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New Economic Perspectives for Resource Allocation in Wireless Networks

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Abstract— **The economic approaches of potential game theory and bargaining theory are applied to the area of power control in CDMA wireless networks. These perspectives help identify suitable equilibrium points, and algorithms that can be shown to converge to them. The bargaining approach also suggests an iterative method for the simultaneous routing and resource allocation problem.**

I. INTRODUCTION

In recent years there has been an increasing trend to adopt models of economic theory to study resource allocation problems in communication networks. In particular, if units in a network can elastically adapt their demand for a given resource (e.g., bandwidth), it is natural to use the language of *utility functions* to study the contention for the scarce supply of this resource [1]. In the case of CDMA wireless networks, transmission *power* is subject to contention in a rather complex way, due to the effect of mutual interference. This has led to recent work [2], [3], [4] applying *game theory* to the power allocation problem.

Taking this as a starting point, the present paper delves further into economic theory to find additional tools that are relevant to the power control problem. In Section II, we show that the theory of *potential games* can apply directly to the setup of [3], [4]: for a similar class of utility functions, we show existence and uniqueness of the Nash equilibrium and decentralized algorithms that converge to it.

As is well known, Nash equilibria are often *inefficient* in the sense that an improved utility for all players could be achieved through collaboration. This "price of anarchy" leads us to consider in Section III the problem from the point of view of *bargaining theory*, seeking Pareto-optimal allocations, and ways to select between these by assigning bargaining powers to the users. These types of solutions depart from the decentralized assumption of the previous section, and the abovementioned literature. They are, however, quite natural in a wireless network in which one often would like to alleviate mobiles from the computational burden of a complex algorithm. Indeed, in the same flavor some recent work [5] has attempted to allocated both power, routing and rate through a global, centralized optimization algorithm. Due to the non-convexities arising from the power control in the CDMA case, [5] is forced to employ an approximate analysis. The bargaining approach of this paper suggests a procedure in which the power control could be studied exactly; some preliminary work in this direction is reported.

Section IV gives conclusions and outlines directions of future research.

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II. POTENTIAL GAMES FOR WIRELESS POWER CONTROL

In this section we analyze a scenario in power allocation decisions in the wireless network which are based on the selfish optimization of each unit's utility function, possibly with a penalty term introduced by the network. This approach has been introduced and studied in [2], [3], [4]. For example, in the last two references, each user is assigned a utility of the form

$$u_i = a\log_2(1+\gamma_i) - c_i p_i$$

where a is a positive constant, and

$$\gamma_i = \frac{W}{B}\frac{h_i p_i}{\sum_{j\neq i} h_j p_j + \sigma}$$

is the signal-to-interference ratio (SIR) of user i in which W and B are system bandwidth and signal unmodulated bandwidth respectively, h_i is user i's channel gain, p_i is his transmission power, and σ is the noise power at receiver. The motivation for this kind of utility is as follows: the first term is proportional to the Gaussian channel capacity of a channel with the given SIR, and can thus be interpreted approximately as the transmission rate allowed by the overall power allocation. The second term is a cost of power, that can be viewed as representing the unit's own battery expenditure, or alternatively as a penalty assigned by the base station for the received power (in that case c_i would be proportional to the channel gain h_i, see [4]).

It has been shown in the above papers that with these utility functions, the resulting game possesses a unique Nash equilibrium, and selfish players synchronously updating their powers to the best response to the others' moves, will converge to the equilibrium.

In this section we will see that utilities of a similar nature give rise to a so-called *potential* game, and thus the above properties can be extended to this class. We first review some definitions and results on potential games, that can be found in [6].

A. Potential Games

We consider a game $\Gamma(u_1, u_2, \cdots, u_l)$ where the set of players is $L = \{1, 2, \cdots, l\}$, and the strategy space is $P = P_1 \times P_2 \times \cdots \times P_l$. Each user i plays a strategy p_i from the set $P_i = [\underline{p}_i, \overline{p}_i]$, and receives a utility determined by the payoff function $u_i : P \mapsto \mathbb{R}$. As is customary, the vector of opponents' strategies is represented by p_{-i}, varying in the space P_{-i} .

