Radar Deception through Phantom Track Generation

D. H. A. Maithripala and Suhada Jayasuriya

Abstract— This paper presents a control algorithm to be used by a team of ECAVs (Electronic Combat Air Vehicle) to deceive a network of radars through the generation of a phantom track. An important feature of the algorithm is that it translates kinematic constraints on the ECAV dynamic system into constraints on the phantom point. The phantom track between two specified waypoints then evolves without violating any of the system constraints. The evolving phantom track in turn generates the actual controls on the ECAVs so that ECAVs have flyable trajectories.

I. Introduction

A radar detects the presence of a target by listening into the echoes of its transmitted radio waves, bouncing off of the target. Measurements of the round-trip time and comparison of the frequency of the transmitted pulses to that of the moving target enables it to determine the range as well as the range-rate of the target. A radar's radiation pattern will give rise to a main-lobe, where most of its radiated power is concentrated, and much weaker side-lobes which are consequences of the cancellation and addition of radar waves. A target has to lie inside the main-lobe or a side-lobe to be identified by the radar. For a more detailed description and comparison of main-lobe and side-lobe deception see [1], [2].

An ECAV (Electronic Combat Aerial Vehicle) is assumed to have the capability of intercepting and appropriately delaying the return of a radar's transmitted pulse thereby making it see a phantom (false) target beyond the actual range of the ECAV. This capability of intercepting and digitally storing and returning encoded pulses is known as range delay. Based on this principle, how a team of four ECAVs can be used to generate a phantom track through range delay techniques to deceive a radar network is illustrated in Fig.1. In this example scenario, there are four fixed ground radars that share information about the phantom track. There are four ECAVs, one assigned to each radar. At the start of the track, each ECAV is in the line of sight joining the corresponding radar location to the phantom position P. The radar pulses received by each ECAV are delayed appropriately so that the perceived range vectors all intersect at P. The ECAVs are then repositioned so that they continuously stay in the radars' line of sight at all subsequent times. This generates the phantom track of the desired speed and heading at each time. Since each of the radars confirms the other's target track, the track is considered valid by the radar network.

The problem is formulated in two dimensions and thus the phantom point will have 2-DOF while each ECAV will have only a single DOF. The loss of a DOF for the ECAV is due to the constraint that it has to be inline with the phantom point and the radar that is assigned to it. Though the actual implementable controls are on the ECAVs, the approach to the problem is to formulate a phantom track that would guarantee existence of feasible controls on the ECAVs for generating this formulated phantom track.

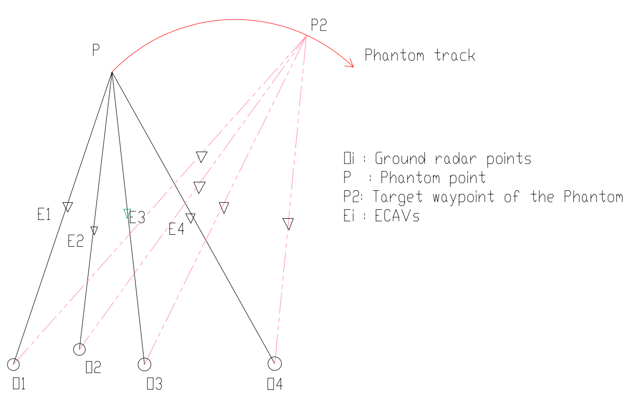

Fig. 1. Phantom track generation through a team of four ECAVs

The paper presents the development of an algorithm for the generation of a phantom track through cooperative control among a team of ECAVs. One additional motivation behind the proposed cooperative control policy is the desire to keep communications between the ECAVs to a minimum. Decentralization naturally requires that the algorithm not be computationally intense.

The paper consists of four sections of which this introduction is Section I. Section-II briefly describes system dynamics developed in an earlier study [1].

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Formulation of the problem, development of the algorithm and some simulation results are given in section III. Section-IV discusses some issues that need further research.

