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SLIDING MODE CONTROL OF A SIMULATED MEMS GYROSCOPE

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Abstract
The micro electro mechanical systems (MEMS) are penetrating more and more into measurement and control problems because of their small size, low cost, and low power consumption. The vibrating gyroscope is one of those MEMS devices that will have significant impact on the stability control systems in transportation industry. This paper studies the design and control of a vibrating gyroscope. The device has been constructed in Pro-E environment and its model has been simulated in finite element domain in order to approximate its dynamic characteristics with a lumped model. A model reference adaptive feedback controller and the sliding mode controller have been considered to guarantee the stability of the device. It is shown that the sliding mode controller of the vibrating proof mass results in better estimate of the unknown angular velocity than that of the model reference adaptive feedback controller.

1. Introduction

Figure 1 shows the operating principle of the vibrating rate gyroscopes. The proof mass is affected by the Coriolis force due to table rotation of angular velocity Ω. The most challenging control issue is the minimization of coupling between the motions in x (actuation) and y (sensing) axes while measuring Ω which is a time varying unknown term. Additionally, the damping and natural frequencies are only approximately known and they are subject to change.

Contribution of this work is to design an adaptive model reference and a sliding mode controller to improve the performance of an existing MEMS design in [2]. We show that the model reference controller can converge under some restrictive assumptions. Specifically we prove that the boundedness of signals does not guarantee a consistent estimate of the unknown angular velocity Ω unless both the actuation and sensing axes contain active control signals as in [3], [10]. The second contribution is the introduction of a sliding mode controller which guarantees stability in the Lyapunov sense despite the bounded uncertainties in the system. Several techniques are proposed to consistently estimate Ω while the system is under the sliding mode controller.

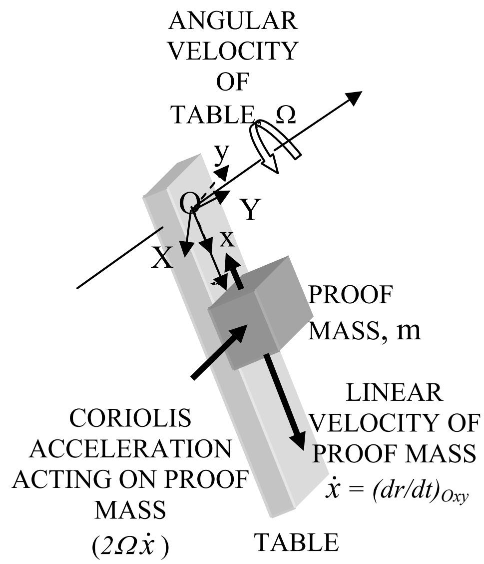

Figure 1. Operating principle of the rate gyroscope

Figure 2a shows the basic structure of the vibrating MEMS gyroscope as described by Yoichi et. al. in [2]. Figure 2b shows the technical drawing of the gyroscope used in this study. The device has essentially two proof masses. Four outer beams suspend the outside proof mass. To increase motion sensitivity in *y*-direction, four inner beams are created by machining four legs thereby creating an inner proof-mass. In our model development we lump these two masses into one. The electro static force generated by the comb actuator vibrates the proof mass in *x*-direction. If the proof mass rotates with respect to z-axis, a Coriolis force is generated in the direction of the *y*-axis,i.e., the sensing direction.

Figure 2a: Assembly of the gyroscope.
(Figure is given in K., Qais, C. Batur," Design and Control of a Vibrating Gyroscpe", pp. 2505-2510. 0-IEEE # 04CH37538C, ISBN: 0-7803-8336-2, Proceedings of the American Control Conference, June 30-July 2, 2004.)

Figure 2b: Technical drawing of the gyroscope. All dimensions are in micron.
(Figure is given in K., Qais, C. Batur," Design and Control of a Vibrating Gyroscpe", pp. 2505-2510. 0-IEEE # 04CH37538C, ISBN: 0-7803-8336-2, Proceedings of the American Control Conference, June 30-July 2, 2004.)

This force moves the proof mass in the direction of the *y*-axis. This displacement is proportional to the angular rate of rotation with respect to z-axis. The displacement is measured by the capacitance change in *y*-direction. Simulated motions of the device in *x* and *y* directions are shown in Figures 3 and 4 [1]. All simulations are performed in standard Pro-E and Pro-Mechanica.

