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The Adaptive Control System of A MEMS Gyroscope with Time-varying Rotation Rate

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Abstract: This paper presents a new adaptive control system to control both axes of a vibrational MEMS gyroscope. Here we suppose the rotation rate of gyro is an unknown time-varying parameter, rather than a constant as in current literature. A recently reported polynomial approximation is used to identify the time-varying rotation rate. A Lyapunov approach is employed to attain both of the control and adaptive laws. The simulation results based on Berkeley's Z-axis gyroscope model verify the controllers.

1. Introduction

A MEMS gyroscope is an angular rate sensor, whose size is several orders smaller than most mechanical gyroscopes. With the advancement of micromachining technology, MEMS gyroscopes have been applied on many areas such as attitude control, homing, control stabilization, consumer electronics, automobile, and yaw and roll control etc. The fabrication design and analysis of MEMS gyroscopes are described in [1]-[3]. Most MEMS gyroscopes are vibratory rate gyroscopes, which have a two degree-of-freedom vibrating structure. Their operation is based on the transfer of energy between two vibrating modes caused by the Coriolis acceleration. The diagram of a vibrating MEMS gyroscope is shown as Fig.1. It can be considered as a proof mass attached to a rigid frame by springs. The mass is driven to resonance along the drive axis. The reference frame rotates about rotation axis, which creates a Coriolis force is created along the sense axis, which is perpendicular to both the drive axis and rotation axis. The Coriolis acceleration coupling along the sense axis is detected. The sense axis vibration provides information of the applied rotation rate. We can determine the rotation rate by measuring sense axis vibration.

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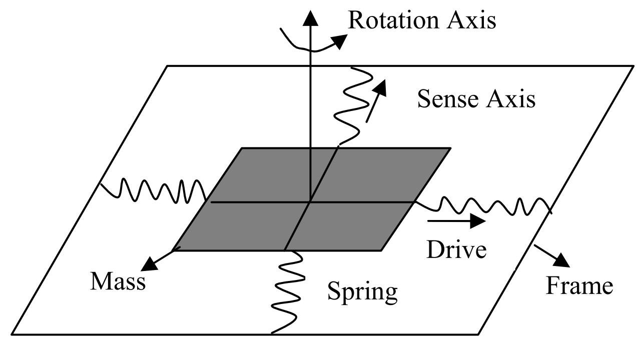

Fig.1: Illustration of MEMS Vibrational Gyroscope

For an ideal vibrational MEMS gyroscope, the drive and sense axes should be mechanically uncoupled, their natural frequencies are matched, and the output of gyro is only sensitive to rotation rate. However in reality, fabrication imperfections and environmental variations result in a frequency mismatch and stiffness and damping couplings between two vibrating modes. These imperfections cause errors in sensing the rotation rate and degrade the performance of gyro. Therefore, a control system is essential for gyro to compensate for the imperfections and improve the performance. Since 1990's, a lot of researchers have been working on the control designs of MEMS gyroscopes. Adaptive controllers are introduced in [4]-[10]. The adaptive controllers reported in [6]-[7] drive both axes of vibration, and control the entire operation of the gyroscope. All of these adaptive controllers are based on the assumption that the rotation rate is constant. But in practice, the rotation rate of gyro is changing with time. In this paper, we build a new adaptive control system for sensing time-varying rotation rates.

This adaptive control system must accomplish four tasks. First, the drive axis must be driven at resonance in order to obtain a large response and to maintain phase synchronization. Second, the output amplitude of the drive axis must be fixed. Third, the quadrature error caused by stiffness coupling between the drive and sense axes must be canceled. Fourth, for gyroscopes operating in force-to-rebalance mode, the sense axis vibration needs to be driven to zero. Simultaneously the time-varying rotation rate must be identified with the knowledge of Coriolis force and sense axis output.

Two types of controllers have been proposed for conventional force-to-rebalance mode of operation. One is the Lyapunov based adaptive controller [4], and the other is the adaptive add-on controller [7]. In [4] and [7], the adaptive controllers are developed based on the constant unknown parameters. Here we take the rotation rate as a generally unknown, piecewise, continuous, time-varying parameter. We build an adaptive controller based on the estimated time-varying rotation rate.

