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Adaptive Output-Feedback For Nonlinear Systems With No A Priori Bounds on Parameters

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***Abstract*— In a recent result [1], we proposed a dynamic high-gain observer/controller architecture for nonlinear systems of the generalized output-feedback canonical form. The design in [1] utilized the dual architecture of a high-gain observer and controller and incorporated a single dynamic scaling. The designed output-feedback controller was shown to be robust to functional and parametric uncertainties coupled with all states. However, a magnitude bound on the unknown parameters was required. In this paper, we propose a time-varying output-feedback controller that can handle time-varying nonlinear parametric uncertainty coupled with all states without requiring any *a priori* magnitude bounds on the unknown parameters. This is achieved using a novel time-varying dynamics of the high-gain scaling parameter. The proposed observer/controller structure provides a globally asymptotically stabilizing output-feedback solution for the benchmark open problem proposed in our earlier work with no magnitude bounds or sign information on the unknown parameter being necessary.**

I. Introduction

We consider systems of the uncertain generalized output-feedback canonical form:

$$\begin{aligned} \dot{x}_i &= \phi_i(t,x,u) + \phi_{(i,i+1)}(x_1)x_{i+1}\ ,\ i = 1,\dots,n-1 \\ \dot{x}_n &= \phi_n(t,x,u) + \mu_0(x_1)u \\ y &= x_1 \end{aligned} \tag{1}$$

where $x = [x_1,\dots,x_n]^T \in \mathcal{R}^n$ is the state, $y \in \mathcal{R}$ the output, and $u \in \mathcal{R}$ the input. $\phi_{(i,i+1)}, i = 1,\dots,n-1$, and μ_0 are known continuous functions of their arguments. $\phi_i, i = 1,\dots,n$, are uncertain functions.

High gain as a technique for controller and observer designs has been investigated extensively in the literature. The well-known adaptive high-gain controller given in its basic form by $u = -ry, \dot{r} = y^2$ is applicable to minimum-phase systems with relative-degree one [2,3]. Static high-gain scaling based observers [4,5] which introduce observer gains $r,\dots,r^n$ with a constant r provide semiglobal solutions. The observer analysis utilizes scaled observer errors $\frac{e_i}{r^i}$ (or $\frac{e_i}{r^{i-1}}$) with e_i being the estimation error of the i^{th} state. In [6], a high-gain observer and a backstepping controller were designed for systems of form (1) with $\phi_{(i,i+1)} = 1, i = 1,\dots,n-1$, and with $\phi_i, i = 1,\dots,n$, being known functions of $x_1,\dots,x_i$ incrementally linear in unmeasured states in the sense that $|\phi_i(x_1,\dots,x_i) - \phi_i(x_1,\hat{x}_2,\dots,\hat{x}_i)| \le \Gamma(x_1)\sum_{j=2}^{i}|\hat{x}_j - x_j|$ with $\Gamma(x_1)$ being a known function. The dynamics of the high-gain parameter r in [6] are given by a scalar differential Riccati equation driven by y guaranteeing boundedness of r if y remains bounded (which is not guaranteed by the dynamics $\dot{r} = y^2$).

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In [1], a dual high-gain observer and controller were designed for systems including nonconstant functions $\phi_{(i,i+1)}$ as long as they satisfied a cascading dominance assumption closely linked to the Cascading Upper Diagonal Dominance (CUDD) condition[1] introduced in [8]. The dual high-gain design in [1] required the solution of a pair of coupled Lyapunov equations which were shown to be always solvable under a cascading dominance assumption on the upper diagonal terms [9,1]. The control law designed in [1] was of an algebraically simple structure requiring no recursive computations and the associated Lyapunov functions were quadratic (with a scaling). The dual high-gain approach introduced in [1] appears to be very flexible and is applicable to systems with appended Input-to-State Stable (ISS) dynamics driven by all states[2] [1], to both state-feedback and output-feedback control of feedforward systems [10,11], and also to state-feedback control of nontriangular polynomially-bounded systems [12]. In [1], the functions $\phi_i, i = 1,\dots,n$, were allowed to contain functional and parametric uncertainties coupled with all the states. It was seen that a complexity of bounds on the uncertain terms ϕ_i does not result in complexity of the Lyapunov function but is instead handled through the dynamics of the high-gain scaling. However, [1] required a magnitude bound on the uncertain parameters in the system. In this paper, we relax this requirement and provide a global output-feedback solution for systems of form (1) with ϕ_i involving parametric uncertainty coupled with all system states without any *a priori* bounds on unknown parameters. The observer and controller designs are similar to [1] with the essential novelty being in the design of the dynamics of the high-gain scaling parameter. The basic idea is to asymptotically guarantee sufficient gain to dominate the unknown parameters while retaining closed-loop stability. This is achieved using time-varying dynamics of the high-gain scaling parameter. This provides a solution to the benchmark open problem proposed in our earlier paper[13]

$$\begin{aligned} \dot{x}_1 &= x_2 \\ \dot{x}_2 &= x_3 \\ \dot{x}_3 &= u + \theta_0 x_1^2 x_3 \\ y &= x_1 \end{aligned} \tag{2}$$

with u being the input, y the output, and θ_0 an uncertain parameter of unknown sign and with no available magnitude bounds. System (2) is of a very simple form with a single nonlinearity and a single unknown parameter. If any of the components of $\theta_0 x_1^2 x_3$ are dropped, a globally asymptotically stabilizing output-feedback solution can be obtained using available techniques. If θ_0 is known, [14] and [9] provide controllers of dynamic orders 9 and 2,

[1]It was shown in [7] that the cascading dominance assumption can be removed using a multiple time-scaling technique utilizing non-successive powers of r in the scaling. However, since the cascading dominance required in the observer and controller contexts are dual (see Remark 2), it was not possible to use a dual high-gain observer/controller in [7]. Instead, a high-gain observer was coupled to a backstepping controller.

[2]Previous results required the ISS appended dynamics to have nonzero gain only from the output y.

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respectively. If x_1^2 is removed, the system is linear. If x_3 is removed, the system is in standard output-feedback canonical form [15,16]. If a magnitude bound on θ_0 is available, a solution is provided by [1]. However, with θ_0 completely unknown, no previous output-feedback control design technique can globally asymptotically stabilize the system and the technique in this paper provides the first solution.

