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Adaptive Resonance Tuning through Feedback

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Abstract— **An adaptive control method that tunes the resonant frequency of a lightly damped second order system to its excitation frequency is investigated. The resonance tuning is achieved by using proportional feedback around the second order system and adaptively controlling the feedback gain using the error between the excitation and resonant frequencies. This error is obtained by a phase detector. Assuming that the parameters of the lightly damped second order system are slowly time-varying, a nonlinear time-varying model that accurately predicts the performance of the resonance tuning system is developed. This developed model is subsequently linearized to obtain a linear time-invariant model that facilitates both analysis and design of the resonance tuning system. Based on the developed linear time-invariant model, guidelines for designing the resonance tuning system are also provided. The results are illustrated by examples.**

Index Terms— **Resonant frequency tuning, phase detectors, adaptive control.**

I. INTRODUCTION

A wide variety of systems, such as ultrasonic motors, piezoelectric transducers, induction heating loads, resonant inverter loads, microelectromechanical gyroscopes, cavity resonators and cyclotrons, can be modelled as lightly damped second order systems (see [1], [2], and the references therein). Such systems must be driven at their resonant frequencies in order to achieve optimal performance. However, even if these systems are driven initially at their resonant frequencies, their excitation or resonant frequencies may vary with time due to environmental changes. These changes may significantly impair their performance and necessitate employment of adaptive resonance tuning control systems that maintain lock between the driving and resonant frequencies of such systems.

Two adaptive resonance tuning methods have been investigated in our previous work [1], [2]. The first method used a phase locked loop to adaptively tune the excitation frequency of a second order system to its resonant frequency, while the second method used a phase detector to adaptively tune the resonant frequency of a second order system to its excitation frequency. This paper considers a third method for resonance tuning in which the resonant frequency of the system is changed using proportional feedback and the feedback gain is adaptively adjusted to tune the resonant frequency of the resulting closed loop system to its excitation frequency. This resonance tuning method was introduced in [3] for tuning the resonant frequency of the drive axis of a vibrational gyroscope.

In [3], a multiplication type phase detector that consists of an analog multiplier and a lowpass filter is used to obtain the error between the excitation frequency and resonant frequency. In that work, however, the importance of the lowpass filter on the overall performance of the system was not recognized and it was completely ignored in the analysis. Although the lowpass filter was included in some simulations with an ad-hoc saturation element at its input to prevent instability at high gains, no explanations are given for how its parameters were chosen and why the instability occurred. As it will be shown in this paper, the lowpass filter plays a fundamental role in the resonance tuning system and determines performance measures such as rise time, settling time, steady-state error, overshoot, robustness and stability. Hence, careful design of the lowpass filter can lead to better resonance tuning performance.

Motivated by the above discussion, the adaptive resonance tuning method introduced in [3] is reconsidered in this paper. However, unlike [3], which considers the resonance tuning problem for a specific vibrational gyroscope, the present paper investigates the problem for a more general generic lightly damped second order time-varying system. Similar to [3], the resonant frequency of the system is changed using a proportional feedback and the feedback gain is adaptively adjusted to tune the resonant frequency of the resulting closed loop system to its excitation frequency. This adaptation is based on the error, measured by a phase detector, between the resonant and driving frequencies. Two types of phase detectors, a multiplication type and an exor type, are considered.

The goal of this paper is to develop analysis and design methods for the adaptive resonance tuning system described above. For this purpose, assuming that the parameters of the lightly damped second order system (including its resonant frequency) and its driving frequency vary slowly with time, a nonlinear time-varying model that accurately predicts the performance of the adaptive resonance tuning system is developed. This developed model is subsequently linearized to obtain a more tractable linear time-invariant model that still predicts the performance of the system very accurately provided that the deviations from the nominal operating point are small. Based on the developed linear time-invariant model, guidelines for designing the adaptive resonance tuning system are provided. The developed results are illustrated by examples including the vibrational gyroscope example considered in [3].

The remainder of the paper is organized as follows. In Section II, the model of the resonance tuning system is presented. In Section III, the resonance tuning system is analyzed. In Section IV, the design of the resonance tuning system is considered. In Section V, the results are illustrated by examples. In Section VI, conclusions are given.

