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Adaptive Schemes for Stable Teleoperation with Communication Delay Based on IOS Small Gain Theorem

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Abstract— The problem of adaptive stabilization of the force-reflecting telerobotic system in presence of time delay in the communication channel is addressed. We propose two adaptive control schemes that make the overall telerobotic system input-to-state stable (ISS) with respect to external forces for any constant communication delay. In particular, in case where the joint velocity measurements are replaced by the estimates obtained using "dirty derivative" filters, we show the input-to-state stability in the semiglobal practical sense. These results are based on a new version of the IOS (input-to-output stability) small gain theorem for functional differential equations.

I. INTRODUCTION

According to [1], teleoperation can be defined as the extension of a person's sensing and manipulation capability to a remote location. In a teleoperator system, two manipulators called master and slave are connected to each other via a communication channel. The master is moved by a human operator, and the information about the master's motion is sent through the communication channel to the remotely located slave. The slave manipulator is designed to follow the motion of the master. In this paper, we consider the so-called force-reflecting or bilateral teleoperator systems, where the contact force due to the environment is transmitted back through the communication channel to the master manipulator without alteration. This makes the human operator feel the interaction of the slave with the environment. In 1966, Ferrell [2] showed experimentally, that existence of even a small delay in the communication channel may cause instability of a force-reflecting teleoperator system. A control scheme that guarantees, under certain assumptions, stability of a bilaterally controlled teleoperator for any communication delay was first presented in [3], [4]. The problem of stabilization of bilaterally controlled teleoperators in presence of communication delay was also addressed in [5], [6], [7], [8], [9], among other papers. In [10], an idea of using the ISS small gain theorem [11] to design a force-reflecting telerobotic system stable for an arbitrary (constant) communication delay was presented. However, the scheme proposed in [10] has several shortcomings in practical implementations. First, it is assumed in [10], that the exact models of both the master and the slave manipulators are available for the designer. In practical situations, however, only nonlinear structure of the dynamical equations of a manipulator expressed in terms of the so called regressor function is usually available, while the mass-inertia parameters are unknown or (and) subject to changes. On the other hand, the algorithm proposed requires the joint velocities to be available for direct measurement, which is not usually the case in practice. In this work, we address the above mentioned issues and present two extensions of the stabilization algorithm proposed in [10]. First, we address a situation where the mass-inertia parameters of both the master and the slave manipulators are unknown, and provide an adaptive version of the stabilization algorithm. Further, we assume that only the joint positions of both the master and the slave manipulators are available for measurement subject to small measurement disturbances. In the latter case, we consider a version of the adaptive stabilization algorithm where the velocity measurements are replaced by the estimates obtained using the so-called "dirty derivative" filters. In both these cases we show that the overall telerobotic system, being considered as a system of functional differential equations, is input-to-state stable (ISS) (in the global practical or semiglobal practical sense) with respect to external forces, for any communication delay. Our main tool is Theorem 1, which is a new version of the IOS (ISS) small gain theorem for functional differential equations.

The paper is organized as follows. In section II, the mathematical model of the force reflecting telerobotic system is given. In Section III, we present a new version of the IOS (ISS) small gain theorem that is used essentially in our proofs of stability of the telerobotic system. In section IV, the adaptive stabilization problem for bilaterally controlled teleoperators with communication delay is addressed under the assumption that joint velocities are available for measurement. The adaptive stabilization problem without velocity measurement is considered in section V. An example of simulation results is presented in section VI. Finally, some concluding remarks are given in section VII. Due to space reasons, all the proofs are omitted.

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II. MATHEMATICAL MODEL OF THE TELEOPERATOR SYSTEM

In this paper, we consider a bilaterally controlled teleoperator system where both the master and the slave are n-dimensional fully actuated manipulators with revolute (rotational) joints that described by the Euler-Lagrange equations as follows

$$\begin{aligned} H_m(q_m)\ddot{q}_m + C_m(q_m,\dot{q}_m)\dot{q}_m + G_m(q_m) \\ = F_h + \hat{F}_e + u_m, \end{aligned} \tag{1}$$

$$H_s(q_s)\ddot{q}_s + C_s(q_s,\dot{q}_s)\dot{q}_s + G_s(q_s) = F_e + u_s. \tag{2}$$

Here $q_m \in \mathbb{T}^n$, $\dot{q}_m \in \mathbb{R}^n$ are position and velocity of the master, $q_s \in \mathbb{T}^n$, $\dot{q}_s \in \mathbb{R}^n$ are position and velocity of the slave, $F_h \in \mathbb{R}^n$ is a force (torque) applied by the human operator, $F_e \in \mathbb{R}^n$ is the contact force (torque) due to environment applied to the slave, $\hat{F}_e \in \mathbb{R}^n$ represents the measurement of F_e transmitted back to the motors of the master, and $u_m, u_s \in \mathbb{R}^n$ are the control inputs of the master and the slave respectively. Let $\tau_1 \geq 0$ be a communication delay of the forward communication channel (from the master to the slave). Then the following signals

$$\hat{q}_m(t) = q_m(t-\tau_1), \tag{3}$$

$$\hat{\dot{q}}_m(t) = \dot{q}_m(t-\tau_1), \tag{4}$$

are available on the slave side. On the other hand, the transmission equation for F_e is as follows

$$\hat{F}_e(t) = F_e(t-\tau_2), \tag{5}$$

where $\tau_2 \geq 0$ is a communication delay in the backward direction (from the slave to the master). Throughout the paper we will make use of the following assumption.

