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Robust Parallel Design of the Subsystems Constituting a Complex System

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Abstract— **The problem of designing complex systems, with performance specifications from multiple disciplines, consisting of several subsystems, and where the subsystems are designed in parallel is addressed. The subsystems interact in the sense that the performance of one subsystem is influenced by the design details of other subsystems. This introduces uncertainty into the design process. A team designing a given subsystem needs design details from other subsystems to evaluate its subsystem performance. If the subsystems are designed in parallel, these design details are generally not readily available.**

In this paper, we formulate the subsystem design problems using a robust design framework. Nonlinear optimization is used to design subsystems that are robust with respect to the uncertainties arising from designing the subsystems in parallel. In addition to its own uncertain design parameters, the uncertain parameters for a given subsystem include the design variables and outputs from other subsystems that are needed in that subsystem analysis. Accounting for these uncertainties allows the subsystems to be designed in parallel while guaranteeing achievement of system-level performance specifications upon assembly of the subsystems. The proposed Robust Parallel Design (RPD) approach is illustrated using a passive suspension design example for a half-car model.

I. Introduction

THE design of complex systems usually involves several design teams working in parallel. This requires that the overall design problem be decomposed into several smaller sub-problems, each small enough to be handled by one of the design teams. Usually performance specifications are known at the system-level. In order to design the subsystems in parallel, these system-level performance specifications need to be cascaded down to subsystem design targets. The subsystems can then be designed in parallel to meet their respective subsystem targets.

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Target cascading should be performed in an "efficient" and "consistent" manner to avoid iterations at later stages of the design process and to ensure that once subsystem targets are met, the desired system-level specifications are achieved [7]. Once the performance specifications for a subsystem are cascaded, design teams work in parallel to achieve their subsystems' design targets. Designing the subsystems in parallel introduces uncertainty to the design process. Since the subsystems are coupled, or interacting, in the sense that outputs from one subsystem may be needed to evaluate the performance of another subsystem, i.e., the performance of a subsystem depends on design variables of other subsystems, in addition to depending on its own design variables and parameters. The values of the design variables of other subsystems are not readily available to the team designing a given subsystem because the subsystems are designed in parallel.

This paper addresses the uncertainty introduced by designing subsystems in parallel. We consider worst-case uncertainties, i.e., each subsystem design team proceeds with their subsystem design assuming worst-case values for the uncertain parameters. This allows us to determine (conservative) bounds for the performance of the overall system. The proposed Robust Parallel Design (RPD) approach can be modified to make use of the statistical distributions of uncertainties (when these are known) to design the different subsystems so that the overall system has a high probability of meeting its performance specifications to a prescribed confidence level.

II. Literature Review

The mathematical statement of a general design problem (GDP) is [8]:

$$\begin{aligned}&\text{Find } \mathbf{x} \in X\\&\text{subject to } g(\mathbf{x}, \mathbf{p}) \le \mathbf{0}\end{aligned} \qquad (1)$$

In this statement, $\mathbf{x}$ represents the vector of design variables, X is the set constraint for the design variables and $g(\mathbf{x}, \mathbf{p})$ represents inequality constraints as functions of design variables $\mathbf{x}$ and design parameters $\mathbf{p}$.

The GDP is transformed into an Optimal Design Problem (ODP) by selecting some performance measures and optimizing the system with respect to these performance measures. These performance measures are weighted functions of the design variables $\mathbf{x}$ and design parameters $\mathbf{p}$. The weighting vector will be denoted by $\mathbf{w}$. This leads to the following statement of the ODP [8]:

$$\begin{aligned}&\text{Minimize } f(\mathbf{x}, \mathbf{p}, \mathbf{w})\\&\quad \mathbf{x} \in X \qquad\qquad (2)\\&\text{subject to } g(\mathbf{x}, \mathbf{p}) \leq \mathbf{0}\end{aligned}$$

The complexity of this optimal design problem is Non-deterministic Polynomial (NP) for many design problems and solution times grow exponentially as the number of design variables increases. Thus, it is desirable to decompose this problem into a set of smaller sub-problems where each sub-problem can be handled more efficiently. The benefits of system decomposition are known within both the design and control communities.