II. Background of Problem

In previous studies on this problem [1],[3],[4] allowable ECAV trajectories were determined for pre-specified phantom tracks and pre-specified initial conditions on the ECAVs. Some of the work therein is summarized below.

The kinematic geometry of a single ECAV along with its victim radar is shown in Fig.2. Non dimensional variables of Fig.2 with $\upsilon_E = 1$, $R_0 = 1$, and $\theta_0 = 0$ are defined as follows:

$$t \to \frac{\upsilon_E}{R_0} t \qquad r \to \frac{r}{R_0} \qquad \alpha \to \frac{\upsilon_T}{\upsilon_E} \qquad R \to \frac{R}{R_0}$$

EOMs of the dynamic system having a single ECAV are,

$$\dot{r} = \cos\phi_E, \qquad r(0) = r_0 \tag{1}$$

$$\dot{\theta} = \frac{1}{r}\sin\phi_E, \quad \theta(0) = 0 \tag{2}$$

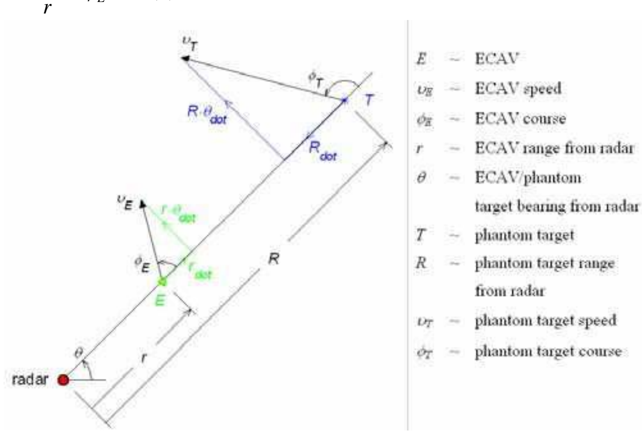

Fig. 2. ECAV and phantom track variables and their relations

$$\dot{R} = \alpha\cos\phi_T, \quad R(0) = 1 \tag{3}$$

$$\dot{\theta} = \alpha\frac{1}{R}\sin\phi_T, \ \ \theta(0) = 0 \tag{4}$$

$$\dot{r} = \sqrt{1-(r\dot{\theta})^2}, \qquad r(0) = r_0 \tag{5}$$

$$\dot{r} = -\sqrt{1-(r\dot{\theta})^2}, \qquad r(0) = r_0 \tag{6}$$

assuming $\ddot{\theta}(t_s) = 0$

For a constant-speed constant-course phantom track, as shown in Fig.3, the system equations then become,

$$R(t) = \sqrt{1+\alpha^2 t^2 - 2\alpha t\cos\psi} \tag{7}$$

$$\theta(t) = \arcsin\left(\frac{\alpha t \sin\psi}{R(t)}\right) \tag{8}$$

$$\dot{\theta}(t) = \frac{\alpha\sin\psi}{1+\alpha^2 t^2 - 2\alpha t\cos\psi} \tag{9}$$

Six different cases are solved for a predetermined generic phantom track by setting one ECAV variable constant in each case.

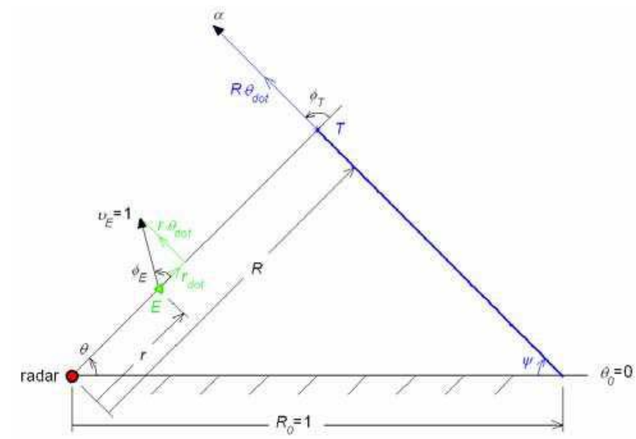

Fig. 3. Variables and relations for a constant speed ECAV and a constant-speed constant-course phantom track

III. Problem Formulation

The following assumptions are made in the problem formulation where the phantom trajectory is not specified except for the initial and final waypoint. Also it is assumed that the velocities are determined by the constraints placed on the control inputs.