The required assumptions are listed in Section II. The dual high-gain design of [1] which is applicable under the assumption of known magnitude bounds on the unknown parameters coupled with unmeasured states is summarized in Section III. The modification of the dynamics of the high-gain parameter to remove the requirement of knowledge of magnitude bounds on unknown parameters and the closed-loop stability analysis are contained in Section IV. Extension of the design to systems with ISS appended dynamics and inverse dynamics is briefly outlined in Section V. The design for system (2) is illustrated in Section VI.

II. Assumptions

Assumption A1: Observability and controllability of system (1), i.e., for all $x_1 \in \mathcal{R}$,

$$\begin{aligned} |\phi_{(i,i+1)}(x_1)| &\geq \sigma > 0\ ,\ 1 \leq i \leq n-1 \\ |\mu_0(x_1)| &\geq \sigma > 0. \end{aligned} \tag{3}$$

Assumption A2: Nonnegative continuous functions $\Gamma_0(x_1)$ and $\Gamma_1(x_1)$ are known such that for all $t \in \mathcal{R}$, $x \in \mathcal{R}^n$, and $u \in \mathcal{R}$,

$$|\phi_i(t,x,u)| \leq \Gamma(x_1)\sum_{j=1}^{i}|x_j|\ ,\ 1 \leq i \leq n \tag{4}$$

$$\Gamma(x_1) = \Gamma_0(x_1) + \theta\Gamma_1(x_1) \tag{5}$$

where θ is an unknown positive parameter (with no known magnitude bounds). Furthermore, a nonnegative continuous function $\overline{\Gamma}_1(x_1)$ exists such that[3] for all $x_1 \in \mathcal{R}$

$$\Gamma_1(x_1) \leq \overline{\Gamma}_1(x_1)|x_1|. \tag{6}$$

Assumption A3: Positive constants $\overline{\rho}_i$ and $\underline{\rho}_i$ exist such that for all $x_1 \in \mathcal{R}$

$$|\phi_{(i,i+1)}(x_1)| \geq \overline{\rho}_i|\phi_{(i-1,i)}(x_1)|, i = 3,\ldots,n-1 \tag{7}$$

$$|\phi_{(i,i+1)}(x_1)| \leq \underline{\rho}_i|\phi_{(i-1,i)}(x_1)|, i = 3,\ldots,n-1. \tag{8}$$

Remark 1: For simplicity, it is assumed that the same function Γ serves in the bounding of all ϕ_i's in (4). To remove possible conservativeness, a different Γ can be utilized for the bound of each ϕ_i yielding a design along the same lines as presented here. Γ can be taken to be a function of both x_1 and the time t as long as it is bounded uniformly in time as a function of x_1. Also, the unknown parameter θ in (5) is taken as a lumped characterization of the parametric uncertainty in the entire system. Note that the uncertain parameters in the system can be several, time-varying, and nonlinearly parametrized as long as a bound of the form given in (4) and (5) can be obtained. Furthermore, uncertain parameters with known magnitude bounds can be incorporated into Γ_0.

Remark 2: Assumption A3 requires ratios of the "upper-diagonal" terms $\phi_{(i,i+1)}$ to be bounded. The condition (7) requires the upper-diagonal terms closer to the input to be larger while condition (8) requires the upper-diagonal terms closer to the output to be larger. The conditions (7) and (8) constitute the cascading dominance assumptions [8] in the observer context and controller context, respectively, and are related to uniform solvability of coupled Lyapunov equations [9,1] which are instrumental in the design of observer and controller gains in a dual dynamic high-gain design. Using Theorems A1 and A2 in [1], Assumptions A1 and A3 are necessary and sufficient for existence of functions $g_2(x_1),\ldots,g_n(x_1),k_2(x_1),\ldots,k_n(x_1)$, symmetric positive-definite matrices P_o and P_c, and positive constants ν_o, $\tilde{\nu}_o$, $\overline{\nu}_o$, $\underline{\nu}_o$, ν_c, $\overline{\nu}_c$, and $\underline{\nu}_c$ to satisfy for all $x_1 \in \mathcal{R}$

$$\begin{aligned} &P_oA_o(x_1) + A_o^T(x_1)P_o \leq -\nu_o I - \tilde{\nu}_o|\phi_{(2,3)}(x_1)|C^TC \\ &\underline{\nu}_o I \leq P_o(D_o - \tfrac{1}{2}I) + (D_o - \tfrac{1}{2}I)P_o \leq \overline{\nu}_o I \end{aligned} \tag{9}$$

and

$$\begin{aligned} &P_cA_c(x_1) + A_c^T(x_1)P_c \leq -\nu_c|\phi_{(2,3)}(x_1)|I \\ &\underline{\nu}_c I \leq P_c(D_c - \tfrac{1}{2}I) + (D_c - \tfrac{1}{2}I)P_c \leq \overline{\nu}_c I \end{aligned} \tag{10}$$

where

$$C = [1,0,\ldots,0] \tag{11}$$

$$D_o = D_c = \text{diag}(1,2,\ldots,n-1), \tag{12}$$

$$A_o = \begin{bmatrix} -g_2 & \phi_{(2,3)} & 0 & 0 & \ldots \\ -g_3 & 0 & \phi_{(3,4)} & 0 & \ldots \\ \vdots & & & \ddots & \\ -g_{n-1} & & & & \phi_{(n-1,n)} \\ -g_n & 0 & & \ldots & 0 \end{bmatrix} \tag{13}$$

$$A_c = \begin{bmatrix} 0 & \phi_{(2,3)} & 0 & 0 & \ldots \\ 0 & 0 & \phi_{(3,4)} & 0 & \ldots \\ \vdots & & & \ddots & \\ 0 & & & & \phi_{(n-1,n)} \\ -k_2 & -k_3 & & \ldots & -k_n \end{bmatrix}. \tag{14}$$

Furthermore, by Theorem A1 in [1], $g_2(x_1),\ldots,g_n(x_1)$ can be picked to be linear constant-coefficient combinations of $\phi_{(2,3)}(x_1),\ldots,\phi_{(n-1,n)}(x_1)$. Hence, using Assumption A3, a positive constant $\overline{G}$ exists such that

$$\sqrt{\sum_{i=2}^{n} g_i^2(x_1)} \leq \overline{G}|\phi_{(2,3)}(x_1)|. \tag{15}$$

III. Dual High-Gain Design [1] Assuming Known Magnitude Bounds on Unknown Parameters Coupled with Unmeasured States

In this section, the dual high-gain observer/controller structure of [1] is summarized. This design is applicable under the assumption that known magnitude bounds are available for unknown parameters coupled with unmeasured states. This is equivalent to replacing Assumption A2 with the requirement that $|\phi_i| \leq \Gamma(x_1)[\theta|x_1| + \sum_{j=2}^{i}|x_j|]$ with θ being an unknown nonnegative constant.