We call a game a *potential game* if there exists a function $f : P \mapsto \mathbb{R}$ with the following property:

$$u_i(p_i^1, p_{-i}) - u_i(p_i^2, p_{-i}) \quad = \quad f(p_i^1, p_{-i}) - f(p_i^2, p_{-i}) \tag{1}$$

for all $i \in L$ and all $p_i^1, p_i^2 \in P_i$ and $p_{-i} \in P_{-i}$. In this case the function f is called the potential function of game Γ.

To obtain strong results about potential games, we restrict the attention to potential functions that satisfy the following conditions:

- *C1*: Potential function f is continuously differentiable on the space of strategies P.
- *C2*: Potential function f is a strictly concave function of the strategies.

The first condition relates to the following useful characterization of potential games from [6]:

Lemma 1: Let Γ be a game with all strategy sets in the form of intervals of real numbers. If all the payoff functions are continuously differentiable, then the function $f : P \mapsto \mathbb{R}$ is a potential function for Γ if and only if f is continuously differentiable and also

$$\frac{\partial u_i}{\partial p_i} \quad = \quad \frac{\partial f}{\partial p_i} \qquad \text{for every} \quad i \in L. \tag{2}$$

The above lemma captures the fact that any unilateral change in strategy is reflected in both the deviator payoff and the potential function in exactly the same way.

The second condition implies that the function f has a unique maximum over the hypercube P; it turns out, this is the Nash equilibrium of the game:

Theorem 2: Given a potential game Γ with potential function f satisfying $C1$ and $C2$, then there is a unique Nash equilibrium p, which coincides with the unique maximizer p^* of the function f over P.

Proof: If $p^* = \text{argmax}_{x \in P} f(x)$ then users cannot increase f through a unilateral change of strategy; so, by (1), they cannot increase their payoff and thus, by definition, p is a Nash equilibrium of Γ. Conversely assume now that p is a Nash equilibrium of the game. By definition, for each $i \in L$ we have:

$$p_i \quad = \quad \text{argmax}_{x \in P_i} f(x, p_{-i}) \tag{3}$$

Since f is concave, its restriction to a line is concave too. Combining this with (3) yields [7]:

$$\frac{\partial f}{\partial p_i}(p) \cdot (x_i - p_i) \quad \leq \quad 0 \qquad \forall i \in L, \forall x_i \in P_i$$

Collecting all the inequalities for $i \in L$ in vector form we obtain: $\nabla f^T(p) \cdot (x - p) \leq 0, \forall x \in P$. Now using concavity of f implies that p is the maximizer of f over P [7]. ■

B. Convergence to Nash Equilibrium

Now that we know sufficient conditions for game Γ to possess a unique Nash equilibrium, it remains to show that myopic players playing in their own self-interest will reach this point. More precisely, if each user updates p_i to be the payoff maximizing strategy under the assumption that p_{-i} is fixed, does this distributed algorithm converge to Nash equilibrium? Since, as we saw in theorem 2, the equilibrium is the unique maximizer of the potential function f, this kind of iteration is in essence a distributed optimization algorithm of the type studied in [10]. In particular, some restrictions apply as to the sequencing of these distributed updates to allow for a convergence result. Among other possibilities, we focus here for concreteness on the *nonlinear Gauss-Seidel algorithm* [10], that works as follows:

At each turn only one player has the right to play, i.e. update his strategy. Being at point $p = (p_1, p_2, \cdots, p_l) \in P$, player i, who has to play now, maximizes the objective function f assuming that every body else's strategy, p_{-i}, is fixed. This routine is repeated in a circular regime. Formally the nonlinear Gauss-Seidel algorithm is defined by:

$$\begin{aligned} p_i(t+1) \quad = \quad & \text{argmax}_{p_i \in P_i} f(p_1(t+1), \cdots, p_{i-1}(t+1), \\ & p_i, p_{i+1}(t), \cdots, p_l(t)) \end{aligned} \tag{4}$$

The following proposition gives the main convergence results of the nonlinear Gauss-Seidel algorithm.

Proposition 3: Suppose that the set $P = P_1 \times P_2 \times \cdots \times P_l$ is bounded where each $P_i, i = 1, \cdots, l$ is a nonempty interval of real numbers. Also assume that the function $f : P \mapsto \mathbb{R}$ satisfies conditions $C1$ and $C2$. Let $\{p(t)\}$ be the sequence generated by the nonlinear Gauss-Seidel algorithm (4); then $\{p(t)\}$ converges to $p^* = \text{argmax}_{x \in P} f(x)$.