Within the design community, it has been recognized that the use of system decomposition and parallel engineering can reduce a product's design-cycle time, reduce a design problem's dimensionality and allow companies to outsource some of their design tasks to their suppliers [7]. Similarly, in the control community, Siljak argued that system decomposition and decentralized control can be used to reduce the dimensionality of a control problem and to satisfy information structure constraints and accommodate uncertainties [19].

The problem of decomposing a large optimal design problem into a set of smaller sub-problems had been addressed in [8] and [16]. There are three main types of system decomposition found in the literature; object, aspect and sequential decomposition. Object decomposition divides the system into different subsystems based on their physical components. Aspect decomposition is based upon the different specialized disciplines used in the modeling of the system. Sequential decomposition is based upon the flow of information within the system.

Tools for system decomposition are available in the literature. For example, Krishnamachari and Papalambros propose an approach for hierarchical decomposition of an engineering system [8]. A survey of system decomposition approaches can be found in [7]. An application of one of the problem decomposition strategies, namely optimal model-based decomposition, to a vehicle powertrain is illustrated in [22].

Several approaches are described in the literature to solve an optimal design problem. All-At-Once (AAO) optimization is used to solve an optimal design problem directly without decomposing the original problem into sub-problems. Alternatively, there are Multidisciplinary Optimization (MDO) approaches based on system decomposition and coordinating the solutions of the resulting subsystem design problems to yield an optimal system design. Examples of MDO approaches include Collaborative Optimization (CO), Target Cascading [7] and Bi-Level Integrated System Synthesis (BLISS) [20]. MDO approaches attempt to derive specifications for all the subsystems and components of the overall system, usually using surrogate models, and assuming full interaction between the different subsystems. For a review of different MDO approaches the reader is referred to [1].

MDO approaches assume full coordination between subsystem optimization problems, i.e., changes in the values of design variables in one subsystem optimization problem are made available to other subsystem optimization problems. Thus MDO approaches are suited to parallel and distributed computing environments where the goal is to utilize several computers to solve a system-level optimization problem.

In this paper, we focus on situations where subsystems are designed independently. This is often the case in an industrial environment where organizational barriers and geographical constraints place a burden on communication between design teams and where decisions taken by one of the design teams may not be fully communicated to other teams [13]. Relevant research includes [11] where exclusive groups perform subsystem design tasks sequentially. A central authority is used to determine the sequence of the design tasks. In [9] and [10], the authors identify two factors that influence the extent to which a design activity can be decomposed into two activities that can be overlapped, or parallelized, successfully: i) the sensitivity of downstream design to upstream activities and ii) the rate of evolution of design details.

Sensitivity to upstream design details refers to the extent to which a design task needs rework if there are changes in another upstream design activity. The rate of evolution of design details refers to the speed with which design details evolve, i.e., fast evolution means that more of the design details become available at en earlier stage of the design process. Parallel design is most suited to routine design tasks with low sensitivity to upstream design activities and high rate of evolution of design details.

Other relevant research includes the work of Chang and Ward [2] and Chang *et al.* [3] where the authors propose that a design team should treat the design variables in other subsystems as noise factors. Taguchi's parameter design approach can then be used to design subsystems that are robust with respect to both physical noise factors and to the "conceptual" noise factors, i.e., the design variables of other subsystem. In their formulation, Chang and Ward [2], assume that the design teams can communicate at certain intervals. A cost of delay is calculated to determine whether a design team should decide on the values of its design variables immediately or wait for more information from other design teams [2].

We propose a new Robust Parallel Design Approach that enables the parallel design of subsystems while accounting for the possible lack of co-ordination between design teams. Target cascading is used to specify subsystem design targets and the nominal values of some of the key design variables using surrogate models [7]. However, the use of higher fidelity models in the actual subsystem design may result in the deviation of some of the design variables from their nominal values.

The deviation of design variables and parameters around their nominal values can be described using statistical properties. In this paper, we assume that design variables and design parameters are bounded around their nominal values and that the values of these bounds are known. The deviation of design variables and parameters is taken into account when determining bounds for the performance of the overall system.

The RPD approach builds on some of the concepts commonly used in the controls community, namely decentralized and robust control. In a decentralized control structure, the control input for a subsystem depends on the states of that subsystem. A decentralized controller is designed using the block diagonals in the state-space matrices corresponding to the subsystem being controlled. The connective stability of a system with stabilizing subsystem controllers can be checked using vector Lyapunov functions. Robust control tools can be used to analyze the performance of the resulting systems [23].

