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Passivity-based Dynamic Visual Feedback Control with Uncertainty of Camera Coordinate Frame

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***Abstract*— In this paper, we consider the dynamic visual feedback control with the uncertainty of the camera coordinate frame based on the passivity. Firstly the brief summary of the nominal visual feedback systems with a fixed camera is given with the fundamental representation of a relative rigid body motion. Secondly we construct the visual feedback system with uncertainty which is not be limited to the orientation around the optical axis. Next, we derive the passivity of the dynamic visual feedback system by combining the manipulator dynamics and the visual feedback system. Based on the passivity, stability and L_2-gain performance analysis are discussed. Finally the validity of the proposed control law can be confirmed by comparing the simulation results.**

I. INTRODUCTION

Robotics and intelligent machines need many information to behave autonomously under dynamical environments. One suitable way to recognize unknown surroundings is to use visual information. Vision based control of robotic systems involves the fusion of robot kinematics, dynamics, and computer vision to control the motion of the robot in an efficient manner. The combination of mechanical control with visual information, so-called visual feedback control or visual servoing, should become extremely important, when we consider a mechanical system working under dynamical environments [1], [2].

Classical visual servoing algorithms assume that the manipulator dynamics is negligible and do not interact with the visual feedback loop. However, this assumption is invalid for high speed tasks, while it holds for kinematic control problems [3]. Additionally, as mentioned in [1], camera calibration problems are long standing research issues in the visual feedback systems with a fixed camera as depicted in Fig. 1. Kelly [3] considered the set-point problems for the dynamic visual feedback system with the uncertainty of the camera orientation. In [4], Bishop $et\ al.$ proposed an inverse dynamics based control law for the position tracking and the camera calibration problems of the dynamic visual feedback system. Recently, Zergeroglu $et\ al.$ developed an adaptive control law for the position tracking and the camera calibration problems of the dynamic visual feedback system with parametric uncertainties in [5]. Although these control laws guarantee the stability of the system based on the Lyapunov method and are effective for the dynamic visual feedback system, robot manipulators are unfortunately limited to the planar type. Moreover, the uncertainty of the camera coordinate frame is limited to the orientation around the optical axis.

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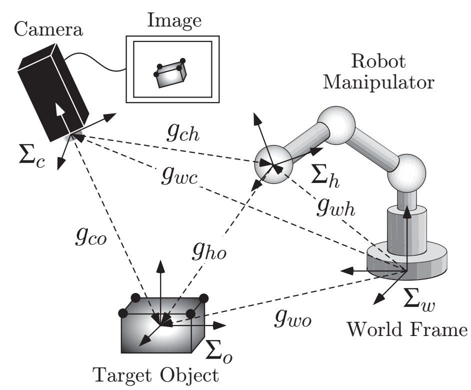

Fig. 1. Visual feedback system with a fixed camera.

In this paper, we consider the dynamic visual feedback control with the uncertainty of the camera coordinate frame. The uncertainty will not be limited to the orientation around the optical axis, although we use a simple camera model. In this work, our previous research [9] is extended to the case of uncertain visual feedback systems. Hence, we can derive that the dynamic visual feedback system preserves the passivity of the visual feedback system by the same strategy in our previous works [6]-[9]. Stability and L_2-gain performance analysis for the dynamic visual feedback system will be discussed based on passivity with an energy function. Comparing the simulation results, the validity of the proposed control law can be confirmed.

Throughout this paper, we use the notation $e^{\hat{\xi}\theta_{ab}} \in \mathcal{R}^{3\times 3}$ to represent the change of the principle axes of a frame Σ_b relative to a frame Σ_a. The notation '$\wedge$' (wedge) is the skew-symmetric operator such that $\hat{\xi}\theta = \xi \times \theta$ for the vector cross-product $\times$ and any vector $\theta \in \mathcal{R}^3$. The notation '$\vee$' (vee) denotes the inverse operator to '$\wedge$': i.e., $so(3) \to \mathcal{R}^3$. $\xi_{ab} \in \mathcal{R}^3$ specifies the direction of rotation and $\theta_{ab} \in \mathcal{R}$ is the angle of rotation. Here $\hat{\xi}\theta_{ab}$ denotes $\hat{\xi}_{ab}\theta_{ab}$ for the simplicity of notation. We use the 4×4 matrix

$$g_{ab} = \begin{bmatrix} e^{\hat{\xi}\theta_{ab}} & p_{ab} \\ 0 & 1 \end{bmatrix} \tag{1}$$