This proof is essentially from [10]: there it is shown, under slightly weaker assumptions on the concavity of f, that every limit point of the sequence $\{p(t)\}$ must be a maximizer of f over P. Under the strict concavity stipulated in $C2$, there is a unique maximizer, therefore only one limit point and the sequence must converge to it.

To interpret this result in terms of the game, we rewrite the distributed optimization algorithm in the following equivalent way:

$$\begin{aligned} p_i(t+1) \quad = \quad & \text{argmax}_{p_i \in P_i} u_i(p_1(t+1), \cdots, p_{i-1}(t+1), \\ & p_i, p_{i+1}(t), \cdots, p_l(t)) \end{aligned} \tag{5}$$

In other words, players update their strategy taking turns in a round-robin fashion, and play the best response to the other players' strategies. The equivalence of the equations 5 and 4 is apparent by definition of potential function. The following theorem follows immediately.

Theorem 4: Assume in game Γ, the strategy sets $P_i, i = 1, \cdots, l$ are bounded nonempty intervals of real numbers. Also assume that Γ is a potential game with a potential function $f : P \mapsto \mathbb{R}$ which satisfies conditions $C1$ and $C2$. Then the circular Gauss-Seidel scheme 5 converges to the unique Nash equilibrium of the game.

Remark: We have focused on the Gauss-Seidel scheme, which imposes a certain order in the game. However there are more general iterations that share the convergence property. The main requirements are: (i) two players should not be allowed to update strategies at exactly the same time; (ii) each must continue to update infinitely often.

The first requirement implies that the sequence of values of f is monotonic, hence by boundedness it must converge to a limit. The second requirement implies that at this limit, no player can improve utilities unilaterally. But then this limit must be the unique Nash equilibrium of the game.

C. Application to CDMA networks

We now return to applying these results to CDMA networks.

Example 1: Assume that user i's utility is defined as:

$$u_i(p) \quad = \quad \log(1 + \frac{h_i p_i}{\sum_{j \neq i} h_j p_j + \sigma}) - a_i p_i, \qquad (6)$$

where all parameters are as defined before, and the positive real number a_i is the per-unit penalty for using power by user i charged by the central authority. This is of the form considered in [4]. Assume also that the strategy space of user i is the closed interval $[0, \overline{p}_i]$ where $\overline{p}_i > 0$. It is easily seen by (2) that the function $f(p) = \log(\sum_{i \in L} h_i p_i + \sigma) - \sum_{i \in L} a_i p_i$ is a potential function for the game above where p is the strategy profile. Since f satisfies *C1* and *C2*, as a result of theorem 4, users will converge to the unique Nash equilibrium of the game p^* which is also the unique maximizer of f on P if they play selfishly and circular as in (5).

Example 2: Assume that the set-up is identical to last example but we allow the penalty term to be an arbitrary convex function of power. Namely,

$$u_i(p) \quad = \quad \log(1 + \frac{h_i p_i}{\sum_{j \neq i} h_j p_j + \sigma}) - b_i c_i(p_i) \qquad (7)$$

where $c_i(p_i)$ is a convex penalty function of user i and b_i is a nonnegative constant. It is not hard to check that $f(p) = \log(\sum_{i \in L} h_i p_i + \sigma) - \sum_{i \in L} b_i c_i(p_i)$ is a potential function for the game. Since f satisfies *C1* and *C2*, the update scheme 5 will result in the convergence of p to the unique Nash equilibrium of the game which maximizes f on P.

As we observed in the two examples above, this way we can analyze a range of utility and price functions as long as *C1* and *C2* are satisfied.

III. A CENTRALIZED BARGAINING APPROACH

In the last section we established existence and uniqueness of the Nash equilibrium, and an algorithm that guarantees convergence to it. The natural question is how efficient the Nash equilibrium is from a social perspective. As is well known (see e.g. [8], [9]), there is often a "price of anarchy" in games of this nature, meaning there may be operating points which are better for all players but are not found through selfish interactions.

