ \mathrm{m/s^2}$, F_H is the force relevant to boat's hulls, L_H is the lift force of the hulls, and I_{yy} is the inertia of moment about Y-axis. θ is pitch angle, and ξ is boat's elevation. x_{fi}, x_{pi}, x_G, x_b and x_H are the coordinates of hydrofoil's lift force, flap's lift force, gravitational force, buoyant force and hull's lift force on X-axis, respectively. Simulation researches are carried out, based on hydrofoil catamaran, HC200B-A1, the principal particulars of which are listed in Table I [1].

Our T-S fuzzy model for hydrofoil catamaran takes boat's speed U_i as language variable. Its membership functions are shown in Fig. 1.

Plant Rule i: IF $U\ is\ U_i$,
THEN $\dot{x}^e(t) = (A_i + \Delta A_i)x^e(t) + (B_i + \Delta B_i)u(t)$;

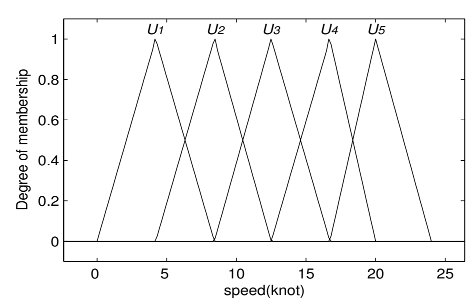

Fig. 1. Membership function

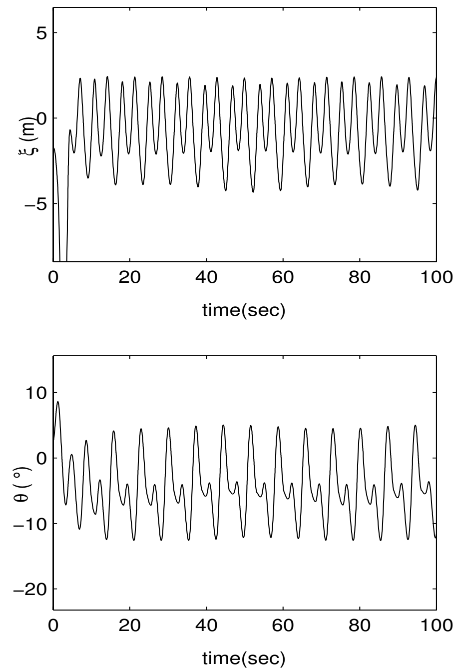

Fig. 2. Time response of HC200B-A1's attitude, i.e. elevation (ξ) and pitch angle (θ) with controller off

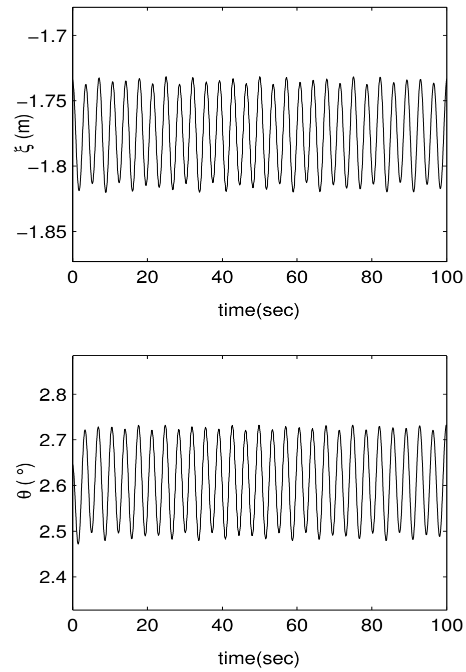

Fig. 3. Time response of HC200B-A1's attitude, i.e. elevation (ξ) and pitch angle (θ) under control

Controller Rule i:

IF U is U_i, THEN $u(t) = K_i x(t)$.

where $x^e(t) = [\dot{\xi}_1^e, \dot{\xi}_2^e, \dot{\theta}_1^e, \dot{\theta}_2^e]$ denotes state vector, and $u(t) = [\alpha_{fp1}, \alpha_{fp2}]$ denotes control input [2].