Assumption 1. The contact force F_e can be represented as follows

$$F_e(t) = F_e^s(t) + F_e^*(t), \tag{6}$$

where F_e^s satisfies the following "finite-gain" condition with respect to the slave variables

$$|F_e^s(t)| \leq \gamma_e(|\dot{q}_s(t)| + |q_s(t)|) \tag{7}$$

for some $\gamma_e > 0$ and for almost all $t \geq 0$, and $F_e^*(t)$ is an arbitrary measurable essentially bounded function. The term F_e^* contains the disturbances and the globally bounded part of the environmental forces.

III. IOS (ISS) SMALL GAIN THEOREM FOR FUNCTIONAL DIFFERENTIAL EQUATIONS

Following the notations of [12], for a given function $x\colon [-t_d,\infty) \to \mathbb{R}^n$, $t_d \geq 0$, and given $t \geq 0$, let us define a function $x_d(t)(\cdot)\colon [0,t_d] \to \mathbb{R}^n$ as $x_d(t)(s) := x(t-s)$. Consider a system described by functional-differential equations (FDE) of the following form

$$\begin{array}{rcl} \dot{x}(t) &=& F(x_d(t), u_d(t), w_d(t)), \\ y(t) &=& H(x_d(t), u_d(t), w_d(t)), \end{array} \tag{8}$$

where $x_d(\cdot)$ is the state, $u \in \mathbb{R}^l$, $w \in \mathbb{R}^m$ are the inputs, and $y \in \mathbb{R}^p$ is the output. We assume that the operator $F(\cdot)$ satisfies assumptions that guarantee, for each initial data $x_d(t_0)$ and each admissible input $w_d(\cdot)$, the existence of a unique maximal solution defined on $[t_0, t_0+t_{max})$ for some $t_{max} > 0$. Additionally, we assume that the operator $H(\cdot)$ is continuous and satisfies $H(0_d, 0_d) = 0$, where 0_d is a function equal to 0 for all $[0,t_d]$. The following notation $|x_d(t)| = \sup_{t-t_d \leq s \leq t} |x(s)|$ will be used throughout the section, and $|u_d(t)|$, $|w_d(t)|$ are defined analogously.

In the definition below, we reformulate the input-to-output stability notion [13] to the case of systems described by functional-differential equations of the form (8 (see also [12]).

Definition 1. System (8) is said to be input-to-output stable (IOS) with IOS gains $\gamma_u, \gamma_w \in \mathcal{K}$, restriction $(\Delta_x, \Delta_u, \Delta_w)$, where $\Delta_x > 0$, $\Delta_u > 0$, $\Delta_w > 0$, and offset $\delta > 0$, if $|x_d(0)| \leq \Delta_x$, $\sup_{s\geq 0} |u_d(s)| \leq \Delta_u$, and $\sup_{s \geq 0} |w_d(s)| \leq \Delta_w$ imply that the solutions of (8) are defined for all $t \in [-t_d, +\infty)$, and the following two properties hold:

i) boundedness: there exists a function $\beta \in \mathcal{K}_\infty$ such that

$$\sup_{t\geq 0} |y(t)| \leq \max \left\{ \begin{array}{c} \beta(|x_d(0)|), \gamma_u\left(\sup_{t\geq 0}|u_d(t)|\right), \\ \gamma_w\left(\sup_{t\geq 0}|w_d(t)|\right), \delta \end{array} \right\};$$

ii) convergence:

$$\limsup_{t\to\infty} |y(t)| \leq \max \left\{ \begin{array}{c} \gamma_u\left(\limsup_{t\to\infty}|u_d(t)|\right), \\ \gamma_w\left(\limsup_{t\to\infty}|w_d(t)|\right), \delta \end{array} \right\}.$$

The system is called input-to-state stable if it is IOS with respect to the output $y = x$. •

To investigate stability properties of the telerobotic system in the subsequent sections, we need to formulate and prove a version of the IOS (ISS) small gain theorem for FDEs (8). Consider two systems of FDE of the following form

$$\Sigma_1\colon \quad \begin{array}{rcl} \dot{x}_1(t) &=& F_1(x_{1d}(t), u_{1d}(t), w_{1d}(t)), \\ y_1(t) &=& H_1(x_{1d}(t), u_{1d}(t), w_{1d}(t)), \end{array} \tag{9}$$

where $t_d = \tau_1 \geq 0$, and

$$\Sigma_2\colon \quad \begin{array}{rcl} \dot{x}_2(t) &=& F_2(x_{2d}(t), u_{2d}(t), w_{2d}(t)), \\ y_2(t) &=& H_2(x_{2d}(t), u_{2d}(t), w_{2d}(t)), \end{array} \tag{10}$$

where $t_d = \tau_2 \geq 0$. We will investigate the behavior of the overall system (9), (10) subject to constraints on inputs u_1, u_2 described as follows: $u_1(t) \equiv u_2(t) \equiv 0$ for $t < 0$, and

$$|u_2(t)| \leq \chi_1(|y_1(t)|) \tag{11}$$

for $t \geq 0$,where $\chi_1(\cdot), \chi_2(\cdot) \in \mathcal{K}$.