More precisely, the Nash equilibrium of the game need not be Pareto optimal. If one wants to operate at these more desirable Pareto optimal points, a more centralized approach to power allocation is required. Note that while decentralized implementations have appeal, in the wireless network context it is also true that one would like to minimize the computational burden on the mobiles; having a centralized solution in which a base station performs the computation and acts as central authority may be more suitable. In this section, we explore this possibility by invoking tools from *bargaining theory*.

A. Review of Bargaining Theory

Some of the definitions and results of this subsection can be found in [11]. An *l-person bargaining problem* is defined as a pair (S, d) where S is a non-empty compact subset of $\mathbb{R}^l$ and $d \in S$ which is called the disagreement point. The set S is the *utility possibility set* and the disagreement point d is a point where the bargaining procedure settles at, in case the bargaining among agents fails. A *bargaining solution* F is a mapping from each pair (S, d) to S. There are some well-known bargaining solutions in the literature such as Nash bargaining solution, egalitarian bargaining solution, Kalai-Smorodinsky solution, etc. Each of those solutions are characterized by a set of desirable properties that one expects from a bargaining solution. Some of these properties together with other necessary definitions are defined below:

- A set $S \in \mathbb{R}^l$ is *d-comprehensive* if $y \in S$ and $d \leq x \leq y$ imply $x \in S$.
- The *ideal point* of a problem (S, d) is defined as:

$$a(S, d) = \left(\max_{x \in S, x \geq d} x^1, \max_{x \in S, x \geq d} x^2, \cdots, \max_{x \in S, x \geq d} x^l \right)$$

- A bargaining solution F is *scale invariant* if $\forall \lambda \in \Lambda^l, F(\lambda(S), \lambda(d)) = \lambda(F(S, d))$ where λ is a linear transformation $\lambda : \mathbb{R}^l \to \mathbb{R}^l$ of the form $\lambda(x) = Bx$ for some diagonal positive matrix $B \in \mathbb{R}^{l \times l}_{++}$ and Λ^l is the set of all such transformations.
- A bargaining solution F is called *restricted monotone* if it meets the following condition: If $S \subset S', d = d'$, and $a(S, d) = a(S', d')$then $F(S', d') \geq F(S, d)$.
- A bargaining solution F is *weak Pareto optimal* if $F(S, d) \in WP(S)$ where $WP(S) = \{x \in S | y \gg x$ implies $y \notin S\}$ and is called the weak Pareto surface of the set S.

Out of the many solutions in the literature, we review here the *extended Kalai-Smorodinsky* bargaining solution [11], which is most suited to our problem.

$$\text{KS}(S, d) \quad = \quad \max_{x \in S} x \in \text{aff}(d, a(S, d)).$$

This solution forms the line connecting d and the ideal point $a(S, d)$ and offers the best point on that line which also lies in the feasible set, as the solution to the bargaining problem.

We will find it useful to further generalize the above solution to allow for users to have different *bargaining powers*; we thus define the *generalized Kalai-Smorodinsky* bargaining solution by

$$\text{KS}(S,d) \quad = \quad \max_{x\in S} x \in \text{aff}(d, Ca(S,d))$$

where $C_{l\times l}$ is the diagonal matrix of whose C_{ii} element is the bargaining power of user i and satisfies $C_{ii} \geq 0$ with at least one C_{ii} positive. Fig. 1 depicts an example to compute KS$(S, 0)$. We see that this version adjusts the ideal point prior to forming the line in which to seek bargaining solution.

To state some properties of KS(S, d) we need some assumptions:

1) S is compact.
2) S is d-comprehensive. As we will see this is a crucial assumption which replaces the convexity condition in the conventional bargaining theory.
3) There is a point $x \in S$ such that $x \gg 0$. This way each participating player has a motivation for diverting from the disagreement point.

From this point on we assume $d = 0$. There are three important properties that KS satisfies. We bring all here and omit the proofs for reader's convenience:

property 1: KS is weak Pareto optimal.
property 2: KS is scale invariant.
property 3: KS is restricted monotone.

Before applying this to networks in the next subsection, it is insightful to analyze the above properties of KS in more detail. The first property satisfies our desire to reach the Pareto surface, something the non-cooperative approach cannot do. Scale invariance means that the bargaining solution is not sensitive to units. Should the utilities get scaled by some positive numbers, the scaled version of the same solution results. Restricted monotonicity is a weaker version of the so-called independence of irrelevant alternatives in economics. Very simply stated, it means as long as the ideal point is fixed, one can not improve any utility in KS by eliminating utility possibilities.

