2005 American Control Conference
June 8-10, 2005. Portland, OR, USA

An Adaptive Lag-Synchronization Method for Time-delay Chaotic Systems

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***Abstract*— Based on the master-slave configuration, an adaptive feedback controller design method is proposed for the time-delay chaotic systems to realize the lag-synchronization. It is shown that the controlled uncertain slave system can adaptively synchronize the master system lagged by the propagation time. The proposed method is successfully applied to the synchronization of time-delay Chua's circuits.**

I. INTRODUCTION

Inspired by the pioneering work of Pecora and Carroll in 1990 [1], there has been increasing interest in achieving the synchronization based on master-slaver configuration. Due to its potential applications for secure communications, many theoretical and experimental synchronization methods have been proposed in the past decade [2-6]. In these synchronization studies, the time-delay chaotic systems have received more attention because of the significant role of delayed states in the dynamics of many physical and biological systems [4-6]. The delay-differential systems often exhibit multi-stability (i.e. the coexistence of several attractors) which enables such systems to act as memory devices [5]. The study of time-delay systems is also motivated by the fact that these systems exhibit high-dimensional chaos and, therefore, can be used in communication systems based on chaotic synchronization, to securely encrypt information into their chaotic outputs [5]. In [6], chaos synchronization is experimentally demonstrated in delay-differential equations involving optical delays. The particular feature is the all-optical chaos subtraction for message extration and the very high masking efficiency.

On the other hand, it is worthy noticing that the propagation delay may exist in the remote communication systems. For the time-delay chaotic systems, the propagation delay might not be equal to the system time delay. The propagation delay problem was reported and studied theoretically in [7-9]. Yalcin et al. [7] initiated the master-slave synchronization scheme for Lur'e type chaotic systems with propagation delay. The same problem was further investigated in [8] where some improving results were presented. In [9], the authors recast it as a more general synchronization problem based on the delayed feedback control (DFC) scheme where the controller time delay is the propagation delay. It should be noted that the propogation delay in [7-9] has to be known a priori. However, it is often difficult to obtain the precise value of the propogation delay in practical systems. Nevertheless, a relationship between the master system and the slave systems may be established. Indeed, it has been demonstrated that there exists a regime where the states of two systems are nearly identical, but one system lags in time to the other. Thus there appears a shifted-in-time coincidence method for the states of the master system and the slave systems, which is the so-called lag-synchronization. To our best of knowledge, it was predicted and observed for the first time in [10] in two non-identical coupled laser chaotic systems. The coupling of the two lasers which are in their own stable states leads to a synchronized chaotic regime, with a constant time delay between the laser intensities. In [11], the chaotic lag-synchronization is analyzed experimentally and numerically between a continous system and a transient chaos resulting from a pulsed information. More recently, the lag-synchronization was studied between two unidirectionally coupled identical time-delay systems in the case that the propagation time is equal to the system delay [12]. The lag-synchronization was implemented in [13] for the unidirectionally coupled semiconductor lasers. In [14], two coupled time-delayed chaotic systems are investigated to realize exponential lag-synchronization. The controlling parameter is exponentially increased with the system time delay. So the system time delay should be also known a priori.

It is worth pointing out that a salient assumption of the aforementioned schemes is that the system equations and the system time delay are completely known or all states are available. Unfortunately, the system parameters might not be known exactly in many practical systems, or not all states are available. To tackle this problem, the observer-based synchronization methods were proposed [15] [16] [17]. In [16], an observer configuration and a subsystem configuration are respectively studied based on linear matrix inequality for input-independent global synchronization. But such a design method can be only implemented for the system which can be partitioned into two specific subsystems. Based on the filter design method, an adaptive observer was presented for chaos synchronization and some unknown costant parameter can be estimated [17]. But this method cannot tackle the synchronization problem for systems with unknown nonlinear function. In [18], an

This work was supported by a grant from the Research Grant Council of the Hong Kong Special Administrative Region, P.R.China with number CityU 1024/02E and a grant from the NSF of China with number 60274023.