A. Observer Design

A reduced-order observer for the system (1) is given by[4], $i = 2,\ldots,n$,

$$\begin{aligned} \dot{\hat{x}}_i = &\ \phi_{(i,i+1)}(x_1)[\hat{x}_{i+1} + r^i f_{i+1}(x_1)] + \mu_{i-n}(x_1)u \\ &-(i-1)\dot{r}r^{i-2}f_i(x_1) - r^{i-1}g_i(x_1)[\hat{x}_2 + rf_2(x_1)] \end{aligned} \tag{16}$$

where r is the dynamic high-gain scaling parameter and

[3] The assumption (6) essentially requires that Γ_1 should vanish at the origin and be $O[s]$ around $s = 0$.

[4] For simplicity of notation, we introduce the dummy variables $\hat{x}_{n+1} = f_{n+1} = 0$ and $\mu_i \equiv 0$ for $i < 0$.

$f_i(x_1)$ are design functions of x_1 of the form $f_i(x_1) = \int_0^{x_1} g_i(s)/\phi_{(1,2)}(s)ds$, $2 \leq i \leq n$ with $g_2, \ldots, g_n$ being functions chosen as in Remark 2. The observer errors e_i and the scaled observer errors ϵ_i are defined as, $2 \leq i \leq n$,

$$e_i = \hat{x}_i + r^{i-1} f_i(x_1) - x_i \; ; \; \epsilon_i = \frac{e_i}{r^{i-1}}. \tag{17}$$

The dynamics of the scaled observer error vector $\epsilon = [\epsilon_2, \ldots, \epsilon_n]^T$ are given by

$$\dot{\epsilon} = rA_o\epsilon - \frac{\dot{r}}{r}D_o\epsilon + \overline{\Phi} \tag{18}$$

where

$$\overline{\Phi} = [\overline{\Phi}_2, \ldots, \overline{\Phi}_n]^T \;,\; \overline{\Phi}_i = -\frac{\phi_i}{r^{i-1}} + g_i\frac{\phi_1}{\phi_{(1,2)}}, \tag{19}$$

and D_o and A_o are defined in (12) and (13), respectively.

B. Controller Design

The control law is given by

$$u = -\frac{r^n}{\mu_0}\sum_{i=2}^{n} k_i(x_1)\eta_i \tag{20}$$

where the controller gain functions $k_2, \ldots, k_n$ are chosen as in Remark 2 and

$$\left.\begin{aligned} \eta_2 &= \tfrac{\hat{x}_2 + rf_2(x_1) + \zeta(x_1,\hat{\theta})}{r} \\ \eta_i &= \tfrac{\hat{x}_i + r^{i-1}f_i(x_1)}{r^{i-1}} \;,\; i = 3, \ldots, n \end{aligned}\right\} \tag{21}$$

$$\zeta(x_1, \hat{\theta}) = (1+\hat{\theta})x_1\zeta_1(x_1) \tag{22}$$

with ζ_1 being a design freedom and $\hat{\theta}$ a parameter estimator. The signals $\eta_i, i = 2, \ldots, n$, are scaled observer estimates of the states x_i with an additional design freedom ζ incorporated into η_2. The dynamics of $\eta = [\eta_2, \ldots, \eta_n]^T$ are

$$\dot{\eta} = rA_c\eta - \frac{\dot{r}}{r}D_c\eta + \Phi - rG\epsilon_2 + H(\eta_2 - \epsilon_2) + \Xi \tag{23}$$

with D_c and A_c defined in (12) and (14), respectively, and

$$\Phi = [\Phi_2, \ldots, \Phi_n]^T \;,\; \Phi_i = g_i\frac{\phi_1}{\phi_{(1,2)}} \tag{24}$$

$$G = [g_2, \ldots, g_n]^T \tag{25}$$

$$H = \left[(1+\hat{\theta})\left\{\zeta_1' x_1 + \zeta_1\right\}\phi_{(1,2)}, 0, \ldots, 0\right]^T \tag{26}$$

$$\Xi = \frac{1}{r}\left[\dot{\hat{\theta}}\zeta_1 x_1 + (1+\hat{\theta})\left\{\zeta_1' x_1 + \zeta_1\right\}\left\{\phi_1 - (1+\hat{\theta})\zeta_1 x_1 \phi_{(1,2)}\right\}, 0, \ldots, 0\right]^T \tag{27}$$

where $\zeta_1'(x_1)$ denotes the partial derivative evaluated at x_1 of ζ_1 with respect to its argument.

C. Dynamics of the High-Gain Scaling Parameter

The dynamics of the high-gain scaling parameter r can be picked to be of the form

$$\dot{r} = r[-a(r-1) + b\gamma(x_1, \hat{\theta})] \tag{28}$$

with a and b being positive constants and γ a continuous positive function. At $r = 1$, the derivative $\dot{r}$ is positive. Hence, initializing r to be greater than 1, $r(t)$ governed by (28) remains greater than 1 for all time t. This ensures that the scaling in (17) and (21) is well-defined.

IV. Dual High-Gain Design Global in Unknown Parameters

The design in Section III is applicable as long as magnitude bounds are available for the unknown parameters which appear coupled with unmeasured states. This requirement is relaxed in this section by generalizing the dynamics of the high-gain scaling parameter to be of the form

$$\dot{r} = r[-a(r-1) + b\gamma(x_{1t}, \hat{\theta}, t)] \; ; \; r(0) > 1 \tag{29}$$

with a and b being positive constants and γ being a positive function. x_{1t} denotes the restriction of the time function $x_1(\tau)$ to the time interval $[0, t]$.