The other useful feature of KS is the bargaining power matrix C. Among all uses of bargaining power we can mention the following: Suppose that the willingness-to-pay of different users of the network is different. In such a situation, the different functionalities that users expect from the network could be reflected in their bargaining powers in a clear quantitative way; For example the diagonal elements of C could be chosen directly proportional to the willingness-to-pays of all users. Another interesting application is discussed in III-C.

B. Centralized Power Control Algorithm

Now that we have studied the necessary results from bargaining theory, we apply them to a centralized power control scheme for the wireless network.

Assume that the network under consideration is comprised of l users together with a base station which is the ruling authority and performs all the computations. The utility of player i is defined as:

$$u_i = B\log_2(1 + \frac{W}{B}\frac{h_i p_i}{\sum_{j\neq i} h_j p_j + \sigma})$$

where all parameters are defined as above. In fact u_i is the capacity of channel from user i to base station. The strategy space for player i is the interval $P_i = [0, \overline{p}_i]$ where $\overline{p}_i > 0, \forall i \in L = \{1, \cdots, l\}$. The overall strategy space is $P = P_1 \times \cdots \times P_l$. The utility possibility set is therefore

$$S = \{(u_1(p), \cdots, u_l(p))|p \in P\}.$$

One crucial observation is that S is not in general a convex set. A typical possibility set is shown in Fig. 1.

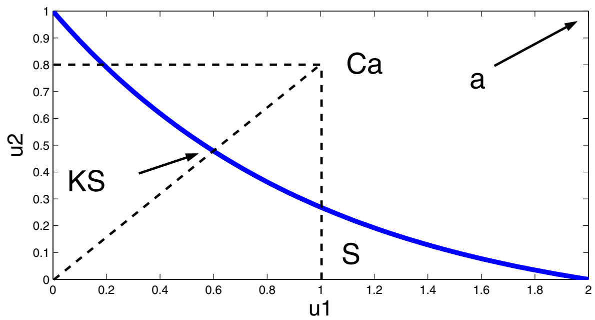

Fig. 1. KS for a sample set S with $C_{11} = 0.5, C_{22} = 0.8$

Now the problem is to assign each user with a power level so that the resulting operating point of the network is efficient. The generalized Kalai-Smorodinsky solution (KS) above uses comprehensiveness of S in place of its convexity which holds in our problem because given a feasible set of link capacities, every underestimate of those link capacities is feasible as well. Besides, compactness of P together with continuity of $u_i(p), i \in L$ (assuming $\sigma > 0$) implies that S is compact. In addition all players have positive power limits. So the three assumptions of last subsection are met by S. We also note that each user can transmit at zero power to pull out of the bargaining. Hence the disagreement point is $d = 0$. As a result we have proven the following result:

In network set-up above, the generalized Kalai-Smorodinsky solution is scale invariant, weak Pareto optimal, and restricted monotone.

As mentioned in the last subsection, in addition to the above properties, KS incorporates the bargaining power feature, which supports all cases from absolute fairness ($C = I$) to ignoring some user completely ($C_{ii} = 0$).

Now it remains to show how to implement KS. The procedure is in fact not more than a line search. To find aff$(0, Ca(S, 0))$, we need to know $a(S, 0)$ for which we have:

$$a_i(S,0) = B\log_2(1 + \frac{W}{B}\frac{h_i\overline{p}_i}{\sigma}).$$

So the base station needs to know the path gains of all users, which could be done by dividing the received power of each user at base station by the transmission power of the same user which has been commanded by base station in the last iteration. Alternatively, if there is any uncertainty about whether or not each user has been able to transmit at the ordered power and also for the very first iteration of the power control, each user can send his transmission power level to the base station.