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adaptive recurrent neural control method is proposed for noisy chaos synchronization. Although some functions of the master system are assumed to be unknown and can be estimated by the recurrent neural network, the slave system is assumed to be completely known. In this paper, a new adaptive lag-synchronization method is proposed for partially known time-delay chaotic systems based on master-slave configuration. Only the output information is available for the slave system to lag-synchronize the master system. The unknown nonlinear functions are assumed to be Lipschitz, but the Lipschitz constants are unknown. The parameter differences between the master system and the slave system are considered as the system uncertainties. They are assumed to be time-varying but norm-bounded. Here, the upper bounds of the uncertainties are not available. Based on Lyapunov-Krasovskii functional, the designed adaptive controller can force the slave system to lag-synchronize the master system. Moreover, the designed adaptive laws may not lead to high-gain phynomenon.

II. PRELIMINARIES

Consider a class of time-delay chaotic systems in the form of

$$\left\{ \begin{array}{l} \dot{x}(t) = Ax(t) + A_d(t)\, x(t-\tau) + Bf(x(t)), \\ y(t) = Cx(t), \\ x(0) = \phi(0), \end{array} \right. \tag{1}$$

where $x(t) \in \mathcal{R}^n$ is the state vector, $x(t-\tau) \in \mathcal{R}^n$ the time-delay state vector, τ the unknown constant system delay, $y(t) \in \mathcal{R}^q$ the output vector, $A \in \mathcal{R}^{n\times n}, B \in \mathcal{R}^n$ and $C \in \mathcal{R}^{q\times n}$ the known system matrices, $A_d(t) \in \mathcal{R}^{n\times n}$ the time-varying continuous matrix function, $\phi(0)$ the initial condition, and $f(\cdot) : \mathcal{R}^n \to \mathcal{R}$ an unknwon nonlinear function satisfying the Lipschitz condition,

$$\|f(x_1) - f(x_2)\| \leq \alpha \|x_1 - x_2\|, \tag{2}$$

where α is an unknown positive scalar.

It is noted that a large class of chaotic systems can be represented in the form of (1), such as the time-delay Chua's circuit [7], the time-delay Logistic system and M-G system [4], to name just a few.

Given the master system (1), one considers the slave system

$$\left\{ \begin{array}{l} \dot{\hat{x}}(t) = (A + \Delta A(t))\, \hat{x}(t) \\ \qquad + (A_d(t) + \Delta A_d(t))\, \hat{x}(t-\tau) \\ \qquad + B(f(\hat{x}(t)) + \Delta f(\hat{x}(t))) + Bu(t), \\ \hat{y} = C\hat{x}(t), \\ \hat{x}(0) = \hat{\phi}(0), \end{array} \right. \tag{3}$$

where the system matrices with symbol Δ are the corresponding uncertainties which represent the time-varying parameter uncertainties of the slave system and the parameter differences between the master system and the slave system. Assume that there exist uniformly continuous matrix functions $N_{Ad}(t)$, $E_A(t)$ and $E_{Ad}(t)$ such that

$$\begin{aligned} A_d(t) &= BN_{Ad}(t), \\ \Delta A(t) &= BE_A(t), \\ \Delta A_d(t) &= BE_{Ad}(t). \end{aligned}$$

These conditions are so-called *matching conditions* [19].

In (3), $\hat{\phi}(0)$ is the initial condition which is different from $\phi(0)$. $u(t) \in \mathcal{R}$ is the control variable vector to be determined such that the system (3) is lag-synchronized to the system (1) with respect to the unknown constant channel propagation time $d > 0$, i.e. $\lim_{t\to\infty} \|\hat{x}(t) - x(t-d)\| \to 0$. It is noted that $d \neq \tau$. This synchronization scheme can be shown in Fig. 1.

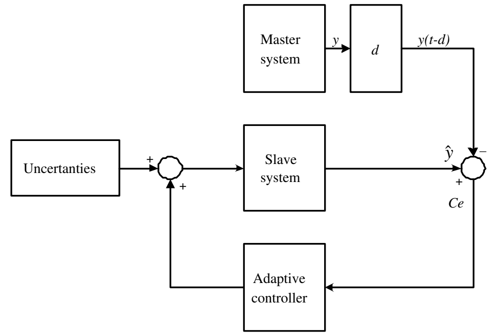

Fig. 1. The lag-synchronization scheme based on adaptive control.