Note that the system (9), (10) subject to constraints (11) is not a feedback system in the classical sense, since u_1 and u_2 are not completely determined by outputs y_2, y_1 respectively. However, for such a system it is still possible to state a small gain result which provides an upper bound for output (state) trajectories in terms of bounds on inputs w_1, w_2 uniformly with respect to any admissible $u_1(\cdot)$, $u_2(\cdot)$ that satisfy the constraints (11). To be precise, let us give the following definition where a version of the IOS notion is presented appropriately for the system of the form (9), (10), (11). Let us denote $x := \left(x_1^T, x_2^T\right)^T$, $y := \left(y_1^T, y_2^T\right)^T$, and let us use a norm of x defined as follows $|x(\cdot)| := \max\{|x_1(\cdot)|, |x_2(\cdot)|\}$, and $|y(\cdot)|$ defined analogously.

Definition 2. System (9), (10) subject to constraints (11) is said to be input-to-output stable (IOS) with IOS gains $\gamma_{w1}, \gamma_{w2} \in \mathcal{K}$, restriction $(\Delta_x, \Delta_{w_1}, \Delta_{w_2})$, Δ_x, Δ_{w_1}, $\Delta_{w_2} > 0$, and offset $\delta > 0$, if $|x_d(0)| \le \Delta_x$, $\sup_{t\ge 0}|w_{1d}(t)| \le \Delta_{w_1}$, $\sup_{t\ge 0}|w_{2d}(t)| \le \Delta_{w_2}$ imply that for any admissible $u_1(\cdot)$, $u_2(\cdot)$ satisfying the constraints (11), the solutions of (9), (10) are defined for all $t \in [-t_d, +\infty)$, and the following two properties hold:

i) boundedness: there exists a function $\beta \in \mathcal{K}_\infty$ such that

$$\sup_{t\ge 0}|y(t)| \le \max\left\{\begin{array}{c} \beta(|x_d(0)|), \gamma_{w_1}\left(\sup_{t\ge 0}|w_{1d}(t)|\right), \\ \gamma_{w2}\left(\sup_{t\ge 0}|w_{2d}(t)|\right), \delta \end{array}\right\};$$

ii) convergence:

$$\limsup_{t\to\infty}|y(t)| \le \max\left\{\begin{array}{c} \gamma_{w_1}\left(\limsup_{t\to\infty}|w_{1d}(t)|\right), \\ \gamma_{w_2}\left(\limsup_{t\to\infty}|w_{2d}(t)|\right), \delta \end{array}\right\}.$$

The notion of input-to-state stability for the system (9), (10), (11) is defined analogously.

The following theorem will be our main tool in the subsequent sections.

Theorem 1. (IOS small gain theorem for FDE). Consider a system (9), (10) subject to constraints (11). Suppose each subsystem Σ_i, $i = 1, 2$, is input-to-output stable with IOS gains $\gamma_{iu}, \gamma_{iw} \in \mathcal{K}$ and restrictions $(\Delta_{ix}, \Delta_{iu}, \Delta_{iw})$. Suppose also that the gains γ_{iu}, χ_i form a strict contraction, i.e.,

$$\chi_1 \circ \gamma_{1u} \circ \chi_2 \circ \gamma_{2u}(s) < s \quad \text{for all } s > 0. \tag{12}$$

Then, given $\Delta_x \le \min\{\Delta_{1x}, \Delta_{2x}\}$, if

$$\Delta_{1u} > \Delta_{1u}^* := \chi_2\left(\max\left\{\begin{array}{c} \beta_2(\Delta_x), \gamma_{2w}(\Delta_{2w}), \\ \gamma_{2u} \circ \chi_1 \circ \beta_1(\Delta_x), \\ \gamma_{2u} \circ \chi_1 \circ \gamma_{1w}(\Delta_{1w}), \end{array}\right\}\right),$$

$$\Delta_{2u} > \Delta_{2u}^* := \chi_1\left(\max\left\{\begin{array}{c} \beta_1(\Delta_x), \gamma_{1w}(\Delta_{1w}), \\ \gamma_{1u} \circ \chi_2 \circ \beta_2(\Delta_x), \\ \gamma_{1u} \circ \chi_2 \circ \gamma_{2w}(\Delta_{2w}), \end{array}\right\}\right),$$

then the system (9), (10), (11) with $t_d = \max\{\tau_1, \tau_2\}$ is input-to-output stable with restriction $(\Delta_x, \Delta_{1w}, \Delta_{2w})$ and IOS gains $\tilde{\gamma}_{1w}(\cdot) := \max\{\gamma_{1w}(\cdot), \gamma_{2u} \circ \chi_1 \circ \gamma_{1w}(\cdot)\}$, $\tilde{\gamma}_{2w}(\cdot) := \max\{\gamma_{2w}(\cdot), \gamma_{1u} \circ \chi_2 \circ \gamma_{2w}(\cdot)\}$, corresponding to inputs w_1, w_2 respectively. •

