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Robustness Margin Maximization for Inaccurate Controller Implementation

Yasuhide Kobayashi and Toru Asai

***Abstract*— This paper presents a synthesis method of controllers of which robustness margin against controller perturbation is maximized, in order to achieve cost reduction of controller implementation by choosing simple realization and/or truncating wordlength of controller coefficients. The proposed method is based on LMIs with one design parameter, which makes the design procedure easier than the existing non-fragile controller design method. Moreover, a design example of active noise control system is shown to illustrate the validity of the proposed method.**

I. INTRODUCTION

Our goal is to implement controllers with cheap hardwares such as microprocessors rather than more expensive DSPs, satisfying various constraints on closed-loop performance such as $\mathcal{H}_\infty$ and $\mathcal{H}_2$ norm constraints, and the regional pole placement constraint.

It is needed for cheap implementation to design low complexity state-space realization of controller, i.e. coefficient matrices whose wordlength is short enough and whose structure is sparse enough so that the control raw is successfully calculated, since such cheap hardwares have neither much memories nor high computational speed. One can find such low complexity realization from a designed realization of controller by truncating wordlength and/or the similarity transformation. However, the closed-loop performance might be destroyed [1]. Therefore, in order to achieve the goal, controllers must be designed by considering not only the constraints on closed-loop performance but also the constraints on implementation.

There are two ways to consider the constraints on both closed-loop performance and implementation. One is the direct method which is to design a low complexity realization of controller which satisfies the constraints on closed-loop performance. The other is the indirect method which is to firstly design a realization of controller which is not restricted to low complexity but is robust against controller perturbation so that the constraints on closed-loop performance hold, secondary design a low complexity realization by exploiting the robustness margin of controller perturbation. The direct method gives the optimal controller, however, it requires significant computation even if the constraints on closed-loop performance are simple [2], since the similarity transformation changes the behavior of the controller when the wordlength of the realization is restricted.

There are two subjects to design controllers by the indirect method:

(S1) design a robust controller which satisfies the constraints on closed-loop performance against controller perturbation.

(S2) design a low complexity realization of given controller such that the constraints on implementation hold.

Each subject has been separately studied to achieve chap implementation of controller without obvious connection to the indirect method. However, it is not sufficient to design controller by solving only one subject: Even if the controller is designed by solving (S1) to have robustness margin against controller perturbation, the closed-loop performance might be destroyed if the realization is not properly chosen; Even if the realization is chosen by solving (S2), the constraints on implementation do not always hold if the robustness margin of given controller is small.

Since the subject (S2) has been sufficiently solved by the sensitivity minimization method [3], [4], [5], we concentrate on the subject (S1) in this paper.

There are two ways to solve (S1): the explicit method and the implicit method, distinguished by whether the norm-bounded controller perturbation is explicitly considered or not. It has been known that some robustness margin against controller perturbation is expected by the implicit method such as the McFarlane & Glover loop-shaping method [6]. However, it is difficult to design controllers systematically by the implicit method since robustness margin of controller perturbation can not be specified in the design process.

The explicit methods of $\mathcal{H}_\infty$ controller have been proposed based on the constantly scaled $\mathcal{H}_\infty$ norm constraint by introducing the maximal singular value to bound the controller perturbation [7], [8]. By using these methods, robustness margin of controller perturbation can be specified in the design process. However, systematical design is difficult by the method of [7] since it depends on many design parameters. In addition, the output feedback controller design is not addressed in the method of [8].

In this paper we propose an explicit method of $\mathcal{H}_\infty$ controller which solves the problems of existing methods: the proposed method depends on only one design parameter; output feedback controllers are designed. In addition, the robustness margin against the controller perturbation is maximized in order to improve the solvability of (S2). The proposed method can be extended to consider the other constraints on closed-loop performance such as $\mathcal{H}_2$ norm

Y. Kobayashi is with Faculty of Engineering, Nagaoka University of Technology, 1603-1 Kamitomioka, Nagaoka, Niigata, JAPAN kobayasi@vos.nagaokaut.ac.jp

T. Asai is with Graduate School of Engineering, Osaka University, 2-1 Yamada-oka, Suita-shi, Osaka, JAPAN tasai@mech.eng.osaka-u.ac.jp

constraint and regional pole placement constraint which are described by LMIs [9], [10]. A design example for an active noise control system is shown to illustrate the validity of the proposed method.