It is worth to note that only the delayed output variable vector $y(t-d)$ is available to the controller in the system (3). Considering the time-delay states as the system perturbation, one can present in this paper the following strictly positive real conditions:

$$P(A - LC) + (A - LC)^T P = -Q, \tag{4}$$

$$B^T P = C, \tag{5}$$

where P and Q are positive definite matrices, and L is a real matrix with an appropriate dimension.

In order to realize the lag-synchronization between the systems (1) and (3), the adaptive controller in the slave system (3) is chosen to be

$$u(t) = \mathcal{K}(t)(\hat{y}(t) - y(t-d)) \tag{6}$$

where $\mathcal{K}(t)$ is the adaptive controller gain.

Subtracting the d-lagged system (1)

$$\left\{ \begin{array}{l} \dot{x}(t-d) = Ax(t-d) + A_d x(t-d-\tau) \\ \qquad + Bf(x(t-d), t), \\ y(t-d) = Cx(t-d), \\ x(t) = \phi(0), t \in [-d, 0] \end{array} \right.$$

from (3) with (6) yields the error dynamical system

$$\left\{ \begin{array}{l} \dot{e}(t) = (A + B\mathcal{K}(t)C)\, e(t) + A_d e(t-\tau) \\ \qquad + B[F(t) + g(\hat{x}(t), \hat{x}(t-\tau), t)], \\ \tilde{y}(t) = \hat{y}(t) - y(t-d) = Ce(t) \end{array} \right. \tag{7}$$

where

$$\begin{aligned}
e(t) &= \hat{x}(t) - x(t-d), \\
e(t-\tau) &= \hat{x}(t-\tau) - x(t-d-\tau), \\
F(t) &= f(\hat{x}(t)) - f(x(t-d)), \\
g(\hat{x}(t), \hat{x}(t-\tau)) &= E_A(t)\hat{x}(t) + E_{Ad}(t) \\
&\quad \times \hat{x}(t-\tau) + \Delta f(\hat{x}(t), t).
\end{aligned}$$

It follows from (2) that $F(t)$ is uniformly bounded and satisfies that $\|F(e(t), t)\| \leq \alpha \|e(t)\|$. Furthermore, introduce the following notations:

$$\begin{aligned}
\beta(t) \quad : &= \sup_{t \in R^+} \{\|g(\hat{x}(t), \hat{x}(t-\tau), t)\| : \\
&\quad \|\hat{x}(t)\| \leq M\}, \qquad (8)
\end{aligned}$$

$$\bar{\beta} := \sup_{t \in R^+} \{\beta(t)\}, \qquad (9)$$

where $M > 0$ is a constant. It is noted that the condition $\|\hat{x}(t)\| \leq M$ in (8) can be easily satisfied since the chaotic system is considered in a region of interest.

The control objective is then to stabilize the system (7) so that $\| e(t) \| \to 0$ as $t \to \infty$.

III. STABILIZATION ANALYSIS

In this section, the feedback adaptive gain $\mathcal{K}(t)$ will be designed so that the error states of (7) decrease asymptotically to zero.

We first introduce the following notations:

$$\begin{aligned}
\rho(t) &= \sup_{t \in R^+} \|N_{Ad}(t)\|, \\
\bar{\rho} &= \sup_{t \in R^+} \{\rho(t)\}, \\
\bar{\varphi} &= \frac{\bar{\rho}^2}{\eta} + \frac{\alpha^2}{\varepsilon_1} + \frac{1}{\varepsilon_2}, \\
\bar{\psi} &= 2\bar{\beta}, \qquad (10)
\end{aligned}$$

where the positive scalars η, ε_1 and ε_2 are chosen to be small enough such that

$$\Gamma := Q - \left(\varepsilon_1 + \varepsilon_2 \left\|L^T P\right\|^2 + \eta\right) I > 0, \qquad (11)$$

and the matrices P, Q and L are defined in (4) and (5). It is noted that all of the constants $\bar{\rho}$, $\bar{\varphi}$ and $\bar{\psi}$ in (10) are unknown.

