Control of an Autonomous Underwater Vehicle Platoon with a Switched Communication Network

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***Abstract*— We examine the stability and behavior of a platoon of autonomous vehicles which operate in a decentralized manner and communicate to achieve formation control objectives. We are interested in communication networks that are time-varying, with each vehicle interacting with different sets of vehicles as the system evolves. We are particularly interested in networks which are disconnected, and in which basic control objectives such as stability cannot be achieved, in a frozen-time sense. Graph theoretical concepts are used to help model the platoon and provide a framework for stability analysis. Periodic fast switching, a tool from the field of switching theory, is adopted to assess stability of systems with time-varying communication networks. We show that if switching is sufficiently fast, then a platoon of autonomous vehicles can be stabilized even when the communication network is disconnected in frozen time.**

I. INTRODUCTION

The properties and behavior of distributed dynamical systems are important areas of research in a number of scientific and technological fields [1]. Communication among components is an essential factor in the operation of many distributed systems, and a significant amount of research in the field of robotics has investigated the effect of communication in vehicle platoon control and estimation activities [2], [3].

For the purpose of illustrating the role of communication in distributed systems, we examine the behavior of an autonomous underwater vehicle (AUV) platoon. It should be emphasized that in this paper, our purpose in examining AUV behavior is neither to develop a realistic application model nor to propose a practical control algorithm, but rather to establish a framework for investigating networked system communication issues. A platoon of AUVs is a particularly compelling framework, since underwater communication bandwidth is severely limited. Practical examples of underwater vehicle platoons are presented in [4] and [5]. In both cases, vehicles communicate only when they surface.

Analysis of networked systems often relies on concepts from the field of graph theory [6]. In this section, graph theoretical concepts suitable for describing communication networks are defined, and the motivation for and contribution of our research is presented.

A. Graph Theoretical Characterization of Communication Networks

Concepts from graph theory [7] are helpful in understanding communication networks. A network of communicating agents can be viewed as a graph $\mathbb{G}$, which is defined by a set of vertices $\mathbb{V}$ and a set of edges $\mathbb{E}$ connecting the vertices. The vertex $v_i \in \mathbb{V}$, with $i \in \{1,\dots,N\}$, corresponds to agent i, and the edge $(v_i, v_j) \in \mathbb{E}$, with $i, j \in \{1,\dots,N\}$, corresponds to an active communication link between agents i and j. The adjacency matrix $\mathcal{A} = [a_{ij}] \in \{0,1\}^{N\times N}$, where

$$a_{ij} = \begin{cases} 0 & (v_i, v_j) \notin \mathbb{E} \\ 1 & (v_i, v_j) \in \mathbb{E} \end{cases}$$

indicates which vertices are connected by an edge, and therefore which agents communicate. Communication between agents is bidirectional if the graph representing the network is undirected, i.e. if $\mathcal{A} = \mathcal{A}^T$.

The following definitions relate properties of communication networks studied in this paper to characteristics of a graph $\mathbb{G}$.

Definition 1.1: A communication network is *time-invariant* if a constant $\mathbb{E}$ defines the graph for all time.

Definition 1.2: A communication network is *time-varying* if $\mathbb{G} = \mathbb{G}(t)$ depends explicitly on time t, a condition which amounts to switching between a finite number of edge sets $\mathbb{E}_1,\dots,\mathbb{E}_r$, with $r \in \mathbb{N}$, as the graph evolves.

Definition 1.3: A communication network is *T-periodic* if $\mathbb{G}(t) = \mathbb{G}(t+T)$ for all t, where $T > 0$.

Definition 1.4: A time-invariant communication network is *connected* if a path exists between every pair of vertices in $\mathbb{G}$, i.e. for each i, j pair, there is an $m \in \{0,\dots,N-1\}$ such that $a_{ij,m} > 0$, where $[a_{ij,m}] = \mathcal{A}^m$.