as the homogeneous representation of $g_{ab} = (p_{ab}, e^{\hat{\xi}\theta_{ab}}) \in SE(3)$ which is the description of the configuration of a frame Σ_b relative to a frame Σ_a. The adjoint transformation associated with g_{ab} is denoted by $\mathrm{Ad}_{(g_{ab})}$ [10]. Let us define the vector form of the rotation matrix as $e_R(e^{\hat{\xi}\theta_{ab}}) := \mathrm{sk}(e^{\hat{\xi}\theta_{ab}})^{\vee}$ where $\mathrm{sk}(e^{\hat{\xi}\theta_{ab}})$ denotes $\frac{1}{2}(e^{\hat{\xi}\theta_{ab}} - e^{-\hat{\xi}\theta_{ab}})$.

II. Passivity-based Visual Feedback System

A. Fundamental Representation for Visual Feedback System

Visual feedback systems typically use four coordinate frames which consist of a world frame Σ_w, a target object frame Σ_o, a camera frame Σ_c and a hand (end-effector) frame Σ_h as in Fig. 1. Then, g_{wh}, g_{wc} and g_{wo} denote the rigid body motions from Σ_w to Σ_h, from Σ_w to Σ_c and from Σ_w to Σ_o, respectively. Similarly, the relative rigid body motions from Σ_c to Σ_h, from Σ_c to Σ_o and from Σ_h to Σ_o can be represented by g_{ch}, g_{co} and g_{ho}, respectively, as shown in Fig. 1.

The relative rigid body motion from Σ_c to Σ_o can be led by using the composition rule for rigid body transformations ([10], Chap. 2, pp. 37, eq. (2.24)) as follows

$$g_{co} = g_{wc}^{-1} g_{wo}. \tag{2}$$

The fundamental representation of the relative rigid body motion involves the velocity of each rigid body. To this aid, let us consider the velocity of a rigid body as described in [10]. Now, we define the body velocity of the camera relative to the world frame Σ_w as

$$\hat{V}_{wc}^b = g_{wc}^{-1}\dot{g}_{wc} = \begin{bmatrix} \hat{\omega}_{wc} & v_{wc} \\ 0 & 0 \end{bmatrix} \quad V_{wc}^b = \begin{bmatrix} v_{wc} \\ \omega_{wc} \end{bmatrix} \tag{3}$$

where v_{wc} and ω_{wc} represent the velocity of the origin and the angular velocity from Σ_w to Σ_c, respectively ([10] Chap. 2, eq. (2.55)).

Then, the fundamental representation of the relative rigid body motion g_{co} is described as follows [8].

$$V_{co}^b = -\mathrm{Ad}_{(g_{co}^{-1})} V_{wc}^b + V_{wo}^b \tag{4}$$

where V_{wo}^b is the body velocity of the target object. The notation $\mathrm{Ad}_{(g_{ab})}$ means the adjoint transformation associated with g_{ab} [10]. Roughly speaking, if both the camera and the target object move, then the relative rigid body motion g_{co} will be derived from the difference between the camera velocity V_{wc}^b and the target object velocity V_{wo}^b. In the case of the fixed camera configuration, the fundamental representation of the relative rigid body motion g_{co} can be rewritten as

$$V_{co}^b = V_{wo}^b, \tag{5}$$

because the camera is static, i.e. $V_{wc}^b = 0$ in the case of the fixed camera configuration.

B. Camera Model

To control the relative rigid body motion using visual information provided by a computer vision system, we derive the model of a pinhole camera with a perspective projection. Let λ be a focal length, $p_{oi} \in \mathcal{R}^3$ and $p_{ci} \in \mathcal{R}^3$ be coordinates of the target object's i-th feature point relative to Σ_o and Σ_c, respectively. Using a transformation of the coordinates, we have

$$p_{ci} = g_{co} p_{oi}, \tag{6}$$

where p_{ci} and p_{oi} should be regarded as $[p_{ci}^T \ 1]^T$ and $[p_{oi}^T \ 1]^T$ via the well-known representation in robotics, respectively (see, e.g. [10]).

The perspective projection of the i-th feature point onto the image plane gives us the image plane coordinate $f_i := [f_{xi} \ f_{yi}]^T \in \mathcal{R}^2$ as follows

$$f_i = \frac{\lambda}{z_{ci}} \begin{bmatrix} x_{ci} \\ y_{ci} \end{bmatrix} \tag{7}$$

where $p_{ci} := [x_{ci} \ y_{ci} \ z_{ci}]^T$. It is straightforward to extend this model to the m image points case by simply stacking the vectors of the image plane coordinate, i.e. $f := [f_1^T \ \cdots \ f_m^T]^T \in \mathcal{R}^{2m}$. We assume that multiple point features on a known object can be used.