- Both the phantom track as well as the ECAVs will have constrained dynamics
- ECAVs are mass-less and their states are completely observable
- All ECAVs are initially in-line with the radar it votes on and the initial waypoint of the phantom track.
- Ground radar locations, initial conditions of the ECAVs, initial and final waypoints of the phantom track are known a priori
- Ground radar locations are fixed
- Creation of the phantom track is through main-lobe deception using range delay techniques

The problem is essentially a constrained optimization leading to a classic two-point boundary value problem. As is well known solving a TPBVP is a difficult task at best and computationally intense. Consequently, it is not likely that one could generate solutions that can be implemented in real time and online. Instead in this paper we propose algorithms that are computationally attractive and are amenable to real time computations. The algorithms are essentially finite dimensional searches which can further be reduced to one dimensional parameter searches.

Considered are two cases.

A. Bounds on Rates

For this case we constrain the system dynamics through bounds on the range rate and angular rate of the ECAVs and the Phantom point. We let the phantom state be (R,θ), ECAV state be (r,θ) and the bounds on the rates be such that

$$\dot{r}_{\min} \le \dot{r} \le \dot{r}_{\max}$$
$$\omega_{\min} \le \dot{\theta} \le \omega_{\max}$$
$$\dot{R}_{\min} \le \dot{R} \le \dot{R}_{\max}$$

These constraints lead to feasible velocity sectors being rectangular regions for both the ECAV and the phantom point as shown in Fig.4. The bounds on the rates of the ECAV's state will be functions of the ECAV's aero-dynamics as well as of its state. The two bounds on the phantom's range rate will be functions of the ECAV's aero-dynamics, ECAV's state and of the dynamics of the range delay.

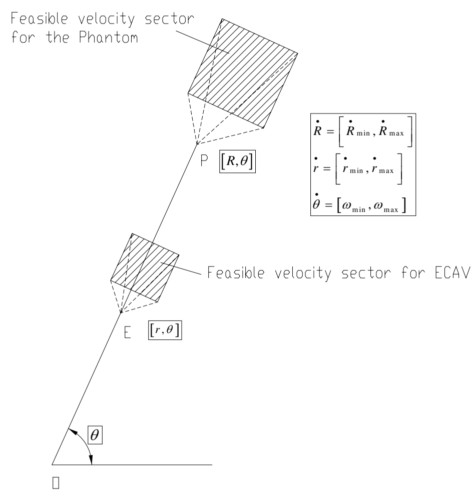

Fig. 4. Feasible velocity sectors for the ECAV and the phantom for bounded rates.

In the absence of adequate knowledge on these bounds, constant bounds are assumed on each of the rates. For the ECAV and the phantom to be contained within their respective velocity rectangles it is clear that each rate has to have bounds of opposite sign. For a set of constant bounds that would result in the phantom not being contained within its velocity rectangle, for any given set of initial conditions there will always be a region the phantom can never reach or enter. The simulation result of an algorithm implemented on MATLAB, which considers two ECAVs, given in Fig.5 is for such a set of constant bounds which does not result in the phantom being contained within its velocity rectangle. As is clear from Fig.5, for the given set of initial conditions the target waypoint of the phantom is in that region the phantom can never reach. It is clear that the time varying nature of each of the bounds is critical for this particular approach to the problem.

B. Bounds on Ground Speed

Now we consider constraints placed through bounds on the ground speeds of the ECAVs and the phantom point. Such constraints lead to the feasible velocity sectors being annular as illustrated in Fig.6.