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II. MODELLING

In this section, the model of the resonance tuning system is presented. A brief overview of the phase detectors used in the paper is also included.

A. Resonance Tuning System

Each one of the resonant systems listed above can be modelled as a lightly damped second order system. The dynamics of such a system are governed by the differential equation

$$\ddot{y}(t)+2\zeta(t)\omega_n(t)\dot{y}(t)+\omega_n^2(t)y(t)=k_g\omega_n^2(t)u(t), \quad (1)$$

where $u(t)$ is the input, $y(t)$ is the output, $\omega_n(t)$ is the resonant (natural) frequency, $\zeta(t)$ is the damping ratio and k_g is the input gain. The input to the system $u(t)$ is assumed to be in the form

$$u(t)=A(t)\cos\left[\omega_0 t+\theta(t)\right], \quad (2)$$

where $A(t)>0$ is the instantaneous amplitude, ω_0 is the nominal angular frequency and $\theta(t)$ is the instantaneous phase of the input. It is further assumed that $\omega_n(t)$, $\zeta(t)$, $A(t)$ and $\theta(t)$ are slowly time varying compared to the time variation of $\omega_0 t$.

In order to achieve optimal performance, this system must be driven at its resonant frequency $\omega_n(t)$. Since the instantaneous frequency [4] of the input is

$$\omega_s(t)=\omega_0+\dot{\theta}(t), \quad (3)$$

the condition for the optimal performance is $\omega_s(t)=\omega_n(t)$. Although it is easy to achieve this condition nominally, both $\omega_s(t)$ and $\omega_n(t)$ may drift in time due to environmental changes and cause loss of performance. Two methods to overcome this problem have been investigated in our earlier work [1], [2]. A third method to overcome the same problem is proposed in [3], where the lightly damped second order system is placed inside a feedback loop as shown in Fig. 1 and the feedback gain is adaptively adjusted to bring the resonant frequency of the closed-loop system shown inside the dashed box to match the frequency of the input.

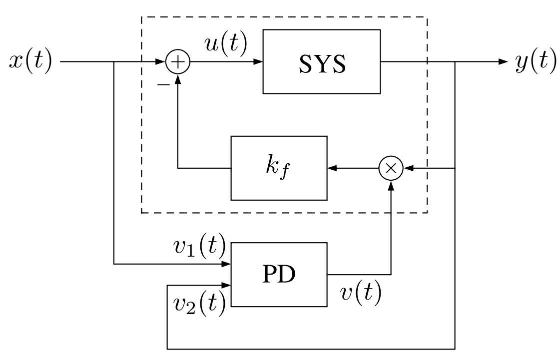

Fig. 1. Model of the resonance tuning system.

In this figure, SYS is the lightly damped second order system, PD is the phase detector, k_f is the feedback gain and $x(t)$ is the external input to the system. It follows from this figure that

$$u(t)=x(t)-k_f v(t)y(t), \quad (4)$$

where $v(t)$ is the error signal generated by the phase detector. Thus, the differential equation governing the closed-loop system inside the dashed box can be written as

$$\begin{aligned}&\ddot{y}(t)+2\zeta(t)\omega_n(t)\dot{y}(t)\\&\quad+\left[1+k_f k_g v(t)\right]\omega_n^2(t)y(t)=k_g\omega_n^2(t)x(t).\end{aligned} \quad (5)$$

The resonant frequency of this system is

$$\omega_c(t)=\omega_n(t)\sqrt{1+k_\omega v(t)}, \quad (6)$$

where $k_\omega=k_f k_g$. Hence, the goal of the resonance tuning system is to keep $\omega_c(t)$ as close to $\omega_s(t)$ as possible through $v(t)$, despite disturbances due to environmental changes.

B. Phase Detectors

A phase detector compares the phases of two signals applied to its inputs and generates an output signal whose average value is related to the phase difference between these input signals [5], [6]. There exist several types of phase detectors with different characteristics. The phase detectors considered in this paper are a multiplication type analog phase detector and an exor type digital phase detector. Below, these two types of phase detectors will be reviewed briefly.

1) Multiplication Phase Detector: A multiplication type phase detector uses an analog multiplier phase comparator followed by a lowpass filter to extract the phase information at its inputs as shown in Fig. 2. In this figure, the signals $v_1(t)$ and $v_2(t)$ are the inputs, $v(t)$ is the output, $F(s)$ is the transfer function of the lowpass filter and $\times$ is the multiplication operation.

