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Optimization-Based PI/PID Control for a Binary Distillation Column

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Abstract — **This paper presents the PI/PID control of a binary distillation column via a genetic searching algorithm (GSA). The time-domain design criterion, expressed as an integral of the squared error, is reformulated in the frequency-domain using the Parseval's relation and Padé approximation. A genetic algorithm is then used to search over the stability region in the controller parameter space for the best settings to minimize the design criterion. Our results indicate that the genetic algorithm can provide better solutions for the PI/PID control schemes as compared to those using the single-loop and multi-loop Ziegler-Nichols tuning methods. We found that the GSA is easier to use than traditional optimization techniques. In addition, no knowledge of complex mathematics is required to use the GSA effectively.**

I. Introduction

The distillation column is probably one of the most popular and important processes studied in chemical engineering literature. Distillation is used in many chemical processes for separating feed streams and for purification of final and intermediate products. It is known that high-purity distillation columns are highly nonlinear and the composition interaction between the stages due to the counter flow of vapor and liquid is also large (e.g., [1], [2]). Thus, the control of columns to give multiple products of constant composition is very difficult. It is perhaps one of the most challenging problems in process control. Various methods of controlling distillation columns have been reported in the literature (e.g., internal model control method [1], ratio control [3], [4], non-interacting control [3]-[5], μ-synthesis method [6], linear-quadratic Gaussian with loop transfer recovery (LQG/LTR) method [7], fuzzy control [8]). Despite many advanced/modern control design techniques developed in recent years, PI and PID controllers still represent the majority of the controllers used in industry. It is, therefore, interesting to develop an appropriate approach to adjust the parameters of the PI and PID controllers for the control of distillation columns.

In this paper, the PI/PID control of a binary distillation column is studied. Although most columns handle multicomponent feeds, many can be approximated by binary (or pseudo binary) mixtures. However, due to the strong cross coupling and significant time delays inherent in the distillation column, the simultaneous control of overhead and bottoms composition using reflux and steam flow as the control variables is still difficult. Attempting to use tuning rules such as the well-known Ziegler-Nichols rule to adjust the PI/PID controller for each individual loop often leads to deteriorated control performance for the overall system. This is because these tuning rules do not take the interaction between the control loops into consideration ([3], [9]).

In this paper, instead of using a tuning rule we focus on using a genetic algorithm to determine the optimal PI/PID control settings for a pilot-scale binary distillation column model. A genetic searching algorithm (GSA) is chosen to test whether such algorithms offer any practical advantages over traditional optimization algorithms. For the test example, we select the Wood and Berry binary distillation model [4] because it was derived from an operational distillation column and is representative of many chemical control processes. Determination of the optimal gain settings for the PI/PID controllers with the Wood and Berry model serves as our optimization problem. A parameter space method [10], [11] is used to determine the stability region in the controller's parameter plane (space), i.e., the search space for the GSA. Limiting the search of the controller parameters to such a region guarantees the system stability. A standard integral of the squared error (ISE) is used as our control criterion, which can be analytically expressed in terms of the controller gains via the Parseval's Theorem. A second-order Padé approximation is also used to approximate the time delays involved in the model. Based on the criterion, a GSA is implemented and used to search over the stability region for the controller gains that achieved the best performance. The simulation results are given, and they are also compared with those using the single/multi-loop Ziegler-Nichols tuning methods. We found that the GSA is easier to use than traditional optimization techniques.

II. The Binary Distillation Column

The binary distillation column we studied basically separates a mixture of two components having different

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boiling points. Moreover, the process is greatly enhanced by forcing separation to occur in stages within the column. One way to accomplish this is by directing a liquid stream of high purity distillate, or reflux, back into the column through an arrangement of sieve trays. For example, consider a feed mixture consisting of component A with a low boiling point and component B with a higher boiling point. As feed enters the column, some of the mixture will immediately vaporize. The remainder will concentrate at the bottom of the column where further boiling is induced by the re-boiler. Consequently, a vapor stream rich in component A is created and rises in opposition to the liquid stream. The sieve tray arrangement creates “pockets” in which the vapor stream’s contact with the liquid stream is maximized. This increases the likelihood that the liquid stream will entrain the residual B component from the vapor stream and direct it back toward the bottom of the column. Most importantly, the sieve tray arrangement provides the mechanism needed for multi-staged boiling. Each sieve tray can be viewed as a separate boiler of increased efficiency as the vapor stream rises toward the top of the column.

