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A Parametric Approach to Robust State and Parameter Estimation for a Certain Class of Nonlinear Systems

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***Abstract*— This work presents an observer design methodology which guarantees stability as well as bounds for performance for simultaneous state and parameter estimation for a certain class of nonlinear systems. The approach is based upon the idea that the unknown parameters can be treated as augmented states of the system and stability of the state and parameter estimator is guaranteed by application of a variation of Kharitonov's theorem. Certain performance criteria can also be met with the design approach via a change of coordinates followed by application of Kharitonov's theorem. The presented technique is illustrated with an example where simultaneous state and parameter estimation is performed.**

I. INTRODUCTION

High-performance estimation and control techniques are needed for chemical processes due to increased demands on productivity, product quality, and environmental requirements. Furthermore, key process parameters frequently represent unknown or poorly known time-varying disturbances that are particularly encountered in areas such as reaction engineering, bioprocess engineering, and environmental engineering [1]. Therefore, there is a need for accurate estimation of unmeasurable process variables and key process parameters [2], [3] which can be performed using an observer. In the case of linear systems without model uncertainty, a Luenberger observer or a Kalman filter can be used for estimating unmeasurable state variables.

The focus of this work is on systems that are linear in their state variables, but where the drift vector field is in general a nonlinear function of a finite-dimensional unknown parameter vector (see eq (4), Section III). A variety of different techniques exist for designing nonlinear closed loop observers for state and parameter estimation [4], [5], [6], [7], [8]. In [5] the authors propose an adaptive nonlinear observer with unknown parameters as augmented states. The major restriction of their approach is that the unknown parameters are required to be affine in the state-space equations of the process. In [6] the estimator is based on a polynomial nonlinearity in the Lyapunov function to guarantee stability and parameter convergence. The major restriction of the aforementioned approach is that the drift vector field of the dynamical system has to be Lipschitz with its state arguments. In [7] the authors construct a nonlinear observer via a system of first-order singular PDEs. Restrictive assumptions and the non-trivial computational effort required for solving the system of PDEs makes this approach less attractive for complex processes. In [8] the authors propose a recursive Kalman filter for uncertain linear systems, which is computationally expensive for online applications.

The methodology presented in this paper uses a parametric approach to guarantee local convergence of the state and parameter estimates. Additionally, it is possible to determine a bound for the rate of convergence for the observer. The technique is significantly less computationally expensive than other approaches, e.g. extended Luenberger observers, while it is applicable to systems where the states and parameters interact in a certain nonlinear fashion.

II. PRELIMINARIES

In Section II.A the design of Luenberger observers for LTI systems is briefly discussed. Required background information about the concept of stability of an interval family of polynomials is presented in Section II.B. This serves as a foundation for the presentation of the new technique in Section III.

A. Luenberger observer for LTI Systems

Consider a linear time invariant system with inputs

$$\begin{aligned} \dot{x} &= Ax + Bu \\ y &= Cx. \end{aligned} \tag{1}$$

where $x \in \Re^n$ is a vector of state variables, $u \in \Re^q$ is a vector of input variables and $y \in \Re^m$ is a vector of output variables, n is the number of states, q refers to the number of input variables and m refers to the number of output variables. A, B and C are matrices of appropriate dimensions. Assuming that the above system is observable, a Luenberger observer [9] for the system can be designed as

$$\begin{aligned} \dot{\tilde{x}} &= A\tilde{x} + L(y - \tilde{y}) + Bu \\ \tilde{y} &= C\tilde{x} \end{aligned} \tag{2}$$

where L is chosen to make the closed loop observer stable and achieve a desired observer dynamics and $\tilde{x} \in \Re^n$

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and $\tilde{y} \in \Re^m$ are the estimates of x and y, respectively.

B. Robust Stability of an Interval Polynomial

Consider a set $\delta(s)$ of real polynomials of degree n of the form

$$\delta(s) = \delta_0 + \delta_1 s + \delta_2 s^2 + \cdots + \delta_n s^n$$

where the coefficients lie within the given ranges:

$$\delta_0 \in [\delta_0^-, \delta_0^+],\ \delta_1 \in [\delta_1^-, \delta_1^+],\ \cdots,\ \delta_n \in [\delta_n^-, \delta_n^+]$$

Denote that

$$\delta = [\delta_0, \delta_1, \ldots, \delta_n]$$

and define a polynomial $\delta(s)$ by its coefficient vector δ. Furthermore, define a hyperrectangle of coefficients as follows:

$$\Omega := \{\delta : \delta \in \Re^{n+1}, \delta_i^- \leq \delta_i \leq \delta_i^+, i = 0, 1, 2, \ldots, n\}$$

Assume that the degree of the polynomial remains invariant over the family, so that $0 \notin [\delta_n^-, \delta_n^+]$. A set of polynomials with above properties is called an interval polynomial family [10]. Kharitonov's theorem provides a necessary and sufficient condition for Hurwitz stability of all members of this family.