Define a uniform continuous function $\upsilon(t) \in \mathcal{R}^+$ which satisfies that

$$\lim_{T \to \infty} \int_0^T \upsilon(t)\, dt \leq \bar{\upsilon} < \infty. \qquad (12)$$

Given the master system (1) and the slave system (3), we have the following result for the lag-synchronization.

Theorem 1: If there exist positive definite matrices P and Q with appropriate dimensions, and a real matrix L satisfying (4)-(5), then the slave system (3) can be lag-synchronized to the master system (1) by the controller (6) with the adaptive controller gains

$$\begin{cases}
\mathcal{K}(t) = -\frac{1}{2}\varphi(t) - \frac{\psi^2(t)}{2[\|\tilde{y}(t)\||\psi(t)|+\upsilon(t)]}, \\
\dot{\varphi}(t) = -\upsilon(t)\varphi(t) + \|\tilde{y}(t)\|^2, \\
\dot{\psi}(t) = -\upsilon(t)\psi(t) + \|\tilde{y}(t)\|.
\end{cases} \qquad (13)$$

Proof: Given the error system (7), construct a Lyapunov-Krasovskii functional candidate of the form

$$\begin{aligned}
V(t) &= e^T(t)Pe(t) + \eta \int_{t-\tau}^{t} e^T(\theta)e(\theta)d\theta \qquad (14) \\
&\quad + \frac{1}{2}\left(\tilde{\varphi}^2(t) + \tilde{\psi}^2(t)\right),
\end{aligned}$$

where P is the same positive definite matrix defined in (4) and (5), η is defined in (11), $\tilde{\varphi}(t) := \varphi(t) - \bar{\varphi}$, $\tilde{\psi}(t) := \psi(t) - \bar{\psi}$, and φ and ψ respectively the estimates of the unknown constant $\bar{\varphi}$ and $\bar{\psi}$.

Thus the derivative of (14) along the trajectories of (7) can be obtained as

$$\begin{aligned}
\dot{V}(t) &= 2e^T(t)P\dot{e}(t) + \eta e^T(t)e(t) - \eta e^T(t-\tau) \\
&\quad \times e(t-\tau) + \tilde{\varphi}(t)\dot{\varphi}(t) + \tilde{\psi}(t)\dot{\psi}(t) \\
&= e^T(t)\left[P(A-LC) + (A-LC)^T P + \eta I\right] e(t) \\
&\quad + 2e^T(t)PLCe(t) + 2e^T(t)PA_d x(t-\tau) \\
&\quad + 2e^T(t)PB\mathcal{K}(t)Ce(t) - \eta e^T(t-\tau)e(t-\tau) \\
&\quad + 2e^T(t)PB(F(t) + g(\hat{x}(t), \hat{x}(t-\tau), t)) \\
&\quad + \tilde{\varphi}(t)\dot{\varphi}(t) + \tilde{\psi}(t)\dot{\psi}(t). \qquad (15)
\end{aligned}$$

In view of the relation $\|F(t)\| \leq \alpha \|e(t)\|$ and the definitions in (10), one has

$$\begin{aligned}
&2e^T(t)PA_d e(t-\tau) \\
\leq\; &\frac{1}{\eta}e^T(t)PA_d A_d^T Pe(t) + \eta e^T(t-\tau)e(t-\tau) \\
=\; &\frac{\rho^2(t)}{\eta}\left\|B^T Pe(t)\right\|^2 + \eta e^T(t-\tau)e(t-\tau) \\
\leq\; &\frac{\bar{\rho}^2}{\eta}\left\|B^T Pe(t)\right\|^2 + \eta e^T(t-\tau)e(t-\tau), \qquad (16)
\end{aligned}$$

$$\begin{aligned}
&2e^T(t)PB(F(t) + g(\hat{x}(t), \hat{x}(t-\tau), t)) \\
\leq\; &2(\|F(t)\| + \|g(\hat{x}(t), \hat{x}(t-\tau), t)\|)\left\|B^T Pe(t)\right\| \\
\leq\; &2\alpha\left\|B^T Pe(t)\right\|\|e(t)\| + 2\beta(t)\left\|B^T Pe(t)\right\| \\
\leq\; &\frac{\alpha^2}{\varepsilon_1}\left\|B^T Pe(t)\right\|^2 + \varepsilon_1\|e(t)\|^2 + 2\bar{\beta}\left\|B^T Pe(t)\right\| \qquad (17)
\end{aligned}$$