The closed-loop stability is analyzed using the Lyapunov function

$$V = cr\epsilon^T P_o\epsilon + r\eta^T P_c\eta + \frac{1}{2}x_1^2 \tag{30}$$

where c is a positive constant satisfying

$$c > \frac{4}{\tilde{\nu}_o\nu_c}\lambda_{max}^2(P_c)\overline{G}^2. \tag{31}$$

Differentiating (30),

$$\begin{aligned} \dot{V} \leq\; & cr^2\epsilon^T(P_oA_o + A_o^TP_o)\epsilon - c\dot{r}\epsilon^T(P_oD_o + D_oP_o)\epsilon \\ & + 2cr\epsilon^TP_o\overline{\Phi} + c\dot{r}\epsilon^TP_o\epsilon + r^2\eta^T(P_cA_c + A_c^TP_c)\eta \\ & - \dot{r}\eta^T(P_cD_c + D_cP_c)\eta + 2r\eta^TP_c\Phi - 2r^2\eta^TP_cG\epsilon_2 \\ & + 2r\eta^TP_cH(\eta_2 - \epsilon_2) + 2r\eta^TP_c\Xi + \dot{r}\eta^TP_c\eta \\ & + x_1\{\phi_1 + \phi_{(1,2)}[-\zeta + r\eta_2 - r\epsilon_2]\}. \end{aligned} \tag{32}$$

The dynamics of the adaptation parameter $\hat{\theta}$ will be designed such that $\hat{\theta}(t)$ is positive for all time t. Using Assumption A2 and the property that $r(t) \geq 1$,

$$|\overline{\Phi}_i| \leq \Gamma\left[\sum_{j=2}^{i}|\eta_j| + \sum_{j=2}^{i}|\epsilon_j| + \frac{|x_1| + |\zeta(x_1,\hat{\theta})|}{r}\right] + \frac{|g_i|\Gamma|x_1|}{|\phi_{(1,2)}|}$$

$$|\overline{\Phi}| \leq n\Gamma[|\eta| + |\epsilon|] + \frac{n^{\frac{1}{2}}\Gamma}{r}\left[1 + (1+\hat{\theta})|\zeta_1|\right]|x_1| + \frac{\overline{G}|\phi_{(2,3)}|\Gamma|x_1|}{|\phi_{(1,2)}|}$$

$$\begin{aligned} 2cr\epsilon^TP_o\overline{\Phi} \leq\; & 3cr\lambda_{max}(P_o)n\Gamma\left[|\epsilon|^2 + |\eta|^2\right] \\ & + rc^2\lambda_{max}^2(P_o)n\Gamma^2\left[1 + (1+\hat{\theta})|\zeta_1|\right]^2|\epsilon|^2 + x_1^2 \\ & + \frac{c\nu_o}{4}r^2|\epsilon|^2 + \frac{4}{\nu_o}c\Gamma^2\lambda_{max}^2(P_o)\frac{\overline{G}^2\phi_{(2,3)}^2}{\phi_{(1,2)}^2}x_1^2. \end{aligned} \tag{33}$$

Similarly,

$$\begin{aligned} 2r\eta^TP_c\Phi \leq\; & \frac{4}{\nu_c}\lambda_{max}^2(P_c)\overline{G}^2\Gamma^2\frac{|\phi_{(2,3)}|}{\phi_{(1,2)}^2}x_1^2 \\ & + \frac{\nu_c}{4}|\phi_{(2,3)}|r^2|\eta|^2 \end{aligned} \tag{34}$$

$$-2r^2\eta^TP_cG\epsilon_2 \leq \frac{\nu_c}{4}r^2|\phi_{(2,3)}||\eta|^2 + cr^2\tilde{\nu}_o|\phi_{(2,3)}|\epsilon_2^2 \tag{35}$$

$$\begin{aligned} 2r\eta^TP_cH(\eta_2 - \epsilon_2) \leq\; & 3r\lambda_{max}(P_c)(1+\hat{\theta})|\zeta_1' x_1 + \zeta_1| \times \\ & \times|\phi_{(1,2)}|[|\eta|^2 + |\epsilon|^2] \end{aligned} \tag{36}$$

$$\begin{aligned} 2r\eta^TP_c\Xi \leq\; & 3r\lambda_{max}^2(P_c)\Big[\dot{\hat{\theta}}^2\zeta_1^2 \\ & + (1+\hat{\theta})^4(\zeta_1' x_1 + \zeta_1)^2\zeta_1^2\phi_{(1,2)}^2 \\ & + (1+\hat{\theta})^2(\zeta_1' x_1 + \zeta_1)^2\Gamma^2\Big]|\eta|^2 + x_1^2 \end{aligned} \tag{37}$$

$$x_1\phi_1 \leq \Gamma x_1^2 \tag{38}$$

$$\begin{aligned} x_1\phi_{(1,2)}[r\eta_2 - r\epsilon_2] \leq\; & \frac{\nu_c}{4}r^2|\phi_{(2,3)}||\eta|^2 + \frac{c\nu_o}{4}r^2|\epsilon|^2 \\ & + \frac{1}{\nu_c}\frac{\phi_{(1,2)}^2}{|\phi_{(2,3)}|}x_1^2 + \frac{1}{c\nu_o}\phi_{(1,2)}^2x_1^2. \end{aligned} \tag{39}$$

Using (9) and (10),

$$\begin{aligned}\epsilon^T(P_oA_o + A_o^TP_o)\epsilon &\leq -\nu_o|\epsilon|^2 - \tilde{\nu}_o|\phi_{(2,3)}|\epsilon_2^2 \\ \eta^T(P_cA_c + A_c^TP_c)\eta &\leq -\nu_c|\phi_{(2,3)}||\eta|^2. \end{aligned} \quad (40)$$