Theorem 1 (Kharitonov's Theorem): Every polynomial in the family $\delta(s)$ is Hurwitz if and only if the following four extreme polynomials are Hurwitz [11].

$$\begin{aligned}
\delta^{--}(s) =& \delta_0^- + \delta_1^- s + \delta_2^+ s^2 + \delta_3^+ s^3 + \delta_4^- s^4 + \\
& \delta_5^- s^5 + \delta_6^+ s^6 + \cdots, \\
\delta^{-+}(s) =& \delta_0^- + \delta_1^+ s + \delta_2^+ s^2 + \delta_3^- s^3 + \delta_4^- s^4 + \\
& \delta_5^+ s^5 + \delta_6^+ s^6 + \cdots, \\
\delta^{+-}(s) =& \delta_0^+ + \delta_1^- s + \delta_2^- s^2 + \delta_3^+ s^3 + \delta_4^+ s^4 + \\
& \delta_5^- s^5 + \delta_6^- s^6 + \cdots, \\
\delta^{++}(s) =& \delta_0^+ + \delta_1^+ s + \delta_2^- s^2 + \delta_3^- s^3 + \delta_4^+ s^4 + \\
& \delta_5^+ s^5 + \delta_6^- s^6 + \cdots,
\end{aligned} \tag{3}$$

While this theorem has been extensively used in parametric approaches to robust control, this work will make use of Kharitonov's theorem for developing robust observer designs that can handle parametric uncertainties in the model.

III. ROBUST OBSERVER DESIGN

A. Problem Formulation

Consider a continuous linear dynamic system with real parametric uncertainties as follows:

$$\begin{aligned}
\dot{x} &= A(\theta)x + Bu \\
y &= Cx + D\theta.
\end{aligned} \tag{4}$$

Here $\theta \in \Re^p$ is a vector of constant real parameters, *a priori* uncertain, and p is the dimension of the unknown parameter vector θ. $A(\theta)$ is a matrix where the entries can be general nonlinear functions of the parameters θ. B, C and D are matrices of appropriate dimensions with the pair $\{A(\theta), C\}$ being observable for a specified θ.

The goal of this paper is to estimate the state vector without prior knowledge of the parameters θ such that $\lim_{t\to\infty}(x - \tilde{x}) = 0$, where $\tilde{x}$ is the estimate of the state vector x.

B. Estimator Design - A Parametric Approach

Without loss of generality, only the autonomous part of the dynamic system given by eq (4) is considered. To avoid ambiguity it is mentioned that the systems given by eq (4) are often referred as Linear Parameter Varying systems (LPV). Considering the parameters, θ, of the system as additional states of the original system results in the following augmented nonlinear system

$$\begin{aligned}
\begin{pmatrix} \dot{x} \\ \dot{\theta} \end{pmatrix} &= \begin{pmatrix} A(\theta) & 0 \\ 0 & 0 \end{pmatrix} \begin{pmatrix} x \\ \theta \end{pmatrix} \\
y &= \begin{pmatrix} C & D \end{pmatrix} \begin{pmatrix} x \\ \theta \end{pmatrix}
\end{aligned} \tag{5}$$

and with a change of notation

$$\begin{aligned}
\bar{x} = \begin{pmatrix} x \\ \theta \end{pmatrix}, \quad \bar{A}(\theta) = \begin{pmatrix} A(\theta) & 0 \\ 0 & 0 \end{pmatrix} \\
\bar{C} = \begin{pmatrix} C & D \end{pmatrix}
\end{aligned} \tag{6}$$

this results in

$$\begin{aligned}
\dot{\bar{x}} =& \bar{A}(\theta)\bar{x} \\
y =& \bar{C}\bar{x}
\end{aligned} \tag{7}$$

It is assumed that each component θ_i of the parameter vector

$$\theta := [\theta_0, \theta_1, \theta_2, \ldots, \theta_{p-1}] \tag{8}$$

can vary independently of the other components and each θ_i lies within an interval where the upper and lower bounds are known:

$$\Pi := \{\theta : \theta_i^- \leq \theta_i \leq \theta_i^+, i = 0, 1, 2, \ldots, p-1\} \tag{9}$$

