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Nonlinear Feedback Stabilization of High-Speed Planing Vessels by A Controllable Transom Flap

Handa Xi and Jing Sun*

Department of Naval Architecture and Marine Engineering, University of Michigan, Ann Arbor, MI 48109

Abstract— **Planing vessels suffer from porpoising instability at high forward speeds. Controllable transom flaps can be used to control the heave/pitch motion of the planing craft. In this paper, a nonlinear controller is designed based on the feedback linearization method to achieve asymptotic stability of the planing boat, thus avoiding porpoising at high speeds. We first show that the full-state nonlinear dynamic model describing the ship motion is not feedback linearizable. A state transformation is then constructed to decompose the model into a linearizable subsystem and a nonlinear internal dynamic subsystem. A reduced order state feedback is shown next to stabilize the planing vessel motion around the equilibrium point. Analysis of the region of attraction is also performed to provide an assessment of the effective safe operating range around the equilibrium point.**

I. INTRODUCTION

A high-speed planing vessel has a substantial portion of its weight supported by the hydrodynamic lift, in contrast to the conventional displacement vessel which is supported primarily by the hydrostatic buoyancy force. Due to the complicated nature of the hydrodynamic forces, high-speed planing boats face dynamic instability problems in both vertical and transverse planes, such as porpoising, chine walking, progressive heeling, unstable pitching-induced rolling, or a combination of these motions [1], [2].

Porpoising might be the most well-known instability phenomenon of the high-speed planing craft. It refers to the periodic, coupled heave/pitch oscillation in the vertical plane that a planing vessel may experience at high speeds. The motion is sustained by the energy derived from the craft's forward speed and the planing lift force.

The study of the vertical-plane motion of high-speed planing vessels can be traced back to the early twentieth century, and the research became a very active and fruitful field during 60's-90's [3], [4], [5], [6], [7], [8]. These earlier works focused mainly on the effects of design parameters, such as the location of the center of gravity, load, forward speed and other geometric parameters of the planing hull, on the characteristics of the craft's motion.

While the research community was focusing on understanding the fundamentals of the dynamics of high-speed planing boats, scientists and engineers had introduced appendages to control the vertical-plane motion of high-speed vessels. In [9], controllable transom flaps and T-foils were adopted to reduce motion sickness of a high-speed ferry. Savitsky and Brown [10] studied the hydrodynamic force induced by a static transom flap and its effects on the running trim, drag, power requirement and porpoising stability of the planing hull. Compared to the substantial research progress on the hydrodynamics and design of the high-speed planing craft, there has been little work published on the vertical-plane motion control for planing boats using controllable appendages.

Recently, a control-oriented nonlinear model has been developed by the authors for high-speed prismatic planing vessels equipped with controllable transom flaps [11]. The effects of preset static deflections of the transom flap on the running attitude and motion characteristics of planing boats are analyzed. A linear feedback controller is also designed for stability enhancement through dynamic feedback control. In this paper, a nonlinear controller based on feedback linearization is designed to establish asymptotic stabilities of planing boats. A state transformation is constructed to transform the system into a partially feedback linearizable form, as it is demonstrated that the system is not fully feedback linearizable. Local asymptotic stability is obtained by designing a stabilizing feedback control for the linear subsystem and verifying the local stability of the nonlinear zero dynamics. With analysis of the internal dynamics, we prove that motion stability of the high-speed planing vessel can be guaranteed by the proposed controller for initial conditions from which the boat's motion trajectory remains in the applicable range of Savitsky's methods.

The organization of this paper is as follows: In Section 2, a nonlinear model is briefly described for the prismatic planing vessel with a controllable transom flap. The effects of the static feedforward deflection of the transom flap on craft's motion stability are also presented. In Section 3, a stabilizing controller, based on the feedback linearization method, is designed to maintain the boat's stability at high speeds. The region of attraction is analyzed in Section 4, before concluding remarks are given in Section 5.

II. NONLINEAR MODEL OF PLANING VESSELS WITH CONTROLLABLE TRANSOM FLAPS AND MOTION STABILITY

A. Control-oriented Nonlinear Model

A control-oriented nonlinear model has been developed by the authors for high-speed prismatic planing vessels equipped with controllable transom flaps, based on Savitsky's methods and experimental results [11]. Due to the space limit, the model is just briefly described here for self containedness. Detailed description can be found in [11].