While the benefits of reflux to the column’s efficiency are certainly to be desired, the resulting dynamics greatly complicates the simultaneous control of the bottoms and overhead product compositions. Consider the case in which it is desired to increase the overhead product purity by increasing reflux flow with steam flow held constant. Increasing reflux will certainly remove more of the residual component B from the vapor stream, but it will also increase the amount of the component A in the bottoms product. It can be seen that there is a dependent relationship between steam flow and reflux flow that affects both product compositions. This dependence is also apparent in the mathematical model of the distillation column, which can be expressed as

$$\begin{bmatrix} x_D(s) \\ x_B(s) \end{bmatrix} = \begin{bmatrix} P_{11}(s) & P_{12}(s) \\ P_{21}(s) & P_{22}(s) \end{bmatrix} \begin{bmatrix} R(s) \\ S(s) \end{bmatrix} \tag{1}$$

where the outputs $x_D(s)$ and $x_B(s)$ are the overhead and bottom product mole fraction of methanol, respectively, while $R(s)$ is the reflux flow rate and $S(s)$ is the re-boiler steam flow rate. The transfer functions $P_{ij}(s) = Ke^{-\lambda s}/\tau s+1$ (i, j =1, 2) have the parameter values shown below:

Parameters	K	λ	τ
$P_{11}(s)$	12.8	1.0	16.7
$P_{12}(s)$	-18.9	3.0	21.0
$P_{21}(s)$	6.60	7.0	10.9
$P_{22}(s)$	-19.4	3.0	14.4

Note that R. K. Wood and M. W. Berry developed the above mathematical model using a pilot scale replica of an actual binary distillation column [4]. Obviously, the control of the distillation column transfer matrix $P(s) = [P_{ij}(s)]$ is complicated by the dependent relationship that exists between the steam flow and reflux flow. The interaction between the two loops makes controller tuning more difficult. To compensate for undesirable control loop interactions, consider the block diagram of the decoupling scheme [pp. 463-467, 3] shown in Fig. 1, where $D_{12}(s)$ and $D_{21}(s)$ are the decouplers used for the decoupling purpose, while the feedback controllers $C_1(s)$ and $C_2(s)$ are to be designed to provide a satisfactory set point tracking for both channels. From Fig. 1, it is easy to see that in order to eliminate the effect of overhead and bottoms composition control actions on the bottoms and overhead composition, respectively; the two decouplers should be chosen as

$$D_{21}(s) = -\frac{P_{21}(s)}{P_{22}(s)}; D_{12}(s) = -\frac{P_{12}(s)}{P_{11}(s)} \tag{2}$$

Note that alternative decoupling techniques are possible (e.g., [2], [3]). In this paper, we use the decoupling scheme shown in Fig. 1 before attempting to design feedback controllers. Using the two decouplers given in Eq. (2), the system equation becomes

$$\begin{bmatrix} x_D(s) \\ x_B(s) \end{bmatrix} = \begin{bmatrix} T_{11}(s) & 0 \\ 0 & T_{22}(s) \end{bmatrix} \begin{bmatrix} u_1(s) \\ u_2(s) \end{bmatrix} \tag{3}$$

where $T_{11}(s)=P_{11}(s)+P_{12}(s)D_{21}(s)$, $T_{22}(s)=P_{22}(s)+P_{21}(s)D_{12}(s)$. Note that in reality, the exact cancellations of coupling can seldom be realized because of the modeling error and various uncertainties in the process. However, Seborg *et al.* claimed that even approximate cancellations could be very beneficial in reducing control loop interactions while simplifying the controllers [3]. Besides that, the approximated “ideal” decouplers should also work fine except for high-purity distillation columns with large relative gains [12]. Therefore, we will use the decoupling scheme described above. Since the purpose of this work is to study the feasibility and advantages of using a genetic algorithm to tune the controller parameters, a perfect decoupling is assumed so that we can compare the available simulation results with some other methods discussed later. After decoupling, we then close the loops and try to design the controllers $C_1(s)$ and $C_2(s)$ to control $x_D(s)$ and $x_B(s)$ independently. Our objective is to design the PI/PID controllers to minimize the integral of the squared error (ISE) in both channels. That is, our tracking performance is formulated as

$$\min_{\{C_1(s),C_2(s)\}} \int_0^\infty e^T(t)e(t)dt \tag{4}$$

where the vector *e(t)* is defined as $e(t)=[e_1(t)\ e_2(t)]^T$ (the superscript T means the transpose) and $e_i(t)$ represents the tracking error in channel i (i =1,2).

