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Robust Sliding Mode Output Feedback Control Design Using ILMI Approach

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Abstract—**In this paper, a new sufficient and necessary condition is proposed for the existence problem of sliding mode output feedback control (SMOFC). This condition only involves original system parameters. An iterative linear matrix inequality (ILMI) approach is developed on such new condition for the robust SMOFC design. The unit norm sliding matrix is proposed to limit the control cost. Finally, a numerical example illustrates the efficacy of the presented ILMI approach.**

I. Introduction

Sling mode output feedback control (SMOFC) has been paid many attentions in recent years, since in many practical applications the full state information is not always available and only the output information is known. The difficulties in the design of SMOFC are focused on the two aspects. One is the existence problem, i.e., to design a switching surface on the output space which is usually of less order than the state space. Zak and Hui [11] and El-Khazali and Decarlo [6] presented two different design method using eigenvalue assignment and eigenvector technique. Edwards and Spurgeon [4] provided a canonical form on which the design problem of SMOFC is converted to a static output feedback control problem. Choi [3] presented a sufficient and necessary condition in terms of linear matrix inequality (LMI) with a matrix equation constraint. The other is the synthesis problem. It is difficult to synthesis a control law only using the output vector, since the derivative of the sliding surface is always related to the unmeasured states. To solve this problem, some additional restrictive conditions are added in [11] and [4], or a simple one-order dynamic assistant function estimating the norm bound of the unmeasured states is introduced in [8] and [9], or high control effort is required to achieve global convergence of the sliding surface [3] [5].

In spite of many significant works on SMOFC have been made, the basic existence problem is still an unsolvable problem. As stated in [5], all various methods (e.g. presented in [11], [6] and [4]) for the existence problem are, in fact, equivalent to a static output feedback stabilization (SOFS) problem which is consequently more difficult to solve. In addition, when there are some mismatched parametric uncertainties, these methods will be not effective any more. The results of Choi [3] work with the parametric uncertain system, but the given constrained LMI is difficult to solve (as stated in [3]) and the presented solution procedure are complex and still required to solve a SOFS problem. In real application, parametric uncertain always exists, so it is necessary and valuable to explore a tractable robust SMOFC design approach.

In this paper, a new sufficient and necessary condition for the existence problem is developed by two matrix inequalities, one of which is bilinear matrix inequality. Then an iterative linear matrix inequality (ILMI) approach is presented to solve such kind of matrix inequalities. The key features are that the proposed method only involves the original system parameters and does not appeal to directly solving a SOFS problem. More importantly, it can be easily extended to robust SMOFC design.

The notations are standard throughout this paper. Both the Euclidean norm of a vector and the induced spectral norm of a matrix are given by $\|\bullet\|$. $\boldsymbol{M}^{\perp}$ denotes the orthogonal complete matrix of full column rank of matrix $\boldsymbol{M}$. The abbreviation of 's. p. d.' stands for symmetric positive definite.

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II. Problem Statement

Consider the following uncertain system

$$\dot{x}(t) = Ax(t) + Bu(t) + Bf(x,t),$$

$$y(t) = Cx(t), \quad (1)$$

where $x \in \Re^n$ is the state vector, $u \in \Re^m$ is the input vector, $y \in \Re^p$ is the output vector, $f \in \Re^m$ represents the matched model uncertainties and outside disturbances. A, B and C are constant matrices of appropriate dimensions. The following conditions are assumed valid.

A1) The matrix pair (A,B) is controllable

A2) $rank(B) = m < n$, $rank(C) = p < n$ and $rank(CB) = rank(B) = m$.

A3) $f(x,t)$ is unknown but bounded by $\|f(x,t)\| \le b + a\|x(t)\|$, where a and b are known constant positive scalars.

The sliding surface can be defined as

$$\sigma(t) = Sy(t) = 0 \quad (2)$$

where $S \in \Re^{m\times p}$ with $rank(S) = m$ is the sliding matrix. Introducing a transformation matrix $T = \begin{bmatrix} B^{\perp T} \\ (B^T B)^{-1} B^T \end{bmatrix}$ and applying $v = Tx$, the system (1) can be expressed in the following regular form:

$$\begin{bmatrix} \dot{v}_1 \\ \dot{v}_2 \end{bmatrix} = \begin{bmatrix} B^{\perp T} AB^{\perp} & B^{\perp T} AB \\ (B^T B)^{-1} B^T AB^{\perp} & (B^T B)^{-1} B^T AB \end{bmatrix} \begin{bmatrix} v_1 \\ v_2 \end{bmatrix} + \begin{bmatrix} 0 \\ I \end{bmatrix} (u + f(x,t)) \quad (3)$$