II. Problem formulation

Problem 1: Let $G(s)$ be a given generalized plant described by

$$G(s) = \left[\begin{array}{c|cc} A & B_1 & B_2 \\ \hline C_1 & D_{11} & D_{12} \\ C_2 & D_{21} & 0 \end{array}\right] \tag{1}$$

where $A \in \mathbf{R}^{n\times n}$, $B_1 \in \mathbf{R}^{n\times m_1}$, $B_2 \in \mathbf{R}^{n\times m_2}$, $C_1 \in \mathbf{R}^{p_1\times n}$, $C_2 \in \mathbf{R}^{p_2\times n}$, $D_{11} \in \mathbf{R}^{p_1\times m_1}$, $D_{12} \in \mathbf{R}^{p_1\times m_2}$ and $D_{21} \in \mathbf{R}^{p_2\times m_1}$ for given positive integer numbers n, m_1, m_2, p_1 and p_2. Let $K(s)$ be a controller described by

$$K(s) = \left[\begin{array}{c|c} A_K + H_1\Delta E_1 & B_K + H_1\Delta E_2 \\ \hline C_K + H_2\Delta E_1 & D_K + H_2\Delta E_2 \end{array}\right] \tag{2}$$

where $\Delta \in \mathbf{R}^{p_3\times m_3}$ is an unknown real matrix for given positive integer numbers p_3 and m_3, and $E_1 \in \mathbf{R}^{m_3\times n}$, $E_2 \in \mathbf{R}^{m_3\times p_2}$, $H_1 \in \mathbf{R}^{n\times p_3}$ and $H_2 \in \mathbf{R}^{m_2\times p_3}$ are given constant real matrices. Let γ be a given positive scalar number. Then, the design problem here is to find matrices $A_K \in \mathbf{R}^{n\times n}$, $B_K \in \mathbf{R}^{n\times p_2}$, $C_K \in \mathbf{R}^{m_2\times n}$ and $D_K \in \mathbf{R}^{m_2\times p_2}$ which minimize a positive scalar α under the following constraints for any uncertainty Δ that satisfies $\bar{\sigma}(\Delta) \le \frac{1}{\alpha}$:

- the closed-loop system composed of $G(s)$ and $K(s)$ is internally stable
- $\mathcal{H}_\infty$ norm of the closed-loop transfer function is less than γ

Equation (2) represents a class of controllers whose coefficient matrices have additive uncertainty. If all the elements in the coefficient matrices have perturbation, one can set for example

$$H_1 = \begin{bmatrix} I \\ 0 \end{bmatrix}^T, H_2 = \begin{bmatrix} 0 \\ I \end{bmatrix}^T, E_1 = \begin{bmatrix} I \\ 0 \end{bmatrix}, E_2 = \begin{bmatrix} 0 \\ I \end{bmatrix}, \tag{3}$$

which leads to

$$K(s) = \left[\begin{array}{c|c} A_K + \Delta_A & B_K + \Delta_B \\ \hline C_K + \Delta_C & D_K + \Delta_D \end{array}\right] \tag{4}$$

with Δ partitioned appropriately as

$$\Delta = \begin{bmatrix} \Delta_A & \Delta_B \\ \Delta_C & \Delta_D \end{bmatrix}. \tag{5}$$

III. Main results

In this section we begin with the analysis problem against Problem 1. The following lemma gives a sufficient condition for a given controller $K(s)$ to be a solution of Problem 1 based on the constantly scaled $\mathcal{H}_\infty$ constraint [11].

Lemma 1: For given matrices A_K, B_K, C_K, and D_K, assume that the closed-loop system composed of $\hat{G}(s)$ and $K_0(s)$ is internally stable and there exists a positive scalar d such that the following condition holds:

$$\left\| \begin{bmatrix} d^{\frac{1}{2}}I & 0 \\ 0 & \gamma^{-\frac{1}{2}}I \end{bmatrix} \mathcal{F}_l(\hat{G}, K_0) \begin{bmatrix} d^{-\frac{1}{2}}\frac{1}{\alpha}I & 0 \\ 0 & \gamma^{-\frac{1}{2}}I \end{bmatrix} \right\|_\infty < 1 \tag{6}$$

where $\hat{G}(s)$ and $K_0(s)$ are given by

$$\hat{G}(s) = \left[\begin{array}{c|cc|cc} A & 0 & B_1 & 0 & B_2 \\ \hline 0 & 0 & 0 & I & 0 \\ C_1 & 0 & D_{11} & 0 & D_{12} \\ \hline 0 & I & 0 & 0 & 0 \\ C_2 & 0 & D_{21} & 0 & 0 \end{array}\right], \tag{7}$$

$$K_0(s) = \left[\begin{array}{c|cc} A_K & H_1 & B_K \\ \hline E_1 & 0 & E_2 \\ C_K & H_2 & D_K \end{array}\right], \tag{8}$$

and $\mathcal{F}_l(\bullet,\bullet)$ denotes the lower LFT. Then, for any Δ that satisfies $\bar{\sigma}(\Delta) \le \frac{1}{\alpha}$, the closed-loop system composed of $G(s)$ and $K(s)$ is internally stable and $\mathcal{H}_\infty$ norm of the closed-loop system is less than γ.