III. Design Criterion Reformulation

After the system is decoupled, the ISE criterion in each loop can be rewritten as

$$J_i = \min_{\{C_i(s)\}} \int_0^{\infty} e_i^2(t)dt \tag{5}$$

From Fig. 1, we also have $e_i(s) = r_i(s)/(1+T_{ii}(s)C_i(s))$ where $T_{ii}(s)$ represents the decoupled plant transfer function in channel i (i =1, 2). By using the Parseval's Theorem [13], Eq. (5) can be expressed in the frequency-domain as

$$ISE = \int_0^{\infty} e_i^2(t)dt = \frac{1}{2\pi j}\int_{-j\infty}^{j\infty} e_i(-s)e_i(s)ds \tag{6}$$

and $e_i(-s)e_i(s)$ can be further expressed as

$$e_i(-s)e_i(s) = \frac{r_i(-s)r_i(s)}{(1+T_{ii}(-s)C_i(-s))(1+T_{ii}(s)C_i(s))} \triangleq \frac{B(-s)B(s)}{A(-s)A(s)} \tag{7}$$

where B(s) and A(s) are Hurwitz polynomials in s. It is clear that the presence of time delays in $T_{ii}(s)$ prevents us from obtaining the last expression in Eq. (7). That is, we cannot find the polynomials A(s) and B(s). To overcome this problem, we can approximate the time delay using a 2nd-order Padé approximation [3], [14] given below

$$e^{-\alpha s} \approx \frac{1-\frac{\alpha s}{2}+\frac{\alpha^2 s^2}{12}}{1+\frac{\alpha s}{2}+\frac{\alpha^2 s^2}{12}} \tag{8}$$

to replace all the time delays involved in $T_{ii}(s)$. After such an approximation, we can then easily obtain B(s) = $\sum_{k=0}^{7} b_k s^k$ and A(s) = $\sum_{k=0}^{8} a_k s^k$, where the coefficients a_i, b_j are functions of the PI/PID gains. Note that the order of the polynomial A(s) is 6 if a first-order Padé approximation is used, and its order becomes 10 for a third-order approximation.

Using [15] with some derivations, the criterion Eq. (6) can be further expressed as ISE = $(-1/2\ a_8)\ (|\Omega_1|/|\Omega|)$, where the notation $|\cdot|$ means the determinant and the matrices Ω_1, Ω are defined, respectively, as

$$\Omega = \begin{bmatrix} a_0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ a_2 & a_1 & a_0 & 0 & 0 & 0 & 0 & 0 \\ a_4 & a_3 & a_2 & a_1 & a_0 & 0 & 0 & 0 \\ a_6 & a_5 & a_4 & a_3 & a_2 & a_1 & a_0 & 0 \\ a_8 & a_7 & a_6 & a_5 & a_4 & a_3 & a_2 & a_1 \\ 0 & 0 & a_8 & a_7 & a_6 & a_5 & a_4 & a_3 \\ 0 & 0 & 0 & 0 & a_8 & a_7 & a_6 & a_5 \\ 0 & 0 & 0 & 0 & 0 & 0 & a_8 & a_7 \end{bmatrix}$$

$$\Omega_1 = \begin{bmatrix} a_0 & 0 & 0 & 0 & 0 & 0 & 0 & d_0 \\ a_2 & a_1 & a_0 & 0 & 0 & 0 & 0 & d_1 \\ a_4 & a_3 & a_2 & a_1 & a_0 & 0 & 0 & d_2 \\ a_6 & a_5 & a_4 & a_3 & a_2 & a_1 & a_0 & d_3 \\ a_8 & a_7 & a_6 & a_5 & a_4 & a_3 & a_2 & d_4 \\ 0 & 0 & a_8 & a_7 & a_6 & a_5 & a_4 & d_5 \\ 0 & 0 & 0 & 0 & a_8 & a_7 & a_6 & d_6 \\ 0 & 0 & 0 & 0 & 0 & 0 & a_8 & d_7 \end{bmatrix} \tag{9}$$

where

$d_0 = b_0^2,\ d_1 = -b_1^2 + 2b_0b_2,\ d_2 = b_2^2 - 2b_1b_3 + 2b_0b_4,$

$d_6 = b_6^2 - 2b_5b_7$

$d_3 = -b_3^2 + 2b_2b_4 - 2b_1b_5 + 2b_0b_6,\ d_4 = b_4^2 - 2b_3b_5 + 2b_2b_6 - 2b_1b_7$