IV. ADAPTIVE STABILIZATION OF THE BILATERALLY CONTROLLED TELEOPERATORS

In this section, we address the problem of adaptive stabilization of the bilaterally controlled telerobotic system with communication delay under the assumption that the velocities of both the master and the slave are available for direct measurement. A standard statement of the adaptive stabilization problem for robotic manipulators utilizes the so called linear parameterization property of the manipulator's dynamics. Using this property, let us denote

$$\begin{array}{r} -H_m(q_m)\sigma_m\dot{q}_m - C_m(q_m, \dot{q}_m)\sigma_m q_m \\ +G_m(q_m) = Y_{cm}(q_m, \dot{q}_m)\theta_m, \end{array} \tag{13}$$

where $Y_{cm}(q_m, \dot{q}_m) \in \mathbb{R}^{n\times r}$ is the regressor of the master manipulator, and $\theta_m \in \mathbb{R}^r$ is the vector of parameters of the master manipulator. For the slave manipulator, let us write

$$\begin{array}{r} H_s(q_s)\sigma_s\left(\dot{\hat{q}}_m - \dot{q}_s\right) + C_s(q_s, \dot{q}_s)\sigma_s(\hat{q}_m - q_s) \\ +G_s(q_s) = Y_{cs}\left(q_s, \dot{q}_s, \hat{q}_m, \dot{\hat{q}}_m\right)\theta_s, \end{array} \tag{14}$$

where $Y_{cs}\left(q_s, \dot{q}_s, \hat{q}_m, \dot{\hat{q}}_m\right) \in \mathbb{R}^{n\times r}$ is the corresponding regressor function of the slave, and $\theta_s \in \mathbb{R}^r$ is the vector of the slave's parameters. In the following, it is assumed that both θ_m, θ_s are unknown but constant. An adaptive version of the stabilization algorithm presented in [10] can be defined by setting $u_m(t) \equiv 0$, $u_s(t) \equiv 0$ for $t < 0$, and

$$u_m = Y_{cm}(q_m, \dot{q}_m)\hat{\theta}_m - K_m(\dot{q}_m + \sigma_m q_m), \tag{15}$$

$$\begin{array}{rcl} u_s & = & Y_{cs}\left(q_s, \dot{q}_s, \hat{q}_m, \dot{\hat{q}}_m\right)\hat{\theta}_s \\ & & -K_s(\dot{q}_s + \sigma_s(q_s - \hat{q}_m)), \end{array} \tag{16}$$

for $t \ge 0$, where $\hat{\theta}_m, \hat{\theta}_s \in \mathbb{R}^r$ are estimates for θ_m and θ_s respectively, that satisfy $\hat{\theta}_m(t) \equiv \hat{\theta}_s(t) \equiv 0$ for $t < 0$, while for $t \ge 0$ are obtained by the following standard adaptation algorithms with forgetting factors

$$\begin{array}{rcl} \dot{\hat{\theta}}_m & = & -\Gamma_m Y_{cm}^T(q_m, \dot{q}_m)(\dot{q}_m + \sigma_m q_m) \\ & & -\epsilon_m\left(\hat{\theta}_m - \theta_m^*\right), \end{array} \tag{17}$$

$$\begin{array}{rcl} \dot{\hat{\theta}}_s & = & -\Gamma_s Y_{cs}^T\left(q_s, \dot{q}_s, \hat{q}_m, \dot{\hat{q}}_m\right)(\dot{q}_s + \sigma_s(q_s - \hat{q}_m)) \\ & & -\epsilon_s\left(\hat{\theta}_s - \theta_s^*\right), \end{array} \tag{18}$$

where Γ_m, Γ_s are symmetric positive definite matrices, $\theta_m^*, \theta_s^* \in \mathbb{R}^r$ are vectors that represents nominal values of parameters θ_m, θ_s respectively, and $\epsilon_m, \epsilon_s > 0$ are arbitrary constants.

To formulate our results, let us denote $\tilde{\theta}_m := \hat{\theta}_m - \theta_m$, $\tilde{\theta}_s := \hat{\theta}_s - \theta_s$, and $\tilde{q}_s = q_s - \hat{q}_m$. A state of the closed-loop telerobotic system is defined as follows

$$\mathbf{x}_d := \left(q_m^T, \dot{q}_m^T, \tilde{\theta}_m^T, \tilde{q}_s^T, \dot{q}_s^T, \tilde{\theta}_s^T\right)_d^T, \tag{19}$$

where $t_d = \max\{\tau_1, \tau_2\}$. Further, for our purposes it is convenient to define the following output

$$y := \left(q_m^T, \dot{q}_m^T, \tilde{q}_s^T, \dot{q}_s^T\right)^T, \tag{20}$$

Now, one can state the following result:

Theorem 2. Given $\delta > 0$, there exist $\kappa_m > 0$, $\kappa_s > 0$, $g_m > 0$, $g_s > 0$, such that if $\lambda_{\min}(K_m) \geq \kappa_m$, $\lambda_{\min}(K_s) \geq \kappa_s$, $\lambda_{\min}(\Gamma_m) \geq g_m$, and $\lambda_{\min}(\Gamma_s) \geq g_s$, then the closed-loop telerobotic system (1)-(6), (15) - (18) with state (19), input (F_h, F_e^*), and output (20) is

i) input-to-state stable with some offset $D \geq 0$,

ii) input-to-output stable with offset less than or equal to δ.