Using RPD, the need for communication between design teams is reduced significantly. Subsystem targets and nominal values for key design variables are determined during the target cascading phase of the design process. Design teams attempt to achieve their design targets while maintaining the values of the local design variables (design variables specific to a subsystem) within a specified range of their nominal value. The need for communication arises whenever meeting the goals is not possible. Thus, the focus of communication between design teams shifts from communicating design details, that may need to be communicated frequently [14], to communicating ranges for design variables, presumably a less demanding task.

The next section presents a formal statement of the problem. Then, the proposed approaches are illustrated using a simple example consisting of a half-car suspension model.

III. Robust Parallel Design (RPD) Approach

Decomposing a system into subsystems reveals interconnections between the subsystems where the outputs of one subsystem are used as inputs to other subsystems. This is illustrated in Figure 1 for a system that is decomposed into three subsystems.

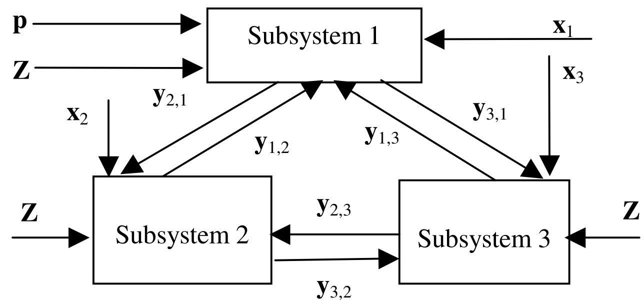

Fig. 1. Multi-Disciplinary Design Problem

In Figure 1, $\mathbf{Z}$ is a vector of shared design variables, $\mathbf{x}_i$ is a vector of design variables for subsystem i and together they comprise the vector $\mathbf{x}$ of (1), $\mathbf{p}$ is a vector of design parameters and $\mathbf{y}_{ij}$ is a vector of outputs to subsystem i from subsystem j. System-level specifications may include outputs from several subsystems, i.e., $f(\mathbf{x}, \mathbf{p}, \mathbf{w})$ in (2) includes outputs from several subsystems.

As mentioned earlier, decomposing a system into several subsystems that are designed in parallel introduces uncertainty in the design process. Local design variables in a subsystem design problem are a subset of the system's vector of design variables $\mathbf{x}$. The values of shared design variables, $\mathbf{Z}$, are determined during the target cascading stage and are not modified by any of the subsystem design teams. Since the subsystems are designed in parallel, the design team working on a particular subsystem design problem treats the design variables in other subsystems as uncertainties.

It is desirable to use the smallest possible uncertainty matrix, in terms of both dimension and magnitude, when designing a subsystem since larger uncertainties lead to more conservative designs. Thus, we assume that system-level and subsystem objectives are aligned, i.e., improving on subsystem objectives improves the system level objective. Having subsystem objectives aligned with system-level objectives can be done by weighting each subsystem objective with the partial derivative of the system-level objective with respect to the respective subsystem objective. In [15], the authors propose an algorithm to efficiently calculate the sensitivity of a Noise-Vibration-Harshness (NVH) system-level objective with respect to subsystem objectives for linear systems.

When system-level and subsystem objective are aligned, changes in the values of the design variables in any subsystem will improve on the overall system-level performance, i.e., the starting nominal value of design variables are worst-case values. This allows us to exclude the design variables of other subsystems from a subsystem's uncertainty matrix and treat them as constants.

In this paper we assume that the subsystem objective is the same as the system-level objective but is minimized with respect to the subsystem design variables in the presence of parameter uncertainty, i.e., the sensitivity of the system-level objective with respect to subsystem objectives is unity. This is similar to the approach adopted in Bi-Level Integrated System Synthesis (BLISS) [20]. The proposed RPD approach differs from BLISS in that it accounts for parameter uncertainties in the subsystem design problem. Figure 2 illustrates the resulting subsystem design problem.