According to Lemma 1, the analysis problem for Problem 1 can be solved as a convex problem respect to the constant scaling d for given α. Moreover, α can be minimized by the bi-section method.

The following theorem provides a necessary and sufficient condition for the condition in Lemma1.

Theorem 1: Let $\hat{G}(s)$ be given by (7), $K_0(s)$ be described by (8) where E_1, E_2, H_1 and H_2 are given matrices. The followings are equivalent:

(i) There exists matrices A_K, B_K, C_K and D_K, and a positive scalar d such that the close-loop system composed of $\hat{G}(s)$ and $K_0(s)$ is internally stable and (6) holds.

(ii) There exists matrices M_{11}, M_{13}, M_{31}, M_{33}, $X > 0$ and $Y > 0$, and a positive scalar d such that the following inequality and (12) (the top of the next page) hold:

$$\begin{bmatrix} X & I \\ I & Y \end{bmatrix} > 0 \tag{9}$$

Moreover, if the inequalities in (ii) have a solution, then $K_0(s)$ with the following matrices stabilizes the closed-loop system and satisfies (6):

$$\begin{bmatrix} A_K & B_K \\ C_K & D_K \end{bmatrix} = \begin{bmatrix} -S^{-1} & S^{-1}YB_2 \\ 0 & I \end{bmatrix}$$
$$\times \left(\begin{bmatrix} M_{11} & M_{13} \\ M_{31} & M_{33} \end{bmatrix} - \begin{bmatrix} YAX & 0 \\ 0 & 0 \end{bmatrix} \right) \begin{bmatrix} X^{-1} & 0 \\ -C_2 & I \end{bmatrix} \tag{10}$$

where $S := Y - X^{-1}$.

Proof: According to the Bounded Real Lemma, (i) is equivalent to the existence of a positive definite matrix $\mathcal{Y} > 0$ such that the following inequality holds:

$$\begin{bmatrix} \mathcal{Y}\mathcal{A} + \mathcal{A}^T\mathcal{Y} & \mathcal{Y}\mathcal{B} & \mathcal{C}^T \\ \mathcal{B}^T\mathcal{Y} & -I & \mathcal{D}^T \\ \mathcal{C} & \mathcal{D} & -I \end{bmatrix} < 0, \tag{11}$$

$$\begin{bmatrix} XA^T + M_{31}^T B_2^T + (*)^T & * & * & * & * & * \\ A^T + C_2^T M_{33}^T B_2^T + M_{11} & A^T Y + C_2^T M_{13}^T + (*)^T & * & * & * & * \\ H_2^T B_2^T & -H_1^T (Y - X^{-1}) + H_2^T B_2^T Y & -\alpha^2 dI & * & * & * \\ B_1^T + D_{21}^T M_{33}^T B_2^T & B_1^T Y + D_{21}^T M_{13}^T & 0 & -\gamma I & * & * \\ d(E_1 + E_2 C_2)X & dE_2 C_2 & 0 & dE_2 D_{21} & -dI & * \\ C_1 X + D_{12} M_{31} & C_1 + D_{12} M_{33} C_2 & D_{12} H_2 & D_{11} + D_{12} M_{33} D_{21} & 0 & -\gamma I \end{bmatrix} < 0 \quad (12)$$

$$\left[\begin{array}{cccccc|c} XA^T + M_{31}^T B_2^T + (*)^T & * & * & * & * & * & * \\ A^T + C_2^T M_{33}^T B_2^T + M_{11} & A^T Y + C_2^T M_{13}^T + (*)^T + Q & * & * & * & * & * \\ H_2^T B_2^T & (H_2^T B_2^T - H_1)^T Y & -\alpha^2 dI & * & * & * & * \\ B_1^T + D_{21}^T M_{33}^T B_2^T & B_1^T Y + D_{21}^T M_{13}^T & 0 & -\gamma I & * & * & * \\ d(E_1 + E_2 C_2)X & dE_2 C_2 & 0 & dE_2 D_{21} & -dI & * & * \\ C_1 X + D_{12} M_{31} & C_1 + D_{12} M_{33} C_2 & D_{12} H_2 & D_{11} + D_{12} M_{33} D_{21} & 0 & -\gamma I & * \\ \hline 0 & 0 & H_1 & 0 & 0 & 0 & -X \end{array}\right] < 0 \quad (13)$$

