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Adaptive Optimal PI Controller for High-Precision Low-Temperature Experiments

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Abstract— **In this paper, we describe an adaptive optimal PI controller for high-precision low-temperature experiments. We used a modified LQR algorithm to obtain the optimal parameters for the PI controller. Since the plant is nonlinear, a gain scheduling method is applied to modify the optimal parameters under different operating conditions. The simulation results show that this controller has good transient response, disturbance rejection ability, and robustness. Furthermore, the controller can accommodate a variety of first-order nonlinear systems. At last, we discuss the design of the optimal PID controller for a class of second order system using the modified LQR algorithm.**

I. INTRODUCTION

THE Critical Dynamics in Microgravity (DYNAMX)[1]-[5] experiment examines the behavior of the liquid helium near the superfluid transition point while being driven away from equilibrium by a heat flux. The experimental apparatus includes five stages connected to each other with stable thermal links. Each stage has a separated controller to regulate the temperature at a desired value and provides a stable temperature platform for the experiment. The temperature control precision is of 1 nano-kelvin (10^{-9} Kelvin).

DYNAMX utilizes the High Resolution of Thermometers (HRTs) as the measurement device in the experiments. It measures the temperature dependent magnetic susceptibility of a dilute alloy of Mn in BaPd Matrix, which serves as the temperature sensing element. PdMn develops a magnetization proportional to its susceptibility when placed in a magnetic filed. A change in the temperature of the PdMn causes a change in its susceptibility and therefore a change in its magnetization. A superconducting pick-up coil is wound around the PdMn, to couple any change in the magnetic field through the PdMn to a dc Superconducting Quantum Interference Device (SQUID). The SQUID system converts the change in magnetic field into voltage, which in turn can be read by conventional digitizing electronics. In order to meet the DYNAMX science requirements, the HRT noise must not exceed 0.3 nK per root-Hz.

In DYNAMX, we choose the Proportional Integral (PI) controller instead of PID because the derivative term may exaggerate the noise and degrade the performance of the experiment.

A PI controller must be tuned to its optimal parameter values for best performance. Conventional PID/PI tuning methods, such as the Ziegler-Nichols tuning, cannot always guarantee satisfactory performance. In this paper, we describe a method to optimally tune the parameters of the PI controller using a modified Linear Quadratic Regulator (LQR) algorithm. This method can guarantee an optimal control performance that minimizes a certain cost function.

Another problem is the nonlinearity of the system. A fixed parameter PI controller cannot always keep the control performance within an acceptable range. We apply the gain scheduling method to adjust parameters of the PI controller for the nonlinear system.

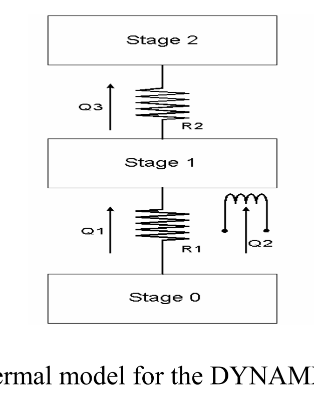

Fig. 1. Thermal model for the DYNAMX apparatus

Fig. 1 shows a simplified three stage thermal model. The temperature of stage 0 and stage 2 are assumed to be constants. We want to develop an optimal PI controller for the middle stage 1.

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According to the model we have the following dynamic equations:

$$(Q_1 + Q_2 - Q_3) = \frac{dT_1}{dt} \cdot C_1 \tag{1.1}$$

$$(T_0 - T_1) = Q_1 \cdot R_1 \tag{1.2}$$

$$(T_1 - T_2) = Q_3 \cdot R_2 \tag{1.3}$$

Q_1 is the heating rate (in watts) from stage 0 to stage 1, Q_2 is the heating rate output from the heater and Q_3 is the heating rate from stage 1 to stage 2. T_0, T_1 and T_2 are the temperatures of stage 0, 1 and 2 respectively. R_1 is the thermal resistance (in Kelvin/Watt) between stage 0 and stage 1, and R_2 is the thermal resistance between stage 1 and stage 2. C_1 is the heat capacity (in Joules/Kelvin) of stage 1.