$d_7 = -b_7^2\ \ d_5 = -b_5^2 + 2b_4b_6 - 2b_3b_7$

By substituting the above two into $-|\Omega_1| / (2a_8)|\Omega|$, we obtain the ISE, which is a function of a_i, b_j and these coefficients are again functions of the PI gains K_p and K_i, (or the PID gains K_p, K_i, and K_d). Therefore, our objective is to use a genetic algorithm to search over the set of PI/PID gains in the controller's parameter plane (space) that maintains the system stability and, at the same time, minimizes the ISE criterion.

III. The Stability Region

Due to the popularity and wide acceptance of PID control among process industries, we use this control for the distillation column. Consider the system shown in Fig. 1 with the controllers $C_1(s)= k_{P1}+k_{I1}/s+k_{D1}s$ and $C_2(s)=k_{P2}+k_{I2}/s+k_{D2}s$ where k_{Pi}, k_{Ii}, and k_{Di} (i=1,2) are the proportional gain, integral gain, and derivative gain, respectively. To achieve the optimal tracking performance (i.e., Eq. (4)), we will find the optimal (k_{Pi}, k_{Ii}, k_{Ii}) (i=1,2) by using a genetic searching algorithm. Before performing the search, we use the Siljak's parameter plane method to find the set of all PID gains that maintain the system stability. That is, for each loop of the decoupled system (i.e., Eq. (3)), we determine the stability region in the k_P k_{Ii} k_{Di}-space after inserting the PID controller in that loop. The search via our genetic algorithm will then be limited to the stability region for optimality of the error criterion. For details about the method to get the stability region in the controller parameter space, please refer to [10,11].

IV. The GenetiC SearchinG Algorithm

A genetic algorithm is a function optimization technique based on the ideas of evolutionary genetics and the natural selection process [16], [17]. Ideally, the algorithm creates new population members (children) who, with each successive generation, are better equipped to succeed in their present environment (or, have a higher degree of fitness). In terms of function optimization, this process equates to incorporating various weighted operators to randomly select values for the independent variable(s) that have a high probability of producing successively higher values of the dependent variable until some global optimum is reached. The implementation of a basic genetic algorithm (GA) can be found in the literature. Our study focuses on using a genetic approach to determine the optimal settings for the classical PID controller. A genetic searching algorithm is chosen to test whether such algorithms offer any practical advantages over traditional optimization algorithms. Desirable properties of the GSA include: (1) No reliance on integral or derivative operators to steer its operation. This allows the GSA to search for a function's global minimum or maximum without regard to the continuity of the function, (2) Many points within the search space are evaluated each generation. This reduces the probability that the algorithm will converge to some local minimum or maximum within the search space, (3) No dependence on the selection of a starting point within the search space to initialize the search process, and (4) Ease of use. The GSA does not require detailed knowledge of complex optimization techniques.

In addition to the basic genetic operators, we choose to incorporate two intuitive operators of our own. The first is designed to eliminate an adverse side effect of elitism, that is, the general population members are excluded from mating with the elite population set. After some experimentation, we found that elitism leads to the average fitness levels of both populations reaching plateau fitness values for many generations. This is minimized by inserting a copy of the fitness elite population member in place of the least fit general population member prior to selection of offspring producing parents. The second operator is designed to force the GSA to focus a small amount of its attention on the search space region immediately surrounding the coordinates of the best elite population member for a particular generation. It makes sense that if a good solution is obtained at a particular point that perhaps a better solution exists close by. Rather than relying solely on the crossover and mutation operators to reach this conclusion randomly, it is much more efficient to strategically manipulate the lower order bit positions of the best elite population member chromosome. This manipulation produces a small number of children whose search space coordinates are very close to the best solution obtained for the previous generation.