We combine the adaptive controller of drive axis [4] with a new adaptive controller for the sense axis which permits time-varying parameters. We allow for the mismatch of natural frequencies between both axes.

We use the polynomial approximation in [11]–[12] to estimate the rotation rate. The idea is to represent it as a polynomial in a finite time interval. The accuracy of approximation depends on the order of polynomial and the width of interval. We divide the estimation time into many small time intervals. During each interval, the coefficients of polynomial can be taken as constant and approximated by special adaptive laws. The adaptive controller with polynomial approximation has been introduced into nonlinear mechanical system such as the two-link robot [12], where the skew-symmetric property must be satisfied for the dynamic model. Here we apply this estimation algorithm on sense axis which can be taken as a second-order linear time-varying system [13]-[14]. We develop a new adaptive controller without placing any restrictions on the system model. A Lyapunov function is explored to design control and the adaptation laws.

2. Dynamic Model of Gyroscope

We disregard damping coupling between two axes. A vibrational gyroscope, with different natural frequencies and stiffness coupling on each axis, is modeled as

$$\begin{aligned} \ddot{x} + 2\varsigma\omega_n\dot{x} + \omega_n^2 x + \omega_{xy} y - 2\Omega\dot{y} &= \frac{1}{m}u_{drive}(t) \\ \ddot{y} + 2\varsigma_y\omega_y\dot{y} + \omega_y^2 y + \omega_{xy} x + 2\Omega\dot{x} &= \frac{1}{m}u_{sense}(t). \end{aligned} \tag{1}$$

In (1), x(t) and y(t) are drive and sense axis outputs. u_{drive} and u_{sense} are control inputs. m is the mass. We define $2\Omega\dot{x}$ and $2\Omega\dot{y}$ as Coriolis forces, where Ω is an unknown time-varying rotation rate. $\omega_{xy}y$ and $\omega_{xy}x$ are stiffness couplings. Rotation sensing is achieved by forcing the drive axis into a fixed amplitude vibration, and measuring the displacement y(t) of sense axis. We take the control signals to be

$$\begin{aligned} u_{drive} &= Ku_{dd} \\ u_{sense} &= Ku_{ss}. \end{aligned} \tag{2}$$

In (2), K is a constant that takes into account sensor, actuator, and amplifier gains. u_{dd} and u_{ss} are basic control inputs for both axes.

3. Representation of time-varying rotation rate

In order to represent a general time-varying function, we need to introduce the following Lemma.

Lemma1: Let I be an open interval in R, and f be a (p+1)-times continuously differentiable function of I into R. Then for any pair of points t_0, t in I

$$\begin{aligned} f(t) = f(t_0) + \frac{(t-t_0)}{1!}f^{(1)}(t_0) + \ldots\ldots\ldots + \frac{(t-t_0)^p}{(p)!}f^{(p)}(t_0) \\ + \frac{(t-t_0)^{p+1}}{(p+1)!}f^{(p+1)}(\xi), \xi \in [t_0, t]. \end{aligned} \tag{3}$$

In (3), $f^{(p)}(\bullet)$ stands for the p-th derivative of the function f(•).

By Lemma 1, the rotation rate Ω can be represented by a polynomial of time with constant coefficients in a small time interval T.

$$\begin{aligned} \Omega(t) &= \sum_{i=0}^{p} h_i (t-t_0)^i + \delta_\Omega, \qquad t_0 < t \le t_0 + T \\ h_i &= \frac{\Omega^{(i)}(t_0)}{i!} \end{aligned} \tag{4}$$

In (4), t_0 is the beginning of time interval, p is the highest order of polynomial, h_i are the constant coefficients of polynomial, and the residue $\delta_\Omega = \frac{(t-t_0)^{p+1}}{(p+1)!}\Omega^{(p+1)}(\xi), \quad \xi \in [t_0, t]$. We suppose the (p+1)-th derivative of Ω is bounded by c_p, then the remainder δ_Ω must be bounded by $|\delta_\Omega| \le \frac{c_p (t-t_0)^{p+1}}{(p+1)!}$. If time interval T is chosen to be small enough, the remainder δ_Ω is negligible.