V. ADAPTIVE STABILIZATION WITHOUT VELOCITY MEASUREMENTS

Now, let us address the adaptive stabilization problem for bilaterally controlled teleoperators in the situation where the joint velocities are not available for measurement. Moreover, we assume that the joint positions of both master and slave manipulators are available for measurement subject to (small) measurement disturbances. More precisely, let

$$\bar{q}_m(t) = q_m(t) + \Omega_m(t) \tag{21}$$

be a vector of measured joint angles of the master manipulator, where q_m is the actual position, and Ω_m are measurement disturbances. In the following, we assume that $\omega_m(t) := d\Omega_m(t)/dt$ exists for almost all t, and $\int_a^b \omega_m = \Omega_m(b) - \Omega_m(a)$. This can be guaranteed, for example, by assumption that $\Omega_m(\cdot)$ is absolutely continuous [14]. Analogously, let the vector

$$\bar{q}_s(t) = q_s(t) + \Omega_s(t) \tag{22}$$

be available for the measurement on the slave side, where q_s is the actual position of the slave, and Ω_s are measurement disturbances on the slave side. To obtain estimates for velocities $\dot{q}_m$, $\dot{q}_s$, one may use the so-called "dirty derivatives" of $\bar{q}_m$ ($\bar{q}_s$) which are provided by the following first order filters

$$T_m(s) := \frac{\rho_m s}{s + \rho_m}, \quad T_s(s) := \frac{\rho_s s}{s + \rho_s}, \tag{23}$$

where $\rho_m > 0$, $\rho_s > 0$ are constants to be determined. Thus, estimates ν_m, ν_s for the master's and slave's velocities can be defined in the Laplace domain as follows

$$\nu_m(s) := T_m(s)\bar{q}_m(s), \tag{24}$$

$$\nu_s(s) := T_s(s)\bar{q}_s(s). \tag{25}$$

The initial conditions of the filters (24), (25) in the time domain are set $\nu_m(-t_d) = \nu_s(-t_d) = 0$, where $t_d \geq 0$ is defined below. Since the velocities are no longer available for measurement, the velocity estimate ν_m must be sent through the communication channel instead of $\dot{q}_m$. Thus, the following signals

$$\hat{q}_m(t) \equiv 0, \quad \hat{\nu}_m(t) \equiv 0 \quad \text{for } t < 0, \tag{26}$$

$$\hat{q}_m(t) = \bar{q}_m(t - \tau_1),\ \hat{\nu}_m(t) = \nu_m(t - \tau_1) \text{ for } t \geq 0, \tag{27}$$

are assumed to be available on the slave side. Substituting the estimates ν_m, ν_s for the velocities in the control law of the previous section, we get the following control algorithm $u_m \equiv 0$, $\quad u_s \equiv 0$ for $t < 0$, and

$$u_m = Y_{cm}(\bar{q}_m, \nu_m)\hat{\theta}_m - K_m(\nu_m + \sigma_m \bar{q}_m), \tag{28}$$

$$u_s = Y_{cs}(\bar{q}_s, \nu_s, \hat{q}_m, \hat{\nu}_m)\hat{\theta}_s - K_s(\nu_s + \sigma_s(\bar{q}_s - \hat{q}_m)) \tag{29}$$

for $t \geq 0$, where $Y_{cm}(\cdot)$, $Y_{cs}(\cdot)$ are regressor matrices defined by (13) and (14) respectively, $\hat{\theta}_m, \hat{\theta}_s \in R^r$ are estimates for θ_m and θ_s respectively, and $u_m(t) \equiv 0$, $u_s(t) \equiv 0$ for $t < 0$.

The estimates $\hat{\theta}_m$, $\hat{\theta}_s$ are assumed to satisfy $\hat{\theta}_m(t) \equiv \hat{\theta}_s(t) \equiv 0$ for $t \leq 0$, and for $t \geq 0$ are obtained by the following adaptation algorithms

$$\begin{aligned} \dot{\hat{\theta}}_m &= -\Gamma_m Y_{cm}^T(\bar{q}_m, \nu_m)(\nu_m + \sigma_m \bar{q}_m) \\ &\quad -\epsilon_m\left(\hat{\theta}_m - \theta_m^*\right), \end{aligned} \tag{30}$$

$$\begin{aligned} \dot{\hat{\theta}}_s &= -\Gamma_s Y_{cs}^T(\bar{q}_s, \nu_s, \hat{q}_m, \hat{\nu}_m)(\nu_s + \sigma_s(\bar{q}_s - \hat{q}_m)) \\ &\quad -\epsilon_s\left(\hat{\theta}_s - \theta_s^*\right), \end{aligned} \tag{31}$$

where Γ_m, Γ_s are symmetric positive definite matrices of the corresponding dimensions, and $\theta_m^*, \theta_s^* \in \mathbb{R}^r$ are nominal (expected) values of the parameters θ_m, θ_s respectively.