Definition 1.5: A time-invariant network is *disconnected* if it is not connected.

Definition 1.6: A time-varying network is *disconnected in frozen time* if $\mathbb{G}(t)$ is disconnected for all t.

Definition 1.7: A time-varying network is *jointly connected* over a time interval $[t_0, t_f]$ if the union of its frozen-time edge sets $\mathbb{E}_1 \cup \ldots \cup \mathbb{E}_r$ during the interval constitutes a connected graph [8].

Note that a network which is disconnected in frozen time may be jointly connected.

B. Research Motivation and Contribution

Much of the existing distributed system research conceives communication among agents as a time-invariant network, or as a time-varying network whose frozen-time connectivity supports satisfactory operation, an example of which is the study of the robustness of connected systems to communication failures [9]. However, it is sometimes not possible for practical systems to meet such frozen-time connectivity requirements. Communication may be constrained by a number of factors, including bandwidth limitations and environmental obstacles, that prevent a network from possessing suitable connectivity at any point of time. At present, the behavior of distributed systems which are disconnected in frozen time is relatively unexplored.

One recent investigation of systems that are disconnected in frozen time is [10], which examines connectivity in evolving networks of agents that communicate expiring messages. Synchronization of oscillators associated with the agents is used to probe network connectivity. A main result of the paper is that robust synchronization can occur if the network evolution generates paths of communication at time scales compatible with message dynamics, even if the frozen-time connectivity is below the threshold required for robust synchronization, and even if groups of agents are unable to communicate at every point in time.

Another relevant investigation is reported in [8], in which a group of mobile agents coordinates direction of movement through communication among neighboring agents. Neighborhoods are allowed to change as agents move relative to one another. The results are shown to apply when the level of connectivity that would be required of a time invariant network does not exist at any point in time. Using a graph theoretical approach, the authors introduce the concept of a jointly connected system, in which the union of the graphs representing the frozen-time communication patterns is equivalent to a connected graph.

In studying the behavior of distributed systems with time-varying communication patterns, results from switched system theory are useful [11], [12]. Switched, or hybrid, systems are characterized by piecewise continuous dynamics, and a switching policy that specifies how the dynamics change at distinct points in time. A number of useful techniques are available for switched system stability analysis, including common Lyapunov function, multiple Lyapunov function, and dwell time techniques. However, many of these strategies are ineffective when applied to distributed dynamical systems which lack frozen-time communication connectivity.

In this paper, we investigate the effect of a switched system technique, fast switching, on the behavior of an AUV platoon with a periodic communication network that is disconnected in frozen time. We show that if the switching period is sufficiently small, the response of the switched system is stable and approximates the response of the time-invariant system whose dynamics are defined by the convex combination of the underlying piecewise continuous dynamics. To illustrate the results of our theoretical investigation, we examine a platoon in which each vehicle communicates with at most one other vehicle at any point in time. We propose a simple decentralized controller for the platoon, due to which vehicles travel in formation while regulating their average position.

Our first step is to introduce fast switching theory.

II. Fast Switching Theory

Consider the linear time-varying system

$$\dot{x} = A(t)x \tag{1}$$

where $x \in \mathbb{R}^n$, and $A(t) \in \mathbb{R}^{n\times n}$ is T-periodic and switches between r constant matrices $A_1, \dots, A_r$, so that

$$A(t) = \sum_{i=1}^{r} A_i \chi_i(t) \tag{2}$$

The indicator function $\chi_i(t)$ with support $[(k+\epsilon_{i-1})T, (k+\epsilon_i)T)$, for $i \in \{1, \dots, r\}$ and $k \in \{0, 1, 2, \dots\}$, specifies which constant matrix A_i applies at a particular point in time. The constant scalars ϵ_i represent switching times and satisfy

$$0 = \epsilon_0 < \epsilon_1 < \dots < \epsilon_{r-1} < \epsilon_r = 1$$

The fraction of time that A_i is active is $\delta_i = \epsilon_i - \epsilon_{i-1}$.