$$\begin{aligned}
&2e^T(t)PLCe(t) \\
\leq\; &2\left\|L^T Pe(t)\right\|\|Ce(t)\| \\
\leq\; &\varepsilon_2\left\|L^T P\right\|^2\|e(t)\|^2 + \frac{1}{\varepsilon_2}\left\|B^T Pe(t)\right\|^2, \qquad (18)
\end{aligned}$$

where ε_1 and ε_2 are defined in (11).

Hence, substituting (4-5), and (16-18) into (15) yields

$$\begin{aligned}\dot{V}(t) &\leq e^T(t)\left[-Q+\left(\varepsilon_1+\varepsilon_2\left\|L^TP\right\|^2+\eta\right)I\right]e(t)\\&\quad+\left(\frac{\bar{\rho}^2}{\eta}+\frac{\alpha^2}{\varepsilon_1}+\frac{1}{\varepsilon_2}\right)\left\|B^TPe(t)\right\|^2\\&\quad+2\bar{\beta}\left\|B^TPe(t)\right\|+2e^T(t)PB\mathcal{K}(t)Ce(t)\\&\quad+\tilde{\varphi}(t)\dot{\varphi}(t)+\tilde{\psi}(t)\dot{\psi}(t)\\&= -e^T(t)\Gamma e(t)+\bar{\varphi}\left\|Ce(t)\right\|^2+\bar{\psi}\left\|Ce(t)\right\|\\&\quad+2e^T(t)PB\mathcal{K}(t)Ce(t)\\&\quad+(\varphi(t)-\bar{\varphi})\dot{\varphi}(t)+\left(\psi(t)-\bar{\psi}\right)\dot{\psi}(t),\end{aligned}\tag{19}$$

where $\bar{\varphi}$ and $\bar{\psi}$ are defined in (10), and $\Gamma>0$ is defined in (11).

Substituting the adaptive laws in (13) into (19) yields that

$$\begin{aligned}\dot{V}(t) &\leq -e^T(t)\Gamma e(t)+\bar{\varphi}\left\|Ce(t)\right\|^2+\bar{\psi}\left\|Ce(t)\right\|\\&\quad+2e^T(t)PB\left(-\frac{1}{2}\varphi(t)\right.\\&\quad\left.-\frac{\psi^2(t)}{2\left[\left\|\tilde{y}(t)\right\|\left|\psi(t)\right|+\upsilon(t)\right]}\right)Ce(t)\\&\quad+(\varphi(t)-\bar{\varphi})\left(-\upsilon(t)\varphi(t)+\left\|\tilde{y}(t)\right\|^2\right)\\&\quad+\left(\psi(t)-\bar{\psi}\right)\left(-\upsilon(t)\psi(t)+\left\|\tilde{y}(t)\right\|\right)\\&= -e^T(t)\Gamma e(t)-\upsilon(t)(\varphi(t)-\bar{\varphi})\varphi(t)\\&\quad-\upsilon(t)\left(\psi(t)-\bar{\psi}\right)\psi(t)+\psi(t)\left\|\tilde{y}(t)\right\|\\&\quad-\frac{\psi^2(t)\left\|\tilde{y}(t)\right\|^2}{\left\|\tilde{y}(t)\right\|\left|\psi(t)\right|+\upsilon(t)}.\end{aligned}\tag{20}$$