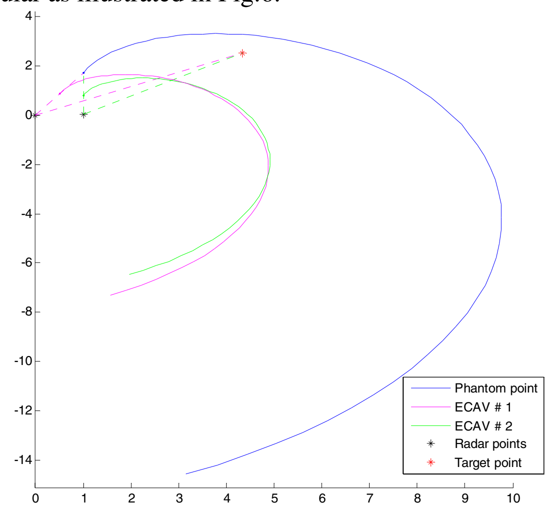

Fig. 5. Simulation results for phantom track generation using two ECAVs when the range rates and angular rates have upper and lower bounds

To allow more flexibility on the initial conditions, the ECAV, the phantom point and the radar (OEP) are constrained to be inline only at the beginning of each time step of the algorithm at which point the direction and speed are set for both the phantom point and ECAV. This collinearity constraint requires

$$\sin\beta = \frac{r(t)}{R(t)}\frac{V}{W}\sin\gamma \tag{10}$$

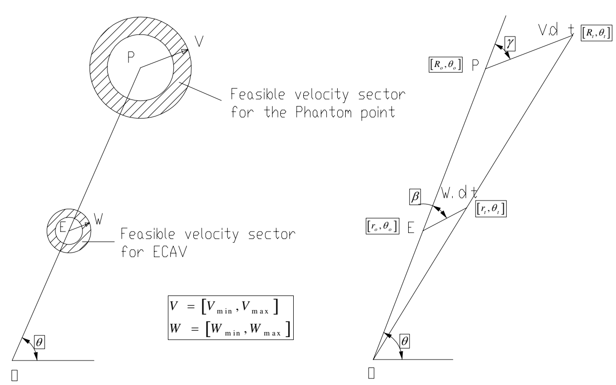

Fig. 6. velocity sectors for the ECAV and the phantom point

A Necessary condition for solutions for (10) to exist is,

$$\frac{r(t)}{R(t)}\frac{V}{W}\sin\gamma \le 1 \tag{11}$$

where V,W are the ground speeds of the phantom and ECAV respectively.

This necessary condition imposes a restriction on the feasible annular velocity sector of the phantom. The hatched area of the annular velocity sector of the phantom in Fig.7 represents this feasible velocity sector restricted through the necessary condition. Here, existence of solutions simply means the existence of a feasible trajectory for the ECAV.

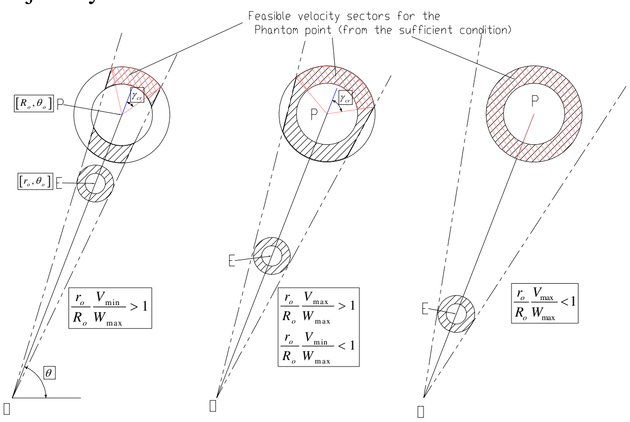

Fig. 7. Geometric representation of necessary and sufficient conditions for different radial locations of the ECAV.