The main result of this section is to show that if

$$\bar{A} = \sum_{i=1}^{r} \delta_i A_i \tag{3}$$

is Hurwitz, then there exists a T^* such that the state equation (1) with coefficient matrix (2) is uniformly asymptotically stable if $0 < T < T^*$. To obtain this result for switched systems, we first establish a result which applies to time-varying systems in general. The following lemma, adapted from [13], provides a bound on the transition matrix of a linear time-varying system.

Lemma 2.1: Consider the linear time-varying system

$$\dot{x} = F(t)x$$

where $x \in \mathbb{R}^n$ and $F(t) \in \mathbb{R}^{n\times n}$, and suppose that

$$\|F\|_\infty = \sup_{t\geq 0} \|F(t)\|$$

exists. Then for all $\Gamma > 0$ the transition matrix associated with $F(t)$ satisfies

$$\Phi(\tau + \Gamma, \tau) = e^{\Gamma F_\Gamma(\tau)} + R(\tau, \Gamma) \tag{4}$$

where

$$F_\Gamma(\tau) = \frac{1}{\Gamma}\int_\tau^{\tau+\Gamma} F(\sigma)d\sigma \tag{5}$$

and

$$\|R(\tau, \Gamma)\| \leq \|F\|_\infty^2 \Gamma^2 e^{\Gamma\|F\|_\infty}$$

Proof: The expression for the transition matrix (4) is determined from its Peano-Baker series representation

$$\begin{aligned}\Phi(\tau+\Gamma,\tau) &= I+\Gamma F_\Gamma(\tau)\\ &\quad+\sum_{k=2}^{\infty}\int_\tau^{\tau+\Gamma}F(\sigma_1)\int_\tau^{\sigma_1}\dots\int_\tau^{\sigma_{k-1}}F(\sigma_k)d\sigma_k\dots d\sigma_1\\ &= e^{\Gamma F_\Gamma(\tau)}+R(\tau,\Gamma)\end{aligned}$$

where

$$\begin{aligned}R(\tau,\Gamma) &= -\sum_{k=2}^{\infty}\frac{\Gamma^k F_\Gamma^k(\tau)}{k!}\\ &\quad+\sum_{k=2}^{\infty}\int_\tau^{\tau+\Gamma}F(\sigma_1)\int_\tau^{\sigma_1}\dots\int_\tau^{\sigma_{k-1}}F(\sigma_k)d\sigma_k\dots d\sigma_1\end{aligned}$$

The norm of each multiple integral satisfies

$$\begin{aligned}&\left\|\int_\tau^{\tau+\Gamma}F(\sigma_1)\int_\tau^{\sigma_1}\dots\int_\tau^{\sigma_{k-1}}F(\sigma_k)d\sigma_k\dots d\sigma_1\right\|\\ &\quad\le\|F\|_\infty^k\int_\tau^{\tau+\Gamma}\int_\tau^{\sigma_1}\dots\int_\tau^{\sigma_{k-1}}d\sigma_k\dots d\sigma_1=\frac{\Gamma^k\|F\|_\infty^k}{k!}\end{aligned}$$

Because $F_\Gamma(\tau)$ represents the average value of $F(t)$ over an interval, $\|F_\Gamma(\tau)\|\le\|F\|_\infty$. Therefore

$$\begin{aligned}\|R(\tau,\Gamma)\| &\le \sum_{k=2}^{\infty}\frac{\Gamma^k\|F\|_\infty^k}{k!}+\sum_{k=2}^{\infty}\frac{\Gamma^k\|F\|_\infty^k}{k!}\\ &\le 2\left(\frac{\Gamma^2\|F\|_\infty^2}{2}\right)\sum_{k=2}^{\infty}\frac{\Gamma^{k-2}\|F\|_\infty^{k-2}}{(k-2)!}\\ &= \Gamma^2\|F\|_\infty^2 e^{\Gamma\|F\|_\infty}\end{aligned}$$