*Corresponding author. jingsun@engin.umich.edu; Ph.: (734)-615-8061; Fax: (734)-936-8820

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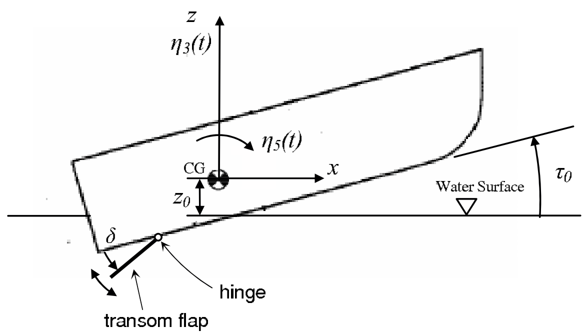

Fig. 1. Coordinate system of the planing vessel

A right-handed coordinate system is defined in Fig.1 for the prismatic planing vessel running in the calm water. At the equilibrium, the trim angle is defined as τ_0, and the vertical distance of the center of gravity (CG) from the water level as z_0. $\eta_3(t)$ and $\eta_5(t)$ are the vertical displacement (i.e., heave) of CG and the rotation (i.e., pitch) of the vessel relative to the inertia axis, respectively. $\eta_3(t)$ is positive upward and $\eta_5(t)$ positive bow down, as shown in Fig.1. The notations and directions are chosen to be consistent with [4], [5]. $\eta_3(t)$ and $\eta_5(t)$ can be expressed as follows, respectively:

$$\eta_3(t) = z(t) - z_0, \tag{1}$$
$$\eta_5(t) = -(\tau(t) - \tau_0) \tag{2}$$

where $\tau(t)$ and $z(t)$ are the effective trim angle and the effective vertical distance of CG from the water level when the craft is in motion, respectively.

For the vertical-plane motion of the planing hull, it is generally accepted that the heave/pitch motion can be decoupled from the surge motion for small trim angles [6], [7]. In this paper, only heave/pitch motions will be considered.

A controllable flap is attached at the stern to affect the vertical-plane motion of the planing vessel as shown in Fig.1. By adding the forces induced by the transom flap to the form used in [4], [5], the motion equation of the high-speed planing boat with a controllable transom flap running in the calm water can be written as follows:

$$A\ddot{\eta} + B\dot{\eta} = F^R(z_0, \tau_0; \eta) + F^F(\delta) \tag{3}$$

where

$$\eta = \begin{bmatrix} \eta_3 \\ \eta_5 \end{bmatrix},$$
$$A = \begin{bmatrix} m + a_{33} & a_{35} \\ a_{53} & I_{55} + a_{55} \end{bmatrix}, \quad B = \begin{bmatrix} b_{33} & b_{35} \\ b_{53} & b_{55} \end{bmatrix},$$
$$F^R(z_0, \tau_0; \eta) = \begin{bmatrix} F_3^R(z_0, \tau_0; \eta) \\ F_5^R(z_0, \tau_0; \eta) \end{bmatrix},$$
$$F^F(\delta) = \begin{bmatrix} F_3^F(\delta) \\ F_5^F(\delta) \end{bmatrix}$$

m is the vessel mass and I_{55} the pitch moment of inertia about CG. a_{ij} and $b_{ij}, i, j = 3, 5$ are the added mass and damping coefficients, respectively. F_3^R and F_5^R are the heave and pitch restoring force, respectively. F_3^F and F_5^F are the forces induced by the transom flap in the heave and pitch direction, respectively. F^F will be used to control the motion of the craft.

The coefficients in A and B are extrapolated from experimental results in [4], depending upon the speed, the equilibrium trim angle and the mean wetted length beam ratio. Savitsky's empirical methods [3], [10] are adopted to calculate $F^R(z_0, \tau_0; \eta)$ and $F^F(\delta)$. The flap lift, $F_3^F(\delta)$, and the flap moment about CG, $F_5^F(\delta)$, can be expressed by (4) and (5), respectively.

$$F_3^F = c_3\delta \tag{4}$$
$$F_5^F = c_5\delta \tag{5}$$

where c_3, c_5 are constant coefficients dependent upon the forward speed.