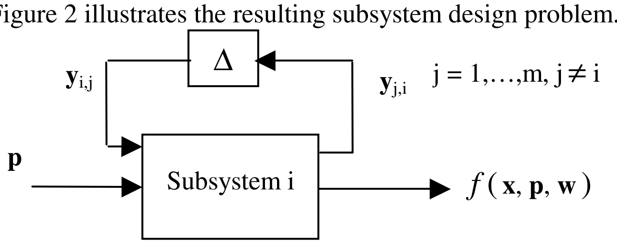

Fig. 2. Robust Subsystem design Problem

In this paper, we consider the worst-case uncertainties encountered when designing the subsystems in parallel. The subsystem design problem, assuming worst-case uncertainties, can be stated as:

$$\min_{x_i} \max_{\Delta} f(\mathbf{x}, \mathbf{p}, \mathbf{w})$$

subject to $\quad g(\mathbf{x}, \mathbf{p}) \le \mathbf{0}$

$(1-\delta)\ \mathbf{x}_0 \le \mathbf{x}_i \le (1+\delta)\ \mathbf{x}_0$

where

- $\mathbf{x}$ vector of design variables with perturbations, δ, about their nominal values, $\mathbf{x}_0$, in the form $\mathbf{x} = (1+\delta)\ \mathbf{x}_0$
- $\mathbf{x}_i$ vector of design variables for subsystem i
- $\mathbf{p}$ design parameters having uncertainties of Δ about their nominal values $\mathbf{p}_0$

Accounting for the worst-case uncertainties implies that the performance index of the overall system is better than the best worst-case performance among all subsystems, i.e., $f(\mathbf{x}, \mathbf{p}, \mathbf{w})$ for the overall system is less than, or equal to, the maximum $f(\mathbf{x}, \mathbf{p}, \mathbf{w})$ evaluated for different subsystem designs. This allows a design problem to be decomposed into several subsystem design problems with confidence as to the expected performance upon assembly of the subsystems.

IV. Case Study: Vehicle Suspension Design

Vehicle suspension systems, active, passive and semi-active, have been studied exhaustively in the literature (e.g., [4], [5], [6], [17], [18], [21] and [25]). Various vehicle models, corner-car, half-car and full-car, are used in these studies. Performance measures for a vehicle suspension include passenger comfort, suspension stroke ("rattlespace") and road handling. These are quantified by the acceleration of the sprung mass, the relative displacement of the sprung and unsprung masses, and the dynamic forces at the tires, respectively. In this paper, the problem of designing a passive suspension is decomposed into two subsystem design problems that can be solved in parallel.

A. Road Excitation Model

A vehicle's suspension is subjected to various sources of excitations (e.g., road roughness, tire-wheel assembly imperfections and engine/transmission excitation). For the purposes of this paper, we consider road excitation as the only source of disturbance. Road roughness is usually modeled as a stationary Gaussian process that can be represented using its Power Spectral Density (PSD). The PSD of a signal measures the power of the signal at a particular frequency. The PSDs of several road profiles, as a function of spatial frequency, are available in the literature (e.g., [6]). Equation (3) is commonly used to approximate the PSD of several road profiles [6] and [17].

$$S(z) = \frac{A_r}{z^2 + z_0^2} \tag{3}$$

where

- A_r is the road roughness coefficient. A value of 16×10^{-7} m^{2}cycle/m is used for a class B road.
- z is the spatial frequency (cycle/m).
- z_0 spatial cutoff frequency to avoid infinite PSD at low frequencies. A value of 0.005 cycle/m is used.

For a vehicle moving at a constant longitudinal speed, V, the temporal frequency, ω (rad/sec), is given by $\omega = 2\pi V z$. In the present work, we assume a longitudinal velocity of 30 mph. Thus, the PSD of road roughness as a function of temporal frequency, ω (rad/sec), is given by:

$$S(\omega) = \frac{2\pi A_r V}{\omega^2 + \omega_0^2} \tag{4}$$

where

$\omega_0 = 2\pi V z_0$

This PSD can be obtained by applying a first-order filter of the form $G(s) = \dfrac{(2\pi A_r V)^{\frac{1}{2}}}{s + \omega_0}$ to unit variance white noise.