$$\begin{bmatrix} XA^T + M_{31}^T B_2^T + (*)^T & * & * & * & * & * \\ A^T + C_2^T M_{33}^T B_2^T + M_{11} & A^T Y + C_2^T M_{13}^T + (*)^T + Q & * & * & * & * \\ H_2^T B_2^T & (H_2^T B_2^T - H_1)^T Y & -\alpha^2 dI + H_1^T X^{-1} H_1 & * & * & * \\ B_1^T + D_{21}^T M_{33}^T B_2^T & B_1^T Y + D_{21}^T M_{13}^T & 0 & -\gamma I & * & * \\ d(E_1 + E_2 C_2)X & dE_2 C_2 & 0 & dE_2 D_{21} & -dI & * \\ C_1 X + D_{12} M_{31} & C_1 + D_{12} M_{33} C_2 & D_{12} H_2 & D_{11} + D_{12} M_{33} D_{21} & 0 & -\gamma I \end{bmatrix} < 0 \quad (14)$$

where the matrices $\mathcal{A}$, $\mathcal{B}$, $\mathcal{C}$ and $\mathcal{D}$ are the state-space realization of the left-hand-side system in (6), which can be written as

$$\begin{bmatrix} \mathcal{A} & \mathcal{B} \\ \mathcal{C} & \mathcal{D} \end{bmatrix} = R + U M_K V \quad (15)$$

$$R = \left[\begin{array}{cc|cc} A & 0 & 0 & \gamma^{-\frac{1}{2}} B_1 \\ 0 & 0 & 0 & 0 \\ \hline 0 & 0 & 0 & 0 \\ \gamma^{-\frac{1}{2}} C_1 & 0 & 0 & \gamma^{-1} D_{11} \end{array}\right]$$

$$V = \left[\begin{array}{cc|cc} 0 & I & 0 & 0 \\ 0 & 0 & d^{-\frac{1}{2}} I & 0 \\ C_2 & 0 & 0 & \gamma^{-\frac{1}{2}} D_{21} \end{array}\right]$$

$$U = \left[\begin{array}{ccc} 0 & 0 & B_2 \\ I & 0 & 0 \\ \hline 0 & d^{\frac{1}{2}} I & 0 \\ 0 & 0 & \gamma^{-\frac{1}{2}} D_{12} \end{array}\right], M_K = \begin{bmatrix} A_K & \frac{1}{\alpha} H_1 & B_K \\ E_1 & 0 & E_2 \\ C_K & \frac{1}{\alpha} H_2 & D_K \end{bmatrix}$$

Now, without loss of generality, $\mathcal{Y}$ can be chosen as

$$\mathcal{Y} = \begin{bmatrix} Y & X^{-1} - Y \\ X^{-1} - Y & Y - X^{-1} \end{bmatrix} \quad (16)$$

with positive definite matrices X and Y which satisfy (9) [12]. Then pre- and post-multiplying the matrices $\mathrm{diag}(\mathcal{T}, I, I)$ and $\mathrm{diag}(\mathcal{T}^T, I, I)$ to (11) respectively where $\mathcal{T} = \begin{bmatrix} X & X \\ I & 0 \end{bmatrix}$, and defining the new variables M_{11}, M_{13}, M_{31} and M_{33} as

$$\begin{bmatrix} M_{11} & M_{13} \\ M_{31} & M_{33} \end{bmatrix} = \begin{bmatrix} X^{-1} - Y & YB_2 \\ 0 & I \end{bmatrix} \begin{bmatrix} A_K & B_K \\ C_K & D_K \end{bmatrix} \times \begin{bmatrix} X & 0 \\ C_2 X & I \end{bmatrix} + \begin{bmatrix} YAX & 0 \\ 0 & 0 \end{bmatrix}, \quad (17)$$

(12) is derived. ■

According to Theorem 1 and Lemma 1, Problem 1 is reduced to minimizing problem of α under the constraint in Theorem 1. However, the constraint in Theorem 1 is non-convex even if the constant scaling d is fixed due to the term X^{-1} in (12), while the scaled $\mathcal{H}_\infty$ control problem is known to become convex if the scaling is fixed, which allows D-K iteration technique to solve the problem. This is the essential difficulty of the non-fragile controller synthesis problem.