Now, let us define a state of the controlled telerobotic system (1), (2), (5), (21)-(31) as follows

$$\mathbf{x}_d := \left({q_m}^T, \dot{q}_m^T, \tilde{\theta}_m^T, \tilde{q}_s^T, \dot{q}_s^T, \tilde{\theta}_s^T, w_m^T, w_s^T\right)_d^T, \tag{32}$$

where $w_m := \nu_m - \dot{q}_m$, $w_s := \nu_s - \dot{q}_s$, and the rest of the notation is introduced in section IV, namely $\tilde{\theta}_m := \hat{\theta}_m - \theta_m$, $\tilde{\theta}_s := \hat{\theta}_s - \theta_s$, $\tilde{q}_s = q_s - \hat{q}_m$, and $t_d = \max\{\tau_1, \tau_2\}$. Moreover, let us define the following output

$$y := \left(q_m^T, \dot{q}_m^T, \tilde{q}_s^T, \dot{q}_s^T, w_m^T, w_s^T\right)^T \tag{33}$$

The main result of this section is as follows.

Theorem 3. Given $\Delta_x \geq 0$, $\Delta_F \geq 0$, $\delta > 0$, there exist constants $\kappa_m > 0$, $\kappa_s > 0$, $g_m > 0$, $g_s > 0$, such that the following holds. Suppose $\lambda_{\min}(K_m) \geq \kappa_m$, $\lambda_{\min}(K_s) \geq \kappa_s$, $\lambda_{\min}(\Gamma_m) \geq g_m$, and $\lambda_{\min}(\Gamma_s) \geq g_s$. Then there exists $\rho_m^* > 0$, $\rho_s^* > 0$ dependent on K_m, K_s, and $\Omega_m^* > 0$, $\Omega_s^* > 0$ dependent on K_m, K_s, Γ_m, Γ_s such that if

$\rho_m \geq \rho_m^*$, $\rho_s \geq \rho_s^*$, and the measurement disturbances satisfy

$$\sup_{t \geq -\max\{\tau_1,\tau_2\}} |\Omega_m(t)| \leq \Omega_m^*, \quad \sup_{t \geq -\max\{\tau_1,\tau_2\}} |\Omega_s(t)| \leq \Omega_s^*,$$

then the overall telerobotic system (1), (2), (5), (21)-(31) with state (32) is input-to-state stable in the sense of definition 2, with restriction (Δ_x, Δ_F), where Δ_x corresponds to state, and Δ_F is a restriction for F_h, F_e^*, and some offset $D > 0$. Moreover, the offset for output (33) is less than or equal to δ.

VI. SIMULATION RESULTS

In this section, an example of computer simulations of the adaptive stabilization scheme is presented. Consider a force reflecting telerobotic system described by (1)–(5) with $H_m(q) = H_s(q) \in \mathbb{R}^{2\times 2}$, where

$$\begin{aligned} h_{11} &= (2l_1 \cos q_2 + l_2)l_2 m_2 + l_1^2(m_1 + m_2), \\ h_{12} &= h_{21} = l_2^2 m_2 + l_1 l_2 m_2 \cos q_2, \\ h_{22} &= l_2^2 m_2, \end{aligned}$$

$C_m(q, \dot{q}) = C_s(q, \dot{q}) \in \mathbb{R}^{2\times 2}$, where

$$\begin{aligned} c_{11} &= -l_1 l_2 m_2 \sin(q_2)\dot{q}_2, \\ c_{12} &= -l_1 l_2 m_2 \sin(q_2)\left(\dot{q}_1 + \dot{q}_2\right), \\ c_{21} &= l_1 l_2 m_2 \sin(q_2), \\ c_{22} &= 0, \end{aligned}$$

and $G_m(q) = G_s(q) \in \mathbb{R}^2$, where

$$\begin{aligned} g_1 &= g\left(m_2 l_2 \sin(q_1 + q_2) + (m_1 + m_2)l_1 \sin(q_1)\right), \\ g_2 &= g m_2 l_2 \sin(q_1 + q_2), \end{aligned}$$

$m_1 = 10$, $m_2 = 5$, $l_1 = 0.7$, $l_2 = 0.5$, $g = 9.81$. The initial conditions are all zeros, i.e. $q_{m1}(0) = \dot{q}_{m1}(0) = q_{m2}(0) = \dot{q}_{m2}(0) = q_{s1}(0) = \dot{q}_{s1}(0) = q_{s2}(0) = \dot{q}_{s2}(0) = 0$. The mass/inertia parameters for both the master and the slave manipulators are chosen as follows $\theta_1 = l_1 l_2 m_2$, $\theta_2 = l_1^2(m_1 + m_2)$, $\theta_3 = l_2^2 m_2$, $\theta_4 = g m_2 l_2$, $\theta_5 = g(m_1+m_2)l_1$. In the simulations below, we assume that the force (torque) applied by the human operator to both first and second joints of the master manipulators is as shown in figure 1.