Substitute Q_1 and Q3 in (1.1) with (1.2) and (1.3), we get the following equation:

$$\dot{T}_1 = (-\frac{1}{R_1 \cdot C_1} - \frac{1}{R_2 \cdot C_1}) \cdot T_1 + \frac{Q_2}{C_1} + \frac{T_0}{C_1 \cdot R_1} + \frac{T_2}{C_1 \cdot R_2} \tag{2}$$

The thermal resistances can be treated as constants, but the heat capacity varies with temperature[7]:

$$C(\widetilde{T}) \cong P \cdot \ln\left|\widetilde{T}\right| + Q, \quad \widetilde{T} = \left|1 - \frac{T}{T_\lambda}\right| \tag{3}$$

where $T_\lambda = 2.1768K$, is the superfluid transition temperature. If $T \ge T_\lambda$, then P=-5.355, Q=-7.77. If $T < T_\lambda$, then P=-5.10, Q=15.52.

II. Controller Design

A. Optimal controller design[8]-[10]

To design an optimal controller, we assume the heat capacity is a constant, therefore a linear time invariant system model can be used.

Since the temperature of the stage 0 and 2 are constants, we can use the constant disturbance w to represent them. Assuming $x = T_1$, u is the output of the controller, we have

$$\dot{x} = ax + bu + w \tag{4}$$

where $a = -\frac{1}{R_1 \cdot C_1} - \frac{1}{R_2 \cdot C_1}$, $w = \frac{T_0}{C_1 \cdot R_1} + \frac{T_2}{C_1 \cdot R_2}$, $b = \frac{\alpha}{C_1}$, α is the gain of the high resolution heater.

The measurement of temperature is y and output equation is:

$$y = cx \tag{5}$$

where c is the gain of the HRT.

Since the reference value is not zero and there are constant disturbances in the state-space equations, we cannot use the standard LQR algorithm directly. However, through the state augmentation, it can be reduced to a typical LQR problem.

Assuming $\widetilde{x}$ is the error between the state x and the desired state x_d, i.e. $\widetilde{x} = x - x_d$, we have

$$\dot{\widetilde{x}} = a\widetilde{x} + bu + (ax_d + w) \tag{6}$$

$$z = y - cx_d = c\widetilde{x} \tag{7}$$

Choose a new state vector $\xi = \begin{pmatrix} \dot{\widetilde{x}} \\ z \end{pmatrix}$, and let $\nu = \dot{u}$,

$$\dot{\xi} = \begin{pmatrix} \ddot{\widetilde{x}} \\ \dot{z} \end{pmatrix} = \begin{pmatrix} a & 0 \\ c & 0 \end{pmatrix} \cdot \begin{pmatrix} \dot{\widetilde{x}} \\ z \end{pmatrix} + \begin{pmatrix} b \\ 0 \end{pmatrix} \cdot \dot{u} \tag{8}$$

$$= \begin{pmatrix} a & 0 \\ c & 0 \end{pmatrix} \cdot \xi + \begin{pmatrix} b \\ 0 \end{pmatrix} \cdot \nu = A_1\xi + B_1\nu$$

where $A_1 = \begin{pmatrix} a & 0 \\ c & 0 \end{pmatrix}$, $B_1 = \begin{pmatrix} b \\ 0 \end{pmatrix}$.

Now our problem is reduced to a standard LQR problem, i.e. for the system (8), we are finding a state feedback controller of the form $\nu = -K \cdot \xi$ to minimize a cost function $\Phi = \int_0^\infty (\xi' M \xi + \nu' N \nu) dt$, where M is the weight matrix of the control accuracy and N is the weight of control effort.

The solution for the above LQR problem is:

$$K = N^{-1} B_1' P \tag{9}$$

where P satisfies the algebraic Riccati equation

$$A_1 P + P A_1 + M - P B_1 N^{-1} B_1' P = 0 \tag{10}$$

It is nontrivial to solve the Riccati equation. Fortunately, we can use the computer to get the solution of K without difficulty. Matlab also has a standard LQR function which can be used to solve this problem.