HC200B-A1's local state-space models in the consequents are obtained through global linearization method, at 5 operating points all covered by the working envelope. The coefficient matrices of each subsystem can be found in [11]. Suppose that all the elements of uncertain matrices ΔB_i are zero. Because zero lift angles in model (19) contain the interactions between fore and aft foils, they tend to vary with time. Herein, zero lift angles are assumed to fluctuate between 20% of their nominal values in this study. Therefore, ΔA_is are obtained by

$$\Delta A_i = A_i \mid_{\alpha_0} - A_i \mid_{(\alpha_0 \times 120\%)} \tag{20}$$

where $A_i \mid_{\alpha_0}$ denotes A_i with the nominal value of zero lift angle α_0, and $A_i \mid_{(\alpha_0 \times 120\%)}$ denotes A_i at its 120% times of the nominal value. Arbitrary positive scalar ε and matrix Q are chosen as 1 and $-3I$, respectively. By use of Matlab LMI Control Toolbox, the local feedback gains K_is and common P are presented in Appendix. To testify the robust fuzzy controller in Section III, an operating point, e.g. 30 knots, about 55.56 km/h, which is selected arbitrarily, which is not one of the above 5 operating points.

Simulation results are presented with attitude controller off in Fig. 2, from which it can be seen that wave-induced heave and pitch motion are too large to tolerate. During the simulation research, zero lift angles fluctuate with time, namely

$$\tilde{\alpha}_0 = \alpha_0(1 + 0.2 \sin t) \tag{21}$$

Wave parameters are that, ahead sea, wavelength is 100 meters, period 8 seconds, and wave height 5 meters. The expected elevation ξ and pitch angle θ are -1.74 meters and $2.65°$, respectively. Fig. 3 shows the simulation results under control. Simulation researches manifest that, the desired attitude can be achieved satisfactorily at the arbitrary speed, and the wave-induced heave and pitch motion can also be attenuated efficiently in comparison with Fig. 2.

From Fig. 3, conclusion can be drawn that the drawbacks of traditional control strategy is overcome successfully by use of the proposed fuzzy model-based robust controller in Section III. Fig. 4 displays the control effects to achieve the performance in Fig. 3, i.e. the variations of fore and aft flap angle, respectively.

V. Conclusions and Future Works

A. Conclusions

In this paper, a novel robust fuzzy controller design has been addressed via fuzzy interpolation of a series of linear systems. The design is featured by less conservativeness for a class of nonlinear system with norm-bounded parametric uncertainties, in which the relationship between the subsystems are much closely taken into account. Furthermore, an example has shown that the effectiveness of the fuzzy model-based robust controller for hydrofoil catamaran.

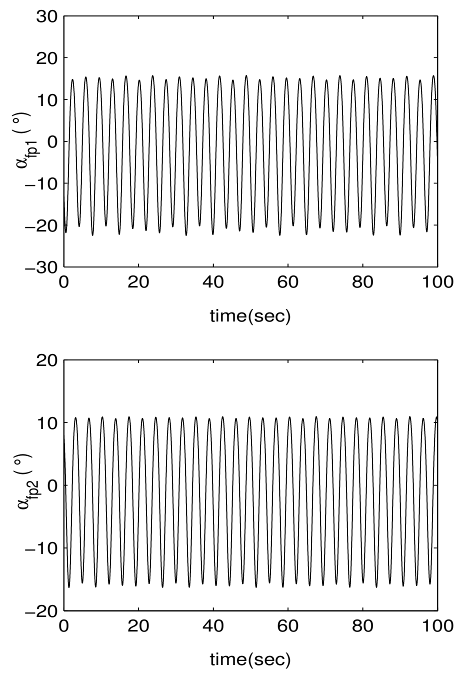

Fig. 4. Time response of fore (α_{fp1}) and aft (α_{fp2}) flap control angles

B. Future Works

There is still a lot of further work left to deal with, e.g. the modelling of six degrees of freedom, how to suppress the rolling motion of hydrofoil catamaran in rough seas, and etc.