We divide the estimation time t into many equal small time intervals. There is a trade-off between the length of interval and the order of polynomial. We need to choose T and P properly. During each interval T, the rotation rate Ω(t) can be approximated by a local polynomial of time. At the beginning of each time interval $t_0 = t_r$ (r=0, 1,2……..), the coefficient h_i needs to be reset. The resetting condition can ensure that the estimated time-varying parameter is continuous and the local polynomial in each interval is different. Fig.2 illustrates the idea of polynomial approximation.

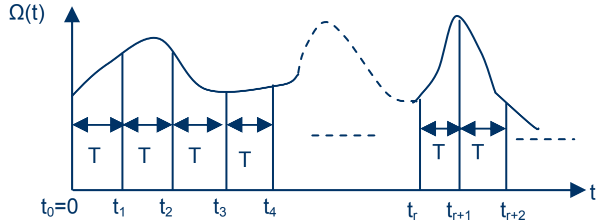

Fig.2: Local Polynomial approximation of continuous Ω(t)

Let t_r be the time when the r-th window begins, and t_{r+1} be the time when the r+1-th window begins. T is the length of window, and T= $(t_{r+1} - t_r)$. Let $[h_0(t_{r+1})\ \ h_1(t_{r+1})\ \dots\ h_p(t_{r+1})]^T$ be the coefficient vector of the estimated parameter Ω(t) during the time interval beginning at t_{r+1}, and $[h_0(t_r)\ \ h_1(t_r)\ \dots\ h_p(t_r)]^T$ be the coefficient vector during the time interval beginning at t_r. As in [13], the resetting condition is given by (5), where B is a transition matrix of order $p \times p$, and b_{ij} is the element of B.

$$\begin{bmatrix} h_0(t_{r+1}) \\ h_1(t_{r+1}) \\ \vdots \\ h_p(t_{r+1}) \end{bmatrix} = B \times \begin{bmatrix} h_0(t_r) \\ h_1(t_r) \\ \vdots \\ h_p(t_r) \end{bmatrix}$$
$$b_{ij} = \frac{i!}{j!(i-j)!} T^{i-j}, \quad i = 0,1,2 \dots\dots p \qquad (5)$$
$$j = 0,1,2 \dots\dots p$$

4. Adaptive control system of sense axis

Before we develop an adaptive controller for sense axis, we need to suppose the drive axis has been stabilized, i.e. the drive axis can output a signal with constant amplitude and resonant frequency. Here we assume the drive axis output x(t)= -Acos(ωt), where ω is the resonance frequency, and A is the amplitude of this output.

4.1 Control Design

The block diagram of adaptive control system is shown as Fig.3. $\hat{\Omega}$ is the estimated rotation rate. $\hat{\omega}_{xy}$ is the estimated ω_{xy}. $\hat{\omega}_{xy}$ and x serve as the input of adaptive controller in order to cancel the quadrature error caused by stiffness coupling. Our control goal is to drive the output y to zero. We also need to identify ω_{xy} and the rotation rate Ω through adaptation mechanism.

Let k_1=K/ωm, the sense axis model can be written as

$$\frac{\ddot{y}}{k_1\omega} = -\omega_y^2 \frac{y}{k_1\omega} - 2\varsigma_y\omega_y v - \omega_{xy}\frac{x}{k_1\omega} - 2\Omega\frac{\dot{x}}{k_1\omega} + u_{ss}. \qquad (6)$$

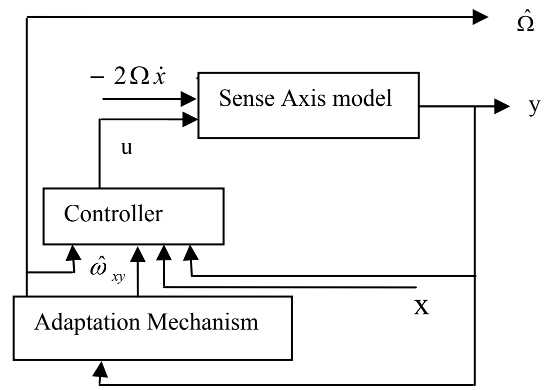

Fig.3: Block diagram of the adaptive controller for sense axis

Since the desired output is zero, the tracking error e= y. Let Λ be a positive gain, then we can take the velocity error as $e_v = \dot{e} + \Lambda e$. We suppose the derivative of y is available. In order to get the derivative of e_v, we need to define $\dot{y}_e$ and $\ddot{y}_e$ first. Let $\dot{y}_e = -\Lambda e$, and $\ddot{y}_e = -\Lambda \dot{e}$. If we choose F_v as positive control gain, we can get the control effort