After the optimal control gain vector K is calculated, the controller becomes:

$$\nu = -K \cdot \xi = -k_1 \cdot \dot{\widetilde{x}} - k_2 \cdot z \tag{11}$$

Integrating the above equation, we get the PI controller

$$u(t) = -k_1 \tilde{x} - k_2 \int_0^t z \cdot dt \tag{12}$$
$$= -k_1 \cdot (x - x_d) - k_2 \cdot \int_0^t (y - cx_d) \cdot dt$$
$$= -\frac{k_1}{c} \cdot e - k_2 \cdot \int_0^t e \cdot dt$$

where $e = y - c \cdot x_d$, is the error between the output and the reference value.

Substituting $R_1 = 1790 K/W$, $R_2 = 6000 K/W$, $C_1 = 2.41 J/K$, $\alpha = 0.05 W/V$ and $c = 1\times10^{-3}\phi_0 / K$ into (4) and (5), we can get the parameters: $a = -3\times10^{-4}$ Hz, $b = 0.02$Hz·K/V, $w = 2\times10^{-4}\cdot T_0 + 7\times10^{-5}\cdot T_2$

In our experiments, since the heater power is typically about $10\mu W$, this is not a challenge. But the control precision is an important parameter. Therefore, we choose $N = 1$, $M = \begin{pmatrix} 0 & 0 \\ 0 & 100 \end{pmatrix}$.

Using above algorithm, we get $K = (k_1 \quad k_2) = (32 \quad 10)$. According to (12), the corresponding optimal PI controller is:

$$u(t) = 32 \cdot e(t) + 10 \cdot \int_0^t e(t) \cdot dt \tag{13}$$

B. Gain Scheduling [11]

In the actual experiment, the heat capacity of the stage 1 is not a constant since it changes with the temperature. If the parameters of the optimal controller are kept unchanged, the performance of the closed-loop system will become degraded, and possibly even unstable. Therefore, we need to modify the parameters in accordance to the temperature change.

Since we know how the dynamics of our system change with the operating conditions of the process, the gain scheduling method can be used to adapt the PI parameters. Obviously, the temperatures of the stages are chosen as the scheduling variables.

When the temperature of stage 1, T_1, changes, the heat capacity C_1 changes in accordance to the rule of (3). Parameters a, b, w within (4) changes. That means the system model changes. The proportional gain and the integral gain in (13) should be changed accordingly. Since we cannot get direct mathematic expressions for the relationships between the temperature T_1 and gains of the optimal PI controller, the problem must be solved in the following two steps:

First, we use the modified LQR algorithm described above to get the optimal PI parameters corresponding to each heat capacity. Fig. 2 shows how the parameters of the PI controller vary with the heat capacities. The proportional gain in the PI controller, k_1 changes almost linearly with the heat capacity and the integral time, k_2, remain constant.

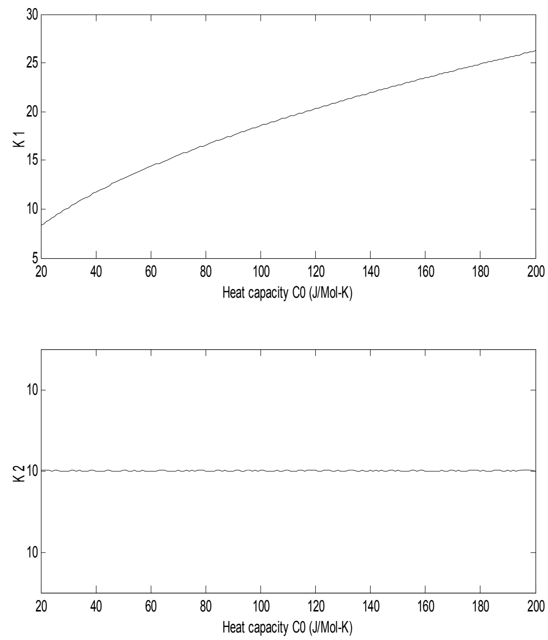

Fig. 2. The changes of the optimal gains to the heat capacity

Next we get the relationship between the parameters of the PI controller and the current temperature value through (3). The relationship between the temperature and the optimal parameters can be saved in a table, so we can use it directly without additional computing cost during the experiment.