By defining the state vector $x = [x_1, x_2, x_3, x_4]^T = [\eta_3, \eta_5, \dot{\eta}_3, \dot{\eta}_5]^T$, the motion equation can be transformed to the state-space form as follows:

$$\dot{x} = f(z_0, \tau_0; x) + b\delta \tag{6}$$

where

$$f(z_0, \tau_0; x) = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & \alpha_1 & \alpha_2 \\ 0 & 0 & \alpha_3 & \alpha_4 \end{bmatrix} x + \begin{bmatrix} 0 \\ 0 \\ \theta_1(z_0, \tau_0; x_1, x_2) \\ \theta_2(z_0, \tau_0; x_1, x_2) \end{bmatrix},$$
$$\begin{bmatrix} \alpha_1 & \alpha_2 \\ \alpha_3 & \alpha_4 \end{bmatrix} = -A^{-1}B,$$
$$\begin{bmatrix} \theta_1(z_0, \tau_0; x_1, x_2) \\ \theta_2(z_0, \tau_0; x_1, x_2) \end{bmatrix} = A^{-1}F^R(z_0, \tau_0; x_1, x_2),$$
$$b = \begin{bmatrix} 0 \\ 0 \\ \gamma_1 \\ \gamma_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ A^{-1}\begin{bmatrix} c_3 \\ c_5 \end{bmatrix} \end{bmatrix}.$$

In general, the restoring forces, F^R, are nonlinear functions of the motion displacement, (η_3, η_5). The expressions of all the nonlinear functions used in (6) are given in [11]. Equation (6) describes a nonlinear model of the planing vessel with a controllable transom flap, and will be used for control system design and analysis in the following sections.

B. Motion Stability with Static Feedforward Control

A simulation of heave/pitch motions of planing boats is shown in Fig.2, where porpoising dynamics are exhibited. The effects of the static feedforward deflection of the transom flap on the craft's equilibrium running attitude and motion stability have been analyzed by the authors in [11].

Fig.3 shows the maximum forward speed coefficient $C_{v_{max}}$ that the planing hull can maintain without inducing porpoising, for different static deflections of the transom flap. Although a small flap deflection can increase the maximum stable speed by a small increment in Fig.3 (for example, $\delta = 1^\circ$ increases $C_{v_{max}}$ from 5.02 to 5.05), larger

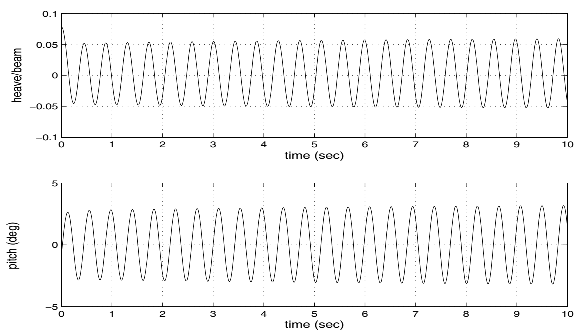

Fig. 2. Heave/pitch motion of the planing vessel when $C_v = 5.1$ and $\delta = 0$.

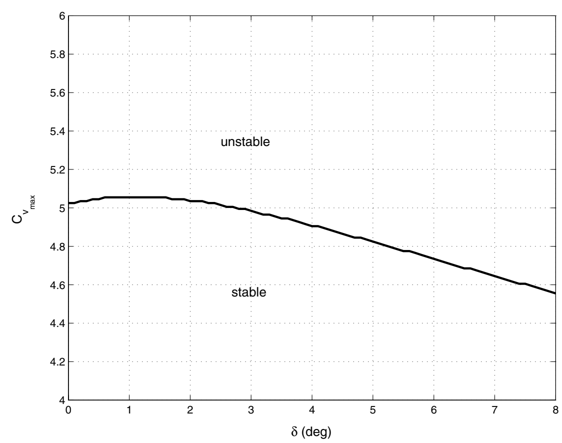

Fig. 3. Maximum speed coefficient where the stability of the planing vessel can maintain.

deflections reduce the stable operating speed range. Setting $\delta = 8°$ lowers $C_{v_{max}}$ down to 4.55. For $C_v > 5.06$, the static transom flap is unable to stabilize the heave/pitch motion of the vessel. To extend the maximum stable operating speed without redesigning the planing hull, nonlinear dynamic feedback stabilization through the controllable transom flap is pursued in the next section.

III. STABILIZATION BASED ON FEEDBACK LINEARIZATION METHOD

A nonlinear controller based on the feedback linearization method is developed in this section to stabilize the planing craft at high speeds. The control scheme is illustrated in Fig.4, where $u_\delta = k(x)$ is the feedback control law to be designed. The deflection of the transom flap, δ, is decomposed into two parts: $\delta = \delta_0 + u_\delta$, where δ_0 is the preset nominal deflection that determines the equilibrium running attitude of the planing craft, and u_δ is the relative deflection about δ_0 as a feedback control input.