B. Half-car model

A four-degree-of-freedom (4 DOF) half-car model is used to illustrate the proposed approach. The 4 DOF model is shown in Figure 3. The equations of motion for the half-car model are:

$$m_{uf}\ \ddot{z}_1 + c_{uf}\left(\dot{z}_1 - \dot{z}_{0f}\right) + k_{uf}\left(z_1 - z_{0f}\right) - c_{sf}\left(\dot{z}_2 - \dot{z}_1\right) - k_{uf}\left(z_2 - z_1\right) = 0 \tag{5}$$

$$m_{ur}\ddot{z}_3 + c_{ur}\left(\dot{z}_3 - \dot{z}_{or}\right) + k_{ur}\left(z_3 - z_{0r}\right) - c_{sr}\left(\dot{z}_4 - \dot{z}_3\right) - k_{ur}\left(z_4 - z_3\right) = 0 \quad (6)$$

$$m_s\ddot{z}_2 + \left(1 + \frac{m_s l_f^{\,2}}{I}\right)\left[c_{sf}\left(\dot{z}_2 - \dot{z}_1\right) + k_{uf}\left(z_2 - z_1\right)\right] + \left(1 - \frac{m_s l_f l_r}{I}\right)\left[c_{sr}\left(\dot{z}_4 - \dot{z}_3\right) + k_{sr}\left(z_4 - z_3\right)\right] = 0 \quad (7)$$

$$m_s\ddot{z}_4 + \left(1 + \frac{m_s l_r^{\,2}}{I}\right)\left[c_{sr}\left(\dot{z}_4 - \dot{z}_3\right) + k_{sr}\left(z_4 - z_3\right)\right] + \left(1 - \frac{m_s l_f l_r}{I}\right)\left[c_{sf}\left(\dot{z}_2 - \dot{z}_1\right) + k_{sf}\left(z_2 - z_1\right)\right] = 0 \quad (8)$$

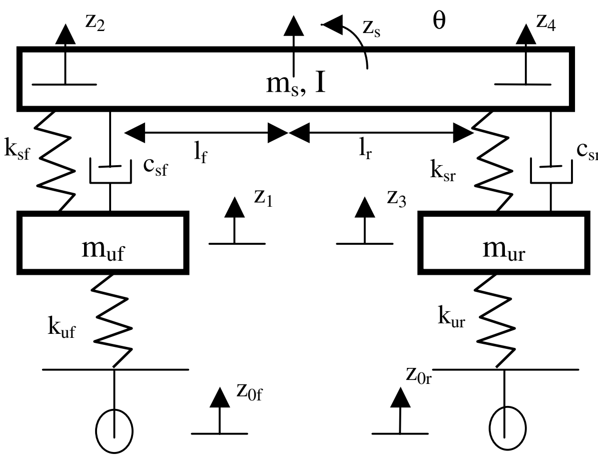

Fig. 3. Half-car suspension model

The nominal values of the different design variables and parameters and the maximum allowed perturbation about these nominal values used in this example are given in Table 1. The nominal values in Table 1 are based on the values used for the full-car model by Zuo and Nayfeh [26]. Design variables and parameters, **x** and **p**, are constrained to lie within the bounds given in Table 1. As mentioned previously, target cascading can be used to obtain nominal values for key design variables. The uncertainties, δ, are estimated based on the maximum expected deviations when using high fidelity models for the actual subsystem designs compared to the surrogate models used during target cascading.

TABLE 1: NOMINAL VALUES AND UNCERTAINTIES FOR DESIGN VARIABLES AND DESIGN PARAMETERS

Description	Symbol	Value
Sprung mass	m_s	688 kg +/- 10 %
Sprung moment of inertia	I	1172 kg.m^2 +/- 10 %
Unsprung masses (front/rear)	m_{uf}/m_{ur}	40/40 kg +/- 10 %
Tire Stiffness (front/rear)	k_{uf}/k_{ur}	182087/182087 N/m +/- 10 %
Suspension stiffness (front/rear)	k_{sf}/k_{sr}	20985/19122 N/m +/- 20 %
Suspension damping (front/rear)	c_{sf}/c_{sr}	1306/1470 N.s/m +/- 44 %
Distance between c.g and front/rear tires	l_f/l_r	1.125/1.511 m