While Theorem 1 can be directly proven by using the technique in [12], [10], another approach in [9] is also available to derive a similar result to Theorem 1, which is also non-convex.

Remark 1: The constraint in Theorem 1 is reduced to convex if (1) no controller perturbation exists (H_1, H_2, E_1, and E_2 are zero), or (2) the state feedback is designed ($C_2 = I$ and $D_{21} = 0$). In the former case, the 3rd row and column of (12) are eliminated, which is equivalent to the constant scaled $\mathcal{H}_\infty$ control problem. In the latter case, the 2nd row and column of (12) are eliminated, since M_{13} can be set by $M_{13} = -\beta I$ with arbitrary large positive scalar β. This convexity is consistent with the result of [8].

In this paper, in order to avoid difficulty in solving the synthesis problem, the term X^{-1} is removed by considering a sufficient condition for (12). The following theorem gives the sufficient condition.

Theorem 2: Let $\hat{G}(s)$ be given by (7), $K_0(s)$ be described by (8) where E_1, E_2, H_1 and H_2 are given matrices, and d be a given positive scalar. Assume that there exists matrices M_{11}, M_{13}, M_{31}, M_{33}, $X > 0$, $Y > 0$ and $Q > 0$

such that the following LMIs and (13) hold.

$$\begin{bmatrix} X & I \\ I & Y \end{bmatrix} > 0, \quad \begin{bmatrix} Q & I \\ I & X \end{bmatrix} > 0 \qquad (18)$$

Then, the matrices M_{11}, M_{13}, M_{31}, M_{33}, $X > 0$, $Y > 0$ satisfy (12) and (9).

Proof: Assume that (13) and (18) have a solution. By applying the Schur complement to (13), (14) holds. Then by adding the negative definite matrix $\mathrm{diag}(I, \begin{bmatrix} -Q & X^{-1}H_1 \\ H_1^T X^{-1} & -H_1^T X^{-1} H_1 \end{bmatrix}, I, I, I)$ to the left-hand-side matrix in (14), (12) holds. ■

Note that the condition in Theorem 2 is convex with respect to the decision variables M_{11}, M_{13}, M_{31}, M_{33}, X, Y, Q, and α^2 if the constant scaling d is fixed.

The condition in Theorem 2 is conservative since it is only a sufficient condition for Theorem 1. However, by using the condition in Theorem 2, non-fragile $\mathcal{H}_\infty$ controllers can be designed via LMIs with one design parameter, d, while the existing method [7] has more design parameters (σ_3, T_a and T_b) to which trial and error tuning might be needed. Moreover, robustness margin against controller perturbation can be maximized by minimizing α.

In addition, the effect of the conservativeness involved by Theorem 2 can be reduced by combination use of Theorem 1 after applying Theorem 2. That is, to execute the following steps for each fixed value of d:

Step.1 Find matrices M_{11}, M_{13}, M_{31}, M_{33}, X, Y and Q in (13) and (18), which minimizes α.

Step.2 Fix the matrix X as in Step.1. Then, find matrices M_{11}, M_{13}, M_{31}, M_{33} and Y in (12) and (9), which minimize α.

Note that for each fixed value of d, the optimized value of α in Step.2, does not become larger than that in Step.1.

IV. Numerical Examples

This section presents two numerical examples: one is from [7] and the other is active noise control system design.

A. Numerical example from [7]

Let $G(s), E_1, E_2, H_1, H_2$ and γ are given by

$$G(s) = \left[\begin{array}{cc|cc:c} -0.8 & 0.5 & 0 & 0 & 0 \\ 0.5 & 1 & 0.5 & 0 & 6 \\ \hline 0.2 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 \\ \hdashline 0.2 & 2 & 0 & 0.5 & 0 \end{array}\right]$$

$$\begin{bmatrix} H_1 \\ H_2 \end{bmatrix} = \left[\begin{array}{cccc} 0 & 0 & \sqrt{0.5} & 0 \\ 0 & \sqrt{0.1} & 0 & \sqrt{0.5} \\ \hdashline \sqrt{0.9} & 0 & 0 & 0 \end{array}\right]$$

$$\begin{bmatrix} E_1^T \\ E_2^T \end{bmatrix} = \left[\begin{array}{ccccc} 1 & 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 & 1 \\ \hdashline 0 & 0 & 1 & 0 & 0 \end{array}\right], \gamma = 1.6.$$