Given δ_0, the model in (6) can be rewritten in the following form:

$$\dot{x} = \hat{f}(x) + bu_\delta \tag{7}$$

where $\hat{f}(x) = f(z_0, \tau_0; x) + b\delta_0$. (z_0, τ_0) is the equilibrium running attitude of the vessel corresponding to the nominal

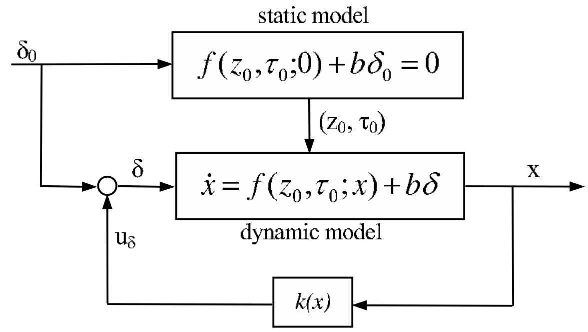

Fig. 4. Control loop of the system

flap deflection, δ_0. The objective is to design a feedback control law, $u_\delta = k(x)$, using feedback linearization to stabilize the system of planing vessels described by the model of (7) at the equilibrium of $x = 0$.

For convenience, the following notations are defined and will be used in the sequel, where θ_1, θ_2 are given in Section II.

$$\left[\begin{array}{c} \hat{\theta}_1(x_1,x_2) \\ \hat{\theta}_2(x_1,x_2) \end{array}\right] := \left[\begin{array}{c} \theta_1(z_0,\tau_0;x_1,x_2) \\ \theta_2(z_0,\tau_0;x_1,x_2) \end{array}\right] + \left[\begin{array}{c} \gamma_1 \\ \gamma_2 \end{array}\right]\delta_0.$$

A. Feedback Linearizability

If the system in (7) can be transformed into a linear form by applying a state transformation and state feedback, then, stabilizing controllers can be easily designed based on this linear form. For the system model under consideration, however, we have:

Proposition 3.1: The model of planing hulls described in (7) is not fully feedback linearizable.

To verify the proposition, we need the following theorem which gives the sufficient and necessary conditions for the system feedback linearizability [12]:

Theorem 3.1: The system $\dot{x} = \varphi(x) + g(x)u$, where x is n-dimensional, is fully feedback linearizable near x^0 if and only if the following conditions are satisfied

(i) the matrix $P = [g(x^0), ad_\varphi g(x^0), ..., ad_\varphi^{n-1} g(x^0)]$ has rank n, where $ad_\varphi g(x) := \frac{\partial g}{\partial x}\varphi(x) - \frac{\partial \varphi}{\partial x} g(x)$;

(ii) the distribution $\Omega = span\{g, ad_\varphi g, ..., ad_\varphi^{n-2} g\}$ is involutive (i.e., $\forall \omega_1, \omega_2 \in \Omega \Rightarrow ad_{\omega_1}\omega_2 \in \Omega$) near x^0.

For the boat system described by (7), $n = 4$, $\varphi(x) = \hat{f}(x)$, $g(x) = b$, $x^0 = 0$. Thus, we obtain

$$\begin{aligned} P &= \left[b, ad_{\hat{f}} b(x^0), ad_{\hat{f}}^2 b(x^0), ad_{\hat{f}}^3 b(x^0)\right] \\ &= \left[\begin{array}{cccc} 0 & -\gamma_1 & \gamma_1\alpha_1 + \gamma_2\alpha_2 & -\phi_1(0,0) \\ 0 & -\gamma_2 & \gamma_1\alpha_3 + \gamma_2\alpha_4 & -\phi_2(0,0) \\ \gamma_1 & -\gamma_1\alpha_1 - \gamma_2\alpha_2 & \phi_1(0,0) & -\phi_3(0,0) \\ \gamma_2 & -\gamma_1\alpha_3 - \gamma_2\alpha_4 & \phi_2(0,0) & -\phi_4(0,0) \end{array}\right], \end{aligned}$$

$$\begin{aligned} \Omega &= span\left\{b, ad_{\hat{f}} b, ad_{\hat{f}}^2 b\right\} \\ &= span\left\{\begin{array}{ccc} 0 & -\gamma_1 & \gamma_1\alpha_1 + \gamma_2\alpha_2 \\ 0 & -\gamma_2 & \gamma_1\alpha_3 + \gamma_2\alpha_4 \\ \gamma_1 & -\gamma_1\alpha_1 - \gamma_2\alpha_2 & \phi_1(x_1,x_2) \\ \gamma_2 & -\gamma_1\alpha_3 - \gamma_2\alpha_4 & \phi_2(x_1,x_2) \end{array}\right\} \end{aligned}$$