III. Simulation Results

Fig. 3 shows the comparison of the step responses between our optimal controller and a PI controller we used in the experiments before. This PI controller was tuned with Ziegler-Nichols method at the same temperature.

The step response of the conventional PI controller has an overshoot 12.88% and the settling time is 53 seconds. For the optimal PI controller the overshoot is 10.69% and the settling time is only 10 seconds.

In table 1, we compare the disturbance rejection ability using the root mean square (RMS) of the steady state error for different controllers. The simulation sampling time is 0.01s. We add a Gaussian process noise w(t) and a measurement noise v(t). The expected values for both are 0. The variance for w(t) is 0.1, for v(t) is 0.003.

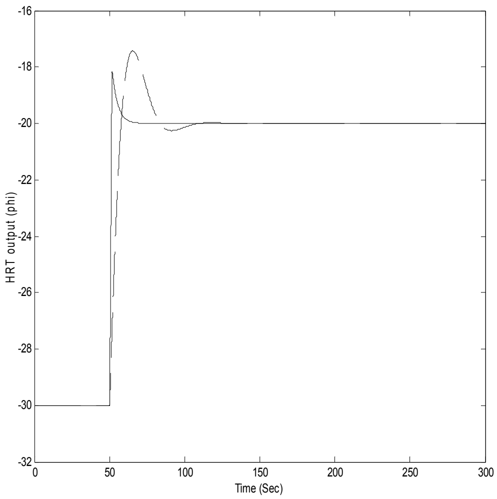

Fig. 3. Step response for the optimal PI controller (solid line) and PI controller tuned with Ziegler-Nichols method (dashed line).

Table1. The comparison of the disturbance rejection ability for four control algorithms

Controller	Parameters	RMS(Φ_0)
PI	D(S)=2*(1+1/8S)	0.05832
PI+Kalman filter	D(S)=2*(1+1/8S); K_k =[0.0722]	0.01953
Optimal PI controller	D(s)=32+10/S	0.05462
Optimal PI controller + Kalman filter	D(s)=32+10/S; K_k =[0.0722];	0.00837

Note: 1) D(S) is the transfer function of the controller, S is the Laplace operator.

2) K_k is the gain of the Kalman filter.

From the table, we can see that the optimal PI controller has a better performance to attenuate the noises than the PI controller tuned with conventional methods with or without the Kalman filter [12].

In fig. 4, we compare the robustness of the between the fixed PI controller and the adaptive PI controller. The three figures on the left column show the step responses of the system with fixed parameters PI controller, $D(s) = 32 + \frac{10}{s}$, when the heat capacities are 20 J/Mol-K, 100 J/Mol-K and 200 J/Mol-K respectively. The figures on the right column show the response of the system with adaptive PI controller under the same operating conditions.

From Fig. 4a, we can see that when the heat capacity is 20J/mol-K, the system is unstable if we use the fixed PI controller. The performance is improved when we use the adaptive PI controller. The gain scheduling method can guarantee not only the closed-loop system stability but also the optimal performance.

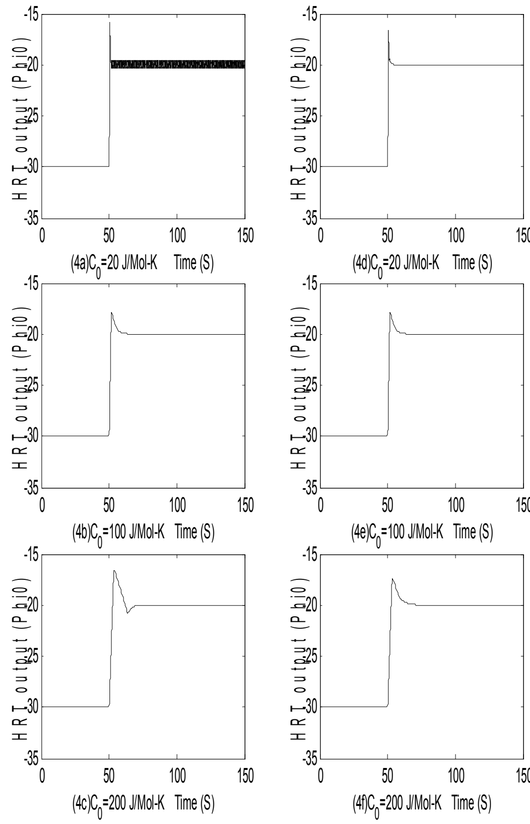

Fig. 4 Comparisons of the step responses between the fixed PI controller (left figures) and the adaptive PI controller (right figures).