APPENDIX

The uncertain matrices for HC200B-A1's state-space models are presented as follows, at 5 operating points.

$$\Delta A_1^T = \begin{bmatrix} 0 & 1.0598 \times 10^{-5} & 0 & 1.1110 \times 10^{-8} \\ 0 & 1.8406 \times 10^{-1} & 0 & 1.9297 \times 10^{-4} \\ 0 & 4.6447 \times 10^{-1} & 0 & 4.8695 \times 10^{-4} \\ 0 & -1.7514 & 0 & 7.7904 \times 10^{-2} \end{bmatrix},$$

$$\Delta A_2^T = \begin{bmatrix} 0 & 6.1913 \times 10^{-5} & 0 & 6.4921 \times 10^{-8} \\ 0 & 3.3546 \times 10^{-1} & 0 & 3.5166 \times 10^{-4} \\ 0 & 2.3548 & 0 & 2.4692 \times 10^{-2} \\ 0 & -15.098 & 0 & 6.5673 \times 10^{-2} \end{bmatrix},$$

$$\Delta A_3^T = \begin{bmatrix} 0 & 6.9025 \times 10^{-4} & 0 & 7.2405 \times 10^{-7} \\ 0 & 1.5138 & 0 & 1.5879 \times 10^{-3} \\ 0 & 12.392 & 0 & 1.2999 \times 10^{-2} \\ 0 & -6.4841 & 0 & 7.7885 \times 10^{-1} \end{bmatrix},$$

$$\Delta A_4^T = \begin{bmatrix} 0 & 9.1868 \times 10^{-3} & 0 & 9.6369 \times 10^{-6} \\ 0 & 2.0359 & 0 & 2.1357 \times 10^{-3} \\ 0 & 28.627 & 0 & 3.0030 \times 10^{-2} \\ 0 & -6.0000 & 0 & 1.0949 \end{bmatrix},$$

$$\Delta A_5 = \begin{bmatrix} 0 & 4.5721 \times 10^{-2} & 0 & 4.7962 \times 10^{-5} \\ 0 & 1.5795 & 0 & 1.6569 \times 10^{-3} \\ 0 & 35.247 & 0 & 3.6974 \times 10^{-2} \\ 0 & -7.9072 & 0 & 0.79277 \end{bmatrix};$$

By use of Matlab LMI Control Toolbox, the local feedback gain K_is and common P for "HC200B-A1" are presented as follows.

$$K_1 = \begin{bmatrix} 25.697 & 112.54 & 754.52 & 661.72 \\ 22.986 & 93.243 & 885.30 & 751.17 \end{bmatrix},$$

$$K_2 = \begin{bmatrix} 21.782 & 96.911 & 602.55 & 534.85 \\ 20.026 & 81.636 & 764.96 & 48.45 \end{bmatrix},$$

$$K_3 = \begin{bmatrix} 18.024 & 76.828 & 591.19 & 510.02 \\ 15.929 & 62.704 & 671.21 & 563.14 \end{bmatrix},$$

$$K_4 = \begin{bmatrix} 6.7346 & 9.3968 & 735.78 & 575.59 \\ 4.3574 & 3.1563 & 559.58 & 435.31 \end{bmatrix},$$

$$K_5 = \begin{bmatrix} 6.6951 & 10.457 & 706.59 & 549.41 \\ 4.2480 & 3.4506 & 537.05 & 416.46 \end{bmatrix};$$

$$P = \begin{bmatrix} 9.9375 \times 10^{-3} & 2.3178 \times 10^{-2} & -5.2587 \times 10^{-2} & -3.8504 \times 10^{-2} \\ 2.3178 \times 10^{-2} & 1.2847 \times 10^{-1} & 1.5426 \times 10^{-2} & -7.1486 \times 10^{-6} \\ 2.3178 \times 10^{-2} & 1.2847 \times 10^{-1} & 7.2226 & 3.7499 \\ -5.2587 \times 10^{-2} & 1.5426 \times 10^{-2} & 3.7499 & 2.9806 \\ -3.8504 \times 10^{-2} & -7.1486 \times 10^{-6} & & \end{bmatrix}.$$

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