where

$$\phi_1(x_1,x_2) = \gamma_1\frac{\partial\hat{\theta}_1}{\partial x_1}+\gamma_2\frac{\partial\hat{\theta}_1}{\partial x_2} +\alpha_1(\gamma_1\alpha_1+\gamma_2\alpha_2)+\alpha_2(\gamma_1\alpha_3+\gamma_2\alpha_4),$$

$$\phi_2(x_1,x_2) = \gamma_1\frac{\partial\hat{\theta}_2}{\partial x_1}+\gamma_2\frac{\partial\hat{\theta}_2}{\partial x_2} +\alpha_3(\gamma_1\alpha_1+\gamma_2\alpha_2)+\alpha_4(\gamma_1\alpha_3+\gamma_2\alpha_4),$$

$$\phi_3(x_1,x_2) = (\gamma_1\alpha_1+\gamma_2\alpha_2)\frac{\partial\hat{\theta}_1}{\partial x_1}+(\gamma_1\alpha_3+\gamma_2\alpha_4)\frac{\partial\hat{\theta}_1}{\partial x_2} +\alpha_1\phi_1(x_1,x_2)+\alpha_2\phi_2(x_1,x_2),$$

$$\phi_4(x_1,x_2) = (\gamma_1\alpha_1+\gamma_2\alpha_2)\frac{\partial\hat{\theta}_2}{\partial x_1}+(\gamma_1\alpha_3+\gamma_2\alpha_4)\frac{\partial\hat{\theta}_2}{\partial x_2} +\alpha_3\phi_1(x_1,x_2)+\alpha_4\phi_2(x_1,x_2).$$

It can be shown that $b, ad_{\hat{f}}b$ and $ad^2_{\hat{f}}b$ are independent in R^4, and

$$ad_{ad_{\hat{f}}b}(ad^2_{\hat{f}}b) = \begin{bmatrix} 0 \\ 0 \\ -\gamma_1^2\left.\frac{\partial^2\hat{\theta}_1}{\partial x_1^2}\right|_0 - 2\gamma_1\gamma_2\left.\frac{\partial^2\hat{\theta}_1}{\partial x_1\partial x_2}\right|_0 - \gamma_2^2\left.\frac{\partial^2\hat{\theta}_1}{\partial x_2^2}\right|_0 \\ -\gamma_1^2\left.\frac{\partial^2\hat{\theta}_2}{\partial x_1^2}\right|_0 - 2\gamma_1\gamma_2\left.\frac{\partial^2\hat{\theta}_2}{\partial x_1\partial x_2}\right|_0 - \gamma_2^2\left.\frac{\partial^2\hat{\theta}_2}{\partial x_2^2}\right|_0 \end{bmatrix}$$

is not in Ω for $\forall x$ near $x^0 = 0$. Hence, Ω is not involutive. According to Theorem 3.1, our model is not fully feedback linearizable. We then look for a state transformation which can partially linearize the system.

Proposition 3.2: For the system in (7), there exists a state transformation, $z = [z_1, z_2, z_3, z_4]^T = \Phi(x)$, such that the system can be transformed into the following form:

$$\dot{\hat{z}} = E\hat{z} + Fv \tag{8}$$

$$\dot{z}_4 = q(z) \tag{9}$$

where

$$\hat{z} = \begin{bmatrix} z_1 \\ z_2 \\ z_3 \end{bmatrix},\ E = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{bmatrix},\ F = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}.$$

Proof: Consider the following virtual output function:

$$h(x) = -\gamma_2 x_1 + \gamma_1 x_2. \tag{10}$$

Define a state transformation $z = \Phi(x)$ based on $h(x)$ as follows:

$$\begin{bmatrix} z_1 \\ z_2 \\ z_3 \\ z_4 \end{bmatrix} = \Phi(x) := \begin{bmatrix} h(x) \\ L_{\hat{f}}h(x) \\ L^2_{\hat{f}}h(x) \\ x_1 \end{bmatrix}, \tag{11}$$

where $L_{\hat{f}}(\cdot)$ is the Lie derivative along $\hat{f}(x)$. Substitute $h(x)$ into (11), we have

$$\Phi(x) = \begin{bmatrix} -\gamma_2 x_1 + \gamma_1 x_2 \\ -\gamma_2 x_3 + \gamma_1 x_4 \\ \beta_1 x_3 + \beta_2 x_4 + \gamma_1\hat{\theta}_2(x_1,x_2) - \gamma_2\hat{\theta}_1(x_1,x_2) \\ x_1 \end{bmatrix}$$