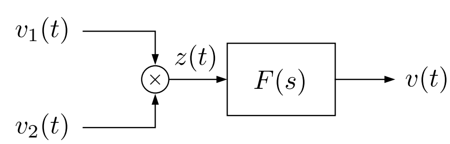

Fig. 2. Model of the multiplication phase detector.

Assuming that $v_1(t)$ and $v_2(t)$ are in the forms

$$v_1(t)=A_1(t)\cos\left[\omega_0 t+\theta_1(t)\right] \quad (7)$$

and

$$v_2(t)=A_2(t)\cos\left[\omega_0 t+\theta_2(t)\right], \quad (8)$$

where $A_1(t)>0$ and $A_2(t)>0$ are the instantaneous amplitudes, $\theta_1(t)$ and $\theta_2(t)$ are the instantaneous phases and ω_0 is the nominal angular frequency, it follows that

$$\begin{aligned}z(t)=&\frac{A_1(t)A_2(t)}{2}\cos\left[\theta_2(t)-\theta_1(t)\right]\\&+\frac{A_1(t)A_2(t)}{2}\cos\left[2\omega_0 t+\theta_2(t)+\theta_1(t)\right].\end{aligned} \quad (9)$$

Assuming further that $A_1(t)$, $A_2(t)$, $\theta_1(t)$ and $\theta_2(t)$ vary slowly with time compared to the time variation of $\omega_0 t$ (i.e., $|\dot{A}_1(t)| \ll \omega_0$, $|\dot{A}_2(t)| \ll \omega_0$, $|\dot{\theta}_1(t)| \ll \omega_0$ and $|\dot{\theta}_2(t)| \ll \omega_0$) and that the lowpass filter completely removes the high frequency term around $2\omega_0$, the phase detector output $v(t)$ can be written as

$$v(t) = f(t) * \varphi_m \left[\theta_2(t) - \theta_1(t)\right], \tag{10}$$

where

$$\varphi_m \left[\theta(t)\right] = \frac{A_1(t)A_2(t)}{2} \cos\left[\theta(t)\right], \tag{11}$$

$f(t)$ is the impulse response of the lowpass filter and $*$ is the convolution operation.

This nonlinear equation describes the operation of the multiplication type phase detector very accurately provided that the assumptions made earlier are satisfied. Note that the amplitudes of both input signals affect the output of the phase detector. This must be accounted for when designing the gains in the system. As will be shown next, the amplitudes of the input signals do not affect the output of the exor type phase detector.

2) Exor Phase Detector: An exor type phase detector uses a digital exor phase comparator followed by a lowpass filter to extract the phase information at its inputs as shown in Fig. 3. In this figure, the signals $v_1(t)$ and $v_2(t)$ are the inputs, $v(t)$ is the output, $F(s)$ is the transfer function of the lowpass filter, $\oplus$ is the exor logic gate with logic 0 and 1 levels being $-V$ and $+V$, respectively, and HLs are hard limiters with limiting levels $\pm V$ that convert the signals $v_1(t)$ and $v_2(t)$ at the input of phase detector into the digital signals $u_1(t)$ and $u_2(t)$.

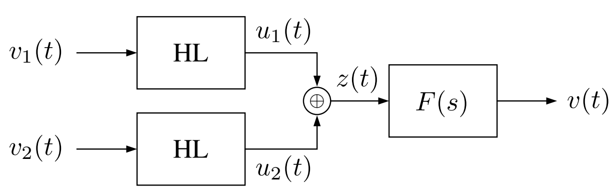

Fig. 3. Model of the exor phase detector.