C. Passenger Comfort

Passenger comfort is proportional to the amount of acceleration experienced by the vehicle passengers. For the purposes of this paper, we consider the acceleration of the c.g. of the sprung mass to be the acceleration experienced by the vehicle passengers. In addition to the magnitude of acceleration forces, human perception of comfort depends on the frequencies of these accelerations. International Standard ISO 2631-1 provides frequency weights that can be used to modify measured accelerations. The frequency-weighted accelerations are a better measure of passenger comfort. Zuo and Nayfeh provide some low-order continuous time filters to approximate the ISO 2631 frequency weighting curves [25]. We use the following second order filter to approximate ISO 2631-1 frequency weighting curves [25], [26].

$$W(s) = \frac{50s + 500}{s^2 + 50s + 1200} \quad (9)$$

D. System-level Optimal Design Problem

The system-level optimal design problem can be expressed as

$$\min_x f(\mathbf{x}, \mathbf{p}, \mathbf{w})$$

subject to Equations (5)-(8)

$$(1-\delta)\,\mathbf{x_0} \le \mathbf{x} \le (1+\delta)\,\mathbf{x_0}$$

where

$\mathbf{x} = \begin{bmatrix} k_{sf} \\ c_{sf} \\ k_{sr} \\ c_{sr} \end{bmatrix}$ vector of design variables

$\mathbf{p} = \begin{bmatrix} k_{uf} \\ k_{ur} \\ m_{uf} \\ m_{ur} \\ m_s \\ I \end{bmatrix}$ vector of design parameters

w vector of weights used to trade-off between objectives

The performance index $f(\mathbf{x}, \mathbf{p}, \mathbf{w})$ consists of a weighted sum of the root-mean-square values of signals of interest. These include; the frequency-weighted acceleration of the sprung mass, the velocity of the sprung mass, the rotational velocity of the sprung mass, the suspension stroke and the tire dynamic forces, i.e.,

$$f(\mathbf{x}, \mathbf{p}, \mathbf{w}) = E\left\{ \begin{array}{l} r_1 \ddot{z}_s^2 + r_2 \dot{z}_s^2 + r_3 \dot{\theta}^2 + r_4 \left[(z_2 - z_1)^2 + (z_4 - z_3)^2 \right] \\ + r_5 \left[k_{uf}^2 (z_1 - z_{0f})^2 + k_{ur}^2 (z_3 - z_{0r})^2 \right] \end{array} \right\} \quad (10)$$

The values of the weights used, obtained from [26], are

Weight	r_1	r_2	r_3	r_4	r_5
Value	1	8.3	120	120	8.3e-3

E. Application of the RPD Approach

The half-car suspension model will now be decomposed into two subsystems to be designed in parallel: front and rear suspension. Subsystem A consists of the front suspension stiffness and damping and subsystem B consists of the rear suspension stiffness and damping. The uncertainty matrix in each subsystem design problem consists of the uncertain design parameters. The subsystem design problems consist of minimizing the subsystem objective function over the range of subsystem design variables in the presence of the uncertainty matrix. In the present work, we are assuming a worst-case uncertainty matrix. The resulting problems take the form of the following mini-max optimization problems:

Subsystem A

$$\min_{x_A} \max_{\Delta_A} f(\mathbf{x}, \mathbf{p}, \mathbf{w})$$

subject to Equations (5)-(8)

$$(1-\delta)\ \mathbf{x}_{A0} \le \mathbf{x}_A \le (1+\delta)\ \mathbf{x}_{A0}$$

where

$\mathbf{x}_A = \begin{bmatrix} k_{sf} \\ c_{sf} \end{bmatrix}$ design variables for subsystem A

$\mathbf{p}$ uncertain design parameters

Subsystem B

$$\min_{x_B} \max_{\Delta_B} f(\mathbf{x}, \mathbf{p}, \mathbf{w})$$

subject to Equations (5)-(8)

$$(1-\delta)\ \mathbf{x}_{B0} \le \mathbf{x}_B \le (1+\delta)\ \mathbf{x}_{B0}$$

where

$\mathbf{x}_B = \begin{bmatrix} k_{sr} \\ c_{sr} \end{bmatrix}$ design variables for subsystem B

$\mathbf{p}$ uncertain design parameters

The mini-max subsystem design problems can be solved using non-linear optimization algorithms, e.g., Sequential Quadratic Programming (SQP). An iterative process is used. Starting with initial vector x, the performance index is maximized over Δ, the uncertainty of the design parameters. The resulting Δ is then substituted back to the system/subsystem and the performance index of the resulting system/subsystem is minimized over x. The new x is used to modify the system/subsystem. This process is repeated until convergence.