Noticing the fact that for any functions $\psi(t)$ and $\upsilon(t)$,

$$\begin{aligned}&-\frac{\psi^2(t)\left\|\tilde{y}(t)\right\|^2}{\left\|\tilde{y}(t)\right\|\left|\psi(t)\right|+\upsilon(t)}\\&\leq -\left\|\tilde{y}(t)\right\|\left|\psi(t)\right|+\upsilon(t)\\&\leq -\left\|\tilde{y}(t)\right\|\psi(t)+\upsilon(t),\end{aligned}$$

one can obtain from (20) that

$$\begin{aligned}\dot{V}(t) &\leq -e^T(t)\Gamma e(t)+\upsilon(t)+\frac{1}{4}\upsilon(t)\left(\bar{\varphi}^2+\bar{\psi}^2\right)\\&\quad-\upsilon(t)\left[\left(\varphi(t)-\frac{1}{2}\bar{\varphi}\right)^2+\left(\psi(t)-\frac{1}{2}\bar{\psi}\right)^2\right]\\&\leq -e^T(t)\Gamma e(t)+\varsigma\upsilon(t),\end{aligned}\tag{21}$$

where

$$\varsigma=1+\frac{1}{4}\bar{\varphi}^2+\frac{1}{4}\bar{\psi}^2.$$

In the following, we aim to show that $e(t)\to 0$ as $t\to\infty$.

Integrating inequality (21) from $t=0$ to any $T>0$ yields that

$$\begin{aligned}&V(T)+\int_0^T e^T(t)\Gamma e(t)\,dt\\&\leq V(0)+\varsigma\int_0^T \upsilon(t)\,dt\end{aligned}\tag{22}$$

For $T\to\infty$, it follows from (22) that

$$\begin{aligned}&\lim_{T\to\infty}\int_0^T e^T(t)\Gamma e(t)\,dt\\&\leq V(0)+\varsigma\lim_{T\to\infty}\int_0^T\upsilon(t)\,dt\\&\leq V(0)+\varsigma\bar{\upsilon}.\end{aligned}$$

It then implies that $e(t)$ is uniformly bounded. The boundednesses of $\varphi(t)$ and $\psi(t)$ can be directly established from the boundedness of $e(t)$. Thus it follows from (6) and (13) that $u(t)$ is bounded. Moreover, the boundedness of $\hat{x}(t)$ is also achieved from (7) since $\hat{x}(t)$ can be expressed in terms of $e(t)$ and the bounded state vector $x(t)$ of the master system (1). Therefore, the right hand part of (7) is bounded, i.e. $\dot{e}(t)\in l_\infty$. It follows from Barbălat's Lemma [20] that $e(t)\to 0$ as $t\to\infty$. The proof is thus completed. ■

It is well known that the estimates $\varphi(t)$ and $\psi(t)$ in (13) without the terms $-\upsilon(t)\varphi(t)$ and $-\upsilon(t)\psi(t)$ may lead to a high-gain controller $\mathcal{K}(t)$. But the adaptive laws (13) can avoid the high-gain phynomenon. Moreover, the designed controller can guarantee that the error states converge uniformly asymptotically to zero, while some modified adaptive laws [21] similar to (13) of this paper can only guarantee the system states to be uniformly ultimate boundedness.

IV. ILLUSTRATIVE EXAMPLE

To illustrate the proposed synchronization approach, we consider a time-delay Chua's circuit described by

$$\left\{\begin{array}{l}\dot{x}_1(t)=a(x_2(t)-m_1x_1)+f_1(x_1)-cx_1(t-\tau),\\\dot{x}_2(t)=x_1(t)-x_2(t)+x_3(t)-cx_1(t-\tau),\\\dot{x}_3(t)=-bx_2(t)+c(2x_1(t-\tau)-x_3(t-\tau)),\\y=5x_1+2x_2+3x_3,\end{array}\right.\tag{23}$$

with the nonlinear characteristics

$$f(x_1)=\frac{1}{2}(|x_1+1|-|x_1-1|),$$

and parameters $m_0=-\frac{1}{7}$, $m_1=\frac{2}{7}$, $a=9$, $b=14.28$, $c=0.1$ and the system time-delay $\tau=1$.

For the master system, the initial condition is set as $\phi(0)=\begin{bmatrix}-0.2 & -0.3 & 0.2\end{bmatrix}^T$.The master system displays a double scroll attractor as shown in Fig. 2.