Assuming that the inputs $v_1(t)$ and $v_2(t)$ are as before, the signals $u_1(t)$ and $u_2(t)$ can be expressed as

$$u_1(t) = V \operatorname{sgn}\left(\cos\left[\omega_0 t + \theta_1(t)\right]\right) \tag{12}$$

and

$$u_2(t) = V \operatorname{sgn}\left(\cos\left[\omega_0 t + \theta_2(t)\right]\right). \tag{13}$$

Thus, it follows that

$$z(t) = u_1(t) \oplus u_2(t). \tag{14}$$

Similar to the previous case, assuming that both $\theta_1(t)$ and $\theta_2(t)$ vary slowly with time compared to the time variation of $\omega_0 t$ (i.e., $|\dot{\theta}_1(t)| \ll \omega_0$ and $|\dot{\theta}_2(t)| \ll \omega_0$), the signal $z(t)$ can be expressed as

$$z(t) = \varphi_e \left[\theta_2(t) - \theta_1(t)\right] + r(t), \tag{15}$$

where $\varphi_e\left[\theta(t)\right]$ as a function of $\theta(t) = \theta_2(t) - \theta_1(t)$ is as shown in Fig. 4 and the term $r(t)$ includes the high frequency components that occur around $2\omega_0$, $4\omega_0$, and so on. Assuming further that the lowpass filter $F(s)$ completely removes the high frequency term $r(t)$, the output of the phase detector $v(t)$ is given by

$$v(t) = f(t) * \varphi_e \left[\theta_2(t) - \theta_1(t)\right], \tag{16}$$

where $f(t)$ is the impulse response of the lowpass filter and $*$ is the convolution operation.

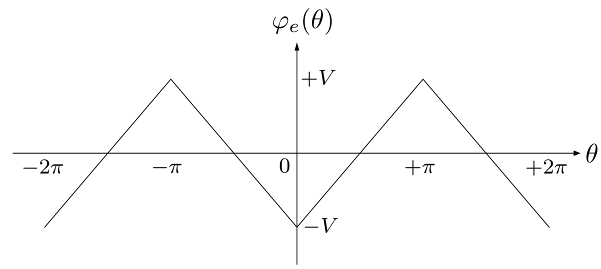

Fig. 4. Exor phase comparator characteristic.

Like the previous case, this nonlinear equation governs the operation of the phase detector very accurately provided that the assumptions stated above are satisfied. Moreover, note that the output of phase detector $v(t)$ is also a slowly time-varying signal.

III. ANALYSIS

In this section, the system shown in Fig. 1 is considered and an analysis method for the resonance tuning system is developed. The approach is very similar to the one used in our earlier work [1], [2].

A. Analysis with Multiplication Phase Detector

Consider the resonance tuning system shown in Fig. 1 and assume that the phase detector is a multiplication type. Assume further that the input $x(t)$ is in the form

$$x(t) = A_1(t) \cos\left[\omega_0 t + \theta_1(t)\right], \tag{17}$$

where $A_1(t) > 0$ is the instantaneous amplitude, ω_0 is the nominal angular frequency and $\theta_1(t)$ is the instantaneous phase of the input $x(t)$. The amplitude $A_1(t)$ and the phase $\theta_1(t)$ are assumed to be slowly time-varying parameters. The instantaneous frequency of the input is

$$\omega_s(t) = \omega_0 + \dot{\theta}_1(t). \tag{18}$$

The output $y(t)$ satisfies the differential equation (5). As this equation is time varying, it is impossible to find $y(t)$ analytically. However, since it is assumed that the parameters $\omega_n(t)$, $\zeta(t)$, $A_1(t)$ and $\theta_1(t)$ vary slowly with time compared to the excitation frequency, $y(t)$ can be approximated using the frozen-time approach. For a frozen time t, the transfer function from $x(t)$ to $y(t)$ is

$$G(s) = \frac{k_g \omega_n^2(t)}{s^2 + 2\zeta(t)\omega_n(t)s + \omega_c^2(t)} \tag{19}$$

and the output $y(t)$ can be approximated using its steady-state part as

$$y(t) = A_2(t)\cos\left[\omega_0 t + \theta_2(t)\right], \tag{20}$$

where $A_2(t) = |G[j\omega_s(t)]|A_1(t)$ and $\theta_2(t) = \theta_1(t) + \angle G[j\omega_s(t)]$. It then follows that

$$\theta_2(t) - \theta_1(t) = \angle G[j\omega_s(t)]. \tag{21}$$

Using this in (11) yields

$$v(t) = f(t) * \varphi_m\left(\angle G[j\omega_s(t)]\right). \tag{22}$$

Thus, the output of the phase detector becomes

$$v(t) = f(t) * \frac{A_1^2(t)}{2}|G[j\omega_s(t)]|\cos\left(\angle G[j\omega_s(t)]\right). \tag{23}$$