$$u_{ss} = [\ddot{y}_e + 2\varsigma_y\omega_y\dot{y}_e + \omega_y^2 y - F_v e_v + 2\dot{x}\hat{\Omega} + x\hat{\omega}_{xy}]/k_1\omega. \qquad (7)$$

According to the analysis above, we can represent the estimated rotation rate and ω_{xy} as (8), where $\hat{h}_i$ and $\hat{j}_i$ are the estimated coefficients of polynomial.

$$\hat{\Omega} = \sum_{i=0}^{p} \hat{h}_i (t - t_0)^i + \delta_\Omega$$
$$\hat{\omega}_{xy} = \sum_{i=0}^{p} \hat{j}_i (t - t_0)^i + \delta_{w_{xy}} \qquad t_0 < t \le t_0 + T \qquad (8)$$

We suppose Coriolis force $2\dot{x}\Omega$ is partly known. Then we can estimate $\hat{h}$ through $2\dot{x}$. Let Γ be a positive gain. During each time interval T, the adaptive law is

$$\dot{\hat{h}}_i = -\Gamma e_v (t - t_0)^i (2\dot{x}) \qquad \Gamma > 0, 0 \le i \le p. \qquad (9)$$

Likewise we can get the adaptive law for $\hat{j}$ shown as

$$\dot{\hat{j}}_i = -\Gamma e_v (t - t_0)^i x \qquad \Gamma > 0, 0 \le i \le p. \qquad (10)$$

4.2 Lyapunov Approach

Let $\tilde{h} = \hat{h} - h, \tilde{j} = \hat{j} - j$. We choose the following Lyapunov candidate, which is positive definite.

$$V(e_v,\tilde{h})=\frac{1}{2}e_v^2+\frac{1}{2}\sum_{i=0}^{p}\Gamma^{-1}\tilde{h}_i^2+\frac{1}{2}\sum_{i=0}^{p}\Gamma^{-1}\tilde{j}_i^2 \quad (11)$$

Differentiate the Lyapunov function, we can get

$$\dot{V}(e_v,\tilde{h})=e_v\dot{e}_v+\sum_{i=0}^{p}\Gamma^{-1}\tilde{h}_i\dot{\tilde{h}}_i+\sum\Gamma^{-1}\tilde{j}_i\dot{\tilde{j}}_i. \quad (12)$$

In order to get the derivative of e_v, we substitute the control input in (6) with (7). Then we get

$$\begin{aligned}\ddot{y}+2\varsigma_y\omega_y\dot{y}=\ddot{y}_e+2\varsigma_y\omega_y\dot{y}_e-F_ve_v+2\dot{x}\hat{\Omega}-2\dot{x}\Omega\\ +x\hat{\omega}_{xy}-x\omega_{xy}.\end{aligned} \quad (13)$$

Since $\ddot{y}-\ddot{y}_e=\dot{e}_v$, and $\dot{y}-\dot{y}_e=e_v$, we can get the representation of $\dot{e}_v$ as in (14).

$$\dot{e}_v=-2\varsigma_y\omega_ye_v-F_ve_v+2\dot{x}\tilde{\Omega}+x\tilde{\omega}_{xy} \quad (14)$$