Also, let $\theta = \theta_{ss} \in \Pi$ be a constant vector of *a priori* uncertain parameters and $(0, \theta_{ss})$ an equilibrium point of eq (5). It is further assumed that the eigenspectrum of $A(\theta)$ lies completely in the open complex LHP $\forall\theta \in \Pi$. In other words, the nonlinear system given by eq (4) is asymptotically stable $\forall\theta \in \Pi$. The augmented system needs to be observable in order to design an observer, which can also estimate the values of parameters. A sufficient condition for local observability of a nonlinear system is if the observability matrix of the augmented system (7) has rank $n + p$ for $\theta = \theta_{ss}$ [12]:

$$W_o(\theta) = \begin{bmatrix} \frac{\partial \bar{C}\bar{x}}{\partial \bar{x}} \\ \frac{\partial L_{\bar{A}(\theta)\bar{x}} \bar{C}\bar{x}}{\partial \bar{x}} \\ \vdots \\ \frac{\partial L_{\bar{A}(\theta)\bar{x}}^{n+p-1} \bar{C}\bar{x}}{\partial \bar{x}} \end{bmatrix} = \begin{bmatrix} \bar{C} \\ \bar{C}\bar{A}(\theta) \\ \vdots \\ \bar{C}\bar{A}^{n+p-1}(\theta) \end{bmatrix} \tag{10}$$

Since the values of the parameters are not exactly known *a priori*, it is required that the rank of $W_o(\theta)$ is checked

$\forall \quad \theta = \theta_{ss} \in \Pi$. In order to proceed, it is assumed that the augmented system is observable over the entire hyperrectangle-like set Π. It is then possible to design an observer for the augmented system

$$\begin{pmatrix} \dot{\tilde{x}} \\ \dot{\tilde{\theta}} \end{pmatrix} = \begin{pmatrix} A(\tilde{\theta}) & 0 \\ 0 & 0 \end{pmatrix} \begin{pmatrix} \tilde{x} \\ \tilde{\theta} \end{pmatrix} + \bar{L}(\tilde{x},\tilde{\theta})(y - \tilde{y})$$
$$\tilde{y} = \begin{pmatrix} C & D \end{pmatrix} \begin{pmatrix} \tilde{x} \\ \tilde{\theta} \end{pmatrix} \tag{11}$$

where $\tilde{x}$ and $\tilde{\theta}$ are the estimates of x and θ, respectively, and $\bar{L}(\tilde{x},\tilde{\theta})$ is a suitably chosen nonlinear observer gain.

1) Determining the family of polynomials for observer design: In this section, the results on Hurwitz stability of an interval family of polynomials from Section II.B are used to determine a methodology for computing the gain $\bar{L}(\tilde{x},\tilde{\theta})$ of the nonlinear observer given by eq (11).

Consider the process model of the augmented system for specific values of the parameters θ_{ss}:

$$\begin{aligned} \dot{\bar{x}} =& \bar{A}(\theta_{ss})\bar{x} \\ y =& \bar{C}\bar{x} \end{aligned} \tag{12}$$

The characteristic polynomial of the system, which determines its stability, is given by

$$\begin{aligned} \delta(s) =& det[sI - \bar{A}(\theta_{ss})] \\ =& \delta_0(\theta_{ss}) + \delta_1(\theta_{ss})s + \cdots + s^n \end{aligned} \tag{13}$$

It can be seen that the coefficients of the characteristic polynomial are nonlinear functions of the parameter vector θ_{ss}. For the case where θ is a scalar, the range within which the coefficients vary can be determined by plotting $\delta_i(\theta_{ss})$ against $\theta_{ss} \ \forall \ i = 0, 1, 2, 3, \cdots, n-1$. Figure 1 shows a typical plot of a coefficient versus a scalar parameter θ. In case of a vector of parameters, advanced NLP algorithms exist that calculate the required bounds on the coefficients. To enforce that the estimation error decays asymptotically

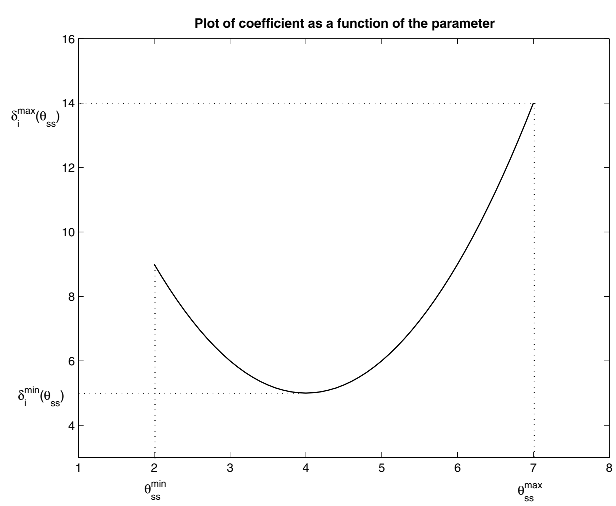

Fig. 1. Sample plot of the coefficient as a function of a scalar parameter.