3. Dynamic Model

The lumped spring-mass-dashpot model of the vibrating gyroscope is assumed to be in the form shown in Figure 5. The equations of motion become:

$$m\ddot{x} + C_{xx}\dot{x} + k_{xx}x + C_{xy}\dot{y} + k_{xy}y = u_x + 2m\Omega\dot{y} \quad (1)$$

$$m\ddot{y} + C_{yy}\dot{y} + k_{yy}y + C_{xy}\dot{x} + k_{xy}x = u_y - 2m\Omega\dot{x} \quad (2)$$

Where *m* is the mass of the proof mass, *x* and *y* are the coordinates of the proof mass relative to the table frame, k_{xx}, k_{yy}, k_{xy}, are the spring coefficients.

Figure 3. Deflection of the outer beams in *x*-direction (drive direction).

Figure 4. Deflection of the inner beams in *y*-direction (sense direction)
(Figures are given in K., Qais, C. Batur," Design and Control of a Vibrating Gyroscpe", pp. 2505-2510. 0-IEEE # 04CH37538C, ISBN: 0-7803-8336-2, Proceedings of the American Control Conference, June 30-July 2, 2004.)

The parameters C_{xx}, C_{yy}, C_{xy} represent the damping, $u_x(t)$, $u_y(t)$, are the electrostatic driving forces, Ω is the unknown rotational speed, and $2m\Omega\dot{y}, -2m\Omega\dot{x}$ are the coupling forces due to Coriolis effect. Typical assumptions made in deriving the lumped

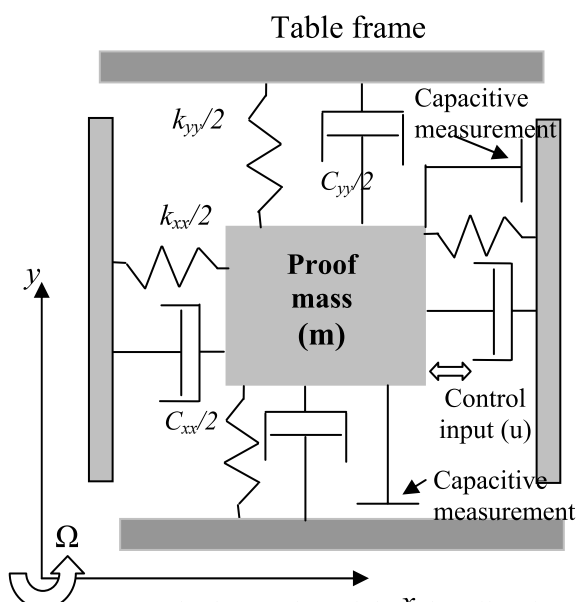

Figure 5. The lumped model of the vibrating gyroscope

dynamic model (1)-(2) are:
$\Omega_x^2 \approx \Omega_y^2 \approx \Omega_z^2 \cong 0$; $\dot{\Omega}_z \equiv \dot{\Omega} \approx 0$ [7].

Friedland and Hutton [13} and Painter and Shekel [14] analyzed the behavior of this coupled system extensively. In order to minimize the coupling between the actuation (*x*) and sense modes (*y*) a four degrees of freedom (two in actuation and two in sensing) has been proposed by Acar and Shkel [12] but we will not use this arrangement in this work. The initial development of MEMS gyroscopes and the controller designs can be traced to the work of Shkel et al [15].

The parameters of the lumped model are estimated by simulating the gyroscope of Figure 2 using Pro-E and Pro-Mechanica. Application of step forces $u_x(t)$ $u_y(t)$ in x and y directions generates step responses of the gyroscope from which the unknown coefficients $[k_{xx}, k_{yy}, k_{xy}, C_{xx}, C_{yy}\ C_{xy}]$ are estimated using second order transfer function models.