C. Visual Feedback System with Fixed Camera

Here the brief summary of our prior work in [9] is given. In order to bring the actual relative rigid body motion g_{ho} to a given reference g_d in Fig. 1, we consider the control and estimation problems in the visual feedback systems. The following dynamic model which just comes from the fundamental representation of the actual relative rigid body motion (5) is considered.

$$\bar{V}_{co}^b = u_e \tag{8}$$

where $\bar{V}_{co}^b := [\bar{v}_{co}^T \ \bar{\omega}_{co}^T]^T$ and $\hat{\bar{V}}_{co}^b := \bar{g}_{co}^{-1}\dot{\bar{g}}_{co}$ mean the estimated body velocity. Here, $\bar{g}_{co} = (\bar{p}_{co}, e^{\hat{\bar{\xi}}\bar{\theta}_{co}})$ denotes the estimated relative rigid body motion. Then, the estimation error of the relative rigid body motion from Σ_c to Σ_o, i.e. the error between $\bar{g}_{co}$ and g_{co}, is defined as

$$g_{ee} = \bar{g}_{co}^{-1} g_{co}, \tag{9}$$

which is called the estimated object error. Using the notation $e_R(e^{\hat{\xi}\theta})$, the vector of the estimated object error is given by $e_e := [p_{ee}^T \ e_R^T(e^{\hat{\xi}\theta_{ee}})]^T$. The estimated object error system is represented by

$$V_{ee}^b = (g_{ee}^{-1}\dot{g}_{ee})^\vee = -\mathrm{Ad}_{(g_{ee}^{-1})} u_e + V_{wo}^b \tag{10}$$

where u_e is the input in order to converge the estimated value to the actual relative rigid body motion.

Similarly, we define the error between g_d and $\bar{g}_{ho}$, which is called the control error, as follows

$$g_{ec} = g_d^{-1} \bar{g}_{ho}, \tag{11}$$

where $\bar{g}_{ho}$ is the estimated relative rigid body motion from Σ_h to Σ_o and obtained from

$$\bar{g}_{ho} = g_{ch}^{-1} \bar{g}_{co}. \tag{12}$$

Here, we assume that g_{ch} is calculated by using the known motion, i.e. g_{wc} and g_{wh}, exactly. The vector of the control error is defined as $e_c := [p_{ec}^T \ e_R^T(e^{\hat{\xi}\theta_{ec}})]^T$. The control error system is described by

$$V_{ec}^b = (g_{ec}^{-1}\dot{g}_{ec})^\vee = -\mathrm{Ad}_{(\bar{g}_{ho}^{-1})} V_{wh}^b + u_e \tag{13}$$

where V_{wh}^b is the body velocity of the hand relative to Σ_w.

Combining (10) and (13), the visual feedback system with a fixed camera is constructed as follows

$$\begin{bmatrix} V_{ec}^b \\ V_{ee}^b \end{bmatrix} = \begin{bmatrix} -\mathrm{Ad}_{(\bar{g}_{ho}^{-1})} & I \\ 0 & -\mathrm{Ad}_{(g_{ee}^{-1})} \end{bmatrix} u_{ce} + \begin{bmatrix} 0 \\ I \end{bmatrix} V_{wo}^b \quad (14)$$

where

$$u_{ce} := \begin{bmatrix} V_{wh}^b \\ u_e \end{bmatrix} \quad (15)$$

denotes the input for the visual feedback system.

Let us define the error vector of the visual feedback system as $e_{ce} := \left[e_c^T \ e_e^T\right]^T$ which contains of the control error vector e_c and the estimation error vector e_e. It should be noted that if the vectors of the control error and the estimation error are equal to zero, then the estimated relative rigid body motion $\bar{g}_{ho}$ equals the reference one g_d and the estimated one $\bar{g}_{co}$ equals the actual one g_{co}, respectively. Moreover, the error between $\bar{g}_{ho}$ and g_{ho} can be also represented as g_{ee}, while g_{ee} are defined as the error between $\bar{g}_{co}$ and g_{co} in (9). Therefore, the actual relative rigid body motion g_{ho} tends to the reference one g_d when $e_{ce} \to 0$.

Now, we show the relation between the input and the output of the visual feedback system.