■

Having established a transition matrix bound for general linear time-varying systems, we next derive a stability result for periodically switched systems. The matrix measure

$$\mu(M)=\lim_{\gamma\downarrow 0}\frac{\|I+\gamma M\|-1}{\gamma}$$

for $M\in\mathbb{R}^{n\times n}$ is used to obtain this result. A useful property of the matrix measure is [14]

$$\|e^{MT}\|\le e^{T\mu(M)}$$

Another useful property is [13]

$$\mu(M)=\frac{1}{2}\bar{\lambda}(P^{1/2}MP^{-1/2}+P^{-1/2}M^TP^{1/2}) \quad (6)$$

where $\bar{\lambda}(Q)$ is the maximum eigenvalue of $Q\in\mathbb{R}^{n\times n}$, and $P\in\mathbb{R}^{n\times n}$ is symmetric positive definite.

The following theorem establishes stability conditions for T-periodic systems with fast switching.

Theorem 2.1: Suppose that (3), defined relative to (2), is Hurwitz. Then there exists a $T^*>0$ such that (1) is uniformly asymptotically stable if $0<T<T^*$.

Proof: $A_T(\tau)$, defined using (5), satisfies

$$A_T(0)=\frac{1}{T}\int_0^T A(\sigma)d\sigma=\sum_{i=1}^{r}\delta_i A_i=\bar{A}$$

By Lemma 2.1, the transition matrix associated with (2) over a switching period is $\Phi(T,0)=e^{\bar{A}T}+R(0,T)$, where

$$\begin{aligned}\|\Phi(T,0)\| &\le \|e^{\bar{A}T}\|+\|R(0,T)\|\\ &\le e^{T\mu(\bar{A})}+\|A\|_\infty^2T^2e^{T\|A\|_\infty} \quad (7)\end{aligned}$$

We next show that $\mu(\bar{A})<0$, following a line of argument similar to one presented in [13]. Because $\bar{A}$ is Hurwitz, there is a symmetric positive definite $P\in\mathbb{R}^{n\times n}$ that satisfies the Lyapunov equation $\bar{A}^TP+P\bar{A}=-I$. Then (6) yields

$$\begin{aligned}\mu(\bar{A}) &= \frac{1}{2}\bar{\lambda}(P^{1/2}\bar{A}P^{-1/2}+P^{-1/2}\bar{A}^TP^{1/2})\\ &= \frac{1}{2}\bar{\lambda}(P^{-1/2}[P\bar{A}+\bar{A}^TP]P^{-1/2})=-\frac{1}{2}\bar{\lambda}(P^{-1})\end{aligned}$$

All of the eigenvalues of P^{-1} are positive, due to the positive definiteness of P. Therefore $\mu(\bar{A})<0$.

Defining

$$g(T)=e^{T\mu(\bar{A})}+\|A\|_\infty^2T^2e^{T\|A\|_\infty}$$

we make the following observations: $g(T)$ is continuous in T; $g(0)=1$; $g(T)\to\infty$ as $T\to\infty$; and $\frac{d}{dT}g(T)|_{T=0}<0$. These observations allow us to conclude that there exists a $T^*>0$ such that $g(T^*)=1$ and $g(T)<1$ for $T\in(0,T^*)$. Therefore $\|\Phi(T,0)\|<1$ for all $T\in(0,T^*)$. ■

The following corollary is a direct extension of Theorem 2.1 to linear time-invariant systems with periodically switched state feedback control laws.

Corollary 2.1: Consider the linear time-invariant system

$$\dot{x}=Ax+Bu$$

with the T-periodic state feedback control $u=K(t)x$, where $K(t)=\sum_{i=1}^{r}K_i\chi_i(t)$ switches between r constant matrices $K_1,\dots,K_r$, and $\chi_i(t)$ is defined as in (2). Suppose that $\bar{K}=\sum_{i=1}^{r}\delta_iK_i$ is such that $A+B\bar{K}$ is Hurwitz. Then there exists a $T^*>0$ such that

$$\dot{x}=[A+BK(t)]\,x$$

is uniformly asymptotically stable for all $T\in(0,T^*)$.