Using (33) - (40), (32) reduces to

$$\begin{aligned}\dot{V} \leq & -\frac{c\nu_o}{2}r^2|\epsilon|^2 - \frac{\nu_c}{4}r^2|\phi_{(2,3)}||\eta|^2 - x_1\phi_{(1,2)}\zeta \\ & +[q_1(x_1) + \theta^* q_2(x_1)]x_1^2 \\ & -c\dot{r}\epsilon^T\left[P_o(D_o - \frac{1}{2}I) + (D_o - \frac{1}{2}I)P_o\right]\epsilon \\ & -\dot{r}\eta^T\left[P_c(D_c - \frac{1}{2}I) + (D_c - \frac{1}{2}I)P_c\right]\eta \\ & +r[w_1(x_1,\hat{\theta},\dot{\hat{\theta}}) + \theta^* w_2(x_1,\hat{\theta})]\{|\epsilon|^2 + |\eta|^2\} \end{aligned} \quad (41)$$

where $\theta^* = \theta + \theta^2$ and

$$\begin{aligned} q_1(x_1) =\ & 2 + \frac{1}{\nu_c}\frac{\phi_{(1,2)}^2}{|\phi_{(2,3)}|} + \frac{1}{c\nu_o}\phi_{(1,2)}^2 \\ & +\frac{8}{\nu_o}c\overline{G}^2\lambda_{max}^2(P_o)\Gamma_0^2\frac{\phi_{(2,3)}^2}{\phi_{(1,2)}^2} \\ & +\frac{8}{\nu_c}\overline{G}^2\lambda_{max}^2(P_c)\Gamma_0^2\frac{|\phi_{(2,3)}|}{\phi_{(1,2)}^2} + \Gamma_0 \end{aligned} \quad (42)$$

$$\begin{aligned} q_2(x_1) =\ & \frac{8}{\nu_o}c\overline{G}^2\lambda_{max}^2(P_o)\Gamma_1^2\frac{\phi_{(2,3)}^2}{\phi_{(1,2)}^2} \\ & +\frac{8}{\nu_c}\overline{G}^2\lambda_{max}^2(P_c)\Gamma_1^2\frac{|\phi_{(2,3)}|}{\phi_{(1,2)}^2} + \Gamma_1 \end{aligned} \quad (43)$$

$$\begin{aligned} w_1(x_1,\hat{\theta},\dot{\hat{\theta}}) =\ & 3\lambda_{max}(P_c)(1+\hat{\theta})|\zeta_1' x_1 + \zeta_1||\phi_{(1,2)}| \\ & +3\lambda_{max}^2(P_c)\left[\dot{\hat{\theta}}^2\zeta_1^2 + (1+\hat{\theta})^4(\zeta_1' x_1+\zeta_1)^2\zeta_1^2\phi_{(1,2)}^2\right] \\ & +3c\lambda_{max}(P_o)n\Gamma_0 \\ & +2c^2\lambda_{max}^2(P_o)n\Gamma_0^2[1 + (1+\hat{\theta})|\zeta_1|]^2 \\ & +6\lambda_{max}^2(P_c)(1+\hat{\theta})^2(\zeta_1' x_1 + \zeta_1)^2\Gamma_0^2 \end{aligned} \quad (44)$$

$$\begin{aligned} w_2(x_1,\hat{\theta}) =\ & 3c\lambda_{max}(P_o)n\Gamma_1 \\ & +2c^2\lambda_{max}^2(P_o)n\Gamma_1^2[1 + (1+\hat{\theta})|\zeta_1|]^2 \\ & +6\lambda_{max}^2(P_c)(1+\hat{\theta})^2(\zeta_1' x_1 + \zeta_1)^2\Gamma_1^2. \end{aligned} \quad (45)$$

Remark 3: By examining the inequalities (33), (34), and (38), it is seen that the terms in $q_2(x_1)$ and the last three terms in $q_1(x_1)$arise from overbounding $|\phi_1|$ by $[\Gamma_0 + \theta\Gamma_1]|x_1|$. In the particular case in which ϕ_1 is known to be free of uncertain parameters, q_2 reduces to zero and the adaptation parameter $\hat{\theta}$ is not required. In the case that q_2 is nonzero, the dynamics of the adaptation parameter are designed as (49).

Using (9), (10), and (29),

$$\begin{aligned} \dot{r}\epsilon^T\left[P_o(D_o-\frac{1}{2}I)+(D_o-\frac{1}{2}I)P_o\right]\epsilon &\geq \{r(a+b\gamma)\underline{\nu}_o - ar^2\overline{\nu}_o\}|\epsilon|^2 \\ \dot{r}\eta^T\left[P_c(D_c-\frac{1}{2}I)+(D_c-\frac{1}{2}I)P_c\right]\eta &\geq \{r(a+b\gamma)\underline{\nu}_c - ar^2\overline{\nu}_c\}|\eta|^2. \end{aligned} \quad (46)$$

Picking b to be an arbitrary positive constant, choose $a > 0$ small enough to ensure that

$$\max(-\frac{c\nu_o}{2} + ac\overline{\nu}_o, -\frac{\sigma\nu_c}{4} + a\overline{\nu}_c) = -a^* < 0, \quad (47)$$

and choose $\zeta_1(x_1)$ such that

$$-\zeta_1(x_1)\phi_{(1,2)}(x_1) + q_1(x_1) + q_2(x_1) \leq -\zeta_1^*(x_1) \quad (48)$$

with ζ_1^* being a positive function of x_1 bounded below by a positive constant $\underline{\zeta}_1^*$. The parameter estimator dynamics are chosen as

$$\dot{\hat{\theta}} = q_2(x_1)x_1^2 \ ; \ \hat{\theta}(0) > 0. \quad (49)$$

Note that the parameter estimate $\hat{\theta}(t)$ with dynamics (49) is a monotonically nondecreasing function of time. A new Lyapunov function is defined including a quadratic of the *parameter estimation error* $(\hat{\theta} - \theta^*)$ as

$$\overline{V} = V + \frac{1}{2}(\hat{\theta} - \theta^*)^2. \quad (50)$$

Differentiating (50) and using (41), (46), (47), and (48),

$$\begin{aligned}\dot{\overline{V}} \leq & -a^*r^2[|\epsilon|^2 + |\eta|^2] - x_1^2\zeta_1^*(x_1) \\ & +r[w_1(x_1,\hat{\theta},\dot{\hat{\theta}}) + \theta^* w_2(x_1,\hat{\theta})]\{|\epsilon|^2 + |\eta|^2\} \\ & -r(a+b\gamma)[c\underline{\nu}_o|\epsilon|^2 + \underline{\nu}_c|\eta|^2]. \end{aligned} \quad (51)$$