Result 1: [9] If $V_{wo}^b = 0$, then the visual feedback system (14) satisfies

$$\int_0^T u_{ce}^T \nu_{ce} d\tau \geq -\beta_{ce}, \quad \forall T > 0 \quad (16)$$

where ν_{ce} is defined as

$$\nu_{ce} := \begin{bmatrix} -\mathrm{Ad}_{(g_d^{-1})}^T & 0 \\ \mathrm{Ad}_{(e^{-\hat{\xi}\theta_{ec}})} & -I \end{bmatrix} e_{ce} \quad (17)$$

and β_{ce} is a positive scalar.

Let us take u_{ce} as the input and ν_{ce} as its output. Then, Result 1 would suggest that the visual feedback system (14) is *passive* from the input u_{ce} to the output ν_{ce} just formally as in the definition in [12].

III. Visual Feedback System with Uncertain Coordinate Frame

A. Uncertainty of Camera Coordinate Frame

In the visual feedback system with a fixed camera, camera calibration problems are long standing research issues [1]. In our approach, the uncertainty of the camera coordinate frame can be regarded as one of the camera calibration problems. Fig. 2 shows the coordinate frames for the visual feedback system with uncertainty of the camera coordinate frame. Let Σ_{c_0} be the nominal camera frame which can be known a priori, while Σ_c denotes the actual one which is unknown. The uncertainty of the camera coordinate frame is denoted as

$$g_\delta = g_{wc_0}^{-1} g_{wc} \quad (18)$$

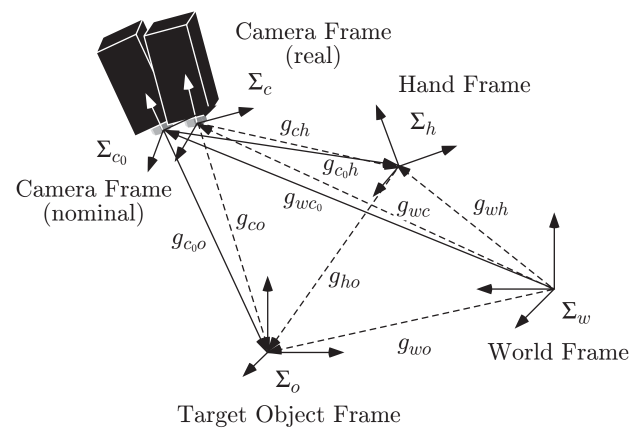

Fig. 2. Coordinate frames for the visual feedback system with uncertainty of the camera frame.

where g_{wc_0} is the rigid body motion from Σ_w to Σ_{c_0}. g_{c_0o} and g_{c_0h} are the relative rigid body motions from Σ_{c_0} to Σ_o and Σ_h, respectively.

From (12), we notice that g_{ho} may not be estimated exactly as follow

$$g_{c_0h}^{-1}\bar{g}_{co} = (g_\delta g_{ch})^{-1}\bar{g}_{co} \neq g_{ch}^{-1}\bar{g}_{co} (= \bar{g}_{ho}), \quad (19)$$

even if the vector of the estimated object error e_e equals to zero. Consequently, g_{ho} will not converge to g_d owing to the uncertainty of the camera coordinate frame, nevertheless e_{ce} tends to zero in the visual feedback system (14) with the passivity-based control law proposed in [9].

B. Estimated Hand Error System

Here we will consider the observer for estimating the relative rigid body motion g_{ch} in order to reduce the effect of uncertainty of the camera coordinate frame.

Based on the fundamental representation of the relative rigid body motion described as (4), the relation among Σ_w, Σ_c and Σ_h can be expressed as

$$V_{ch}^b = -\mathrm{Ad}_{(g_{ch}^{-1})} V_{wc}^b + V_{wh}^b = V_{wh}^b, \quad (20)$$

because the camera is static in the fixed camera configuration. We shall consider the following model from (20)

$$\bar{V}_{ch}^b = u_h \quad (21)$$

where u_h is the new input for reducing the estimation error between g_{ch} and $\bar{g}_{ch}$ which represents the estimated relative rigid body motion from Σ_c to Σ_h.

Similarly to the camera model and the image information mentioned in II-B, we can discuss the image information concerned with g_{ch} by replacing the target object frame Σ_o with the hand frame Σ_h. Hence, we define the image information of the hand and the estimated one as f_h and $\bar{f}_h$, respectively. It is assumed that both the image information of the target object and of the hand, i.e. f and f_h, are available from a single camera.