for the linearized system, the observer gains have to be chosen to satisfy the following condition

$$\lambda(\bar{A}(\theta_{ss}) - \bar{L}(0,\theta_{ss})\bar{C}) \in \mathcal{C}^-, \forall\theta_{ss} \in \Pi \tag{14}$$

where $\lambda(.)$ refers to the eigenvalues of the matrix. The following subsection focuses on computing appropriate gains $\bar{L}$ by making use of Kharitonov's theorem.

2) Observer gain computation: Since it is assumed that the augmented system given by eq (5) is observable over the entire hyperrectangle-like set Π and the equilibrium points corresponding to these parameter values, it is possible to find an invertible transformation $\bar{T}(\theta_{ss})$ such that the system given by eq (12) can be transformed into an observable canonical form [13], considering one output at a time. The transformation of the original system into a canonical form is fundamentally similar to the technique used in [14]

$$\begin{aligned} \dot{\bar{z}} =& \breve{A}(\theta_{ss})\bar{z} \\ y =& \breve{C}\bar{z} \end{aligned} \tag{15}$$

where, $\bar{z} = \bar{T}(\theta_{ss})\bar{x}$, $\breve{A}(\theta_{ss}) = \bar{T}(\theta_{ss})\bar{A}(\theta_{ss})\bar{T}^{-1}(\theta_{ss})$, $\breve{C} = \bar{C}\bar{T}^{-1}(\theta_{ss})$ and

$$\breve{A}(\theta_{ss}) = \begin{bmatrix} \delta_{n-1}(\theta_{ss}) & 1 & 0 & \cdots & 0 & 0 \\ \delta_{n-2}(\theta_{ss}) & 0 & 1 & \cdots & 0 & 0 \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ \delta_2(\theta_{ss}) & 0 & \cdots & 0 & 1 & 0 \\ \delta_1(\theta_{ss}) & 0 & \cdots & 0 & 0 & 1 \\ \delta_0(\theta_{ss}) & 0 & \cdots & 0 & 0 & 0 \end{bmatrix} \tag{16}$$

$$\breve{C} = \begin{bmatrix} 1 & 0 & \cdots & 0 & 0 & 0 \end{bmatrix}$$

The characteristic polynomial of $\breve{A}(\theta_{ss})$ takes the following form:

$$\delta(s) = \delta_0(\theta_{ss}) + \delta_1(\theta_{ss})s + \delta_2(\theta_{ss})s^2 + \cdots + s^n \tag{17}$$

Since, the coefficients of the above characteristic polynomial in s are continuous and nonlinear functions of θ_{ss}, the hyperrectangle within which these coefficients can vary independently from one another can be evaluated from the method discussed in Section III.B.1.

The following analysis provides a method to compute a constant gain vector l for the case of a single output system such that:

$$\lambda(\breve{A}(\theta_{ss}) - l\breve{C}) \in \mathcal{C}^-, \forall\theta_{ss} \in \Pi$$

Consider the set of n nominal parameters $\{\delta_0^0, \delta_1^0, \delta_2^0, \ldots, \delta_{n-1}^0\}$ together with a set of a priori uncertainty ranges $\Delta\delta_0, \Delta\delta_1, \ldots, \Delta\delta_{n-1}$, which is given by $\Delta\delta_i = \delta_i^+ - \delta_i^-$, $i = 0, 1, \ldots, n-1$. Furthermore, consider the family $\delta(s)$ of polynomials,

$$\delta(s) = \delta_0 + \delta_1 s + \delta_2 s^2 + \delta_3 s^3 + \ldots + \delta_{n-1}s^{n-1} + s^n$$

where the coefficients of the polynomial can vary independently from one another and lie within the given ranges.

$$\left\{\delta : \delta_i^0 - \frac{\Delta\delta_i}{2} \leq \delta_i \leq \delta_i^0 + \frac{\Delta\delta_i}{2}\right\}$$

Further let there be n free parameters $l = (l_0, l_1, l_2, \ldots, l_{n-1})$ so as to transform the family $\delta(s)$ into the family described by

$$\begin{aligned}\gamma(s) =&(\delta_0 + l_0) + (\delta_1 + l_1)s + \ldots + (\delta_{n-1}+\\ &l_{n-1})s^{n-1} + s^n\end{aligned} \tag{18}$$

The above problem arises, when it is required to suitably place the closed-loop observer poles for a single output system where the system matrices A and C are in observable canonical form and the coefficients of the characteristic polynomial of A are subject to bounded perturbations.