where $v_1 \in \Re^{(n-m)}$ and $v_2 \in \Re^m$ are the partition of v. The sliding surface (2) in new coordinates can be expressed by

$$\sigma = SCB^{\perp} v_1 + SCBv_2 = 0 . \quad (4)$$

Since S is design parameter, the property that SCB is nonsingular is not limiting [5]. Thus, the reduced-order system on the sliding surface can be obtained by $\sigma = 0$,

$$\dot{v}_1 = B^{\perp T} A\Theta v_1 , \quad (5)$$

where $\Theta = B^{\perp} - B(SCB)^{-1} SCB^{\perp}$. If $C = I_n$, the above system (5) is equivalent to the reduced-order system under the state feedback and many present methods can be used to design the suitable sliding matrix S . Under the output feedback, although in style it is very similar to that under state feedback, the sliding matrix is a surprisingly hard to select to make the system (5) asymptotically stable.

The control objectives are to determine a sliding matrix S, which is called as the existence problem, such that the system (5) is asymptotically stable, and to design a control law, which is called as the synthesis problem, such that the sliding surface (2) can be reached in finite time and remained subsequently.

III. New Condition for Existence Problem

Theorem 1: There exists a sliding surface (2) such that the system (5) is asymptotically stable if and only if there are symmetric matrices $X_1 \in \Re^{(n-m)\times(n-m)}$ and $X_2 \in \Re^{p\times p}$ satisfying the following matrix inequalities:

$$B^{\perp} X_1 B^{\perp T} + C^T X_2 C > 0 , \quad (6)$$

$$(B^{\perp} X_1 B^{\perp T} + C^T X_2 C)A + A^T (B^{\perp} X_1 B^{\perp T} + C^T X_2 C) - C^T X_2 CBB^T C^T X_2 C < 0 . \quad (7)$$

Furthermore, the sliding matrix can be obtained by $S = B^T C^T X_2$.

Proof: (Sufficiency) Suppose that (6) and (7) are feasible and the sliding matrix is set to $S = B^T C^T X_2$. Define $X = B^{\perp} X_1 B^{\perp T} + C^T X_2 C$, then $X > 0$ and $SC = B^T X$. Noting that $I - B(B^T XB)^{-1} B^T X = X^{-1} B^{\perp} (B^{\perp T} X^{-1} B^{\perp})^{-1} B^{\perp T}$, the system (5) can be written as

$$\dot{v}_1 = B^{\perp T} AX^{-1} B^{\perp} (B^{\perp T} X^{-1} B^{\perp})^{-1} v_1 . \quad (8)$$

Consider the Lyapunov function $V = v_1^T (B^{\perp T} X^{-1} B^{\perp})^{-1} v_1$. Its derivative along the system (8) is

$$\dot{V} = v_1^T (B^{\perp T} X^{-1} B^{\perp})^{-1} B^{\perp T} (AX^{-1} + X^{-1} A^T) B^{\perp} (B^{\perp T} X^{-1} B^{\perp})^{-1} v_1 . \quad (9)$$

It can be seen that $\dot{V} < 0$ if and only if $B^{\perp T} (AX^{-1} + X^{-1} A^T) B^{\perp} < 0$, which can be obtained by pre- and post-multiplying (7) by $B^{\perp T} X^{-1}$ and $X^{-1} B^{\perp}$, respectively. So it is concluded that the system (5) is asymptotically stable.

(Necessity) Since the system (5) is asymptotically stable, there exists a s. p. d. matrix $P_0 \in \Re^{(n-m)\times(n-m)}$ such that

$$B^{\perp T} A\Theta P_0 + P_0 \Theta^T A^T B^{\perp} < 0 \quad (10)$$

Define a s. p. d. matrix

$$P = \begin{bmatrix} \Theta & B \end{bmatrix} \begin{bmatrix} P_0 & 0 \\ 0 & I \end{bmatrix} \begin{bmatrix} \Theta^T \\ B^T \end{bmatrix} . \quad (11)$$