In order to compose the estimation error system between $\bar{g}_{ch}$ and g_{ch}, we call the estimated hand error system in this paper, the estimated hand error between $\bar{g}_{ch}$ and g_{ch} is defined as

$$g_{eh} = \bar{g}_{ch}^{-1} g_{ch}. \tag{22}$$

Using the notation $e_R(e^{\hat{\xi}\theta})$, the vector of the estimated hand error is given by $e_h := [p_{eh}^T \ e_R^T(e^{\hat{\xi}\theta_{eh}})]^T$. Then, the relationship between f_h and $\bar{f}_h$ can be given by

$$f_h - \bar{f}_h = J(\bar{g}_{ch}) e_h, \tag{23}$$

where $J(\bar{g}_{ch})$ plays the role of well-known image Jacobian and is defined in [8].

Differentiating (22) and multiplying it by g_{eh}^{-1}, we can obtain

$$\begin{aligned} g_{eh}^{-1}\dot{g}_{eh} &= -g_{eh}^{-1}\hat{\bar{V}}_{ch}^b \bar{g}_{ch}^{-1} g_{ch} + g_{eh}^{-1}\bar{g}_{ch}^{-1} g_{ch} \hat{V}_{ch}^b \\ &= -g_{eh}^{-1}\hat{u}_h g_{eh} + \hat{V}_{wh}^b. \end{aligned} \tag{24}$$

Furthermore, using the property concerning the adjoint transformation, the above equation can be transformed into the following

$$V_{eh}^b = V_{wh}^b - \mathrm{Ad}_{(g_{eh}^{-1})} u_h. \tag{25}$$

Eq. (25) represents the estimated hand error system.

C. Property of Visual Feedback System with Uncertain Coordinate Frame

If g_{co} and g_{ch} are measured, g_{ho} can be obtained without the effect of uncertainty of the camera coordinate frame. In order to reduce the effect of uncertainty, the estimated relative rigid body motion from Σ_h to Σ_o described in (12) should be replaced by

$$\bar{g}_{ho} = \bar{g}_{ch}^{-1} \bar{g}_{co}. \tag{26}$$

Then, the control error system is transformed into

$$V_{ec}^b = u_e - \mathrm{Ad}_{(\bar{g}_{ho}^{-1})} u_h. \tag{27}$$

Moreover, we redefine the estimated object error as

$$g_{ee} = \bar{g}_{co}^{-1} g_{co}. \tag{28}$$

Where the estimated object error system is same as (10).

Combining (10), (25) and (27), the visual feedback system with a fixed camera is constructed as follows

$$\begin{bmatrix} V_{ec}^b \\ V_{ee}^b \\ V_{eh}^b \end{bmatrix} = \begin{bmatrix} 0 & I & -\mathrm{Ad}_{(\bar{g}_{ho}^{-1})} \\ 0 & -\mathrm{Ad}_{(g_{ee}^{-1})} & 0 \\ I & 0 & -\mathrm{Ad}_{(g_{eh}^{-1})} \end{bmatrix} u_{ceh} + \begin{bmatrix} 0 \\ V_{wo}^b \\ 0 \end{bmatrix} \tag{29}$$

where

$$u_{ceh} := \begin{bmatrix} V_{wh}^b \\ u_e \\ u_h \end{bmatrix} \tag{30}$$

denotes the input for the visual feedback system.

Let us define the error vector of the visual feedback system (29) as $e := \begin{bmatrix} e_c^T & e_e^T & e_h^T \end{bmatrix}^T$. It should be noted that if the vectors of the control error, the estimated object error and the estimated hand error are equal to zero, then $\bar{g}_{ho}$, $\bar{g}_{co}$ and $\bar{g}_{ch}$ equal g_d, g_{co} and g_{ch}, respectively. Therefore, g_{ho} tends to g_d when $e \to 0$, while the visual feedback system has the uncertainty of the camera coordinate frame.

Lemma 1: If $V_{wo}^b = 0$, then the visual feedback system (29) satisfies

$$\int_0^T u_{ceh}^T \nu_{ceh} d\tau \geq -\beta_{ceh}, \quad \forall T > 0 \tag{31}$$

where ν_{ceh} is defined as

$$\nu_{ceh} := \begin{bmatrix} 0 & 0 & \mathrm{Ad}_{(e^{-\hat{\xi}\theta_{eh}})} \\ \mathrm{Ad}_{(e^{-\hat{\xi}\theta_{ec}})} & -I & 0 \\ -\mathrm{Ad}_{(g_d^{-1})}^T & 0 & -I \end{bmatrix} e \tag{32}$$

and β_{ceh} is a positive scalar.