III. AUV Platoon Control

For the purpose of illustration, we consider a simple one-dimensional model of a platoon of AUVs which are tasked with traveling in formation. The objective for the platoon is twofold. First, the mean position of the vehicles should follow a specified trajectory. Second, the relative positions of the vehicles should match a desired configuration.

Platoon control is performed in a decentralized manner, with each vehicle ultimately responsible for controlling its own movement. However, vehicles do not operate independently, because communication between vehicles is used to achieve the formation control objectives. Each vehicle measures its own position, and it shares that information with other vehicles.

A. Time-Invariant, Connected Communication Network

We first consider control of a platoon whose communication network is time-invariant and connected. We consider this case initially because it establishes results that are useful in addressing the switched communication problem. Additionally, simulations of the time-invariant, connected system provide a reference against which simulations of the switched system may be evaluated.

As a simplification, we assume that vehicle movement in one dimension is decoupled from movement in all other dimensions. The vehicles are modeled as integrators in a single dimensional coordinate system. Defining $q = [q_1 \ \dots \ q_N]^T \in \mathbb{R}^N$ as the vector of vehicle positions and $u = [u_1 \ \dots \ u_N]^T \in \mathbb{R}^N$ as the vector of velocity command inputs, the state equation for the platoon is written as

$$\dot{q} = u \tag{8}$$

The scalar $\tilde{h}$ is defined as the desired average position of the platoon, and $\tilde{q}_h = [\tilde{h} \ \dots \ \tilde{h}]^T \in \mathbb{R}^N$ consists of $\tilde{h}$ repeated N times. The actual mean position of the vehicles is $h = (1/N)\sum_{i=0}^{N} q_i$, and $q_h = [h \ \dots \ h]^T \in \mathbb{R}^N$ is defined similarly to $\tilde{q}_h$.

Because control is decentralized, vehicles calculate the average position of the platoon individually. If a vehicle communicates with every other vehicle, then it can compute the true average position. In the absence of full communication, a vehicle must calculate an approximation of the mean, using the positions of the vehicles about which it has information. The vector $\hat{q}_h \in \mathbb{R}^N$ contains the average platoon position computed by each vehicle. The equation

$$\hat{q}_h = Gq \tag{9}$$

indicates how the calculation is performed. The matrix $G = [g_{ij}] \in \mathbb{R}^{N\times N}$ is nonnegative, i.e. $g_{ij} \geq 0$ for all $i, j \in \{1, \dots, N\}$. Because vehicles use their own position measurements to calculate the average position, $g_{ii} > 0$. If a communication link exists between vehicles i and j, then $g_{ij} > 0$ and $g_{ji} > 0$. Otherwise, $g_{ij} = g_{ji} = 0$. It is not necessarily the case, though, that $g_{ij} = g_{ji}$ when a link exists, i.e. $G \neq G^T$ in general. The asymmetry of G is due to the fact that each vehicle's calculation of the average depends on the number of vehicles with which it communicates. The nonzero elements in each row g_i are equal, and $\sum_{j=1}^{N} g_{ij} = 1$ for all i. For example, for a platoon of three vehicles in which communication exists between vehicles 1 and 2 and between vehicles 1 and 3

$$G = \begin{bmatrix} \frac{1}{3} & \frac{1}{3} & \frac{1}{3} \\ \frac{1}{2} & \frac{1}{2} & 0 \\ \frac{1}{2} & 0 & \frac{1}{2} \end{bmatrix}$$

The vector $\tilde{q}_d \in \mathbb{R}^N$ contains the desired offsets of the vehicle positions from the mean position. Consistency requires that $\sum_{i=1}^{N} \tilde{q}_{di} = 0$, where $\tilde{q}_{di}$ is the desired offset for vehicle i. The vector $d = q - q_h$ contains the actual set of relative distances, and $q_d = q - \hat{q}_h$ contains each vehicle's approximation of its distance from the average position.