The bargaining power matrix C is also a parameter known by base station. After finding $Ca(S, 0)$, finding KS could be done by a line search. In each iteration of the line search, a feasibility check is needed to test whether the point is in the utility possibility set. This amounts to solving the system of linear equations in transmission powers p. For this purpose define $\gamma_i = 2^{\frac{u_i}{B}} - 1$ as the signal to interference ratio of user i at receiver as a function of corresponding rate. Then p must satisfy $\Lambda p = v$ where $\Lambda \in \mathbb{R}^{l \times l}$ is defined by: $\Lambda_{ii} = -Wh_i, \Lambda_{ij} = B\gamma_i h_j$ and $v \in \mathbb{R}^l$ is defined as $v_i = -B\gamma_i \sigma$. If a solution $p \in P$ to the above linear equations exists then u is feasible, and otherwise not.

It is worth mentioning that with this method we can directly trade off some fraction of the total network capacity against a huge save in the total power usage in the network. In fact instead of forcing all users to transmit at a power corresponding to KS we can alternatively retract from the Pareto surface inwards by a few percents, gaining a significant power saving. Explicitly, assume that the result of the line search above is the utility vector u. If we instead use αu where $\alpha \in [0, 1]$ we are in fact giving up a fraction of the total capacity which is exactly $(1 - \alpha) \sum u_i$. For each α we can also compute the vector of powers resulting in αu. Fig. 2 shows a typical situation where the sum of the powers and the maximum power in the network are depicted. By giving up 5% of the whole capacity the total power consumption has decreased by a factor of 0.25. Using KS gives designer the ability to explicitly buy huge power savings for a determined percentage of total capacity.

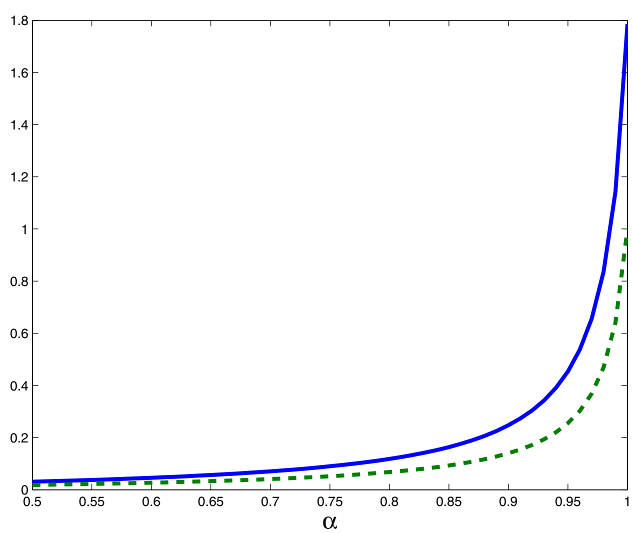

Fig. 2. A typical maximum and total power consumption vs. α

C. A Heuristic Approach to Simultaneous Routing and Resource Allocation

In this subsection we study a heuristic application of the central bargaining method which has been applied to the simultaneous routing and resource allocation (SRRA) problem in a wireless CDMA network. Assume that there is a wireless CDMA network with n nodes and l links. Define vector x as the flow variables including the flows destined for each destination originated from each source and also the flows for each destination carried by each link in the network. Also define vector t as the vector of total flows on links, p as the vector of powers of transmitters, and $\overline{p}$ as the vector of maximum powers feasible to transmitters.

There is a path gain corresponding to each link as in our previous model. The main difference between this model and previous models of this paper is that different agents can directly transmit data between themselves and as a result routing comes into the scene for the first time in this paper. The objective is to maximize the overall throughput of the network. An example of such a network is depicted in Fig. 3 where $n = 6, l = 20$, and the objective is to maximize the total throughput from node 6 to 1 plus 5 to 2. This network model is completely introduced in [5] in which the whole model is cast as an optimization program of the form

$$\begin{aligned} \max \quad & h(x,t) \\ \text{s.t.} \quad & h_1(x,t) \le 0 \\ & 0 \le p \le \overline{p} \\ & t \le c(p) \end{aligned} \tag{8}$$

where $h(x, t)$ is a concave function of x, t that captures total throughput from sources to destinations in the network and $h_1(x, t)$ is a convex function of x and t capturing the conservation of flow and also non-negativity of flows; while $c(p)$ gives the vector of link capacities as a function of all powers. This last constraint (8) makes the problem non-convex; [5] proposes a convex approximation to solve it.