Proof: Consider the following positive definite function

$$V_{ceh} = E(g_{ec}) + E(g_{ee}) + E(g_{eh}) \tag{33}$$

where $E(g) := \frac{1}{2}\|p\|^2 + \phi(e^{\hat{\xi}\theta})$ and $\phi(e^{\hat{\xi}\theta}) := \frac{1}{2}\mathrm{tr}(I - e^{\hat{\xi}\theta})$ which is the error function of the rotation matrix (see e.g. [11]). Differentiating (33) with respect to time yields

$$\dot{V}_{ceh} = e^T \begin{bmatrix} \mathrm{Ad}_{(e^{\hat{\xi}\theta_{ec}})} & 0 & 0 \\ 0 & \mathrm{Ad}_{(e^{\hat{\xi}\theta_{ee}})} & 0 \\ 0 & 0 & \mathrm{Ad}_{(e^{\hat{\xi}\theta_{eh}})} \end{bmatrix} \begin{bmatrix} V_{ec}^b \\ V_{ee}^b \\ V_{eh}^b \end{bmatrix} \tag{34}$$

where we use the property $\dot{\phi}(e^{\hat{\xi}\theta}) := e^{\hat{\xi}\theta}\omega$. Observing the skew-symmetry of the matrices $\hat{p}_{ec}$, $\hat{p}_{ee}$ and $\hat{p}_{eh}$, the above equation along the trajectories of the system (29) can be transformed into

$$\begin{aligned} \dot{V}_{ceh} &= e^T \begin{bmatrix} 0 & \mathrm{Ad}_{(e^{\hat{\xi}\theta_{ec}})} & -\mathrm{Ad}_{(g_d^{-1})} \\ 0 & -I & 0 \\ \mathrm{Ad}_{(e^{\hat{\xi}\theta_{eh}})} & 0 & -I \end{bmatrix} u_{ce} \\ &= u_{ceh}^T \nu_{ceh}. \end{aligned} \tag{35}$$

Integrating (35) from 0 to T, we can obtain

$$\int_0^T u_{ceh}^T \nu_{ceh} d\tau \geq -V_{ceh}(0) := -\beta_{ceh} \tag{36}$$

where β_{ceh} is the positive scalar which only depends on the initial states of g_{ec}, g_{ee} and g_{eh}. ■

Remark 1: In the visual feedback system, $p_{ec}^T \hat{\omega}_{wc} p_{ec} = 0$, $p_{ee}^T \hat{\omega}_{ue} p_{ee} = 0$, $p_{eh}^T \hat{\omega}_{uh} p_{eh} = 0$ holds. This skew-symmetric property is analogous to the one of the robot dynamics, i.e. $x^T(\dot{M} - 2C)x = 0$, $\forall x \in \mathcal{R}^n$ (where $M \in \mathcal{R}^{n\times n}$ is the manipulator inertia matrix and $C \in \mathcal{R}^{n\times n}$ is the Coriolis matrix [10]). Thus, Lemma 1 suggests that the visual feedback system (29) is *passive* from the input u_{ceh} to the output ν_{ceh} as in the definition in [12].

IV. Dynamic Visual Feedback Control

A. Dynamic Visual Feedback System

The manipulator dynamics can be written as

$$M(q)\ddot{q} + C(q,\dot{q})\dot{q} + g(q) = \tau + \tau_d \quad (37)$$

where q, $\dot{q}$ and $\ddot{q}$ are the joint angles, velocities and accelerations, respectively. τ is the vector of the input torques and τ_d represents a disturbance input.

The body velocity of the hand V_{wh}^b is given by

$$V_{wh}^b = J_b(q)\dot{q} \quad (38)$$

where $J_b(q)$ is the manipulator body Jacobian [10]. We define the reference of the joint velocities as $\dot{q}_d := J_b^{\dagger}(q)u_d$ where u_d represents the desired body velocity of the hand. Thus, V_{wh}^b in (30) should be replaced by u_d.

Let us define the error vector with respect to the joint velocities of the manipulator dynamics as $\xi := \dot{q} - \dot{q}_d$. Now, we consider the passivity-based dynamic visual feedback control law as follows.

$$\tau = M(q)\ddot{q}_d + C(q,\dot{q})\dot{q}_d + g(q) + u_\xi + J_b^T(q)\left(\mathrm{Ad}^T_{(g_d^{-1})}e_c + \mathrm{Ad}_{(e^{-\hat{\xi}\theta_{eh}})}e_h\right). \quad (39)$$

The new input u_ξ is to be determined in order to achieve the control objectives.