where $\beta_1 = (\gamma_1\alpha_3 - \gamma_2\alpha_1)$, $\beta_2 = (\gamma_1\alpha_4 - \gamma_2\alpha_2)$. Then, the system in (7) can be transformed into

$$\dot{\hat{z}} = E\hat{z} + F\left[L^3_{\hat{f}}h(x) + L_bL^2_{\hat{f}}h(x)u_\delta\right], \tag{12}$$

$$\dot{z}_4 = x_3. \tag{13}$$

It can be shown that the Jacobion matrix of $\Phi(x)$ is nonsingular at $x = 0$. Hence, the transformation defined in (11) is a diffeomorphism and its inverse mapping, $x = \Phi^{-1}(z)$, exists.

Let

$$v = L^3_{\hat{f}}h(x) + L_bL^2_{\hat{f}}h(x)u_\delta, \tag{14}$$

$$q(z) = x_3, \tag{15}$$

and express x_3 in (15) with z using the inverse mapping, $x = \Phi^{-1}(z)$, we obtain the state space model in the new coordinates in the form given in Proposition 3.2.

Apparently, $\Phi(0) = 0$, and thus $z = 0$ is the corresponding equilibrium of the system in the new coordinates.

Viewing v as the new control input, the model described in (8)-(9) decomposes the system into two parts in the new coordinates: a linear part described by (8), and a nonlinear internal dynamics by (9).

If v is designed to stabilize the whole system including the internal dynamics, the stabilizing control input, u_δ, in the original coordinate of x can be obtained by

$$u_\delta(x) = \frac{v - L^3_{\hat{f}}h(x)}{L_bL^2_{\hat{f}}h(x)}, \tag{16}$$

where $L_bL^2_{\hat{f}}h(x) = \beta_1\gamma_1 + \beta_2\gamma_2 > 0$ for the speed range of $C_v = 2.0 \sim 7.0$ considered in our study.

B. Zero Dynamics and Local Stability of System

While linear control theory can be applied to design the control input, v, to stabilize the linear part described by (8), the stability of the internal dynamics in (9) is required to establish the stability for the whole system of the planing vessel. The effect of the internal dynamics on the system stability can be analyzed through the so-called zero dynamics.

Setting $z_1 = z_2 = z_3 = 0$ in the internal dynamics of (9) results in

$$\dot{z}_4 = q([0,0,0,z_4]^T) \tag{17}$$

which corresponds to the *zero dynamics*. The following lemma can be derived for the system stability from the feedback linearization theory [12], and the proof is omitted here.

Lemma 3.1: Suppose the equilibrium $z_4 = 0$ of the zero dynamics (17) is locally asymptotically stable and $v = -K\hat{z}$ where K is designed such that $(E - FK)$ is Hurwitz. Then the feedback law in (16) locally asymptotically stabilizes the original system (7) at the equilibrium of $x = 0$.

From Lemma 3.1, the local stability of the zero dynamics is critical to establish the local stability of the whole system

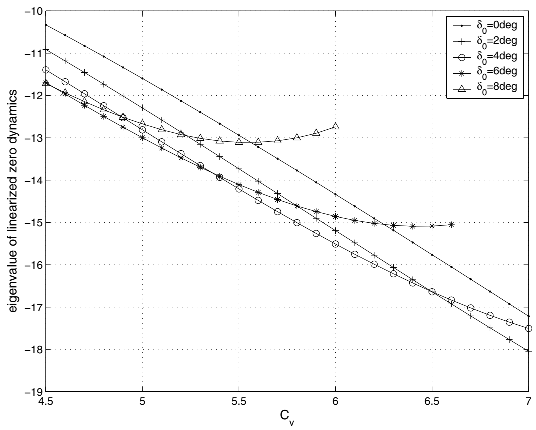

Fig. 5. Eigenvalue of linearized zero dynamics

using the feedback linearization approach based on (8) and (9). The linearized zero dynamics at $z_4 = 0$ is

$$\dot{z}_4 = \left.\frac{\partial q([0,0,0,z_4]^T)}{\partial z_4}\right|_{z_4=0} z_4. \tag{18}$$

Fig.5 shows that the eigenvalue of the linearized zero dynamics of the planing craft is negative for different operating conditions of C_v and δ_0, and therefore, the zero dynamics are locally asymptotically stable at $z_4 = 0$. The local stability of the composite system is then established by Lemma 3.1, when $v = -K\hat{z}$ is chosen such that $(E - FK)$ is Hurwitz.