V. RESULTS

The passive suspension design problem is solved by determining values for the design variables of subsystems A and B, respectively.

Subsystem A

For subsystem A the design variables are the front suspension stiffness and damping. Solving the mini-max optimization problem for subsystem A results in the following values for the front suspension stiffness and damping.

k_{sf} = 20,275 N/m, c_{sf} = 1,881 N.s/m

These new values of the design variables for subsystem A result in a 6.8% reduction of the performance index (from 1,322 to 1,232).

The main motivation for the present work is to be able to decompose a system into subsystems, design the subsystems independently and guarantee achieving satisfactory system-level performance upon assembly of the subsystem. Thus, we are not only interested in the minimum value of the performance index but also the worst-case value of that performance index given the present uncertainties. The worst-case performance index for subsystem A is 1,539.

The worst-case performance depends on the bounds on the uncertainty matrix. If there are no uncertainties (i.e., the uncertainty matrix has an upper bound of zero) then the worst-case performance is equal to the minimum value of the performance index achieved by varying the design variables while keeping other variables and parameters fixed at their nominal values. The bounds on the uncertainty matrix, Δ_A, are varied by multiplying Δ_A by a scalar multiple. As the upper bound on the uncertainty matrix increases, the worst-case performance increases. This relationship is shown in Figure 4.

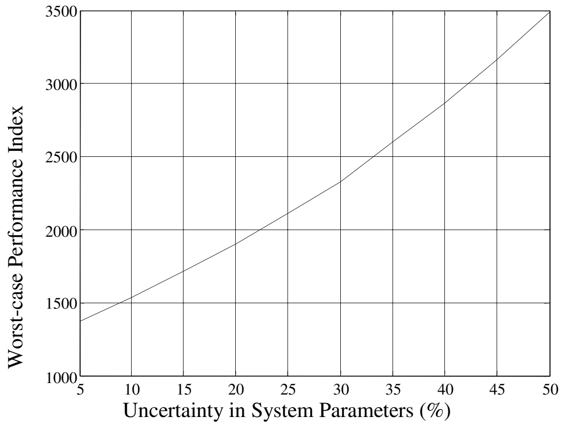

Fig. 4. Worst-Case Performance Index for Subsystem A vs. Changes in Parameter Uncertainty

Subsystem B

For subsystem B the design variables are the rear suspension stiffness and damping. Solving the mini-max optimization problem for subsystem B results in the following values for the rear suspension stiffness and damping.

k_{sr} = 19,122 N/m, c_{sr} = 2,117 N.s/m

The new values of the design variables for subsystem B result in a 10.9% reduction of the performance index (from 1,322 to 1,178). The worst-case performance index for subsystem B is 1,589. The relationship between the worst-case performance index and the upper bound on the uncertainty matrix for subsystem B is similar to the one displayed in Figure 4 for subsystem A and is not included here for the sake of brevity.

The worst-case performance index for the overall system is guaranteed to be less than, or equal to, the minimum of the worst-case performance indices for subsystems A and B, i.e., $\max_{\Delta} f(\mathbf{x}, \mathbf{p}, \mathbf{w}) \leq \min(1539, 1589)$ where **x** includes solutions from different subsystem design problems. This allows a design problem to be decomposed into several subsystem design problems with confidence as to the expected performance upon assembly of the subsystems.

In Table 2, the solution obtained using the RPD approach is compared to the solutions of an All-At-Once (AAO) optimization and Parallel Design (PD), i.e., designing subsystems in parallel without accounting for uncertainties, in terms of both the nominal and worst-case performance indices. The AAO solution is obtained for the case when design variables are not constrained by the bounds in Table 1. The worst-case performance index reported in Table 2 is for the overall system, i.e., $\max_{\Delta} f(\mathbf{x}, \mathbf{p}, \mathbf{w})$ where **x** includes the design variables obtained from the solutions of all subsystem design problems for the cases of RPD and PD.