A sufficient condition for the existence of solutions for (10) is simply,

$\gamma \le \gamma_{cr}$ where γ_{cr} satisfies,

$\tan\left[a\sin\left(\frac{W\cdot\delta t}{r_0}\right)\right] = \frac{V\cdot\delta t\cdot\sin\gamma_{cr}}{R_0 + V\cdot\delta t\cdot\cos\gamma_{cr}}$ where δt is the time-step of the algorithm.

In Fig.7, the area cross-hatched corresponds to this sufficient condition and is as expected a subset of the area corresponding to the necessary condition. The proposed algorithm implements this sufficient condition in place of the more difficult to implement necessary condition. It should be noted here that practical values of the bounds on the speeds make these annuli very narrow compared to their radii and hence the implementation of the sufficient condition in place of the necessary condition does not significantly reduce the freedom allowed for the phantom.

Thus the condition for the existence of solutions is translated as a constraint on the phantom's feasible velocity sector and this can be easily extended to the case of multiple ECAVs. If more than one ECAV imposes a restriction on the phantom's feasible velocity sector, the phantom's feasible velocity sector which would ensure the existence of allowable trajectories for each and every ECAV is then the union of the feasible velocity sectors corresponding to each of the ECAVs.

It is worth noting that each ECAV needs to communicate only two pieces of information, namely the γ_{cr} angle and its orientation.

For any number of ECAVs, for all realistic initial conditions there will always be a feasible velocity sector for the phantom which would guarantee feasible trajectories for the ECAVs. The algorithm would then pick a velocity for the phantom from its feasible velocity sector which would make it travel towards the final waypoint in minimum time.

If any one of the ECAV's states is such that it imposes a restriction on the phantom's movement through the sufficient condition, it is interesting to note that no matter what velocity heading is picked for the phantom, the ECAV can travel only in a direction that would relax the restriction placed by it at the phantom. Fig.8 illustrates this point. Here $r_0 V_{max} > R_0 W_{max}$, this implies $\alpha < \beta$ for all $\alpha \le \gamma_{cr}$

It can be then shown that,

$$\frac{r_t}{R_t} < \frac{r_0}{R_0} \tag{12}$$

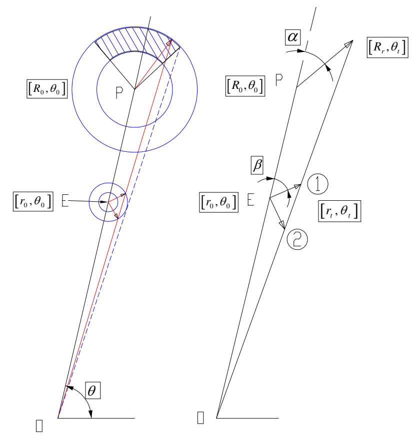

Fig. 8. ECAV velocities when the ECAV imposes a restriction on the phantom's velocity annulus.

Hence in the absence of additional constraints, any restriction placed by ECAVs on the velocity annulus of the phantom only relaxes with travel time and ultimately would cease to apply.

It can be shown that the condition for an ECAV to be collinear with the phantom and the radar for all time and not just at the end of time steps of the algorithm is,

$$\frac{W_{min}}{V_{max}} \le \frac{r_0}{R_0} \le \frac{W_{max}}{V_{min}} \tag{13}$$

The initial conditions that all ECAVs should satisfy to guarantee a straight line path for the phantom from its first waypoint to the targeted waypoint is,

$$\frac{r_{0,i}}{R_{0,i}} \le \frac{W_{max}}{V_{min}} \tag{14}$$

where i indexes the ECAVs.

When the ECAVs are free to pick one of two velocity headings as shown in the right hand diagram of Fig.8, the algorithm picks the velocity that would take the ECAV to the range given in (13).

The fact that the ECAVs will, for most times, have two velocities to pick from, allows the algorithm to easily implement a lower bound on the range of the ECAVs from their respective radars.