When the slave follows the resulting trajectory of the master, it contacts a rigid obstacle which is located at $x = 0.2$ m. The obstacle is modeled by the following equations

$$F_e = 0, \quad \text{if } x < 0.2 \text{ m},$$

and

$$F_e = -B\dot{x} - K\left(x - 0.2\right), \quad \text{if } x \geq 0.2 \text{ m},$$

where $B > 0$, and $K > 0$ are damping and stiffness of the environment, respectively. In the simulations below, we put $B = 1$, and $K = 10000$, i.e., a contact with a very stiff environment is investigated. The parameters of control law (15)–(18) are taken as follows: $K_m = 60 \cdot \mathbb{I}_{2\times 2}$, $\sigma_m = \mathbb{I}_{2\times 2}$, $\Gamma_m = 10 \cdot \mathbb{I}_{5\times 5}$, $\epsilon_m = 1$, $K_s = 10 \cdot \mathbb{I}_{2\times 2}$, $\sigma_s = diag\{4, 3\}$, $\Gamma_s = diag\{20, 20, 20, 40, 40\}$,

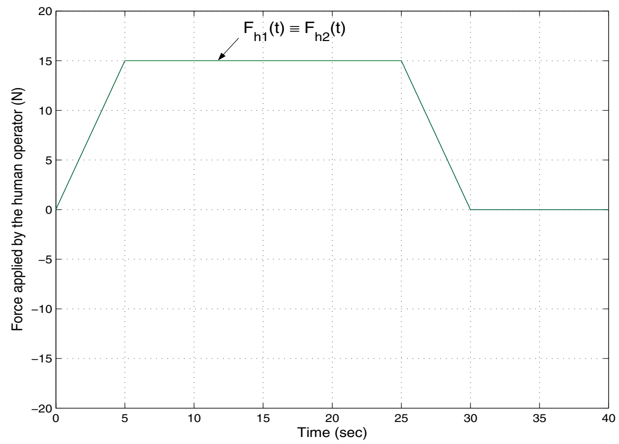

Fig. 1. Forces applied by the human operator, $F_{h1}(t) \equiv F_{h2}(t)$

and $\epsilon_s = 0.5$. Here, by $\mathbb{I}_{n\times n}$ the identity $n \times n$-matrix is denoted. All the initial conditions are set to be zero.

In the set of simulations presented, the communication delays are set $\tau_1 = \tau_2 = 1$ sec. Also, we assume that the nominal values of mass/inertia parameters θ_m^*, θ_s^*, which are used in the adaptation algorithms (17), (18), are different than the nominal ones. Namely, the actual mass of the second link of the slave manipulator is increased to $m_2 = 7kg$, while the nominal values of the slave parameters θ_s^* are still calculated for $m_2 = 5kg$. The corresponding simulations results are shown in figures 2, 3. Namely, in figure 2, a) the X-coordinates of the delayed master and slave trajectories are shown. We see that the slave experiences a contact with the obstacle at $x = 0.2$ m. The X-component of the corresponding contact force is shown in figure 2, b). In figures 3, the corresponding trajectories of the parameter estimates $\hat{\theta}_m$, $\hat{\theta}_s$ are plotted.

The simulations presented show that the adaptive control scheme proposed in this paper provides a stable contact with the obstacle for different values of communication delays as well as in the presence of parametric uncertainty. However, one can note that the tracking properties of the proposed scheme needs some improvements. This will be a topic for future research.

VII. CONCLUSIONS

In this paper, we have presented adaptive schemes for stabilization of force-reflecting telerobotic systems with communication delay. We have demonstrated that the proposed algorithms make the telerobotic system input-to-state stable with respect to external forces regardless of the delay in the communication channel. It seems worth trying to extend these results to the practically important case of time-varying communication delay, which commonly occurs, for example, in the teleoperation through Internet. This is a topic for future research.

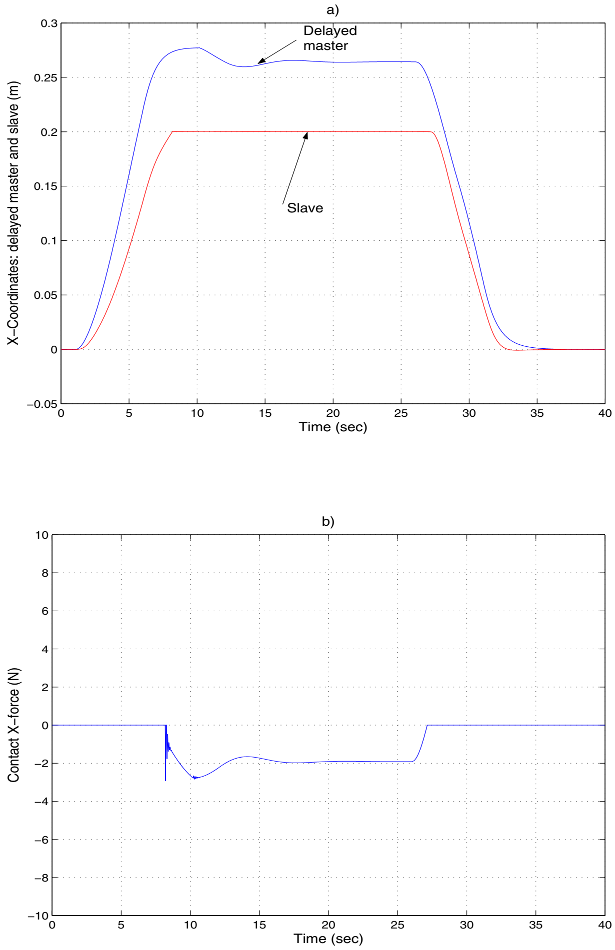

Fig. 2. 2nd set of simulations, $\tau_1 = \tau_2 = 1$ sec, $\theta_s \neq \theta_s^*$.
a) X-coordinates of the delayed master and the slave;
b) Environmental X-forces applied to the slave.