Replacing the $\dot{e}_v$ in (12) with (14), we get the derivative of Lyapunov function

$$\begin{aligned}\dot{V}=-(F_v+2\varsigma_y\omega_y)e_v^2+2\dot{x}\tilde{\Omega}e_v+x\tilde{\omega}_{xy}e_v+\sum_{i=0}^{p}\Gamma^{-1}\tilde{h}_i\dot{\tilde{h}}_i\\ +\sum_{i=0}^{p}\Gamma^{-1}\tilde{j}_i\dot{\tilde{j}}_i,\end{aligned} \quad (15)$$

$$\begin{aligned} where\quad \tilde{\Omega}&=\sum_{i=0}^{p}\tilde{h}_i(t-t_0)^i\\ \tilde{\omega}_{xy}&=\sum_{i=0}^{p}\tilde{j}_i(t-t_0)^i\\ \dot{\tilde{h}}&=\dot{\hat{h}}=-\Gamma(t-t_0)^ie_v(2\dot{x})\\ \dot{\tilde{j}}&=\dot{\hat{j}}=-\Gamma(t-t_0)^ie_vx.\end{aligned}$$

Therefore, we can represent the derivative of Lyapunov function as

$$\dot{V}=-(F_v+2\varsigma_y\omega_y)e_v^2. \quad (16)$$

We choose $F_v \geq 0$. Since $2\varsigma_y\omega_y \geq 0$, the derivative of Lyapunov function $\dot{V}$ is negative semi definite. This implies that e_v , $\tilde{j}$ and $\tilde{h}$ are bounded. From (14), we can see that $\dot{e}_v$ is bounded. Hence $\ddot{V}$ is bounded, which implies that $\dot{V}$ is uniformly continuous. Invoking Barbalat's Lemma, $\dot{V}$ will converge to zero as t goes to infinite. Therefore, e_v asymptotically converges to zero. Since $e_v=\dot{e}+\Lambda e$, both e(t) and $\dot{e}(t)$ asymptotically converge to zero. Since e=y, we can conclude that the output y will converge to zero as well.

5. Adaptive control system of drive axis

Before we design the adaptive controller for drive axis, we need to suppose the sense axis has been stabilized, i.e. the output of sense axis is almost zero. We take control signal u_{dd} as

$$u_{dd}=\hat{A}\sin(\omega t)+\hat{C}y(t)-\hat{K}x(t). \quad (17)$$

In (17), the $\hat{A}\sin(\omega t)$ term implements the automatic gain control loop in order to keep the amplitude of drive axis output constant. The $\hat{C}y(t)$ term is used to cancel the quadrature error $\omega_{xy}y$. Since y has already been driven to zero, the term $\hat{C}y(t)$ can be neglected. The $\hat{K}x(t)$ term is used to drive the natural frequency of the drive axis, $\sqrt{\omega_n^2+K\hat{K}/M}$ to the input frequency ω. As in [4], the Lyapunov methods are used to obtain PI like adaptation laws for $\hat{A},\hat{C}$ and $\hat{K}$. Let u(t)= $\dot{x}/\omega$, y=0, and $\dot{y}=0$. We can rewrite the drive axis model as

$$\begin{aligned}\dot{x}&=\omega u\\ \dot{u}&=-\frac{\omega_n^2}{\omega}x-2\varsigma\omega_nu+k_1(\hat{A}\sin(\omega t)-\hat{K}x(t)).\end{aligned} \quad (18)$$

According to the method of averaging [4], we can isolate the high and low frequency terms by using a coordinate transformation

$$x=a\cos(\omega t)+b\sin(\omega t),\ u=-a\sin(\omega t)+b\cos(\omega t). \quad (19)$$

If we disregard the high frequency terms, we can yield the following "slow" system using averaging approximation, where $\beta=\frac{K}{2\omega m}$.

$$\begin{aligned}\dot{a}&=-\varsigma\omega_na+\beta\tilde{K}b-\beta\tilde{A}\\ \dot{b}&=-\varsigma\omega_nb-\beta\tilde{k}a\end{aligned} \quad (20)$$

We want x(t)=-Acos(ωt). This corresponds to a=-A, and b=0.

Let the difference between real amplitude and ideal amplitude be $\tilde{A}$, and similarly for $\tilde{K}$.