At this point it is worth mentioning that there exists a theorem by Frazer and Duncan [15] which provides necessary and sufficient conditions for stability of an interval polynomial given by eq (13) whose coefficients are dependent on each other. It has been shown in [10] and [16] that it is possible to determine a vector l such that the entire family $\gamma(s)$ is stable. The synthesis of such a vector l is shown in Appendix A. The result is an observer given by eq (11), which estimates the states and parameters of the system given by eq (4) by use of an observer gain $\bar{L}(\tilde{x},\tilde{\theta})$=$\bar{T}^{-1}(\tilde{x},\tilde{\theta})l$. The presented approach yields an analytical expression for the observer gains irrespective of the dimension of the system. This is a significant advantage over other methods, e.g. an extended Luenberger observer where the gains are recomputed after each time step [17]. Therefore, the current methodology is less computationally demanding than other state and parameter estimation techniques while it guarantees local convergence of the error dynamics. It can be seen that there exist an infinite number of vectors l such that the given interval family of polynomials can be transformed into another family $\gamma(s)$ such that $\gamma(s)$ is Hurwitz. The following section discusses how the observer gain computation technique can be extended to also meet certain performance objectives of the closed-loop observer in addition to stability requirements.

IV. OBSERVER GAIN COMPUTATION FOR DESIRED RATE OF CONVERGENCE OF ESTIMATION ERROR

Often it is desired to choose l such that the real part of the eigenvalues of the family of polynomials $\gamma(s)$ have an upper limit $-\alpha$, where $\alpha \geq 0$ is a predefined real number. This problem arises when it is desired to regulate the rate of convergence of an estimation error for observer design.

In order to address this type of problem, apply a coordinate transformation $s = s' - \alpha$ to the interval polynomial family given by eq (18), resulting in

$$\begin{aligned}\gamma(s'-\alpha) =&(\delta_0 + l_0) + (\delta_1 + l_1)(s'-\alpha) + \ldots +\\ &(\delta_{n-1} + l_{n-1})(s'-\alpha)^{n-1} + (s'-\alpha)^n\end{aligned} \tag{19}$$

This polynomial family can then be rewritten explicitly in terms of s'

$$\begin{aligned}\bar{\gamma}(s') = &(\xi_0 + \beta_0) + (\xi_1 + \beta_1)s' + \cdots + (\xi_{n-1}\\ &+ \beta_{n-1})s'^{n-1} + s'^{n}\end{aligned} \tag{20}$$

where, ξ_i is a linear function of $(\delta_0, \delta_0, \cdots, \delta_{n-1})$, $\forall i = 0, 1, 2, \cdots, n-1$ and $\beta = (\beta_0, \beta_1, \beta_2, \cdots, \beta_{n-1})$ are n free parameters that can be chosen such that the roots of the polynomial given by eq (20) lie in the open left half complex plane $s' = 0$. Moreover, it can be shown that the following relationship exists

$$l_{n-r} = \sum_{k=1}^{r} \binom{n-k}{n-r} \alpha^{r-k} \beta_{n-k}, r = 1, 2, 3, \ldots, n \tag{21}$$

The above choice of l will ensure that the roots of the closed-loop polynomial $\gamma(s)$ always lie to the left of $Re(s) = -\alpha$. Hence, α can be chosen such that the slowest root of the family of closed-loop polynomials is at least 3-5 times larger in magnitude than the slowest root of the open-loop characteristic polynomial. Such a design will ensure a certain rate of convergence of the observer even under the influence of parametric uncertainty.