Control must address the two platoon objectives specified above. In terms of the quantities that have been defined, tracking the desired trajectory with the mean vehicle position amounts to driving the error $h - \tilde{h}$ to zero, and maintaining the desired arrangement around the average position amounts to driving the error $d - \tilde{q}_d$ to zero. Each vehicle attempts to control its movement in a manner that reduces these errors, using its knowledge of the system state.

The following feedback control law for (8) is proposed

$$u = -k_h \left(\hat{q}_h - \tilde{q}_h\right) - k_d \left(q_d - \tilde{q}_d\right)$$

where k_h and k_d are scalar gains. The closed loop system can then be written as

$$\dot{q} = -\left[k_h G + k_d (I - G)\right] q + k_h \tilde{q}_h + k_d \tilde{q}_d \tag{10}$$

The feedback algorithm is implemented in a decentralized manner, because the control input associated with each vehicle depends only on the information available to that vehicle. The following lemma provides sufficient stability conditions.

Lemma 3.1: If the gains k_h and k_d satisfy

$$k_d \geq k_h > 0 \tag{11}$$

then the system (10) is uniformly asymptotically stable.

Proof: Uniform asymptotic stability is guaranteed if the coefficient matrix

$$-\left[k_h G + k_d (I - G)\right] = -k_d \left[\left(\frac{k_h}{k_d} - 1\right) G + I\right] \tag{12}$$

is Hurwitz. Defining $\kappa = k_h/k_d - 1$, if the condition (11) is satisfied, then $-1 < \kappa \leq 0$. Recalling the characteristics of G, $\kappa G + I = [b_{ij}]$ is a matrix for which $b_{ii} > 0$, $-1 < b_{ij} \leq 0$ for $i \neq j$, and $\sum_{j=1}^{N} b_{ij} = \kappa + 1 > 0$ for all i.

The Geršgorin disc theorem [15] states that the eigenvalues of $D = [d_{ij}] \in \mathbb{R}^{n\times n}$ are located in the union of discs

$$\bigcup_{i=1}^{n} \{z \in \mathbb{C} \,:\, |z - d_{ii}| \leq s_i(D)\}$$

where

$$s_i(D) = \sum_{j=1,\, j\neq i}^{n} |d_{ij}|\,, \quad 1 \leq i \leq n$$

is the deleted absolute row sum of D for row i. The properties of $B = \kappa G + I$ indicate that $b_{ii} > s_i(B)$ for every row. Therefore the eigenvalues of B are all located in the positive real half-plane, and the coefficient matrix (12) is Hurwitz. ∎

B. Time-Varying, Disconnected in Frozen Time Communication Network

We now apply fast switching theory to the platoon formation control problem. We consider the case where the communication network is disconnected in frozen time, but periodically time-varying and jointly connected across time.

We define a time-varying matrix

$$G(t) = \sum_{i=1}^{r} G_i \chi_i(t) \tag{13}$$

to replace the time-invariant matrix in (9). The matrix $G(t)$ indicates how the vehicles communicate and calculate the average position of the platoon as the network configuration switches. One of the constant matrices $G_1, \ldots, G_r$ is active at any point of time.