Note that $\Theta^T B^{\perp} = I$. The inequality (10) is equivalent to

$$B^{\perp T} (AP + PA^T) B^{\perp} < 0 . \quad (12)$$

By Finsler Lemma [1], there exists a scalar $\varepsilon > 0$ such that the following inequality holds,

$$AP + PA^T - \varepsilon BB^T < 0 . \quad (13)$$

By $\begin{bmatrix} B^{\perp T} \\ (SCB)^{-1}SC \end{bmatrix}\begin{bmatrix}\Theta & B\end{bmatrix} = I$, one can obtain the inverse matrix of P,

$$P^{-1} = B^{\perp}P_0^{-1}B^{\perp T} + C^T S^T (SCB)^{-T}(SCB)^{-1}SC. \quad (14)$$

Multiplying both side of (13) by P^{-1} and letting $\hat{P} = \varepsilon P^{-1}$, we have

$$\hat{P}A + A^T\hat{P} - \hat{P}BB^T\hat{P} < 0. \quad (15)$$

Chose $X_1 = \varepsilon P_0^{-1}$ and $X_2 = \varepsilon S^T(SCB)^{-T}(SCB)^{-1}S$, then the property of that X_1 and X_2 are symmetric matrices obviously follows, and further the inequalities (6) and (7) can be straightforwardly obtained from $\hat{P} > 0$ and the inequality (15). This completes the proof.

Remark 1: Let $X = B^{\perp}X_1B^{\perp T} + C^T X_2 C$, then it can be easily obtained from (6) and (7) that

$$X > 0,\ XA + A^T X - XBB^T X < 0,\ B^T X = SC, \quad (16)$$

which is the just result presented in [3]. However, due to the existence of the matrix equation, the third term in (16), the above constrained LMI (16) is not tractable. Theorem 1 releases the matrix equation constraint using a specially built s. p. d. matrix (6).

IV. Robust SMOFC design

In this section, the results developed in the above section will be extended to robust SMOFC design for a class of more general uncertain system, whose dynamics function is

$$\dot{x}(t) = (A + \Delta A(t))x(t) + (B + \Delta B(t))u(t) + Bf(x,t),$$
$$y(t) = Cx(t), \quad (17)$$

where all the parametric matrices and vector function except for $\Delta A(t)$ and $\Delta B(t)$ are the same as the definition of the system (1) and the assumptions A1)~A3) are also assumed valid. The additional assumptions for $\Delta A(t)$ and $\Delta B(t)$ are shown as follows:

A4) The mismatched parametric uncertainty $\Delta A(t)$ is unknown but bounded and satisfies $\|\Delta A(t)\| \le \alpha$, where $\alpha > 0$ is a known scalar.

A5) The input distribution matrix uncertainty $\Delta B(t)$ satisfies the matching condition, i.e., there exists a unknown but bounded function $F(t)$ such that $\Delta B(t) = BF(t)$, where $\|F(t)\| \le \psi < 1$ with ψ a known positive scalar.

A. Existence problem

Theorem 2: If there exists s.p.d. matrices $X_1 \in \Re^{(n-m)\times(n-m)}$, $X_2 \in \Re^{p\times p}$ and positive scalar ε such that the following matrix inequalities hold

$$X = B^{\perp}X_1B^{\perp T} + C^T X_2 C > 0, \quad (18)$$

$$XA + A^T X + \varepsilon\alpha^2 I + \varepsilon^{-1}X^2 - C^T X_2 CBB^T C^T X_2 C < 0, \quad (19)$$

then the reduced-order sliding mode dynamics of system (17) restricted on the sliding surface $\sigma = B^T C^T X_2 y(t)$ is asymptotically stable.

Proof: Note that $\Delta AX + X(\Delta A)^T \le \varepsilon\alpha^2 I + \varepsilon^{-1}X^2$ with $\varepsilon > 0$ and replace A in Theorem 1 by $A + \Delta A(t)$, then Theorem 2 can be trivially derived.

Remark 2: It is difficult to design the sliding surface by solving SOFS problem [4]of the reduced-order subsystem of the transformed system when the mismatched parametric uncertainty exists, since the subsystem not only owns system matrix uncertainty, but also input matrix uncertainty. In such case, more techniques should be needed and more conversations will be added for SOFS problem.