V. CASE STUDY

In this section, the robust observer design technique is illustrated with an example. Consider the following system with three states, one parameter, and one output,

$$\begin{aligned}\begin{bmatrix} \dot{x_1} \\ \dot{x_2} \\ \dot{x_3} \end{bmatrix} &= \begin{bmatrix} -2.5-0.35\theta & -0.5-0.05\theta & 0.5+0.05\theta \\ -0.1\theta-1 & -0.3\theta-2 & 0.1\theta+1 \\ -0.5-0.05\theta & 0.5+0.05\theta & -0.25\theta-1.5 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}\\ y&=x_1+x_2+2\theta\end{aligned} \tag{22}$$

where θ is a real, scalar parameter. The uncertainty set Π is defined as $\{1.5 \leq \theta \leq 2.5\}$. Using Kharitonov's theorem [11] it can be verified that the above linear system is asymptotically stable for $\forall\ \theta \in \Pi$. To estimate θ, the linear system given by eq (22) can be augmented:

$$\begin{aligned}\dot{\mathbf{v}} &= \begin{bmatrix} -2.5-0.35\theta & -0.5-0.05\theta & 0.5+0.05\theta & 0 \\ -0.1\theta-1 & -0.3\theta-2 & 0.1\theta+1 & 0 \\ -0.5-0.05\theta & 0.5+0.05\theta & -0.25\theta-1.5 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \mathbf{v}\\ \mathbf{v} &= \begin{bmatrix} x_1 & x_2 & x_3 & \theta \end{bmatrix}^T\\ y &= \begin{bmatrix} 1 & 1 & 0 & 2 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ \theta \end{bmatrix}\end{aligned} \tag{23}$$

with

$$\begin{aligned}\bar{A}(\theta) &= \begin{bmatrix} -2.5-0.35\theta & -0.5-0.05\theta & 0.5+0.05\theta & 0 \\ -0.1\theta-1 & -0.3\theta-2 & 0.1\theta+1 & 0 \\ -0.5-0.05\theta & 0.5+0.05\theta & -0.25\theta-1.5 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}\\ \bar{C} &= \begin{bmatrix} 1 & 1 & 0 & 2 \end{bmatrix}\end{aligned} \tag{24}$$

The characteristic polynomial of $\bar{A}(\theta)$ then results in:

$$(6+\frac{3}{125}\theta^3+\frac{23}{50}\theta^2+\frac{29}{10}\theta)s+(\frac{17}{5}\theta+11+\frac{13}{50}\theta^2)s^2 + (6+\frac{9}{10}\theta)s^3+s^4 \tag{25}$$

As θ varies in $[1.5, 2.5]$, the characteristic polynomial becomes an interval polynomial of the following form:

$$\delta(s) = \delta_0 + \delta_1 s + \delta_2 s^2 + \delta_3 s^3 + s^4$$

where $\delta_0 \in [0, 0]$, $\delta_1 \in [11.466, 16.5]$, $\delta_2 \in [16.685, 21.125]$, $\delta_3 \in [7.35, 8.25]$ and $\delta_4 \in [1, 1]$. The pair $\{\bar{A}(\theta), \bar{C}\}$ can be transformed into an observable canonical form:

Let the transformation matrix be represented by column vectors as follows

$$T = \left[\begin{array}{cccc} t_1 & t_2 & t_3 & t_4 \end{array}\right] \tag{26}$$

where the columns for this example are computed to be

$$\begin{aligned} t_1 &= \bar{C}^T \\ t_2 &= (6+\frac{9}{10})\bar{C}^T + \bar{A}^T(\theta)\bar{C}^T \\ t_3 &= (\frac{17}{5}\theta+11+\frac{13}{50}\theta^2)\bar{C}^T + (6+\frac{9}{10})\bar{A}^T\theta\bar{C}^T \\ &\quad + (\bar{A}^T(\theta))^2\bar{C}^T \\ t_4 &= (6+\frac{3}{125}\theta^3+\frac{23}{50}\theta^2+\frac{29}{10}\theta)\bar{C}^T + (6+\frac{9}{10})(\bar{A}^T(\theta))^2\bar{C}^T \\ &\quad + (\frac{17}{5}\theta+11+\frac{13}{50}\theta^2)\bar{A}^T(\theta)\bar{C}^T + (\bar{A}^T(\theta))^3\bar{C}^T \end{aligned}$$

by the method shown in [13].

In a first step, the observer is designed for robust stability with $\alpha = 0$. The observer gain is then computed to be:

$$\bar{L}(\tilde{\theta}) = T^{-T}(\tilde{\theta})\left[\begin{array}{c} -3.8 \\ -6.905 \\ -1.983 \\ 4 \end{array}\right]$$

Figure 2 shows the plots of estimates of the states and of the parameter of the system given by eq (22) for the case of $\theta = 1.95$.

It can be seen that observer gain must be chosen such that the estimation error decays significantly faster than the dominant mode of the open-loop system to not result in sluggish performance. As a rule of thumb, the slowest eigenmode of the closed-loop observer should be at least 4 times faster than the slowest eigenmode of the open-loop system. This will be investigated next.