Picking the design function γ to be of the form

$$\begin{aligned} \gamma(x_{1t},\hat{\theta},t) &= \frac{1}{b\min(c\underline{\nu}_o,\underline{\nu}_c)}[\gamma_1(x_1,\hat{\theta}) + \gamma_2(x_{1t},\hat{\theta},t)] \\ \gamma_1(x_1,\hat{\theta}) &= w_1(x_1,\hat{\theta},q_2(x_1)x_1^2) \\ \gamma_2(x_{1t},\hat{\theta},t) &= [1 + t + \hat{\theta} + \sup_{\tau\in[0,t]} x_1^2(\tau)]w_2(x_1,\hat{\theta}), \end{aligned} \quad (52)$$

(51) reduces to

$$\dot{\overline{V}} \leq \{-a^*r^2 + r\Delta(x_{1t},\hat{\theta},t)\}[|\epsilon|^2+|\eta|^2] - x_1^2\zeta_1^*(x_1) \quad (53)$$

where

$$\Delta(x_{1t},\hat{\theta},t) = \Delta_0(x_{1t},\hat{\theta},t)w_2(x_1,\hat{\theta}) \quad (54)$$

$$\Delta_0(x_{1t},\hat{\theta},t) = \{\theta^* - [1 + t + \hat{\theta}(t) + \sup_{\tau\in[0,t]} x_1^2(\tau)]\}. \quad (55)$$

Closed-loop stability is proved through a sequence of facts below. Local existence and uniqueness of solutions is guaranteed by the assumptions on the functions ϕ_i and $\phi_{(i,i+1)}$. Let the maximal interval of existence of solutions be $[0, t_f)$. Theorem 1 utilizes Facts 1-6 to infer that $t_f = \infty$ (i.e., unique solution exists for all time) and that in the limit as $t \to \infty$, the states $x_1, \ldots, x_n$, the observer errors $e_2, \ldots, e_n$, and the control input u converge to zero.

Fact 1: A class $\mathcal{K}$ function $\overline{\Delta}$ exists such that

$$\Delta(x_{1t},\hat{\theta},t) \leq (1+t)\overline{\Delta}(|x_{1t}| + \hat{\theta}) \quad (56)$$

where $|x_{1t}| \stackrel{\triangle}{=} \sup_{\tau\in[0,t]}|x_1(\tau)|$.

Proof of Fact 1: Using (54) and (55), (56) is satisfied with

$$\overline{\Delta}(s) = [1 + \theta^* + s + s^2]\sup_{|s_1|\leq s} w_2(s_1, s). \quad (57)$$

Note that for every given x_1, the function $w_2(x_1,\hat{\theta})$ is monotonically nondecreasing in its second argument.

Fact 2: If a positive constant M exists such that for some time $T \in [0, t_f]$, the inequality $(|x_{1T}| + \hat{\theta}(T)) \leq M$ holds, then

$$\overline{V}(t) \leq \overline{V}(0)e^{\alpha_1(1+T)\overline{\Delta}(M)t} \quad \forall t \in [0, T) \quad (58)$$

with α_1 being a positive constant independent of T and M. Furthermore, $\sup_{t\in[0,T)} r(t) < \infty$.

Proof of Fact 2: Using (53) and Fact 1,

$$\begin{aligned}\dot{\overline{V}} &\leq r(1+t)\overline{\Delta}(|x_{1t}| + \hat{\theta})[|\epsilon|^2 + |\eta|^2] \\ &\leq \alpha_1(1+T)\overline{\Delta}(M)\overline{V} \quad \forall t \in [0, T) \end{aligned} \quad (59)$$

with

$$\alpha_1 = \frac{1}{\min(c\lambda_{min}(P_o), \lambda_{min}(P_c))}. \quad (60)$$

Using the Comparison Lemma, (59) yields (58). By the dynamics (29), boundedness of $r(t)$ follows from boundedness of x_{1t} and $\hat{\theta}(t)$ for any finite time t.

Fact 3: A positive constant M_0 exists such that if

$$[|x_{1t_0}| + \hat{\theta}(t_0)] \quad \geq \quad M_0 \tag{61}$$

for some time $t_0 < t_f$, then $\Delta(x_{1t}, \hat{\theta}, t) \leq 0$ for all times $t \in [t_0, t_f)$.

Proof of Fact 3: Letting $M_0 = 2\max(\sqrt{\theta^*}, \theta^*)$,

$$\theta^* - [1 + t + \hat{\theta}(t) + \sup_{\tau\in[0,t]} x_1^2(\tau)] \quad \leq \quad 0 \tag{62}$$

if $[|x_{1t_0}|+\hat{\theta}(t_0)] \geq M_0$ and $t \geq t_0$. Note that $[|x_{1t}|+\hat{\theta}(t)]$ is a monotonically nondecreasing function of t for $t \in [0, t_f)$. Since w_2 is nonnegative for all arguments, the statement of Fact 3 follows from (62).

Fact 4: If for some time $T \in [0, t_f)$, the inequality $[|x_{1T}| + \hat{\theta}(T)] \geq M_0$ holds, then

$$\dot{\overline{V}}(t) \quad \leq \quad -\alpha_2 V(t) \;\; \forall t \in [T, t_f) \tag{63}$$

with α_2 being a positive constant independent of T.

Proof of Fact 4: Using (53) and Fact 3, for all $t \in [T, t_f)$,

$$\dot{\overline{V}} \quad \leq \quad -a^* r^2[|\epsilon|^2 + |\eta|^2] - x_1^2 \underline{\zeta}_1^* \tag{64}$$

and (63) follows with

$$\alpha_2 \quad = \quad \min\left(2\zeta_1^*, \frac{a^*}{c\lambda_{max}(P_o)}, \frac{a^*}{\lambda_{max}(P_c)}\right). \tag{65}$$

Fact 5: If $t_f = \infty$, then $\lim_{t\to\infty} V(t) = 0$.

Proof of Fact 5: If $t \geq \theta^*$, then $\Delta(x_{1t}, \hat{\theta}, t) \leq 0$. Hence, for $t \in [\theta^*, \infty)$,

$$\dot{\overline{V}}(t) \leq -\alpha_2 V(t) \tag{66}$$

with α_2 given by (65). Applying Barbalat's Lemma, the statement of Fact 5 follows.