The state equation for the platoon is the time-varying version of (10)

$$\dot{q} = -\left[k_h G(t) + k_d(I - G(t))\right] q + k_h \tilde{q}_h + k_d \tilde{q}_d \tag{14}$$

Control design consists of selecting the communication patterns and switching times in such a way that the coefficient matrix $-\left[k_h \bar{G} + k_d(I - \bar{G})\right]$ is Hurwitz, where $\bar{G} = \sum_{i=1}^{r} \delta_i G_i$ and δ_i has the same meaning as in (3). Control design also involves choosing the gains k_h and k_d, and sufficiently small switching period T, for an asymptotically stable response. For a given switching period T, the bound on the norm of the transition matrix $\Phi(T, 0)$ associated with (14) can be calculated using (7). As discussed in Section II, uniform asymptotic stability is guaranteed if $\|\Phi(T, 0)\| < 1$, given that (14) is T-periodic. So (7) can be used to identify the switching period T^* below which stability is assured.

IV. AUV Platoon Simulations

In this section, we simulate the AUV platoon using the model and control laws developed in Section III. The simulated platoon consists of three vehicles, each of which may communicate with at most one other vehicle at any point in time. The simulations investigate the movement of the vehicles in a single dimension.

We choose the matrix $G(t)$ in (13) so that the jointly connected communication network represents a system in which each vehicle communicates with every other vehicle. With three vehicles, three constant matrices G_1, G_2, G_3 are required

$$G_1 = \begin{bmatrix} \frac{1}{2} & 1 & 0 \\ 1 & \frac{1}{2} & 0 \\ 0 & 0 & 0 \end{bmatrix}, G_2 = \begin{bmatrix} 0 & 0 & 0 \\ 0 & \frac{1}{2} & 1 \\ 0 & 1 & \frac{1}{2} \end{bmatrix}, G_3 = \begin{bmatrix} \frac{1}{2} & 0 & 1 \\ 0 & 0 & 0 \\ 1 & 0 & \frac{1}{2} \end{bmatrix}$$

Choosing the duty cycle associated with the G_i as $\delta_1 = \delta_2 = \delta_3 = 1/3$, the equivalent time-invariant matrix is

$$\bar{G} = \sum_{i=1}^{3} \delta_i G_i = \frac{1}{3} \begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix}$$

The control gains are selected as $k_h = 0.5$ and $k_d = 1$. The desired average position of the platoon is defined as constant $\tilde{h} = 10$, and the desired offsets from the mean as constant $\tilde{q}_d = [-15 \;\; 5 \;\; 10]^T$. The initial vehicle positions are $q(0) = [10 \;\; 20 \;\; -25]^T$. Using (7), the switching period below which stability is guaranteed is calculated as $T^* = 256$ milliseconds.

The simulations demonstrate the effect of switching on the behavior of the system. The first simulation corresponds to the time-invariant communication associated with $\bar{G}$. Figure 1 shows the trajectory of the three vehicles when no switching is involved.

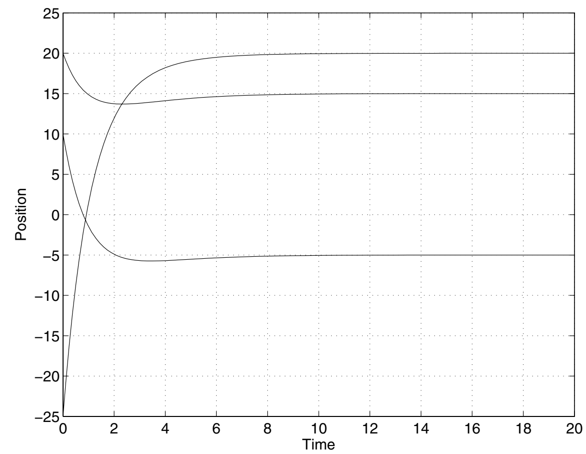

Fig. 1. Vehicle Trajectories, Time Invariant Communication.