For the above generalized plant, the proposed method in the previous section is applied to solve Problem 1. As the result, $\alpha = 0.69$ is obtained for $d = 2.9$. [1] (the detail of the one-dimensional search for d is mentioned later.) Note that in [7] the singular value of Δ is set to 1 (not maximized) which corresponds to $\alpha = 1$ in Problem 1. Therefore, $\alpha = 0.69$ means that our designed controller allows larger perturbation than [7] in this example. Table I shows stability analysis result and $\mathcal{H}_\infty$ norm of the closed-loop system by the designed controller, together with the result of [7] for comparison. Note that Table I shows two 'standard' cases which stand for $\alpha = 0$, since our result for 'standard' case differs from that in [7], while our 'standard' result is calculated by MATLAB *hinfsyn* function with default options. The rows 'without error' and 'with error' show results for $\Delta = 0$ and $\Delta_0 = \mathrm{diag}\left(\begin{bmatrix} 0 & 0.94 \end{bmatrix}, -0.31, -0.7, -0.7\right)$ respectively.

From Table I it can be seen that the proposed method in this paper has similar advantages to the method in [7]: closed-loop stability is maintained against controller perturbation, and the closed-loop $\mathcal{H}_\infty$ norm is bounded by the prescribed value $\gamma = 1.6$. In addition, in order to estimate the worst closed-loop $\mathcal{H}_\infty$ norm against controller perturbation, the analysis problem in Lemma1 is solved by minimizing γ. The minimized value of γ is 1.55. It follows that the conservativeness in Theorem 2 is not serious problem in this example, since 1.55 is close to the prescribed value 1.6.

Fig. 1 shows the detail of the one-dimensional search for d: for each fixed value of d, α is minimized by Theorem 2 (shown by $\circ$) firstly, then minimized by Theorem 1 (shown by $*$). It can be seen that the combination use of two

TABLE I

CLOSED-LOOP CHARACTERISTIC

		Yang & Wang [7]		by this study	
		standard	proposed	standard	proposed
without error	stability	○	○	○	○
	$\mathcal{H}_\infty$ norm	0.41	0.65	0.88	0.83
with error	stability	×	○	×	○
	$\mathcal{H}_\infty$ norm	—	0.47	—	0.72

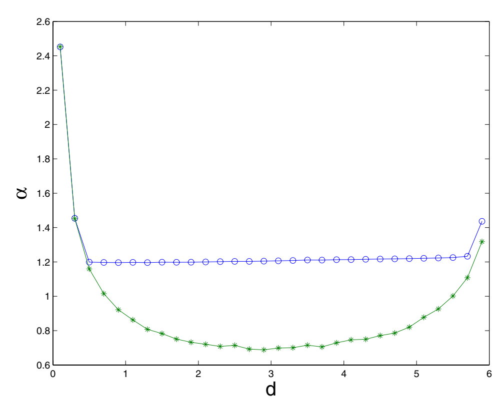


Fig. 1. one-dimensional search for d ($\circ$: minimized α by Theorem 2; $*$: minimized α by Theorem 2 and 1)

[1] MATLAB LMI toolbox and *feasp* function are used with default options

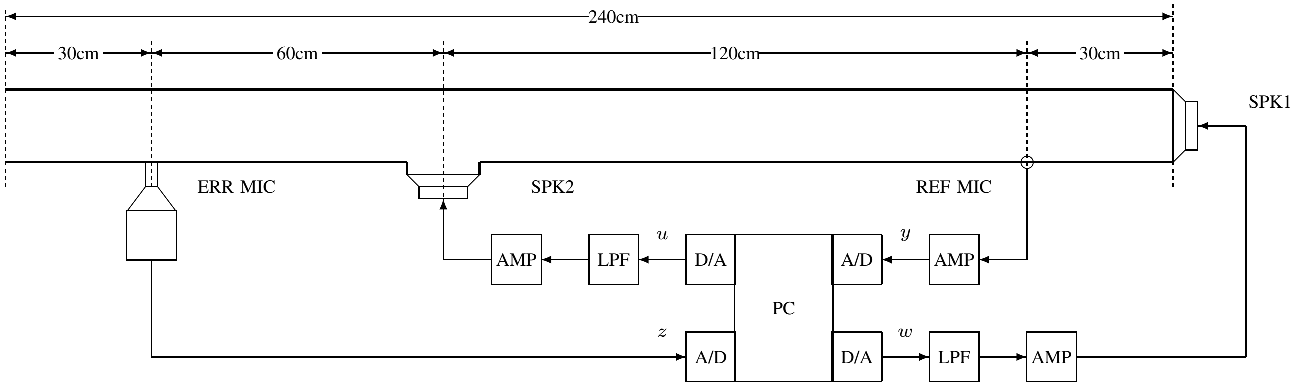


Fig. 2. Experimental apparatus

Theorems is valid to minimize α.