Since

$$|G[j\omega_s(t)]|\cos\left(\angle G[j\omega_s(t)]\right) = \mathrm{Re}\{G[j\omega_s(t)]\}, \tag{24}$$

the phase detector output can be rewritten as

$$\begin{aligned} v(t) &= f(t) \\ &* \frac{A_1^2(t)k_g\omega_n^2(t)\left[\omega_c^2(t) - \omega_s^2(t)\right]}{2\left(\left[\omega_c^2(t) - \omega_s^2(t)\right]^2 + \left[2\zeta(t)\omega_n(t)\omega_s(t)\right]^2\right)}. \end{aligned} \tag{25}$$

Next, solving (6) for $v(t)$ and substituting into (25), it follows that

$$\begin{aligned} \frac{\omega_c^2(t) - \omega_n^2(t)}{\omega_n^2(t)} &= f(t) \\ &* \frac{k_\omega k_g A_1^2(t)\omega_n^2(t)\left[\omega_c^2(t) - \omega_s^2(t)\right]}{2\left(\left[\omega_c^2(t) - \omega_s^2(t)\right]^2 + \left[2\zeta(t)\omega_n(t)\omega_s(t)\right]^2\right)}, \end{aligned} \tag{26}$$

which describes the dynamics of the resonance tuning system with a multiplication type phase detector.

Equation (26) is a nonlinear model of the resonance tuning system. Although simulations show that it represents the actual resonance tuning system very accurately, the nonlinear nature of this equation reduces its value from a design perspective. Therefore, this equation is linearized about the nominal angular frequency ω_0. For this purpose, let $\omega_s(t) = \omega_0 + \delta\omega_s(t)$, $\omega_c(t) = \omega_0 + \delta\omega_c(t)$ and $\omega_n(t) = \omega_0 + \delta\omega_n(t)$. Assume further that $\zeta(t)$ and $A_1(t)$ are equal to their respective nominal values ζ_0 and A_0. Then, linearization yields

$$\delta\omega_c(t) - \delta\omega_n(t) = f(t) * \frac{k_\omega k_\theta}{2\zeta_0}\left[\delta\omega_s(t) - \delta\omega_c(t)\right], \tag{27}$$

where $k_\theta = -k_g A_0^2/(4\zeta_0)$. Finally, letting $k = k_\omega k_\theta/(2\zeta_0)$, the simplified linear time-invariant model of the resonance tuning system becomes

$$\delta\omega_c(t) - \delta\omega_n(t) = kf(t) * \left[\delta\omega_s(t) - \delta\omega_c(t)\right]. \tag{28}$$

It should be noted that this approximation is quite accurate provided that

$$\frac{|\delta\omega_s(t) - \delta\omega_c(t)|}{\zeta_0\omega_0} \le 1. \tag{29}$$

The block diagram of the system described by (28) is shown in Fig. 5. It is evident from this figure that the lowpass filter plays an important role in the resonance tuning system.

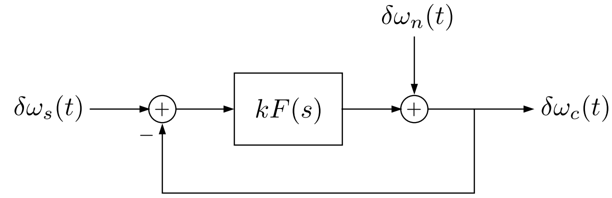

Fig. 5. Simplified model of the resonance tuning system.

B. Analysis with Exor Phase Detector

Consider again the resonance tuning system shown in Fig. 1 with the assumptions of the previous subsection except that now the phase detector is an exor type. Proceeding similarly, the output of the phase detector for this case can be expressed as

$$v(t) = f(t) * \varphi_e\left(\angle G[j\omega_s(t)]\right). \tag{30}$$

Since $-\pi \le \angle G[j\omega_s(t)] \le 0$, it follows from the phase comparator characteristic that

$$v(t) = f(t) * k_\theta\left(-\angle G[j\omega_s(t)] - \pi/2\right), \tag{31}$$

where $k_\theta = 2V/\pi$ is the gain of the phase detector. Thus, using

$$\angle G[j\omega_s(t)] = -\arctan\left[\frac{2\zeta(t)\omega_n(t)\omega_s(t)}{\omega_c^2(t) - \omega_s^2(t)}\right] \tag{32}$$

in (31) yields

$$v(t) = f(t) * k_\theta \arctan\left[\frac{\omega_s^2(t) - \omega_c^2(t)}{2\zeta(t)\omega_n(t)\omega_s(t)}\right]. \tag{33}$$