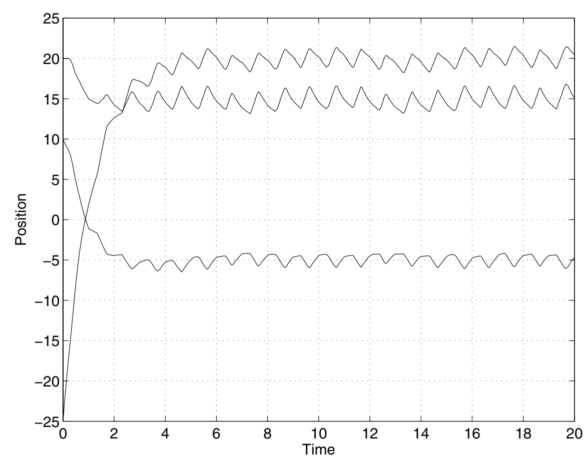

Fig. 2. Vehicle Trajectories, Switched Communication, Period = 1 sec.

Figures 2 and 3 show the vehicle trajectories for systems with switched network configurations. Apart from the switching itself, these two simulations are run under conditions identical to the time-invariant system simulation. The only difference between the two time-varying simulations is the switching frequency. The switching period in the simulation corresponding to Figure 2 is $T = 1$ second. Figure 3 demonstrates the effect of much faster switching, with $T = 1$ millisecond.

There are several notable aspects of the switched system behavior. Primarily, the vehicle trajectories are bounded in

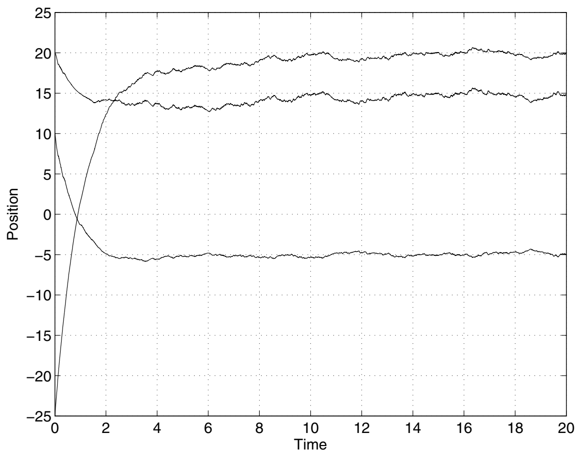

Fig. 3. Vehicle Trajectories, Switched Communication, Period = 1 msec.

the time-varying simulations, and they approach the steady state trajectories of the vehicles in the time-invariant simulation, indicating that a switched communication network can support stable behavior. However, the time-varying simulations reveal oscillations in the vehicle movements, which are pronounced in the system with larger switching period. Deviations from the time-invariant system behavior highlight the role of switching frequency, and suggest that considerably faster switching is needed to reduce error between the desired and actual vehicle trajectories.

V. Conclusion

The theory and simulations presented in this paper demonstrate that periodic fast switching can be applied effectively to distributed dynamical systems, such as autonomous vehicle platoons, in which communication is an essential aspect of control, and in which frozen-time communication capabilities do not support stable behavior. The theory establishes the existence of a switching period that guarantees an asymptotically stable response. It also provides an outline of a procedure for determining an appropriate period. For practical systems, the size of the switching period is a relevant issue, one that influences whether this control strategy is implementable.

Under the heading of time average control, [16] discusses periodically switched systems and comments that the switching period must be an order of magnitude faster than the fastest frozen-time system time constants. It is apparent from the simulations in this paper that if the control gains are scaled by a common factor, that the same qualitative system response can be obtained by adjusting the switching frequency proportionally. So selecting an appropriate switching period depends heavily on the dynamics of the system being controlled. The question of switching period magnitude remains an open area of investigation.

Another current area of research involves applying switched system techniques to more complex, possibly nonlinear, control problems. Autonomous vehicles are often given the task of measuring an environmental process, and controlling their movement based on those measurements. It is easy to imagine how the simple platoon control objectives considered in this paper could be placed in the context of an environmental process. Moreover, whereas this paper has focused on applying switching techniques to a decentralized control problem, the theory is equally applicable to estimation problems. The effectiveness of switching in distributed systems that perform estimation or combined estimation and control is another open area of investigation.

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