Let γ_A, γ_k, λ_A, and λ_k be positive gains. We take the Lyapunov function as (21).

$$V = \frac{1}{2}[(a+A)^2 + b^2] + \frac{\beta}{2}[\frac{1}{\gamma_A}(\tilde{A} - \lambda_A(a+A))^2 + \frac{A}{\gamma_k}(\tilde{K} + \lambda_k b)^2] \quad (21)$$

The adaptation laws are shown in (22).

$$\hat{A}(t) = \hat{A}(0) + \lambda_A(a(t)+A) + \int_0^t \gamma_A(a(\tau)+A)d\tau$$
$$\hat{K}(t) = -\lambda_k b(t) - \int_0^t \gamma_k b(\tau)d\tau. \quad (22)$$

Then

$$\dot{V} = -\varsigma\omega_n[(a+A)^2 + b^2] - \beta[\lambda_A(a+A)^2 + A\lambda_k b^2] \quad (23)$$

The derivative of Lyapunov function is negative semi-definite. This shows that the parameters are driven to their ideal values.

6. Simulation Results

We simulated the adaptive control systems on a model of the Berkeley Z-axis gyroscope. The parameters of gyroscope model are ω_n=81681.4 rad/s, ω_y=80864.6 rad/s, $\zeta=4.5455\times10^{-5}$, $\zeta_y=3.125\times10^{-4}$, m=$2\times10^{-9}$ kg, ω=84194.7rad/s, A=50, ω_{xy}=14856, K=0.8338. For sense axis, we take the real rotation rate $\Omega=0.1\sin(2\pi f_{rate}t)$. We take the time interval of polynomial T=0.01ms. The order of polynomial approximation for rotation rate is p=1, and the order of polynomial for estimating ω_{xy} is p=0. We choose the positive gain F_v as 100. Then we get (24).

$$\hat{\Omega} = \hat{h}_0 + \hat{h}_1(t-t_0)$$
$$\hat{\omega}_{xy} = \hat{j}_0. \quad (24)$$

The adaptive laws are

$$\dot{\hat{h}}_0 = -\Gamma e_v(2\dot{x})$$
$$\dot{\hat{h}}_1 = -\Gamma e_v(t-t_0)(2\dot{x}) \quad (25)$$
$$\dot{\hat{j}}_0 = -\Gamma e_v x$$

The control law is

$$u_{ss} = [\ddot{y}_e + 2\varsigma\omega_n\dot{y}_e + \omega_n^2 y - F_v e_v + 2\dot{x}\hat{h}_0 + 2\dot{x}(t-t_0)\hat{h}_1 + x\hat{j}_0]/k_1\omega. \quad (26)$$

For drive axis, we choose the time constant of low pass filter τ as 0.0005. The parameters of our adaptation law are

$$\lambda_A = 0.03, \gamma_A = 0.5, \lambda_k = 0.064, \gamma_k = 64.$$

The control input for drive axis is

$$u_{dd} = k_1(\hat{A}\sin(\omega t) - \hat{K}x(t)). \quad (27)$$

We take f_{rate} as 50 Hz. Fig.5 shows the real rotation rate and the estimated rotation rate. Fig.6 shows $\omega_{xy}/(k_1\omega)$ and its estimate. Fig.7 shows the estimation error between real rotation rate and its estimate. The peak error is about 0.3%. Fig. 8 shows the output of sense axis y. The stabilized amplitude of y is about 1.4×10^{-7}, which is almost 0.01% of the original amplitude without adaptive control. Fig.9 shows the output of drive axis x. We can see that under adaptive control, drive axis outputs a signal with constant amplitude of 50 and resonant frequency of ω. The simulation results verified the whole control system.