TABLE 2: COMPARISON OF RPD, AAO OPTIMIZATION AND PD

	Nominal	**AAO**	**RPD**	**PD**
k_{sf} (N/m)	20,985	15,260	20,275	16,788
c_{sf} (N.s/m)	1,306	2,334	1,881	1,881
k_{sr} (N/m)	19,122	19,213	19,122	19,122
c_{sr} (N.s/m)	1,470	2,228	2,117	2,117
P.I (% change)	**1,322**	**1,163** **(-12)**	**1,181** **(-10.6)**	**1,178** **(-10.9)**
W.C P.I (% change)	**1,674**	**1,432** **(-14.5)**	**1,459** **(-12.9)**	**1,461** **(-12.8)**

The results in Table 2 show that while the Robust Parallel Design Approach has slightly worse nominal performance compared to Parallel Design, it has slightly better worst-case performance. The All-At-Once design performs better than both RPDA and PD in terms of both nominal and worst-case performances. This is possibly due to not constraining the values of the design variables for the AAO case while constraining these values by the bounds in Table 2 for both the RPDA and PD cases. The present work addresses situations where subsystems need to be designed in parallel assuming lack of collaboration between design teams, i.e., the use of an AAO approach is not possible.

Although RPD outperforms PD in term of the worst-case system-level performance, the difference is quite small for the case reported in Table 2, where the magnitude of the uncertainty is 10 %. We expect the difference between RPD and PD to become more significant as the magnitude of uncertainty increases. To verify this, we solved the suspension design example for the cases when the magnitudes of the uncertainties are 25 % and 50 %. The results are reported in Tables 3 and 4, respectively. The results in Tables 3 and 4 confirm our initial conclusion that as the magnitudes of the uncertainties increase the difference in worst-case performance between RPD and PD, with RPD outperforming PD, becomes more significant.

TABLE 3 COMPARISON OF RPD, AAO OPTIMIZATION AND PD WHEN UNCERTAINTY = 25 %

	Nominal	**AAO**	**RPD**	**PD**
k_{sf} (N/m)	20,985	15,260	19,078	16,788
c_{sf} (N.s/m)	1,306	2,334	1,881	1,881
k_{sr} (N/m)	19,122	19,213	20,732	19,122
c_{sr} (N.s/m)	1,470	2,228	2,117	2,117
P.I	**1,322**	**1,163**	**1,180**	**1,178**
(% change)		**(-12.0)**	**(-10.8)**	**(-10.9)**
W.C P.I	**2,328**	**1,937**	**1,982**	**1,993**
(% change)		**(-16.8)**	**(-14.9)**	**(-14.4)**

TABLE 4 COMPARISON OF RPD, AAO OPTIMIZATION AND PD WHEN UNCERTAINTY = 50 %

	Nominal	AAO	RPD	PD
k_{sf} (N/m)	20,985	15,260	20,139	16,788
c_{sf} (N.s/m)	1,306	2,334	1,881	1,881
k_{sr} (N/m)	19,122	19,213	27,536	19,122
c_{sr} (N.s/m)	1,470	2,228	2,117	2,117
P.I	**1,322**	**1,163**	**1,189**	**1,178**
(% change)		**(-12.0)**	**(-10.1)**	**(-10.9)**
W.C P.I	**3,806**	**3,163**	**3,189**	**3,276**
(% change)		**(-16.9)**	**(-16.2)**	**(-13.9)**

VI. Conclusions

The proposed RPD approach allows a system design task to be decomposed into several subsystem design tasks that can be performed in parallel. A half-car example was used to illustrate the proposed approach. The half-car suspension design problem was decomposed into two subsystem design problems that were solved in parallel. The bounds on the system-level performance depend on the bounds placed on the uncertainty matrix. Larger bounds on the uncertainty matrix result in worse values of guaranteed system-level performance.

Using the worst-case uncertainties may lead to conservative designs. This conservatism can be reduced if the statistical distributions of uncertainties are available. Alternative approaches that use knowledge of the statistical distributions of uncertainties and potentially more efficient solution techniques are currently being investigated.

In addition to being conservative, the use of worst-case uncertainties leads to min-max optimization problems that are computationally intractable. As an extension to this work, we propose using heuristic optimization techniques, e.g., simulated annealing, to solve problems with higher dimensionality.

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