Combining this equation with (6) results in

$$\begin{aligned} \frac{\omega_c^2(t) - \omega_n^2(t)}{\omega_n^2(t)} &= f(t) \\ &* k_\omega k_\theta \arctan\left[\frac{\omega_s^2(t) - \omega_c^2(t)}{2\zeta(t)\omega_n(t)\omega_s(t)}\right], \end{aligned} \tag{34}$$

which describes the dynamics of the resonance tuning system with an exor type phase detector.

Simulations show that (34) represents the actual resonance tuning system very accurately. However, a simpler model is still desired for the purpose of design. Therefore, to proceed, the left side of (34) and the argument of the arctangent function are linearized about the nominal natural frequency ω_0. For this purpose, again let $\omega_s(t) = \omega_0 + \delta\omega_s(t)$, $\omega_c(t) = \omega_0 + \delta\omega_c(t)$ and $\omega_n(t) = \omega_0 + \delta\omega_n(t)$. Moreover, assume that $\zeta(t)$ is equal to its nominal value ζ_0. Then, the resulting expression is

$$\begin{aligned} \frac{2}{\omega_0}\left[\delta\omega_c(t) - \delta\omega_n(t)\right] &= f(t) \\ &* k_\omega k_\theta \arctan\left[\frac{\delta\omega_s(t) - \delta\omega_c(t)}{\zeta_0\omega_0}\right]. \end{aligned} \tag{35}$$

This nonlinear equation can be further approximated accurately by the linear equation

$$\delta\omega_c(t) - \delta\omega_n(t) = f(t) * \frac{k_\omega k_\theta}{2\zeta_0}\left[\delta\omega_s(t) - \delta\omega_c(t)\right], \quad (36)$$

provided that

$$\frac{|\delta\omega_s(t) - \delta\omega_c(t)|}{\zeta_0\omega_0} \leq 1. \quad (37)$$

Finally, letting $k = k_\omega k_\theta/(2\zeta_0)$, the simplified linear time-invariant model of the resonance tuning system becomes

$$\delta\omega_c(t) - \delta\omega_n(t) = kf(t) * \left[\delta\omega_s(t) - \delta\omega_c(t)\right]. \quad (38)$$

The block diagram of the system described by this equation is shown in Fig. 5. Like the previous case, it is evident that the lowpass filter plays an important role in the resonance tuning system.

IV. DESIGN

In this section, the design of the resonance tuning system is considered. Specifically, guidelines are provided for choosing the order and parameters of the lowpass filter.

Based on the developed linear time-invariant models, the resonance tuning system can be designed using standard control system design methods. In the design, the main goal is to find a "controller" $F(s)$ so that the "output" $\delta\omega_c(t)$ tracks the "reference" $\delta\omega_s(t)$ satisfactorily and the effect of the "disturbance" $\delta\omega_n(t)$ on the "output" $\delta\omega_c(t)$ is minimized.

In designing the resonance tuning system, certain guidelines should be taken into consideration. For instance, as a direct consequence of the internal model principle [7], the filter $F(s)$ must contain the model of the reference $\delta\omega_s(t)$ to achieve perfect asymptotic reference tracking. Similarly, the filter $F(s)$ must contain the model of the disturbance $\delta\omega_n(t)$ to achieve perfect asymptotic disturbance rejection. In particular, for $\delta\omega_c(t)$ to asymptotically track any step change in $\delta\omega_s(t)$ and asymptotically reject any step change in $\delta\omega_n(t)$, the filter $F(s)$ must contain at least one integrator. Moreover, the bandwidth of the lowpass filter $F(s)$ should be sufficiently small to filter out the high frequency components at the output of the phase comparator. In addition, the bandwidth of $kF(s)/[1 + kF(s)]$ should be sufficiently large for good reference tracking whereas the gain of $1/[1 + kF(s)]$ should be sufficiently small for good disturbance rejection.