Fact 6: If $t_f = \infty$ and positive constants M_1, M_2, and M_3 exist such that for all time $t \in [0, \infty)$,

$$|x_1(t)| \leq M_1 e^{-M_2 t} \; , \; \hat{\theta} \leq M_3, \tag{67}$$

then

$$\lim_{t\to\infty} \gamma_2(x_{1t}, \hat{\theta}, t) = 0. \tag{68}$$

Proof of Fact 6: Using Assumption A2 and (45), the inequality $w_2(x_1, \hat{\theta}) \leq \overline{w}_2(x_1, \hat{\theta})|x_1|$ is satisfied with

$$\begin{aligned}\overline{w}_2(x_1, \hat{\theta}) = \Big[&3c\lambda_{max}(P_o)n\overline{\Gamma}_1 \\ &+2c^2\lambda_{max}^2(P_o)n\overline{\Gamma}_1^2|x_1|[1 + (1 + \hat{\theta})|\zeta_1|]^2 \\ &+6\lambda_{max}^2(P_c)(1 + \hat{\theta})^2(\zeta_1' x_1 + \zeta_1)^2\overline{\Gamma}_1^2|x_1|\Big].\end{aligned} \tag{69}$$

Using (67),

$$\gamma_2(x_{1t}, \hat{\theta}, t) \leq (1+t+M_3+M_1^2) \sup_{|s_1|\leq M_1} \overline{w}_2(s_1, M_3)M_1 e^{-M_2 t}$$

and (68) follows.

Theorem 1: Under Assumptions A1-A3, the designed dynamic controller guarantees global uniform boundedness of all closed-loop signals. Furthermore, the states $x_1, \ldots, x_n$, the observer errors $e_2, \ldots, e_n$, and the control input u asymptotically converge to zero as $t \to \infty$.

Proof of Theorem 1: With the maximal interval of existence of solutions being $[0, t_f)$, consider the two cases:

<u>Case $\mathcal{A}1$:</u> $\sup_{t\in[0,t_f)}(|x_{1t}| + \hat{\theta}) < \infty$
<u>Case $\mathcal{A}2$:</u> $\sup_{t\in[0,t_f)}(|x_{1t}| + \hat{\theta}) = \infty$.

Under Case $\mathcal{A}1$, if $t_f < \infty$, using Fact 2, $\sup_{t\in[0,t_f)} \overline{V}(t) < \infty$ and $\sup_{t\in[0,t_f)} r(t) < \infty$ so that all closed-loop signals are uniformly bounded on $[0, t_f)$. Hence, solutions exist beyond time t_f contradicting the assumption that $[0, t_f)$ is the maximal interval of existence of solutions. Therefore, $t_f = \infty$. Under Case $\mathcal{A}2$, a time $T < t_f$ exists such that $[x_{1T} + \hat{\theta}(T)] \geq M_0$. Using Fact 4, $\overline{V}(t)$ is a monotonically nonincreasing function of time on $[T, t_f)$. Hence, using Fact 2, all closed-loop signals are uniformly bounded on $[0, t_f)$ and if $t_f < \infty$, solutions exist beyond time t_f. By contradiction, $t_f = \infty$. By consideration of the Cases $\mathcal{A}1$ and $\mathcal{A}2$, it follows that $t_f = \infty$, i.e., unique solution exists for all time. Using Fact 5, $\lim_{t\to\infty} V(t) = 0$. Hence, the states x_1 and $\hat{\theta}$ are uniformly bounded on $[0, \infty)$. Consider the two cases:

<u>Case $\mathcal{B}1$:</u> $\lim_{t\to\infty} \hat{\theta}(t) > \theta^*$
<u>Case $\mathcal{B}2$:</u> $\lim_{t\to\infty} \hat{\theta}(t) \leq \theta^*$.

Under Case $\mathcal{B}1$, using the fact that $\dot{\hat{\theta}} \geq 0$, we have $(\hat{\theta}(t) - \theta^*)\dot{\hat{\theta}}(t) \geq 0$ for all t bigger than some finite time T_θ. Since

$$\dot{V} = \dot{\overline{V}} - (\hat{\theta} - \theta^*)\dot{\hat{\theta}}, \tag{70}$$

it follows using (63) that $\dot{V}(t) \leq -\alpha_2 V(t)$ for all $t \geq \max(T_\theta, \theta^*)$ so that $V(t)$ goes to zero exponentially as $t \to \infty$. Under Case $\mathcal{B}2$, using (6) and (43) , it is seen that $|(\hat{\theta} - \theta^*)q_2 x_1^2| \leq \theta^* \overline{q}_2(x_1)x_1^3$ with $\overline{q}_2$ being a continuous nonnegative function. Since, as proved above, $\lim_{t\to\infty} V(t) = 0$, it follows from (70) that $V(t)$ converges to zero exponentially. Thus, in both Cases $\mathcal{B}1$ and $\mathcal{B}2$, x_1 converges exponentially to zero as $t \to \infty$. Using Fact 6, $\lim_{t\to\infty} \gamma_2(x_{1t}, \hat{\theta}, t) = 0$. By uniform boundedness of x_1 and $\hat{\theta}$, uniform boundedness of $\gamma(x_{1t}, \hat{\theta}, t)$ on $t \in [0, \infty)$ follows. Hence, $r(t)$ is uniformly bounded on $[0, \infty)$. Using (66) and uniform boundedness of $r(t)$, all closed-loop signals are seen to be uniformly bounded on $[0, \infty)$. Furthermore, using Fact 5, $V(t)$ and hence $(x_1, \eta_2, \ldots, \eta_n, \epsilon_2, \ldots, \epsilon_n)$ converge asymptotically to zero as $t \to \infty$. Using the uniform boundedness of r and the properties that $\zeta(0, \hat{\theta}) = 0$ and $f_i(0) = 0, i = 2, \ldots, n$, it follows that the states $x_1, \ldots, x_n$, the observer errors $e_2, \ldots, e_n$, and the observer estimates $\hat{x}_2, \ldots, \hat{x}_n$ converge asymptotically to zero. Using (20), the control input u also converges asymptotically to zero. ◇