B. Synthesis problem

Theorem 3: For the uncertain system (17), if there exist solutions (X_1, X_2, ε) of the inequalities (18) and (19), then sliding motion on the sliding surface $\hat{\sigma} = \rho^{-1}B^T C^T X_2 y(t)$ with $\rho = \|B^T C^T X_2\|$, can be induced in finite time by the following static output feedback control law

$$u = -\frac{0.5\rho}{1-\psi}(1 + \gamma_1^{-1}a^2)\hat{\sigma} - \frac{1}{1-\psi}(b + \gamma_2)\hat{\sigma}/\|\hat{\sigma}\| \quad (20)$$

where γ_1 and γ_2 are positive scalar.

Proof: From (18) and (19), it is true for a very small positive scalar γ_1 that

$$X(A + \Delta A) + (A + \Delta A)^T X + \gamma_1 I + \beta X - XBB^T X < 0. \quad (21)$$

Consider the candidate Lyapunov function $V = x^T Xx$. Its time derivative along the system (17) is given by

$$\begin{aligned}\dot{V} = x^T\left(X(A + \Delta A) + (A + \Delta A)X\right)x + 2x^T XBu(t) \\ +2x^T X\Delta B(t)u(t) + 2x^T XBf(x,t)\end{aligned}. \quad (22)$$

Let $\sigma = \rho\hat{\sigma}$, then $B^T Xx = B^T C^T X_2 y = \sigma$ and

$$u = -\frac{0.5}{1-\psi}(1 + \gamma_1^{-1}a^2)\sigma - \frac{1}{1-\psi}(b + \gamma_2)\sigma/\|\sigma\|. \quad (23)$$

Enlarge the last two terms of (22) by

$$2x^T XBf(x,t) \le \gamma_1^{-1} a^2 \sigma^T \sigma + \gamma_1 x^T x + 2b\|\sigma\|, \quad (24)$$

$$2x^T X \Delta B(t) u(t) \le \frac{\psi}{1-\psi} \sigma^T \sigma + \frac{\psi}{1-\psi} 2(b+\gamma_2)\|\sigma\|. \quad (25)$$

Substituting control law (23) into (22) and using the inequalities (21) (24) and (25), one can obtain that

$$\dot{V} \le -\beta x^T Xx, \quad (26)$$

which implies the asymptotical stability of the system (17).

On the other hand, consider another Lyapunov function $V_\sigma = \sigma^T Q \sigma$, where $Q = \left(B^T C^T X_2 CB\right)^{-1}$. Its derivative with respect to time along the system (17) is

$$\begin{aligned} \dot{V}_\sigma = 2\sigma^T QB^T C^T X_2 C(A+\Delta A)x + 2\sigma^T f(x,t) \\ +2\sigma^T F(t)u(t) + 2\sigma^T u(t) \end{aligned}. \quad (27)$$

Using the control law (20) and noting (23) and assumptions A3) and A4), $\dot{V}_\sigma$ can be enlarged by

$$\begin{aligned} \dot{V}_\sigma \le 2\|\sigma\| \left\{ \left(\|QB^T C^T X_2 C\| \left(\|A\| + \alpha \right) + a \right) \|x\| - \gamma_2 \right\} \\ -(1+\gamma_1^{-1} a^2)\|\sigma\|^2 \end{aligned}. \quad (28)$$

Define a state space domain

$$\Omega = \left\{ x \in \Re^n : \left(\|QB^T C^T X_2 C\| \left(\|A\| + \alpha \right) + a \right) \|x\| < \gamma_2 - \gamma_3 \right\},$$

where $0 < \gamma_3 < \gamma_2$. Since the asymptotical stability, the system states will in finite time enter into the domain Ω, in which $\dot{V}_\sigma \le -2\gamma_3 \|\sigma\|$. Since $\sigma = 0$ and $\hat{\sigma} = 0$ is the same hyperplane in the state space, this inequality shows that the sliding surface $\hat{\sigma} = 0$ can be reached in finite time and remained subsequently. The proof is complete.

Remark 3: Here, we introduce a positive scalar ρ to make the norm of sliding matrix of the utilized sliding surface $\hat{\sigma} = 0$ is 1. The advantages of introducing the unit norm sliding matrix are

I). the control cost can be easily estimated by

$$\|u\| \le \frac{0.5\rho}{1-\psi}(1+\gamma_1^{-1}a^2)\|y(t)\| + \frac{1}{1-\psi}(b+\gamma_2), \quad (29)$$

which is mainly depending on ρ if the $\gamma_1 \gg a^2$. The relationship between the control cost and sliding matrix is very explicitly shown.