C. Simulations and Discussions

The feedback stabilizing control law in the new coordinate, $v = -K\hat{z}$, can be designed by using the pole placement method. Fig.6 shows two examples where the planing vessel is stabilized and porpoising eliminated. The poles of $(E - FK)$ are placed at $[-4 + 4j, -4 - 4j, -8]^T$ by properly chosen K.

When there are uncertainties in the model, which is inevitable for this application, the robustness of the controller resulted from feedback linearization is always a

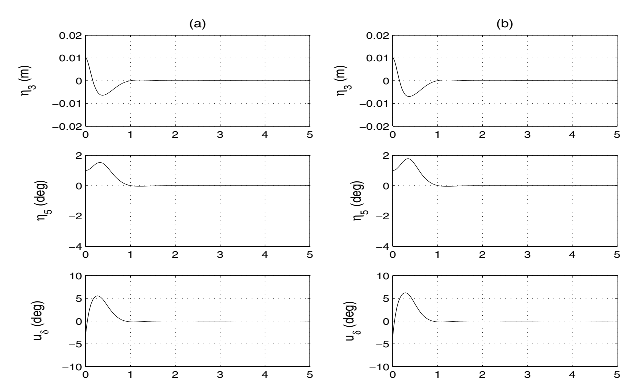

Fig. 6. Response and control input of porpoising stabilization. $\delta_0 = 3deg$ and $x(0) = [0.01m, 1deg, 0, 0]^T$. (a) $C_v = 5.5$; (b) $C_v = 6.0$.

concern since such controller requires "exact" cancellation of nonlinearities. For the planing vessel, the uncertainties in the model, whether associated with A, B, F^R or F^F, will manifest as extra terms in the state equation of (12) and (13) for $\dot{z}_3$ and $\dot{z}_4$. Note that the uncertainty term in (12) is the so-called "matched" uncertainty as it enters the system through the same channel as the control input. The matched uncertainties are usually considered as un-destructive, since many linear/nonlinear control design schemes are available to address them and to enforce system robustness. For the uncertainties that appear in (13), as long as they do not change the property of $\frac{\partial q([0,0,0,z_4]^T)}{\partial z_4}|_{z_4=0} < 0$, the stability and robustness of the equilibrium for the closed loop system will not be affected.

IV. ANALYSIS ON THE REGION OF ATTRACTION

The controller designed in the previous section establishes local asymptotic stability for the planing vessel at high speeds. But the analysis does not specify the region of attraction. In other words, the results of Section III establish that if the initial condition is "sufficiently" close to the equilibrium, then a stable motion is guaranteed. However, a quantitative measure of sufficient closeness is not specified. For practical purpose, it is of interest to investigate conditions under which the system can be led to the equilibrium at the origin. This section is devoted to assessing the region of attraction for the equilibrium point and to define the safe operating range of the high-speed planing boat.

Given that the state $\hat{z}$, consisting of z_1, z_2 and z_3, is governed by a linear subsystem, the region of attraction for the stablilized equilibrium point, $z = 0$ of the system (8) and (9), with $v = -K\hat{z}$ and $(E - FK)$ being Hurwitz, is primarily dictated by the nonlinear internal dynamics defined by (9). Note that (9) has the form of

$$\dot{z}_4 = \tilde{f}(z_1, z_4) + s_1 z_2 + s_2 z_3 \tag{19}$$

where

$$\begin{aligned}\tilde{f}(z_1, z_4) &= -\frac{1}{\beta}\Bigg[\gamma_1^2\hat{\theta}_2(z_4, \frac{1}{\gamma_1}(z_1 + \gamma_2 z_4)) \\ &\qquad -\gamma_1\gamma_2\hat{\theta}_1(z_4, \frac{1}{\gamma_1}(z_1 + \gamma_2 z_4))\Bigg], \\ s_1 &= -\frac{\beta_1}{\beta}, s_2 = \frac{\gamma_1}{\beta}, \\ \beta &= \gamma_1(\gamma_1\alpha_3 - \gamma_2\alpha_1) + \gamma_2(\gamma_1\alpha_4 - \gamma_2\alpha_2).\end{aligned}$$

By defining

$$\tilde{f}_{\hat{z}}(\hat{z}) = \tilde{f}(z_1, 0) + s_1 z_2 + s_2 z_3, \tag{20}$$

$$\tilde{f}_{z_4}(z_1, z_4) = \tilde{f}(z_1, z_4) - \tilde{f}(z_1, 0), \tag{21}$$

we can rewrite (9) in the form of

$$\dot{z}_4 = \tilde{f}_{z_4}(z_1, z_4) + \tilde{f}_{\hat{z}}(\hat{z}). \tag{22}$$