In most practical applications, a first order filter of the form

$$F(s) = \frac{\beta}{s}, \quad (39)$$

where β is a design parameter, usually gives adequate results. With this filter, the settling time for $\delta\omega_c(t)$ calculated from the linearized model is $\tau = 4/(k\beta)$. If the desired performance is not achievable by a first order filter, a second order filter of the form

$$F(s) = \frac{\beta}{s(s+\alpha)}, \quad (40)$$

where α and β are design parameters, may be used. For this case, the settling time for $\delta\omega_c(t)$ calculated from the linearized model is $\tau = 8/\alpha$ provided that $\alpha^2 \leq 4k\beta$.

It should also be noted that the linearized system given in (28) or (38) is stable if all the roots of the characteristic equation $1+kF(s) = 0$ have negative real parts. Under this condition, it can be concluded that the original nonlinear and time-varying resonance tuning system given in Fig. 1 is also stable provided that the rate of variations of the slowly varying parameters are sufficiently small [8].

V. EXAMPLES

In this section, the developed analysis and design methods are applied to two examples. All simulations were performed in MATLAB/Simulink.

The first example illustrates the salient features of the developed methods with the resonance tuning system for the vibrational gyroscope considered in [3]. From the data given in [3], the parameters of the resonance tuning system are as follows: $\omega_n(t) = 63881.1$ rad/s, $\omega_s(t) = 65973.4$ rad/s, $\zeta(t) = 0.0005$, $k_g = 0.0666$ and $A_1(t) = 1$ V. Moreover, the nominal frequency ω_0, the feedback gain k_f and the logic voltage level V were selected as $\omega_0 = 65973.4$ rad/s, $k_f = 100$ and $V = 2.5$ V, respectively.

In [3], the resonance tuning system was first simulated with a multiplication phase detector. For comparison, a simulation was performed using the multiplication phase detector with the first order low-pass filter

$$F(s) = -\frac{0.05}{s}, \quad (41)$$

which was designed using the method developed in this paper. The results of this simulation are shown in Fig. 6. Here, the solid curve is for the system designed in [3], the dashed curve is for the actual system shown in Fig. 1 and the dotted curve is for the nonlinear time-varying model (26). The instantaneous input frequency $\omega_s(t)$ is also shown by the solid horizontal line for convenience.

Later in [3], the resonance tuning system was simulated with the addition of saturation and a lowpass filter. For comparison with this case, a second simulation was performed using the exor phase detector with the filter

$$F(s) = \frac{0.048}{s}, \quad (42)$$

which was again designed using the method developed in this paper. The results of this simulation are shown in Fig. 7. Here, the solid curve is for the system designed in [3], the dashed curve is for the actual system shown in Fig. 1 and the dotted curve is for the nonlinear time-varying model (34). The solid horizontal line is again the instantaneous input frequency $\omega_s(t)$.

It is evident from both Fig. 6 and Fig. 7 that a better performance can be achieved using the design methods developed in this paper. It should also be pointed out that the developed nonlinear models approximate their respective actual systems very accurately. The responses of the

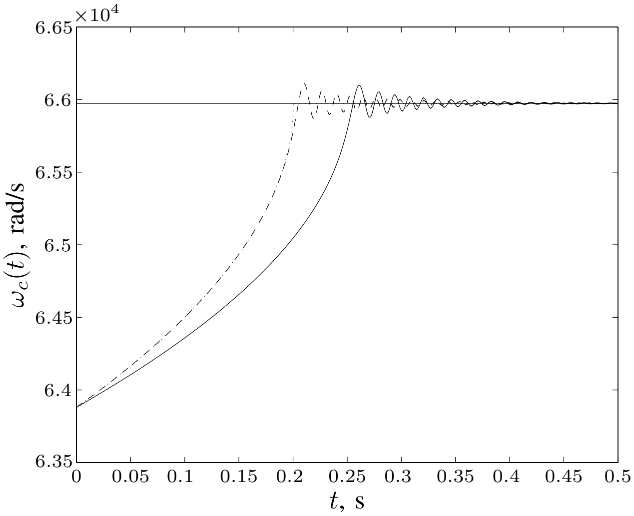

Fig. 6. Simulation results with multiplication phase detector.