We first rewrite the system (23) as (1) with

$$\begin{aligned}A &= \begin{bmatrix}-\frac{18}{7} & 9 & 0\\1 & -1 & 1\\0 & -14.28 & 0\end{bmatrix},\\A_d &= \begin{bmatrix}-0.1 & 0.2 & -0.1\\0 & 0 & 0\\0 & 0 & 0\end{bmatrix},B=\begin{bmatrix}\frac{27}{7}\\0\\0\end{bmatrix},\\C &= \begin{bmatrix}5 & 2 & 3\end{bmatrix}.\end{aligned}$$

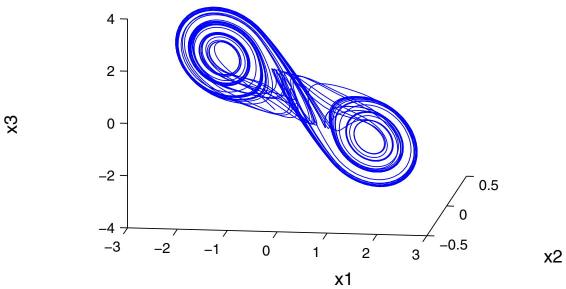

Fig. 2. The phase portrait of the master Chua's circuit.

In the slave system, the uncertainties in (3) are given as

$$\Delta A(t) = \begin{bmatrix} 0.8\sin(2t) & 1+0.3\sin(2t) & 1-0.5\cos(t) \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix},$$

$$\Delta A_d(t) = \begin{bmatrix} 1+\sin(3t) & 0.4 & \cos(3t) \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix},$$

$$\Delta f(\hat{x}(t), t) = \frac{1}{2}\left(|0.1\hat{x}_1(t)+0.1| - |0.1\hat{x}_1(t)-0.1|\right).$$

Obviously, $\Delta A(t)$ and $\Delta A_d(t)$ can be expressed in the forms of $\Delta A(t) = BE_A(t)$ and $\Delta A_d(t) = BE_{Ad}(t)$.

The propogation delay d is 0.5, but it is assumed to be unknown in the simulation. The initial condition of the slave system is $\hat{\phi}(0) = \begin{bmatrix} 0.1 & 0.1 & 0.1 \end{bmatrix}^T$.

For the controller design, consider the constraint (5) and choose

$$P = \begin{bmatrix} 1.3 & 0.52 & 0.78 \\ 0.52 & 3.4 & 0.2 \\ 0.78 & 0.2 & 0.55 \end{bmatrix}.$$

Using the linear matrix inequality (LMI) technique, it is easy to get feasible solutions for (4) as

$$L = \begin{bmatrix} -3.8 & 0.3854 & 5.72 \end{bmatrix}^T,$$

$$Q = \begin{bmatrix} 1.4286 & -0.1318 & 0.8740 \\ -0.1318 & 2.5354 & -0.2037 \\ 0.8740 & -0.2037 & 0.5794 \end{bmatrix}.$$

Next, for the adaptive laws and controller gain in (13), choose $\varepsilon_1 = \varepsilon_2 = \eta = 0.01$ such that (11) holds. Let $\upsilon(t) = 6e^{-0.2t}$, $\varphi(0) = 0$ and $\psi(0) = 0$. Thus an adaptive controller (6) can be obtained.

With the obtained adaptive controller (6), the control result is shown in Fig. 3-Fig. 8. In the simulation, the adaptive controller is switched on at $t = 25\,\mathrm{sec}$. It is observed from Fig. 3-Fig. 5 that the error states of (7) is asymptotically stabilized to 0. It then implies that the slave system is controlled to asymptotically lag-synchronize the master system. It is shown in Fig. 7 and Fig. 8 that the estimating variables $\varphi(t)$ and $\psi(t)$ also converge. It is also observed that there is no high-gain phynomenon.

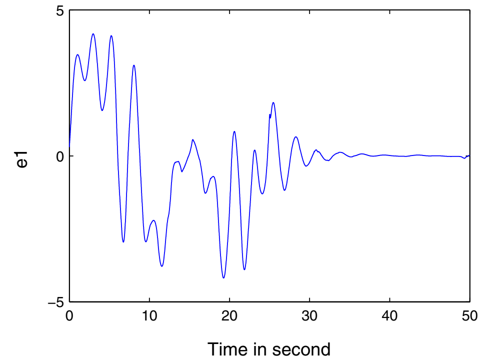

Fig. 3. The time response of $e_1(t)$.