V. Extension to Systems with Appended ISS Dynamics and Inverse Dynamics

By combining the high-gain scaling dynamics design technique in this paper with the technique for handling ISS appended dynamics driven by all states in [1], the proposed results can be extended to the more general class of systems

$$\begin{aligned}\dot{z}_i \;&=\; q_i(z, x, t) \; , \; i = 1, \ldots, n \\ \dot{x}_i \;&=\; \phi_i(t, z, x, u) + \phi_{(i,i+1)}(x_1)x_{i+1} \\ &\quad\; +\psi_i(z, x, t) \; , \; i = 1, \ldots, s-1 \\ \dot{x}_i \;&=\; \phi_i(t, z, x, u) + \phi_{(i,i+1)}(x_1)x_{i+1} + \mu_{i-s}(x_1)u \\ &\quad\; +\psi_i(z, x, t) \; , \; i = s, \ldots, n \\ y \;&=\; x_1\end{aligned} \tag{71}$$

where $z_i \in \mathcal{R}^{n_{z_i}}$ are the (unmeasurable) states of appended Input-to-State Stable (ISS) dynamics[17] and $z = [z_1^T, \ldots, z_n^T]^T$. s is the relative degree of the system and $[x_{s+1}, \ldots, x_n]^T$ is the state of the inverse dynamics. The appended dynamics are driven by all the system states with a triangular structure of ISS interconnections, i.e., z_i is allowed to have nonzero nonlinear gains from states $x_1, \ldots, x_i$. The uncertain functions ϕ_i are required to be bounded by the product of an uncertain parameter θ, a nonlinear function $\Gamma(x_1)$, and a linear combination of $|x_1|, \ldots, |x_i|, |z_2|, \ldots, |z_i|$, and a nonlinear function of $|z_1|$. The inverse dynamics subsystem is assumed to be ISS with nonzero nonlinear gains from $x_1, \ldots, x_s, z_1, \ldots, z_s$. While previous techniques [18] required ISS dynamics and inverse dynamics to be driven only by x_1, [1] provided a method using the dynamic high-gain scaling approach to handle ISS appended dynamics and inverse dynamics driven by all the system states. The design in [1] utilized dynamics of the high-gain scaling parameter of the form $\dot{r} = \lambda(R(x_1, \hat{\theta}, \dot{\hat{\theta}}) - r)\Omega(r, x_1, \hat{\theta}, \dot{\hat{\theta}})$ with R, λ, and Ω being suitably chosen functions. The Lyapunov function in [1] incorporates appropriately scaled versions of the ISS Lyapunov functions of the inverse dynamics and the appended dynamics. By using the techniques in [1] and the time-varying design of the high-gain scaling dynamics in this paper, the proposed controller can be extended to obtain global output-feedback results for (71). The details are omitted here for brevity.

VI. An Illustrative Example

The system (2) belongs to the class of systems (1) with $n = 3$, $\phi_1 = \phi_2 = 0$, $\phi_3 = \theta_0 x_1^2 x_3$, and $\phi_{(1,2)} = \phi_{(2,3)} = \mu_0 = 1$. It is easily seen that Assumptions A1-A3 are satisfied with $\sigma = 1$, $\Gamma_0 = 0$, $\Gamma_1(x_1) = x_1^2$, and $\theta = |\theta_0|$. The inequalities (9) and (10) are satisfied with $g_2 = 2, g_3 = 1, k_2 = 1, k_3 = 2$, and

$$P_o = \begin{bmatrix} 1 & -1 \\ -1 & 2 \end{bmatrix}, P_c = \begin{bmatrix} 2 & 1 \\ 1 & 1 \end{bmatrix}. \quad (72)$$

In this case, $\nu_o = 0.5$, $\tilde{\nu}_o = 0.8$, $\nu_c = 1$, $\underline{\nu}_o = 0.2984$, $\overline{\nu}_o = 6.7016$, $\underline{\nu}_c = 0.4384$, and $\overline{\nu}_c = 4.5616$. A reduced-order observer is constructed for this system as

$$\begin{aligned} \dot{\hat{x}}_2 &= \hat{x}_3 + r^2 g_3 x_1 - r g_2(\hat{x}_2 + r g_2 x_1) - \dot{r} g_2 x_1 \\ \dot{\hat{x}}_3 &= -r^2 g_3(\hat{x}_2 + r g_2 x_1) - 2\dot{r} r g_3 x_1 + u. \end{aligned} \quad (73)$$

Define $\eta_2 = \frac{\hat{x}_2 + r g_2 x_1 + \zeta_1 x_1}{r}$ and $\eta_3 = \frac{\hat{x}_3 + r^2 g_3 x_1}{r^2}$ with $\zeta_1 = 3.1$. By Remark 3, the parameter estimator $\hat{\theta}$ is not needed since $\phi_1 = 0$. The control law for u and the dynamics of the high-gain scaling parameter are picked as

$$\begin{aligned} u &= r^3(-k_2\eta_2 - k_3\eta_3) && (74) \\ \dot{r} &= r[-0.03(r-1) + \gamma(x_{1t}, t)] && (75) \\ \gamma(x_{1t}, t) &= 1481 + 3070 x_1^2[1 + t + \sup_{\tau \in [0,t]} x_1^2(\tau)]. && (76) \end{aligned}$$

The dynamic controller given by (73), (74), and (75) guarantees global uniform boundedness of all closed-loop signals and all the closed-loop states (except r) go to zero asymptotically as $t \to \infty$.

VII. Conclusion

We have proposed a global high-gain scaling based observer/controller for systems in uncertain generalized output-feedback canonical form. Time-varying nonlinear parametric uncertainty is allowed to occur throughout the system coupled with any state in the system dynamics. No magnitude bounds on unknown parameters are necessary. The design utilizes the dual architecture of a high-gain observer and controller. We have introduced a new time-varying dynamics of the high-gain scaling parameter. The design is essentially based on asymptotically providing sufficient gain to dominate the unknown parameter while retaining closed-loop stability. The proposed observer/controller structure provides a globally asymptotically stabilizing feedback for the benchmark open problem (2) proposed in our earlier work [13] with no magnitude bounds or sign information on the parameter. It remains a topic of further research to ascertain whether a global output-feedback controller for (2) can be designed without requiring time-varying scaling dynamics.

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