V. Results analysis

1. Tuning Based on the GSA

Based on the implemented GSA described in Section IV, the algorithm searched for the optimal (k_P, k_i, k_d) in the stability region to maximize the fitness function. This is applied to each loop separately. Results were obtained after performing five consecutive searches for each control loop with the number of generations set at 50 and population size set at 100. For the PI control scheme (i.e., $k_d = 0$), we found that the GSA has no difficulty converging to the same solution during separate runs of the algorithm. However, the GSA showed less repeatability for the PID control scheme, giving three possible solutions for T_{11} and two possible solutions for T_{22}. Despite the loss of repeatability, each of these PID settings yields better performance than optimally tuned PI controllers as expected. The results we tested can be summarized as follows:

- PI Controller Results

Parameters	Kp	Ki	ISE
T_{11} - loop	0.5524	0.07478	2.0284
T_{22} - loop	-0.1651	-0.02118	4.5179

- PID Controller Results

Example 1				
Parameters	Kp	Ki	Kd	ISE
T_{11} - loop	0.6212	0.1569	0.4647	1.4348
T_{22} - loop	-0.1825	-0.04167	-0.3139	3.3318
Example 2				
Parameters	Kp	Ki	Kd	ISE
T_{11} - loop	0.5994	0.1474	0.4216	1.4389
T_{22} - loop	-0.1826	-0.03533	-0.2728	3.3479
Example 3				
Parameters	Kp	Ki	Kd	ISE
T_{11} - loop	0.6306	0.1373	0.3736	1.4535
T_{22} - loop	N/A	N/A	N/A	N/A

Error response plots for the best gain setting obtained for each control scheme are shown in Fig. 2 and Fig. 3.

2. Comparison Study

An accurate comparison of the GSA solutions to results obtained by others is difficult for a number of reasons. There are a number of criteria that can be used to evaluate a controller's performance. Gain settings that provide optimal performance for the ISE criterion may not provide the same performance for some other criterion. Differences in test methods also make comparison difficult. For example, Wood and Berry evaluated the performance of their PI controllers empirically using a different performance criterion [4]. A "rough" comparison of the GSA PI controller results to the single-loop Ziegler-Nichols method (single-loop/Z-N) and multi-loop Ziegler-Nichols method (multi-loop/Z-N) [3], using the same ISE criterion, is shown in the following table. The results are also shown in Figs. 4 and 5. These comparisons show that our results seem better.

Tuning Method	ISE (T_{11} – loop)	ISE (T_{22} – loop)
GSA	2.028	4.518
Single-loop/Z-N	2.069	5.009
Multi-loop/Z-N	10.839	4.653

VI. CONCLUSION

PI and PID control of the Wood and Berry distillation column via a genetic searching algorithm has been presented in this study. The 2x2 multivariable plant is first decoupled by two decouplers so that the system can be converted into two independent SISO subsystems. We use a PI/PID controller to control each loop separately. The time-domain tracking error criterion (ISE) is reformulated by using Parseval's theorem, and the set of the controller gains to maintain each subsystem's stability is also identified in the parameter plane (space). We then search for the optimal PI/PID gains in the stability region to minimize the ISE criterion. A genetic algorithm was implemented and used in the searching process. The simulations show small tracking errors for both channels. Our results also give a better performance when compared with the single-loop/Z-N and multiloop/Z-N approaches. The proposed method can be easily used to control systems where the controller structure is fixed with several adjustable parameters to be determined.

We are able to show that the GSA is certainly capable of tuning PI and PID controllers for this particular application. Our GSA gives good results and is very easy to use. Once stability regions and a performance criterion are established, the operator needs only to initialize the GSA parameters and perform the search. No knowledge of complex mathematics is required to use the GSA effectively. However, we did not find enough evidence to prove or disprove the hypothesis that GSAs provide superior search results when compared to other optimization techniques due to the limited search space in this example. While we are able to obtain better performance using PID controllers tuned with the GSA, we are not able to guarantee that these are the best possible gain settings.

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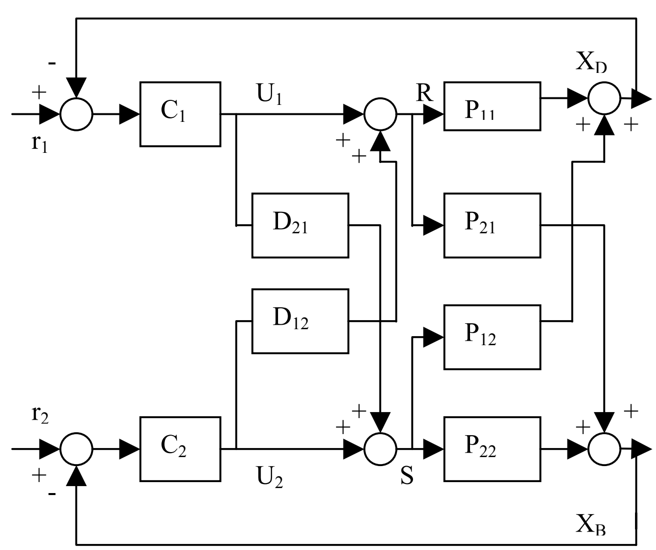

Fig. 1 Block diagram of the model showing the controllers and decouplers.