II). The physical meaning of sliding mode layer width $\|\hat{\sigma}\| \le \delta$ can be clearly shown as $\|y\| \le \delta$, where δ is a small positive scalar and represents the sliding mode layer width by

$$u = -\frac{0.5\rho}{1-\psi}(1+\gamma_1^{-1}a^2)\hat{\sigma} - \frac{1}{1-\psi}(b+\gamma_2)\frac{\hat{\sigma}}{\|\hat{\sigma}\| + \delta}, \quad (30)$$

which is the usual way [10] to smooth the unit control vector to avoid the chattering. In contrast, if the norm of the sliding matrix is not unit scalar we can not get anything from $\|\sigma\| \le \delta$, since the distance between the state vector and the sliding surface may be very great with a very small sliding surface matrix even if $\delta \to 0$

V. ILMI APPROACH

Noting the inequality (19) can not be converted to an LMI, it is difficult to solve the matrix inequalities (18) and (19). In this section, we will propose an ILMI approach to solve (18) and (19). The ILMI approach is first proposed by Cao *et. al.,* [2] to solve a SOFS problem.

Theorem 4: There exists solution pair (X_1, X_2) satisfying the matrix inequalities (18) and (19) if and only if there exist $X_1 \in \Re^{(n-m)\times(n-m)}$, $X_2 \in \Re^{p\times p}$, $W \in \Re^{p\times p}$ and positive scalar β such that the following matrix inequalities hold

$$X = B^\perp X_1 B^{\perp T} + C^T X_2 C > 0, \quad (31)$$

$$\begin{aligned} XA + A^T X + \beta X - C^T X_2 CBB^T C^T WC \\ -C^T WCBB^T C^T X_2 C + C^T WCBB^T C^T WC < 0 \end{aligned}. \quad (32)$$

Proof: It can be easily proved by the fact that $C^T(X_2 - W)CBB^T C^T (X_2 - W)C \ge 0$. Its details can refer to [2] and hence are omitted here.

Note that in Theorem 3 the positive scalar γ_1 is needed to cope with the matching term, so we should add a term $\gamma_1 I$ at the left side of the inequality (32) before constructing the ILMI algorithm

$$\begin{aligned} XA + A^T X + \beta X + \gamma_1 I - C^T X_2 CBB^T C^T WC \\ -C^T WCBB^T C^T X_2 C + C^T WCBB^T C^T WC < 0 \end{aligned} \quad (33)$$

Thus, the following algorithm can be established.

Iterative linear matrix inequality (ILMI) algorithm:

Step1) Chose a suitable positive scalar γ_1 and solve ARE

$$PA + A^T P - PBB^T P + \gamma_1 I = I \quad (34)$$

Step2) $W = CB(B^T C^T CB)^{-1} B^T PB(B^T C^T CB)^{-1} B^T C^T$ is set as the initial value.

Step3) Solve the following optimization problem to get the maximum value β^* of β

OPI: maximize β, subject to the LMIs (31) and (33).

Step4) Fix $\beta = \beta^*$ and solve the following optimization problem,

OPII: minimize δ, subject to

$$\begin{bmatrix} -\delta & X_2 \\ X_2 & -\delta \end{bmatrix} < 0, \quad (35)$$

and the LMIs (31)(33).

Step5) If $\|X_2 - W\| < \zeta$, a predetermined tolerance, go to Step 6); else let $W = X_2$ go to Step2).

Step6) If $\beta^* \geq 0$, then the solution pair (X_1, X_2) of the OPII is the feasible solution of the matrix inequality (18) and (19); else the matrix inequalities (18) and (19) can not be solved by this algorithm. STOP.

Remark 4: The convergence of the above ILMI algorithm can refer to [2], where the arising problem is somewhat different in style, but has the same essence as the problem met here.

Remark 5: If γ_1 is set a very small value, then the control input will be very large when $\hat{\sigma} \neq 0$. If γ_1 is set a large value the algorithm maybe infeasible. So there is a tradeoff on setting γ_1 between the control cost and the feasibility of algorithm. Moreover, when $\gamma_1 \geq a^2$ the OPII of the algorithm means to find a sub-minimal value of ρ. This in some sense sub-minimizes the control cost.

Remark 6: It can be straightforwardly concluded from the inequality (26) that the decay rate of the system (17) with the control law (20) is more than $\beta/2$. So we can change the judge condition of Step6) by $\beta^* \geq \beta_0$ such that the closed-loop system get a decay rate more than $\beta_0/2$.