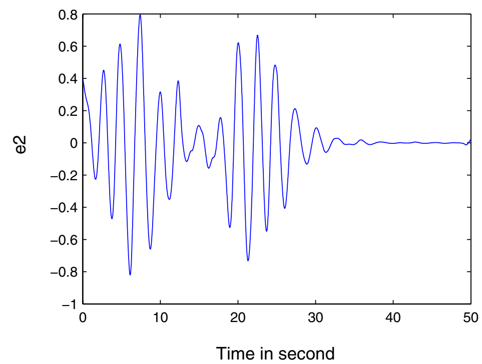

Fig. 4. The time response of $e_2(t)$.

V. CONCLUSION

An analytical method for adaptive feedback controller design method is proposed to achieve the lag-synchronization in the master-slaver configuration with unknown system delay and propogation delay. The time-varying uncertainties in the slave system and differences between the two systems are considered. Here, the upper bounds of the unknown system nonlinear function, time-delay state matrix function and uncertainties are all assumed to be unknown. It is shown that the obtained adaptive controller can force the slave

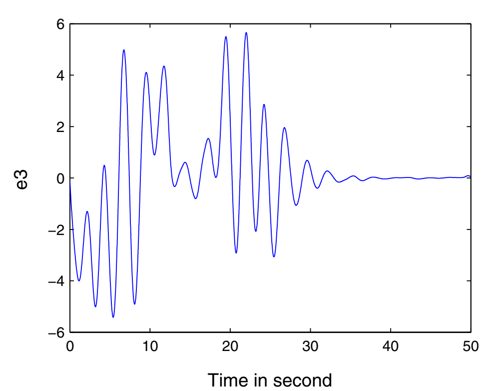

Fig. 5. The time response of $e_3(t)$.

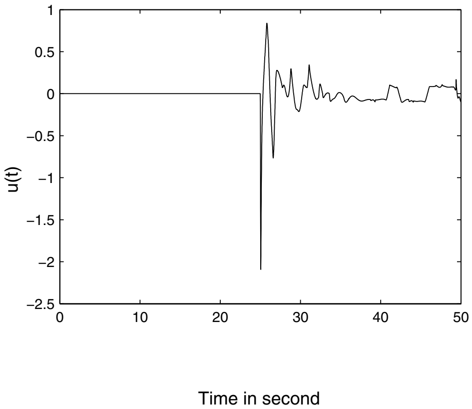

Fig. 6. The control variable.

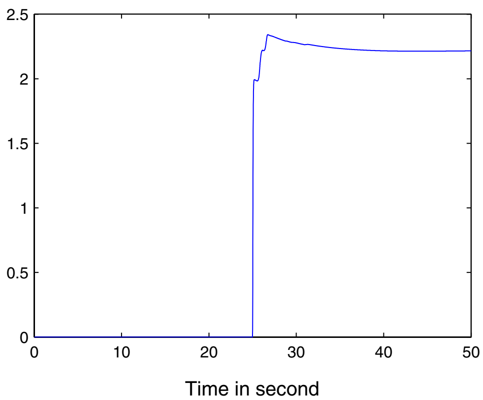

Fig. 7. The updating parameter $\varphi(t)$.

system asymptotically lag-synchronize the master system. The effectiveness of the proposed method is demonstrated by the time-delay Chua's circuit lag-synchronization.

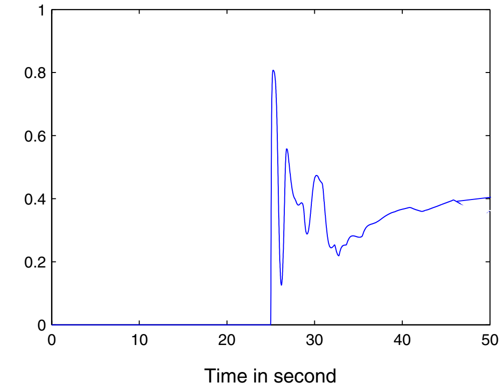

Fig. 8. The updating parameter $\psi(t)$.

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