B. Active noise control system design

In this section we consider a design problem for active noise control system as an example of high-order plant. In addition, the companion form is utilized in order to consider the situation where computational load reduction is essentially important, although the companion form is usually avoided as controller realization since it tends to be very sensitive to parameter variations [6].

Fig. 2 shows the experimental apparatus. (for explanations in detail, see [13].) The original design problem here is to find a controller $K(s)$ which maximizes $k > 0$ to improve the performance under the following constraints: (i) the closed-loop system in Fig. 3 is internally stable and, (ii) $\mathcal{H}_\infty$ norm of the closed-loop transfer function is less than 1 with some constant scaling $d_0 > 0$, where $\bar{G}$ and W in Fig. 3 are a given nominal plant (20th order) and a weight function (2nd order) respectively. Fig. 4 shows the result of frequency response experiment and the approximated nominal plant, $\bar{G}$.

To solve the original problem above, *hinfsyn* of MATLAB is used. As the result, the possible greatest value of k reached to 1.4 for $d_0 = 0.72$, and a full order controller (22nd order) was obtained. Then it is discretized by the Tustin transformation with the sampling period used in control experiments (0.25 msec) and is implemented. Fig. 5 (a) shows the experimental result of the original controller where the controller is started at 5 sec. In this case, the desired performance is obtained.

Next, we proceed to the companion form. Again *hinfsyn* is used as a standard method for comparison with the proposed method, where the common $d_0 = 0.72$ is used for simplicity and the smaller value of $k = 1.0 < 1.4$ is chosen to expect some robustness margin. However, the large oscillation is occurred as shown in Fig. 5 (b). On the other hand, the controller designed by the proposed method works well as shown in (c), while in the design procedure the matrices E_1, E_2, H_1 and H_2 are given by (3), and the whole system without controller $K(s)$ in Fig. 3 is regarded as $G(s)$ for which Problem 1 is solved. The

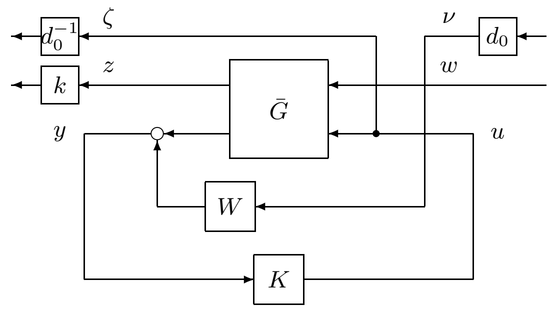
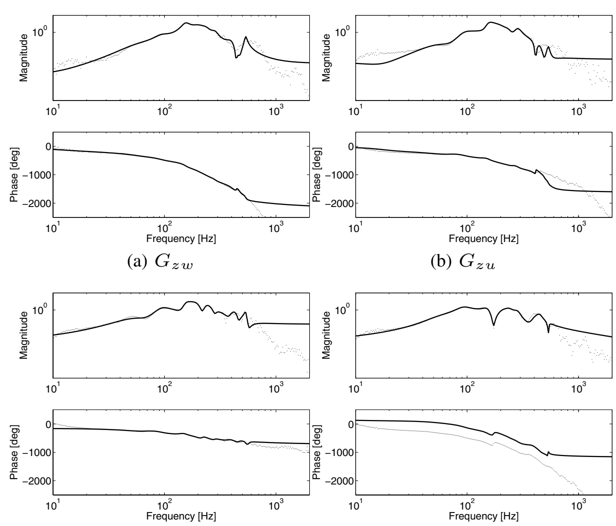