Using (29), (37) and (39), the visual feedback system with manipulator dynamics (we call the dynamic visual feedback system) can be derived as follows

$$\begin{bmatrix} \dot{\xi} \\ V_{ec}^b \\ V_{ee}^b \\ V_{eh}^b \end{bmatrix} = \begin{bmatrix} -M^{-1}C\xi + M^{-1}J_b^T\mathrm{Ad}^T_{(g_d^{-1})}e_c \\ \quad +M^{-1}J_b^T\mathrm{Ad}_{(g_{eh}^{-1})}e_h \\ -\mathrm{Ad}_{(\bar{g}_{ho}^{-1})}J_b\xi \\ 0 \\ -J_b\xi \end{bmatrix} + \begin{bmatrix} M^{-1} & 0 & 0 & 0 \\ 0 & 0 & I & -\mathrm{Ad}_{(\bar{g}_{ho}^{-1})} \\ 0 & 0 & -\mathrm{Ad}_{(g_{ee}^{-1})} & 0 \\ 0 & I & 0 & -\mathrm{Ad}_{(g_{eh}^{-1})} \end{bmatrix} u + \begin{bmatrix} M^{-1} & 0 \\ 0 & 0 \\ 0 & I \\ 0 & 0 \end{bmatrix} w \quad (40)$$

where $x := [\xi^T \ e^T]^T$ and $u := [u_\xi^T \ u_d^T \ u_e^T \ u_h^T]^T$. We define the disturbance of dynamic visual feedback system as $w := \left[\tau_d^T \ (V_{wo}^b)^T\right]^T$. Before constructing the dynamic visual feedback control law, we derive an important lemma.

Lemma 2: If $w = 0$, then the dynamic visual feedback system (40) satisfies

$$\int_0^T u^T\nu d\tau \geq -\beta, \quad \forall T > 0 \quad (41)$$

where

$$\nu := Nx, \ N := \begin{bmatrix} I & 0 & 0 & 0 \\ 0 & 0 & 0 & \mathrm{Ad}_{(e^{-\hat{\xi}\theta_{eh}})} \\ 0 & \mathrm{Ad}_{(e^{-\hat{\xi}\theta_{ec}})} & -I & 0 \\ 0 & -\mathrm{Ad}^T_{(g_d^{-1})} & 0 & -I \end{bmatrix}.$$

Due to space limitations, the proof is only sketched. By using the following positive definite function, the proof can be completed by invoking Lemma 1

$$V(x) = \frac{1}{2}\xi^T M\xi + E(g_{ec}) + E(g_{ee}) + E(g_{eh}). \quad (42)$$

Remark 2: Similarly to Lemma 1, Lemma 2 would suggest that the dynamic visual feedback system is *passive* from the input u to the output ν just formally. From Lemma 2, we can state that the dynamic visual feedback system (40) preserves the passivity of the visual feedback system (29). This is one of main contributions of this work.

B. Stability Analysis for Dynamic Visual Feedback System

It is well known that there is a direct link between passivity and Lyapunov stability. Thus, we propose the following control input.

$$u = -K\nu = -KNx, \ K := \begin{bmatrix} K_\xi & 0 & 0 & 0 \\ 0 & K_c & 0 & 0 \\ 0 & 0 & K_e & 0 \\ 0 & 0 & 0 & K_h \end{bmatrix} \quad (43)$$

where $K_\xi := \mathrm{diag}\{k_{\xi 1}, \cdots, k_{\xi n}\}$ denotes the positive gain matrix for each joint axis. $K_c := \mathrm{diag}\{k_{c1}, \cdots, k_{c6}\}$, $K_e := \mathrm{diag}\{k_{e1}, \cdots, k_{e6}\}$ and $K_h := \mathrm{diag}\{k_{h1}, \cdots, k_{h6}\}$ are the positive gain matrices of x, y and z axes of the translation and the rotation for the control error, the estimated object one and the estimated hand one, respectively. The result with respect to asymptotic stability of the proposed control input (43) can be established as follows.

Theorem 1: If $w = 0$, then the equilibrium point $x = 0$ for the closed-loop system (40) and (43) is asymptotic stable.

It can be proved by the energy function (42) as a Lyapunov function. We omit the proof due to space limitations. Considering the manipulator dynamics, Theorem 1 shows the stability via Lyapunov method for the full 3D dynamic visual feedback system. It is interesting to note that stability analysis is based on the passivity as described in (41).

C. L_2-gain Performance Analysis for Dynamic Visual Feedback System

Based on the dissipative systems theory, we consider L_2-gain performance analysis for the dynamic visual feedback system (40) in one of the typical problems, i.e. the disturbance attenuation problem. Now, let us define

$$P := N^T KN - \frac{1}{2\gamma^2}W - \frac{1}{2}I \quad (44)$$

where $\gamma \in \mathcal{R}$ is positive and $W := \mathrm{diag}\{I, 0, I, 0\}$. Then we have the following theorem.