VI. Numerical Example

Consider the following system [3]

$$A = \begin{bmatrix} -3 & 0 & 1 \\ 1 & 2 & 0 \\ 0 & 1 & -2 \end{bmatrix},\ B = \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix},\ C = \begin{bmatrix} 0 & 1 & 0 \\ 1 & 1 & 0 \end{bmatrix},$$

$$\Delta A = \begin{bmatrix} r_1(t) & 0 & r_2(t) \\ 0 & 0 & 0 \\ 0 & r_4(t) & 0 \end{bmatrix},$$

$$\Delta B(t) = BF(t) = 0.5Br_5(t),$$

$$f(x,t) = r_3(t)[1\ 1\ 0]x(t) + r_6(t)$$

$$x(0) = \begin{bmatrix} 1 & 0 & -1 \end{bmatrix}^T,\ r_i(t) = \sin t,\ i = 1, \cdots, 6. \quad (36)$$

From the above datas, one can obtain that $a = 1.5$, $b = 1$, $\psi = 0.5$ and $\alpha = 1$. As stated in [3], the previous methods [4][5][6][9][11] can not be applied here since the existence of the uncertain parameter matrix ΔA. Although, a solution of the above system (32) is given out in [3], its details are not legible. How to solve a SOFS problem with a LMI restraint, which is the key problem and is the first step in the algorithm provided in [3], was not presented.

By using the ILMI algorithm with $\gamma_1 = \alpha^2$ and $\beta_0 = 2$, we can get the solution

$$X_2 = \begin{bmatrix} -1.1250 & 7.0619 \\ 7.0619 & 1.6230 \end{bmatrix}$$

So the sliding surface is $\hat{\sigma}(t) = \begin{bmatrix} 0.5643 & 0.8255 \end{bmatrix} y(t)$ with $\rho = 10.5203$. Set $\gamma_2 = 1$ and $\delta = 0.01$, then the smoothed control law (30) is obtained,

$$u = -21.0406\hat{\sigma} - \frac{4\hat{\sigma}}{\|\hat{\sigma}\| + 0.01}. \quad (37)$$

which spends less control cost than that in [3].

Figure 1 shows the simulation results, from which one can see that the sliding surface is reached in finite time and remained subsequently, and the closed-loop uncertain system is asymptotically stable under the control law (37).

VII. Conclusion.

In this paper, the robust SMOFC design problem is studied for a class of system with mismatched parametric uncertainty. For the existence problem, we first present a new sufficient and necessary condition consisting of two matrix inequalities, one of which is bilinear matrix inequality. An ILMI approach is then put forward to solve the two inequalities. The interesting feature is that the ILMI approach only involves the original system matrices and can be easily extended to robust SMOFC design. For the synthesis problem, the sub-optimal design is preliminarily explored to minimize the control cost.

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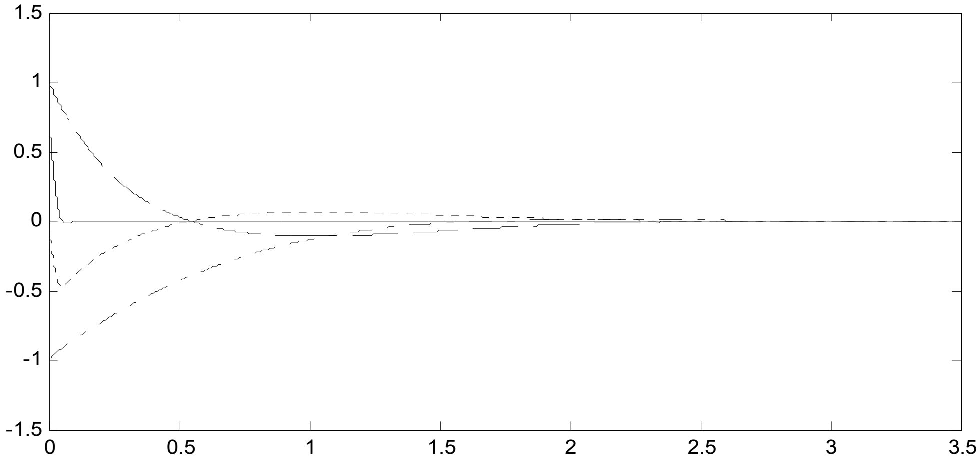

Figure 1. The trajectories of the state vector and the sliding surface: $\hat{\sigma}$ solid line, x_1 dashed line, x_2 dotted line, x_3 dash-dot line.

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