Here we model each link as an agent in the bargaining procedure. The whole bargaining procedure among the links is performed with initial bargaining powers. Based on the result of the bargaining each link is assigned a capacity. We call the vector of capacities after i^{th} iteration c^i. These capacities will appear as constants in the routing optimization program which is *convex* and is of the form:

$$\begin{aligned} \max \quad & h(x,t) \\ \text{s.t.} \quad & h_1(x,t) \le 0 \\ & t \le c^i \end{aligned} \tag{9}$$

When the program is solved, at the optimal solution some links are congested and some are not in general. Now we return to the bargaining procedure and redo it; However this time with *updated bargaining powers*. Intuitively we should increase the power of congested links compared to those that are not congested. Mathematically put, it means

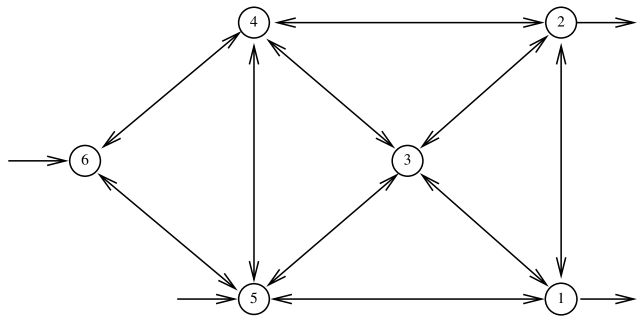

Fig. 3. An example of network studied for SRRA

that we adjust all bargaining powers at iteration i, shown by BP^i, according to Lagrange dual multipliers corresponding to (9): $BP^{i+1} = F(BP^i, \lambda^i)$. We then close the loop and repeat the bargaining-optimization sequence until some termination condition is met. A simulated example of this iterative bargaining power update is shown in Fig. 4 where for the network of Fig. 3, bargaining powers are updated as: $BP^{i+1} = BP^i + 0.5\lambda^i$. The observation through simulations was that over the first few iterations, there is usually a significant improvement in the total flow while after that, this improvement slows down and in some cases starts making slight fluctuations.

IV. CONCLUSION AND FUTURE WORK

In this paper we considered two economic approaches to the resource allocation problem in wireless CDMA communication networks. The first set-up emphasizes decentralized decision making by the mobiles, and the problem was modeled as a potential game. Sufficient conditions on the potential functions were found that give a unique Nash equilibrium point, and a decentralized algorithm that converges to it. In the second set-up, a fully-centralized scheme based on bargaining theory was applied and results from the bargaining problems without convexity were exploited. We also included bargaining powers, and showed a heuristic procedure that uses them to study the combined power and rate control problem.

The price of decentralized implementations is that one must sacrifice Pareto optimality; on the other hand, a fully centralized scheme requires the base station to know all details including the users' utility, which may be excessive. Thus, one could think of a middle ground in which power updates are done by users, and the network uses price incentive to coax users to a Pareto optimal point. We are currently studying alternatives to achieve this, combining ideas from these two extreme approaches. Other future directions for this work involve the problem of simultaneous routing and resource allocation; we gave some initial ideas and simulation results, mathematical results will be sought in the future.

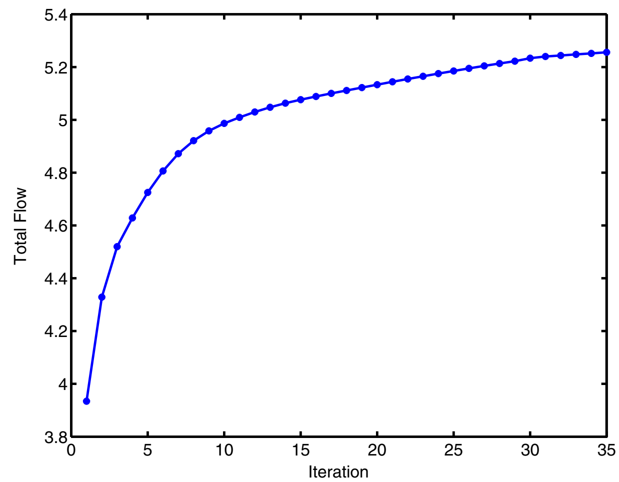

Fig. 4. Simulation result for SRRA

V. ACKNOWLEDGMENTS

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