IV. CONCLUSIONS

In this paper, we describe an adaptive optimal PI controller for the low temperature experiment DYNAMX.

From the simulation results, we can see that the closed loop system with the optimal PI controller has better transient response and disturbance rejection ability than that with the PI controller tuned with the traditional Ziegler-Nichols methods. Moreover, by combining with the gain scheduling algorithms, the controller becomes robust without loss of its good performances. Since all the calculations can be performed offline, the control algorithm doesn’t have additional computing cost during the experiment.

Without loss of generality, the controller we attain from the above design can accommodate a variety of nonlinear plants of one order form:

$$\dot{y} = A(y) \cdot y + B(y) \cdot u + W(y) \tag{14}$$

APPENDIX

An optimal PID controller can be derived using the above modified LQE algorithm if the system model is in the form $D(S) = \frac{c}{S^2 + aS + b}$, where a, b and c are constants.

The transfer function can be written as the following state space equations:

$$\dot{X} = \begin{pmatrix} \dot{x}_1 \\ \dot{x}_2 \end{pmatrix} = \begin{pmatrix} 0 & 1 \\ -b & -a \end{pmatrix} \cdot \begin{pmatrix} x_1 \\ x_2 \end{pmatrix} + \begin{pmatrix} 0 \\ c \end{pmatrix} \cdot u = AX + Bu \tag{15}$$

$$y = \begin{pmatrix} 0 & 1 \end{pmatrix} \cdot \begin{pmatrix} x_1 \\ x_2 \end{pmatrix} = CX \tag{16}$$

Since the desired values are not zeros, assume $\tilde{X}$ is the error between the state vector X and the desired value vector X_d, i.e. $\tilde{X} = X - X_d = \begin{pmatrix} x_1 - x_{1d} \\ x_2 - x_{2d} \end{pmatrix}$, where x_{1d} and x_{2d} are the desired value for x_1 and x_2 respectively.

$$\dot{\tilde{X}} = A\tilde{X} + Bu + AX_d \tag{17}$$

$$z = y - CX_d = C\tilde{X} \tag{18}$$

Choose a new state vector $\xi = \begin{pmatrix} \dot{\tilde{X}} \\ z \end{pmatrix}$, and let $v = \dot{u}$,

$$\dot{\xi} = \begin{pmatrix} \ddot{\tilde{X}} \\ \dot{z} \end{pmatrix} = \begin{pmatrix} A & 0 \\ C & 0 \end{pmatrix} \cdot \begin{pmatrix} \dot{\tilde{x}} \\ z \end{pmatrix} + \begin{pmatrix} B \\ 0 \end{pmatrix} \cdot v \tag{19}$$

$$= \begin{pmatrix} A & 0 \\ C & 0 \end{pmatrix} \cdot \xi + \begin{pmatrix} B \\ 0 \end{pmatrix} \cdot v$$

Now we can have a standard LQR problem. Using the Matlab function LQR, we can find the solution of K and the optimal controller can be expressed as follows:

$$v = -K \cdot \xi = \begin{pmatrix} k_{11} & k_{12} & k_2 \end{pmatrix} \cdot \begin{pmatrix} \tilde{x}_1 \\ \tilde{x}_2 \\ z \end{pmatrix} \tag{20}$$

Integrating the above equation, we get the final PID control law

$$u(t) = -k_{11} \cdot (x_1 - x_{1d}) - k_{12} \cdot (x_2 - x_{2d}) \tag{21}$$

$$- k_2 \cdot \int_0^t (y - x_{2d})$$

$$= -k_{11} \cdot \dot{e} - k_{12} \cdot e - k_2 \cdot \int_0^t e \cdot dt$$

where $e = y - c \cdot x_{2d} = x_2 - x_{2d}$.

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