The model (1) and (2) are further transformed into forms in terms of the quality factors and the natural frequencies as below,

$$x^{\bullet\bullet} + \frac{\omega_n}{q}x^{\bullet} + \omega_n^2 x + \frac{\omega_{n2}}{q_2}y^{\bullet} + \omega_{n2}^2 y = u + 2\Omega y^{\bullet} \qquad (3)$$

$$y^{\bullet\bullet} + \frac{\omega_{n1}}{q_1}y^{\bullet} + \omega_{n1}^2 y + \frac{\omega_{n2}}{q_2}x^{\bullet} + \omega_{n2}^2 x = \frac{u_y}{m} - 2\Omega x^{\bullet} \qquad (4)$$

Where:

$\frac{\omega_n}{q} = \frac{C_{xx}}{m}$, $\omega_n = \sqrt{\frac{k_{xx}}{m}}$, $\frac{\omega_{n1}}{q_1} = \frac{C_{yy}}{m}$, $\omega_{n1} = \sqrt{\frac{k_{yy}}{m}}$, $\frac{\omega_{n2}}{q_2} = \frac{C_{xy}}{m}$, and $\omega_{n2} = \sqrt{\frac{k_{xy}}{m}}$ is the natural frequency due to coupling, $u = u_x / m$ and the parameters (q, q_1, q_2) are the quality factors in x and y direction and coupling respectively. Ω is the unknown constant angular velocity that needs to be determined. In fact Ω is precisely the quantity of interest on this angular velocity measurement device.

Since a typical finite element model only considers the structural damping, the damping coefficients $[C_{xx}, C_{yy}\ C_{xy}]$ are modified using the Couette and squeeze film damping models [8]-[9]. These additional effects are negligible if the proof mass is operating in vacuum.

3. Model Reference Controller

(This section has been removed to save space, interested reader is referred to K., Qais, C. Batur," Design and Control of a Vibrating Gyroscpe", pp. 2505-2510. 0-IEEE # 04CH37538C, ISBN: 0-7803-8336-2, Proceedings of the American Control Conference, June 30-July 2, 2004.

Sliding Mode Controller

The sliding mode controller is also designed to maintain the proof mass to oscillate in x-direction at a given frequency and amplitude, i.e., $x = x_d = A \sin \omega_n t$ despite all bounded uncertainties in the gyroscope parameters. The bounded uncertainties are assumed to be in the following form:

$$B_k^{lower} < \left| \frac{K_{xy}}{m} - \frac{\hat{K}_{xy}}{m} \right| < B_K^{upper}$$

$$B_c^{lower} < \left| \frac{C_{xy}}{m} - \frac{\hat{C}_{xy}}{m} \right| < B_c^{upper}$$

$$B_\Omega^{lower} < \left| \Omega - \hat{\Omega} \right| < B_\Omega^{upper}$$

m	0.57e-8 kg
K_{xx}	80.98 N/m
C_{xx}	0.429e-6 Ns/m
K_{yy}	71.62 N/m
C_{yy}	0.687e-3 N/m
K_{xy}	5 N/m
C_{xy}	0.0429e-6 Ns/m
$\hat{K}_{xy}$	1.25 N/m
$\hat{C}_{xy}$	0.0004 Ns/m
Ω	2.0 rad/sec
B_k^{lower}	0
B_k^{upper}	100e8
B_c^{lower}	0
B_c^{upper}	8e3
B_Ω^{lower}	0
B_Ω^{upper}	100

Table 1 Parameters of the MEMS Gyroscope

The design criterion for the sliding mode control is based on the following Lyapunov function

$$V_{SMC} = \frac{1}{2} s^2 \qquad (24)$$

$sy > 0$ *and* $s\dot{y} > 0$ $u_n = \dfrac{-m\eta\,\mathrm{sgn}(s) + m\lambda B_k^{lower}\, y + m\lambda B_c^{lower}\, \dot{y} - 2m\lambda B_\Omega^{upper}\, \dot{y}}{\lambda}$
If $sy < 0$ and $s\dot{y} < 0$ $u_n = \dfrac{-m\eta\,\mathrm{sgn}(s) + m\lambda B_k^{upper}\, y + m\lambda B_c^{upper}\, \dot{y} - 2m\lambda B_\Omega^{lower}\, \dot{y}}{\lambda}$
If $sy > 0$ is $s\dot{y} < 0$ $u_n = \dfrac{-m\eta\,\mathrm{sgn}(s) + m\lambda B_k^{lower}\, y + m\lambda B_c^{upper}\, \dot{y} - 2m\lambda B_\Omega^{lower}\, \dot{y}}{\lambda}$
$sy < 0$ and $s\dot{y} > 0$ $u_n = \dfrac{-m\eta\,\mathrm{sgn}(s) + m\lambda B_k^{upper}\, y + m\lambda B_c^{lower}\, \dot{y} - 2m\lambda B_\Omega^{upper}\, \dot{y}}{\lambda}$