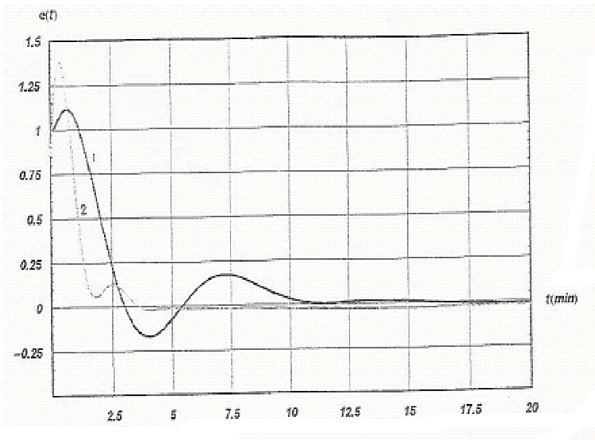

Fig 2 PI versus PID control for overhead loop (PI: curve labeled 1; PID: curve labeled 2).

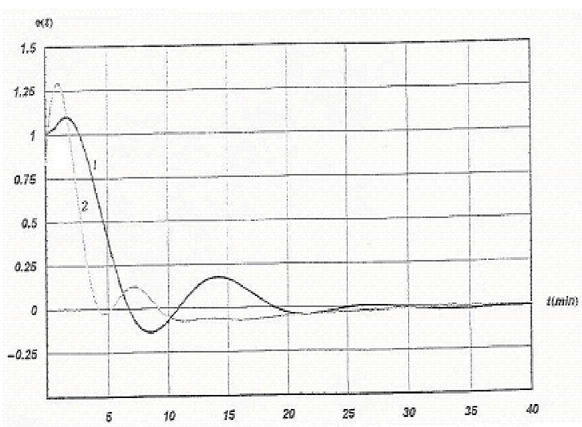

Fig. 3 PI versus PID control for bottom loop (PI: curve labeled 1; PID: curve labeled 2).

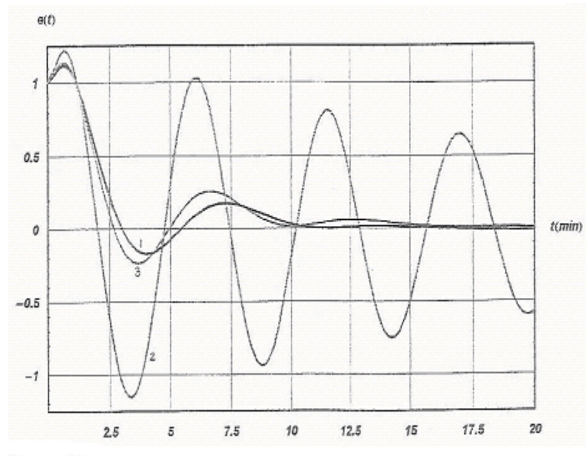

Fig. 4 Performance comparison of PI tuning for overhead (GSA-1; Single-loop/Z-N: 2; Multi-loop/Z-N: 3).

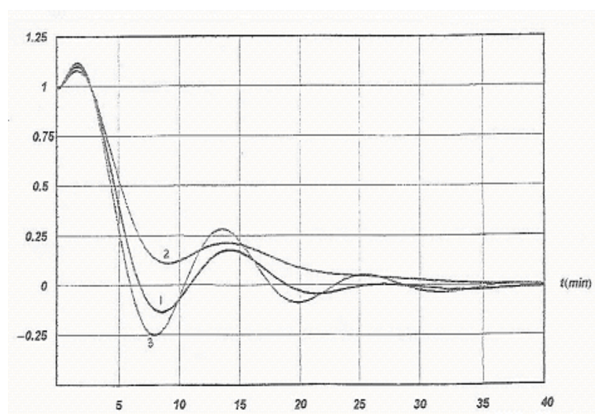

Fig. 5 Performance comparison of PI tuning for bottoms loop (GSA-1; Single-loop/Z-N: 2; Multi-loop/Z-N: 3).