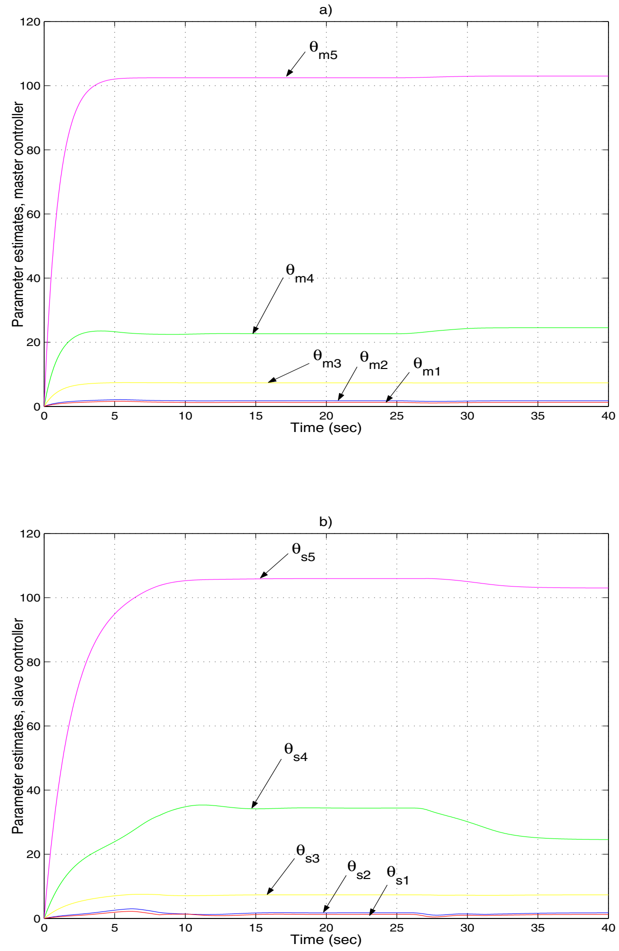

Fig. 3. 2nd set of simulations, $\tau_1 = \tau_2 = 1$ sec, $\theta_s \neq \theta_s^*$.
a) Parameter estimates of the master controller $\hat{\theta}_m$;
b) Parameter estimates of the slave controller $\hat{\theta}_s$.

References

[1] T. B. Sheridan, "Telerobotics," Automatica, vol. 25, no. 4, pp. 487–507, 1989.

[2] W. R. Ferrell, "Delayed force feedback," Human Factors, vol. 8, pp. 449–455, 1966.

[3] R. J. Anderson and M. W. Spong, "Bilateral control of teleoperators with time delay," IEEE Trans. Aut.Contr., vol. AC-34, pp. 494–501, May 1989.

[4] R. J. Anderson and M. W. Spong, "Asymptotic stability of force reflecting teleoperators with time delay," The International Journal of Robotics Research, vol. 11, pp. 135–149, Apr. 1992.

[5] G. M. Leung, B. A. Francis, and J. Apkarian, "Bilateral controller for teleoperators with time delay via mu-synthesis," IEEE Trans. Robotics and Automation, vol. 11, pp. 105–116, 1995.

[6] J. Alvarez-Gallegos, D. de Cortes Rodriguez, and M. W. Spong, "A stable control scheme for teleoperators with time delay," International Journal of Robotics and Automation, vol. 12, no. 3, pp. 73–79, 1997.

[7] W.-H. Zhu and S. E. Salcudean, "Stability guaranteed teleoperation: an adaptive motion/force control approach," IEEE Trans. Aut.Contr., vol. AC-45, pp. 1951–1969, Nov. 2000.

[8] P. Arcara and C. Melchiorri, "Control schemes for teleoperation with time delay: a comparative study," Robotics and Autonomous Systems, vol. 38, pp. 49–64, Jan. 2002.

[9] R. Lozano, N.Chopra, and M.W.Spong, "Passivation of force reflecting bilateral teleoperators with time varying delay," in Proceedings of Mechatronics'02, (Enschede, The Netherlands), June 2002.

[10] I. G. Polushin and H. J. Marquez, "The ISS small gain approach to stabilization of bilaterally controlled teleoperators with communication delay," in Proceedings of the 7th European Control Conference, (University of Cambridge, UK), Sept. 2003.

[11] Z.-P. Jiang, A. R. Teel, and L. Praly, "Small-gain theorem for ISS systems and applications," Mathematics of Control, Signals, and Systems, vol. 7, pp. 95–120, 1994.

[12] A. R. Teel, "Connections between Razumikhin-type theorems and the ISS nonlinear small gain theorem," IEEE Transactions on Automatic Control, vol. AC-43, pp. 960–964, July 1998.

[13] E. D. Sontag and Y. Wang, "Notions of input-to-output stability," Systems & Control Letters, vol. 38, pp. 235–248, 1999.

[14] G. E. Shilov, Elementary Functional Analysis. Cambridge, Mass.: MIT Press, 1974. translated and edited by Richard A. Silverman.