where the sliding line is defined by

$$s = e + \lambda \dot{e} = 0 \qquad (25)$$

with $\mathrm{e} = \mathrm{x} - \mathrm{x}_\mathrm{d}$. The sliding mode controller forces $\dot{s} \to 0$ *and* $s \to 0$ by imposing

$$\dot{s} = -\eta\ \mathrm{sgn}(s) \qquad (26)$$

and therefore guaranteeing stability in the sense of error (e) since

$$\dot{V}_{SMC} = s\dot{s} \le -\eta|s| \qquad (27)$$

Comparison of (25) with (12) shows that the sliding mode controller is a reduced order controller. For convenience the control signal u can be divided into two parts: the linear part u_l and the non-linear (robustness) part u_n, i.e., ($u = u_l + u_n$.) From (25) and (3) the dynamics of the sliding line is expressed as

$$\dot{s} = \dot{x} - \dot{x}_d - \lambda(C_{xx}/m)\dot{x} - \lambda(K_{xx}/m)x - \lambda(C_{xy}/m)\dot{y}$$
$$- \lambda(K_{xy}/m)y + \lambda(u_l/m) + \lambda(u_n/m) + \lambda(2\Omega)\dot{y} - \lambda\ddot{x}_d \qquad (28)$$

The linear part of the control signal is constructed using the best estimate of the parameters as

$$\frac{u_l}{m} = \frac{(-\dot{x} + \dot{x}_d + (\lambda C_{xx}/m)\dot{x} + (\lambda K_{xx}/m)x + (\lambda\hat{K}_{xy}/m)y + (\lambda\hat{C}_{xy}/m - \lambda 2\hat{\Omega})\dot{y} + \lambda\ddot{x}_d)}{\lambda} \qquad (29)$$

Once the linear part of the control signal is substituted into (28) the dynamics becomes,

$$\dot{s} = (\lambda u_n/m) - \lambda(K_{xy}/m - \hat{K}_{xy}/m)y - \lambda(C_{xy}/m - \hat{C}_{xy}/m)\dot{y} + 2\lambda(\Omega - \hat{\Omega})\dot{y} \qquad 30)$$

The nonlinear part of the control signal u_n has to be determined so that the Lyapunov condition (27) is satisfied. Considering the uncertainty bounds of Table 1, the robust part of the control signal can be determined as shown on Table 2. Figure 10 shows the motion of the proof mass while it is following a reference signal of 110.595 *10^3 rad/sec.

The unknown angular velocity Ω is estimated using the force balance technique of [10] where a control signal u_y is introduced in the sensing direction i.e.,

$$m\ddot{y} + C_{yy}\dot{y} + k_{yy}y + C_{xy}\dot{x} + k_{xy}x = u_y - 2m\Omega\dot{x} \qquad (31)$$

Table 2. The robust part of the control signal (not all rules are shown)

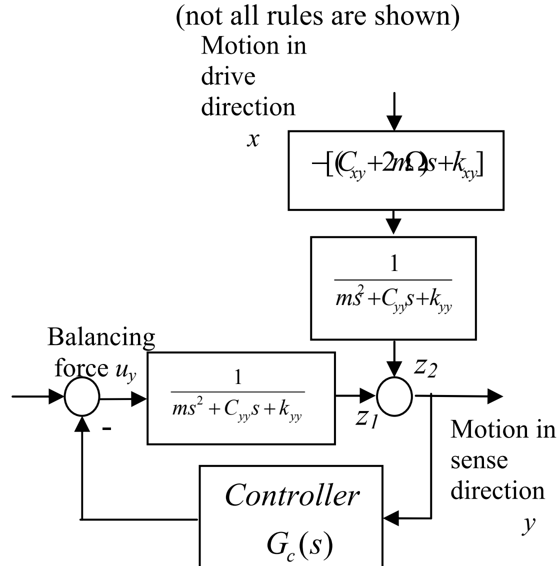

Figure 9. Force balancing controller

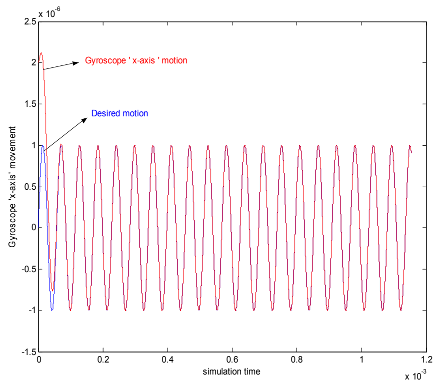

Figure 10. Proof mass motion while following the set point.