The eigenvalues of the system given by eq (22) with $\theta = 2$, i.e. the mean value of the uncertainty interval of the parameter, are computed to be {-1.4,-2.6,-3.8}. To make the observer faster than the system, the value of α is chosen to

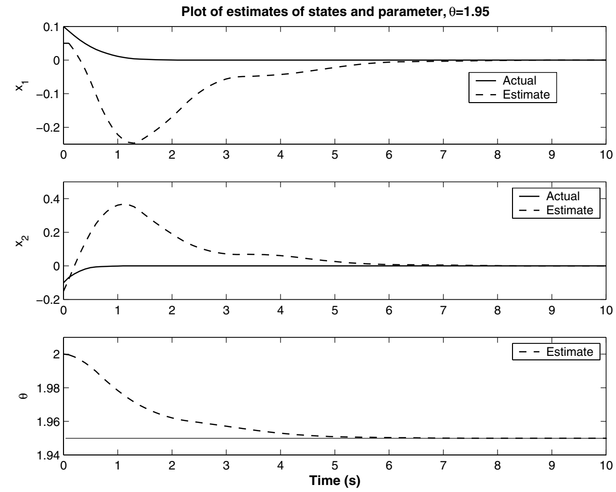

Fig. 2. Plot of the estimate of states with time, $\theta = 1.95$, $\alpha = 0$.

be equal to 4.5. The new computed observer gain vector is

$$\bar{L}(\tilde{\theta}) = T^{-T}(\tilde{\theta})\left[\begin{array}{c} 94.2 \\ 1366.595 \\ 6688.017 \\ 10988 \end{array}\right]$$

Figure 3 shows the plot of the estimates of the states and of the parameter of the system given by eq (22) for the case where $\theta = 1.95$ and where the observer is designed with a value of $\alpha = 4.5$.

These results illustrate the improvement in the rate of convergence of the observer by placing the eigenvalues of the system to the left of a pre-specified bound for the entire family of systems.

Future research will address that the eigenvalues of the entire family of polynomials will be placed within upper and lower bounds on their real part.

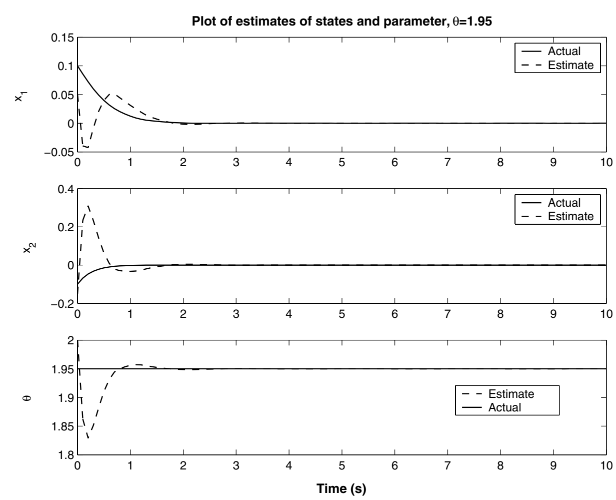

Fig. 3. Plot of the estimate of states with time, $\theta = 1.95$, $\alpha = 4.5$.

VI. CONCLUSIONS

A new observer design framework for a certain class of nonlinear systems has been presented. The resulting observers are capable of simultaneously estimating states and unknown parameters of continuous dynamic systems. A parametric approach was used for the observer design which resulted in an implicit expression for computation of the observer gain. The design technique is based upon stabilization of an interval polynomial family by the application of a variation of Kharitonov's theorem. The presented approach is computationally efficient when compared to other methods, e.g. an extended Luenberger observer, as the observer gains only requires the offline computation of an analytical expression. Additionally, it is possible to guarantee a certain level of performance for convergence of the states and parameters. Further research is underway to also address other performance criteria within the presented observer design framework.