Fig. 3. Robust performance problem

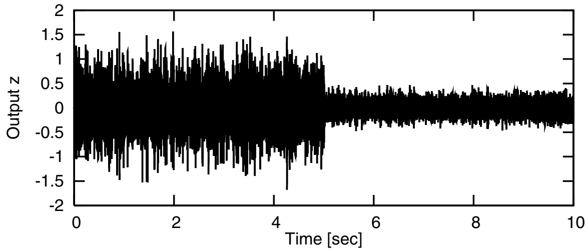


(a) G_{zw} (b) G_{zu} (c) G_{yw} (d) G_{yu}

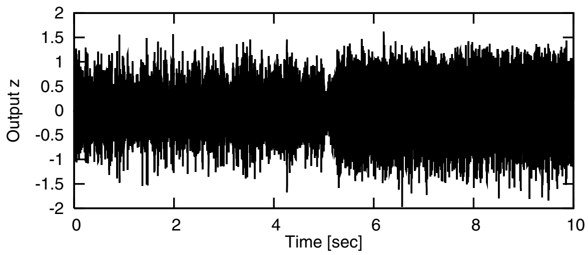
Fig. 4. Frequency response: measured (dots), approximated nominal plant (curve)

possible smallest value of α is 94 for $d = 0.02$ while the common values of $d_0 = 0.72$ and $k = 1.0$ are used.

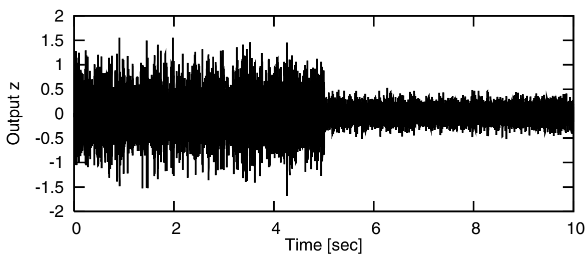
Fig. 6 shows Bode plots of the controllers designed above. It can be seen that the original controller and the companion form controller by the proposed method have similar characteristic in the frequency range from about 100 [Hz] to 300 [Hz] which is the dominant frequency range of the plant since the gain of the open-loop transfer function, G_{zw}, is relatively large in the frequency range. (see Fig. 4 (a).) On the other hand, the companion form controller by



(a) original controller ($k = 1.4$)



(b) the companion form controller by standard method ($k = 1.0$)



(c) the companion form controller by proposed method ($k = 1.0$)

Fig. 5. Error microphone output z

the standard method differs from the others. In addition, the oscillation frequency in Fig. 5 (b) is about 80 [Hz], at which the companion form controller by the standard method also shows large difference.

V. Conclusions

In this paper we proposed a synthesis method of controllers of which robustness margin against controller perturbations is maximized, which allows one to achieve cost reduction of controller implementation by transferring controller realization to simple one e.g. the companion form, and/or by truncating wordlength of controller coefficients. By using the proposed method, non-fragile dynamic output feedback $\mathcal{H}_\infty$ controllers can be designed via LMIs with one design parameter, constant scaling, which makes the design procedure simpler than the existing design method in [7] which has more design parameters.

Moreover, an application of active noise control was shown to illustrate the validity of the proposed method without detailed discussion. Therefore, it might be needed to examine how large controller perturbation is allowed for the final discretized controller by considering the effect of the similarity transformation to get the companion form, and the Tustin transformation to discretize the controller. Those above are the future issues, however, it can be said that the application shows possibility of the proposed method in practical use.

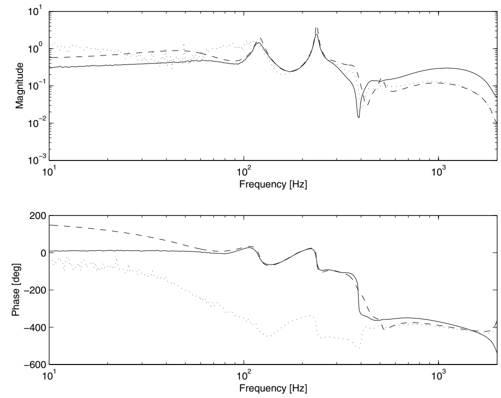


Fig. 6. Controllers (**broken curve** : original controller (k = 1.4); **dotted curve** : the companion form controller by standard method (k = 1.0); **solid curve** : the companion form controller by proposed method (k = 1.0)

It might be empirically carried out to introduce some margin into LMIs for synthesis problem in order to prevent designed controllers being fragile. It is, for example, to solve $\mathcal{L}(x)+\epsilon I < 0$ instead of $\mathcal{L}(x) < 0$ with some positive scalar ϵ. However, such heuristic method has no guarantee how large perturbation is allowed for designed controllers. It can be considered that the proposed method shows the magnitude and the structure that LMIs for synthesis problem should have for the purpose.

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