If the control signal u_y can stabilize the dynamics in the sensing direction, i.e., $y, \dot{y}, \ddot{y} \to 0$ in the steady state, then from (31) and (5) u_y can be written as

$$\begin{aligned} u_y(t) &= k_{xy}x + (C_{xy} + 2m\Omega)x^{\bullet} \\ &= k_{xy}A\sin\omega_n t + (C_{xy} + 2m\Omega)A\omega_n \cos\omega_n t \end{aligned} \quad (32)$$

which shows that u_y can be used to estimate the unknown quantities (Ω, k_{xy}). The feedback control system to stabilize the loop in the sense direction (y) is shown in Figure 9. The controller $G_c(s)$ is designed to maintain $z_1 = -z_2$ at the operating frequency in the drive direction therefore the effect of the motion in the drive direction can be cancelled. Under this condition, one obtains (32) in the steady state. Since the signal u_y is available in the control system, (32) can be used to determine the unknown cross coupling coefficient k_{xy} and the angular velocity Ω provided that the cross coupling term C_{xy} is negligible. Two different techniques can be used to estimate (Ω, k_{xy}):

Least Squares Identification

Once the steady state condition is established (32) can be written for $t=t_1, t_2, ..t_N$ as

Ω *rad/s*	$\hat{\Omega}$ (LS)	$\hat{\Omega}$ (DM)	K_{xy}	$\hat{K}_{xy}$ (LS)	$\hat{K}_{xy}$ (DM)
0.944	0.944	0.944	2.0	2.0	2.0
5.0	5.019	4.998	1.25	1.25	1.25
8.0	8.000	7.995	0.0	0.0	0.0
0.5	0.500	0.497	10.0	10.0	9.999
10.0	9.999	9.992	5.0	5.0	4.999

Table 3. True and estimated values of angular velocity Ω and K_{xy}.
LS=Least Squares, DM = De-modulation

$$u_y(t_i) = \hat{k}_{xy} A \sin \omega_n t_i + 2m\hat{\Omega} A \omega_n \cos \omega_n t_i + \varepsilon_i$$

Minimizing $\sum_{t1}^{tN} \varepsilon_i^2$ results in the Least Squares estimates ($\hat{\Omega}, \hat{k}_{xy}$) of unknowns ($\Omega$, k_{xy}).

De-modulation

Multiplying (32) by $(\sin\omega_n t)$ *and* $(\cos\omega_n t)$ and integrating for one period result in the estimates

$$\hat{k}_{xy} = \frac{\omega_n}{A\pi} \int_0^{2\pi/\omega_n} u_y(t)\sin\omega_n(t)dt\,;$$

$$\hat{\Omega} = \frac{1}{2mA\pi} \int_0^{2\pi/\omega_n} u_y(t)\cos\omega_n(t)dt$$

Table 3 shows that both techniques are equally capable to estimate the unknown angular velocity Ω and the cross coupling spring constant k_{xy}.

5. CONCLUSIONS

There is an inevitable cross-coupling between the orthogonal axes of a vibrating MEMS gyroscope. The quantity to be measured, i.e. the angular velocity of the proof mass is an unknown disturbance term to the controller which has to maintain the proof mass to vibrate at fixed amplitude and frequency. An adaptive model reference and a sliding mode controller have been proposed to estimate the unknown angular velocity and control the motion of the proof mass.

It has been shown both analytically and through numerical simulations that a Lyapunov function based model reference controller can maintain the proof mass at the desired amplitude and frequency therefore the tracking error goes to zero. All signals within the control system, including the estimation error, remain bounded. However, estimation of the angular velocity will approach to the true value under certain assumptions since the Lyapunov stability theorem can only guarantee the boundedness of signals.

If the uncertainties of the system can be estimated, then sliding mode controller on the drive direction and a force balance controller on the sense direction can be constructed to consistently estimate the unknown angular velocity.

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