Theorem 2: Given a positive scalar γ and consider the control input (43) with the gains K_ξ, K_c, K_e and K_h such that the matrix P is positive semi-definite, then the closed-loop system (40) and (43) has L_2-gain $\leq \gamma$.

The proof is omitted due to space limitations. Theorem 2 can be proved using the energy function (42) as a

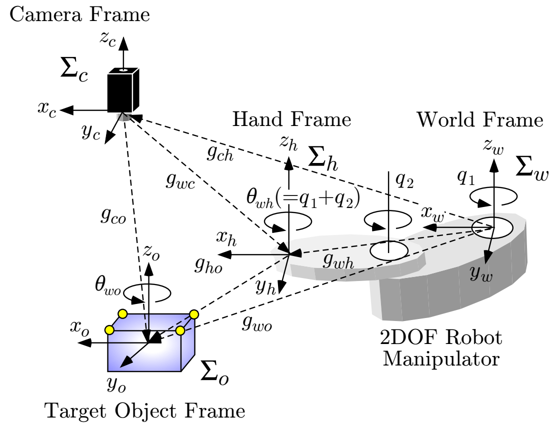

Fig. 3. Coordinate frames for dynamic visual feedback system with two degree of freedom manipulator.

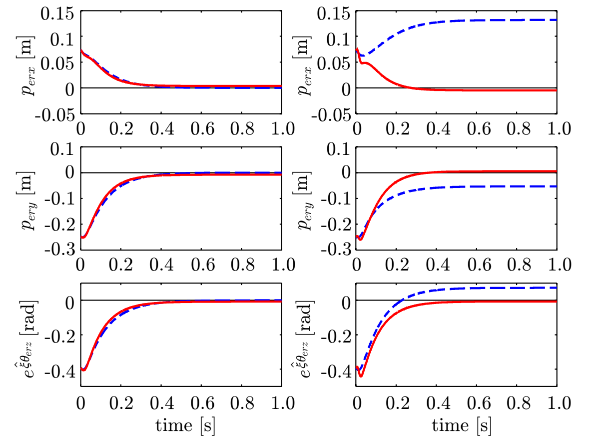

Fig. 4. Errors for the control objective with the proposed control law (solid) and with the previous control law in [9] (dashed). Left side: without the uncertainty, right side: with the uncertainty.

storage function for L_2-gain performance analysis. The L_2-gain performance analysis of the dynamic visual feedback system is discussed via the dissipative systems theory. In H_∞-type control, we can consider some problems by establishing the adequate generalized plant. This paper has discussed L_2-gain performance analysis for the disturbance attenuation problem. The proposed strategy can be extended for the other-type of generalized plants of the dynamic visual feedback systems.

V. Simulation

The simulation results on the two degree-of-freedom manipulator as depicted in Fig. 3 are shown in order to understand our proposed method simply, though it is valid for 3D visual feedback systems. We use the reference of the relative rigid body motion as a constant value, i.e. $p_d = [0\ 0\ \ -0.81]^T$ and $e^{\hat{\xi}\theta_d} = I$. Specifically, we consider the set point problem, i.e. the target object is static, in order to compare the performance of the proposed control law and the previous one discussed in [9] clearly. The uncertainty of the camera coordinate frame is chosen as $g_\delta = ([0.05\ \ -0.02\ 0.04]^T, e^{([-\pi/24\ \ \pi/25\ \ \pi/18]^T)^\wedge})$.

Fig. 5 shows the error between g_{ho} and g_d which is defined as $g_{er} := g_d^{-1} g_{ho}$. The errors for the control objective with the proposed control law tend to zero in both cases, while the previous control law with the uncertainty can not achieve the control objective. Hence, the proposed control law is valid for the uncertainty of the camera coordinate frame. In the case of the tracking problem with the moving target object, it can be confirmed by the same simulation in [9].

VI. Conclusions

This paper dealt with the dynamic visual feedback control with the uncertainty of the camera coordinate frame, which is not limited to the orientation around the optical axis. We can derive that the dynamic visual feedback system preserves the passivity of the visual feedback system by the same strategy in our previous works [6]-[9]. Stability and L_2-gain performance analysis for the dynamic visual feedback system are discussed based on passivity with the energy function. The validity of the proposed control law is confirmed by comparing the simulation resluts.

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