C. *Simulation Results of the Algorithm for the case with Bounds on Ground Speed*

The algorithm was coded in MATLAB and Fig. 9 gives the simulation results for the case of four ECAVs when the initial conditions are such that initially restrictions are imposed on the annular velocity sector of the phantom. Simulation results are for a phantom speed of $400 \pm 40 m/s$, ECAV speeds of $100 \pm 15 m/s$ and a step time of one second. The phantom track makes a sudden change in its course and starts traveling along a straight line towards the final waypoint when all restrictions placed on its annular velocity sector get relaxed with time. Fig.10 shows the same results but when an upper bound is imposed on the phantom's turn rate. It is obvious that if the initial conditions are such that no restrictions are placed on the annular velocity sector of the phantom point, the algorithm would generate a straight line path for the phantom from the first waypoint to the second.

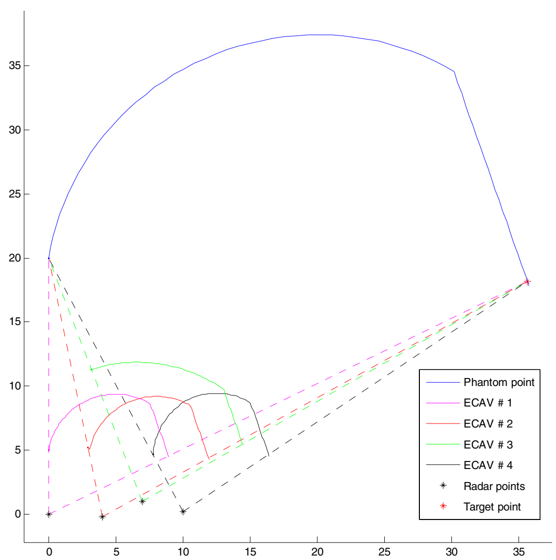

Fig. 9. Simulation results for a team of four ECAVs when initial conditions imposes restrictions at the phantom

IV. Discussion

The decentralized algorithm which allows for computational savings is successful for the problem restricted to the plane and the next major step would be to generalize it to three dimensions. Presently only an upper bound on the phantom's turn rate is implemented, and this has to be extended to include upper bounds on the ECAV's turn rate. Again the proposed approach is to translate these added constraints into constraints on the allowable velocity sector of the phantom.

Exactly how the ECAVs attain the initial conditions that satisfy the required assumptions necessary for the successful implementation of the algorithm is not addressed in this study and remains to be investigated. Though a host

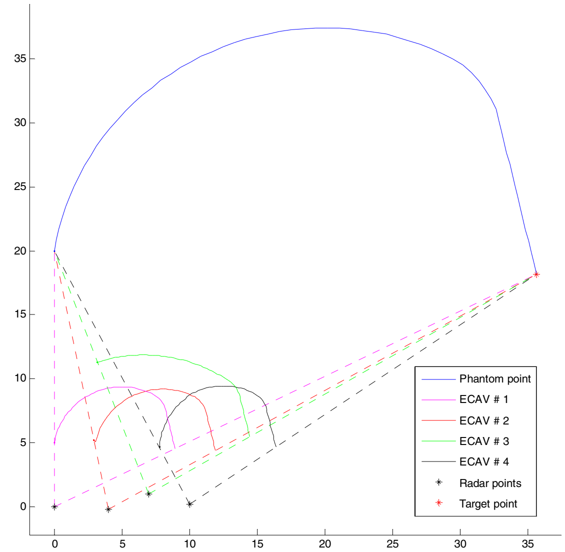

Fig. 10. Simulation results when a bound of 15 degrees per second is placed on the phantom's turn rate.

of other issues ranging from additional constraints of staying within the radars' range, maintaining network connectivity through constraints on maximum allowable distance between ECAVs, collision avoidance, uncertainty of state and radar observations, communication time delays to electronic requirements for generating returns with sufficiently accurate range rate and Doppler frequency information are yet to be tackled, this algorithm provides a helpful platform to develop a final control strategy. A study of cooperative control of ECAVs with network connectivity constraints and collision avoidance can be found in [5]

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