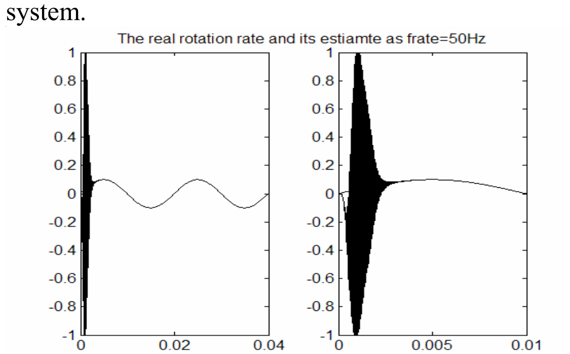

Fig.5: Rotation rate and its estimate as f_{rate}=50Hz

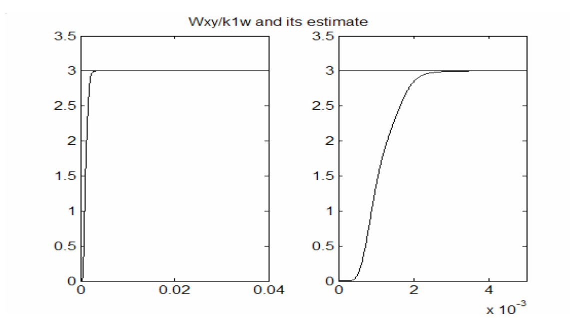

Fig.6: The $\omega_{xy}/k_1\omega$ and its estimate

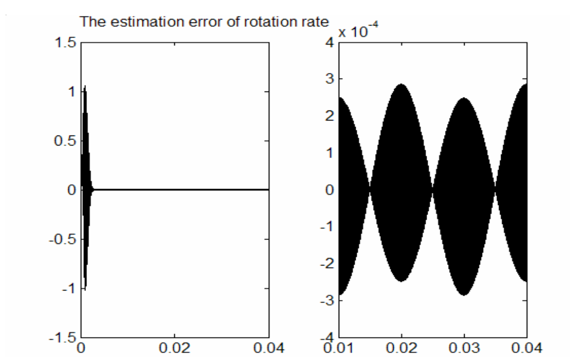

Fig.7: The estimation error of rotation rate

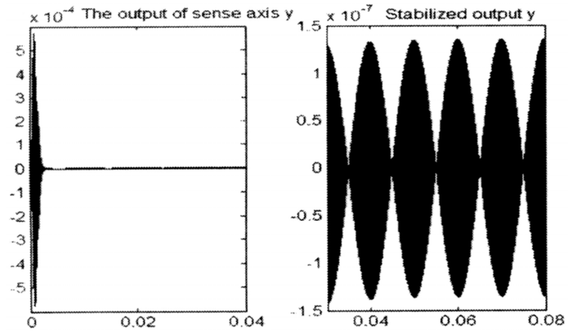

Fig.7: The output of sense axis y

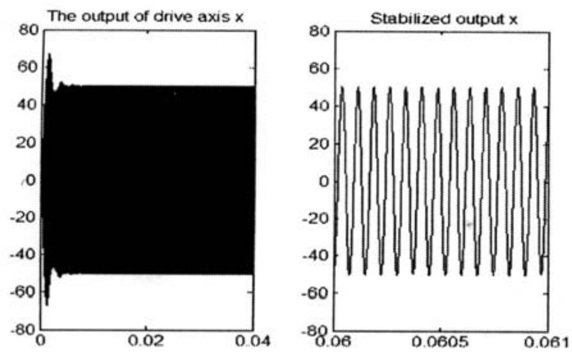

Fig.8: The stabilized output of drive axis x

7. Conclusion

In this paper, we described a new adaptive control system for both of sense axis and drive axis of MEMS gyroscope. We employ force-to-rebalance mode of operation on sense axis. An adaptive controller is designed based on the estimated rotation rate and stiffness coupling. We use Lyapunov-based adaptive controller for drive axis. The PI adaptation laws are introduced to attain adaptation laws. The two controllers are designed separately. We develop the adaptive controller for sense axis assuming the drive axis has been stabilized and vise versa.

We assume the rotation rate is a time-varying unknown parameter but not a constant value. The polynomial approximation method is used to identify the time-varying parameter.

Finally, simulation results indicate that our controllers work successfully on the Berkeley Z-axis gyroscope model [2]. In future work, we will improve the adaptive controller to track faster time-varying rotation rate. We also plan to demonstrate the stability of the whole control system combining two independent controllers. The whole gyroscope model can be taken as being consisted of two independent second-order linear subsystems of drive and sense axes with interconnections. Here the interconnections are Coriolis forces and stiffness couplings between drive and sense axes. According to [15]-[16], the complete system is "asymptotically stable in the large" (ASIL) if all of the isolated subsystems are ASIL, and the interconnections are bounded in the terms of state variables. We plan to prove this theoretically in the later research.

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