VII. ACKNOWLEDGMENTS

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APPENDIX

A.OBSERVER GAIN COMPUTATION

Consider a polynomial

$$\delta(s) = \delta_0 + \delta_1 s + \delta_2 s^2 + \delta_3 s^3 + \ldots + \delta_{n-1} s^{n-1} + s^n$$

whose coefficients can vary independently within a given uncertainty range as follows

$$\left\{ \delta : \delta_i^0 - \frac{\Delta\delta_i}{2} \leq \delta_i \leq \delta_i^0 + \frac{\Delta\delta_i}{2} \right\},$$
$$\Delta\delta_i = \delta_i^+ - \delta_i^-, i = 0, 1, 2, \ldots, n-1$$

The aim is to find a constant vector $l = (l_0, l_1, l_2, \ldots, l_{n-1})$ to transform the interval polynomial family described by

$$\gamma(s) = (\delta_0 + l_0) + (\delta_1 + l_1)s + \ldots + (\delta_{n-1} + l_{n-1})s^{n-1} + s^n$$

such that the entire family $\gamma(s)$ remains Hurwitz. This is achieved by the following:

1) Consider any stable polynomial $R(s)$ of degree $n-1$. Let $\rho(R(s))$ be the radius of the largest stability hypersphere [10] around $R(s)$. It can be shown that $\rho(\alpha R(s)) = \alpha\rho(R(s))$, for any positive real number α.
2) Thus it is possible to find a polynomial $\alpha R(s)$ such that

$$\alpha > \frac{\sqrt{\sum_{i=0}^{n-1} \frac{(\Delta\delta_i)^2}{4}}}{\rho(R(s))}$$

3) Denoting $R(s) = r_0 + r_1 s + r_2 s^2 + \ldots + r_{n-1} s^{n-1} + s^n$, the constant vector l is calculated as follows:

$$\{l : l_i = \alpha r_i - \delta_i^0, i = 0, 1, 2, \ldots, n-1\}$$

Thereby it is possible to compute the observer gains l_i given a chosen polynomial $R(s)$ and α.

REFERENCES

[1] G. Bastin; and D. Dochain. *Online Estimation and Adaptive Control of Bioreactors*. Elsevier, New York, 1990.
[2] M. Soroush. Nonlinear state-observer design with application to reactors. *Chem. Eng. Sci*, 52:387, 1997.
[3] S. Rajaraman; J. Hahn; and M.S. Mannan. A Methodology for Fault Detection, Isolation and Identification for Nonlinear Processes with Parametric Uncertainties. *Ind. Eng. Chem. Res.*, pages 6774–6786, 2004.
[4] M. Zeitz. The extended Luenberger observer for nonlinear systems. *Systems & Control Letters*, 9:149, 1987.
[5] G. Bastin; and M.R. Gevers. Stable adaptive observers for nonlinear time-varying systems. *IEEE Trans. Automatic Control*, 7:650–658, 1988.
[6] C. Cao; and A. Annaswamy. A Polynomial Adaptive Estimator for Nonlinearly Paramterized Systems. Proc. Of The 2004th American Control Conference., page 584, Boston, Massachusetts,USA, 2004.
[7] C. Kravaris; V. Sotiropoulos; C. Georgiou; N. Kazantzis; M. Xiao; and A.J. Krener. Nonlinear Design for State and Disturbation Estimation. Proc. Of The 2004th American Control Conference., page 2931, Boston, Massachusetts,USA, 2004.
[8] J.Y. Ishihara; M.H. Terra; and J.C.T. Campos. Robust Kalman Filter for Descriptor Systems. Proc. Of The 2004th American Control Conference., page 195, Boston, Massachusetts,USA, 2004.
[9] D.G. Luenberger. Observing the state of a Linear system. *IEEE Trans. Milit. Electron.*, 8:74, 1963.
[10] S.P. Bhattacharyya; H. Chapellat; and L.H. Keel. *Robust Control: The Parametric Approach*. Prentice Hall PTR, Upper Saddle River, New Jersey, 1995.
[11] V.L. Kharitonov. Asymptotic stability of an equilibrium position of family of systems of Linear differential equations. *Differential Uraven*, 14:2086, 1978.
[12] R. Hermann; and A.J. Krener. Nonlinear controllability and observability. *IEEE.Trans.Autom.Control*, AC-22:728, 1977.
[13] F.W. Fairman. *Linear Control Theory*. John Wiley and Sons, New York, 1998.
[14] V. Hagenmeyer. Robust tracking of linear systems under parametric uncertainty. Fourteenth International Symposium of Mathematical Theory of Networks and Systems, pages 68–78, Perpignan, France, 2000.
[15] R. Frazer; and W. Duncan. On the criteria for the stability of small motions. *Proc. Royal Society A*, 124:642–654, 1929.
[16] H. Chappelat; and S.P. Bhattacharyya. *Robust stability and stabilization of interval plants*, page 207. Robustness in Identification and control. Plenum Press, New York, 1989.
[17] M.J. Kurtz; and M.A. Henson. State and disturbance estimation for nonlinear system affine in the unmeasurable variables. *Computers Chem. Engng*, 22(10):1441, 1998.