We then have the following proposition concerning the stability of the planing craft motion with different initial conditions:

Proposition 4.1: If $\tilde{f}_{\hat{z}}$ and $\tilde{f}_{z_4}$ given by (20) and (21) satisfy the following conditions:

$$|\tilde{f}_{\hat{z}}(\hat{z})| \leq k_1|\hat{z}| \tag{23}$$

and

$$z_4\tilde{f}_{z_4}(z_1, z_4) \leq -k_2 z_4^2, \tag{24}$$

for some constant $k_1, k_2 > 0$, and the control law $v = -K\hat{z}$ is designed such that $(E - FK)$ is Hurwitz, then

$$z(t) \rightarrow 0 \ as \ t \rightarrow +\infty.$$

Proof: Define a nonnegative function $V(z_4)$ as follows

$$V(z_4) = z_4^2, \tag{25}$$

then, with (23) and (24) being satisfied, we have

$$\begin{aligned} \dot{V} &= 2z_4\left[\tilde{f}_{z_4}(z_1, z_4) + \tilde{f}_{\hat{z}}(\hat{z})\right] \\ &\leq -2k_2 z_4^2 + 2|z_4|k_1|\hat{z}|. \end{aligned}$$

Since $(E - FK)$ is designed to be Hurwitz, there exist positive constants p and q such that

$$k_1|\hat{z}| \leq pe^{-qt} \tag{26}$$

where q is dependent upon the controller gain, K, and p on k_1, K and the initial condition, $\hat{z}(0)$. Thus,

$$\begin{aligned} \dot{V} &\leq -2k_2 z_4^2 + 2|z_4|pe^{-qt} \\ &= -2k_2 V + 2\sqrt{V}pe^{-qt}. \end{aligned} \tag{27}$$

Let $\tilde{V} = \sqrt{V} = |z_4|$, we obtain from (27) that

$$\dot{\tilde{V}} \leq -k_2\tilde{V} + pe^{-qt}. \tag{28}$$

After some manipulations, one can show that

$$|z_4(t)| = \tilde{V}(t) \leq |z_4(0)|e^{-k_2 t} - \frac{p}{q - k_2}(e^{-qt} - e^{-k_2 t}). \tag{29}$$

Therefore, $z(t) \rightarrow 0$ as $t \rightarrow +\infty$ follows by noting that $(E - FK)$ is Hurwitz.

For the planing vessel model described in Section II, conditions (23) and (24) are satisfied for the states within the range covered by Savitsky's empirical methods. To validate (23), we note that since other two terms in $\tilde{f}_{\hat{z}}$ are linear, the Lipschitz condition for $\tilde{f}_{\hat{z}}$ is satisfied if $\tilde{f}(z_1, 0)$ is Lipschitz in z_1. The latter is true by examining Fig.7 (a), which plots $\tilde{f}(z_1, 0)$ for all values of z_1 that are within the applicable range of the model.

For condition (24), we plot $\tilde{f}_{z_4}$ as a function of z_4 for all possible (z_1, z_4) combinations covered by the Savitsky's method, as given in Fig.7 (b). It is clear that (24) is also satisfied for the model of the planing vessel.

The above analysis indicates that, if the initial condition $z(0)$, and the controller gain, K, are chosen such that the motion trajectory of the planing boat remains in the applicable range of the Savitsky's method, then motion stability is guaranteed and porpoising is eliminated.

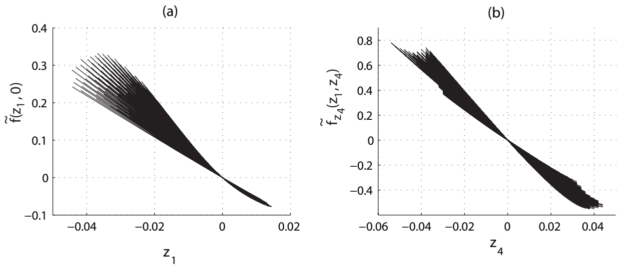

Fig. 7. Plots of $\tilde{f}(z_1, 0)$ and $\tilde{f}_{z_4}(z_1, z_4)$ in the model applicable range.

V. CONCLUSIONS

A nonlinear controller based on the feedback linearization method is designed to enhance the vertical-plane stability of the high-speed planing vessel using a controllable transom flap. A state transformation is constructed to transform the system into a partially linear form, and local asymptotic stability is obtained by verifying the local stability of zero dynamics. We also show that the proposed stabilizing controller guarantees motion stability if the boat's movement remains in the applicable range of the model.

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