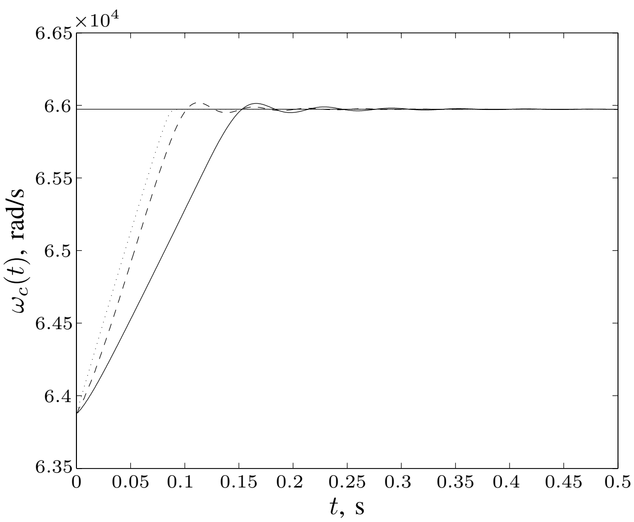

Fig. 7. Simulation results with exor phase detector.

corresponding linear time-invariant models are, on the other hand, quite different since the conditions (29) and (37) are not satisfied. Although this discrepancy reduces the values of the developed linear time-invariant models, they are still quite useful as starting points in the design process.

The second example illustrates the accuracy of the developed linear time-invariant models when the conditions (29) and (37) are satisfied. The parameters of the resonance tuning system were chosen as follows: $\omega_n(t) = 1000$ rad/s, $\omega_s(t) = 1050$ rad/s, $\zeta(t) = 0.05$, $k_g = 1$, $k_f = 1$ and $\omega_0 = 1050$ rad/s. Moreover, V and $A_1(t)$ were chosen as $V = 2.5$ V and $A_1(t) = 0.5642$ V, respectively, so that the gains of the linearized models are the same. A third simulation was performed using the multiplication and exor phase detectors with the filter

$$F(s) = \frac{10}{s\,(s+20)}. \tag{43}$$

The results of this simulation are shown in Fig. 8. Here, the solid and dashed curves are for the actual systems

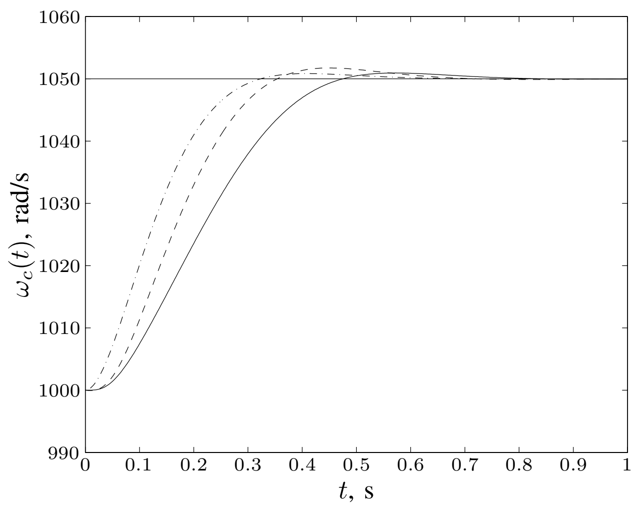

Fig. 8. Simulation results for the second example.

shown in Fig. 1 with the multiplication and exor type phase detectors, respectively, and the dashdot curve is for the simplified linear time-invariant models (28) and (38). The instantaneous input frequency $\omega_s(t)$ is also shown by the solid horizontal line for convenience.

It is clear from Fig. 8 that the developed linear time-invariant models predict the performance of the actual resonance tuning system quite accurately. Hence, when the conditions (29) and (37) are satisfied, the developed linear time-invariant models can be used for both analysis and design of the adaptive resonance tuning system. It should also be pointed out that the linearized model with exor phase detector is more accurate compared to the linearized model with multiplication phase detector.

VI. CONCLUSION

An adaptive control method that tunes the resonant frequency of a lightly damped second order system to its excitation frequency is investigated. A nonlinear time-varying model that accurately predicts the tuning performance of the system is developed. A simple linear time-invariant model that facilitates both analysis and design of the resonance tuning system is also obtained. Simulation results show the effectiveness of the developed models.

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