2005 American Control Conference
June 8-10, 2005. Portland, OR, USA

External Model-Based Disturbance Rejection in Tracking Control of Euler-Lagrange Systems

Garo Zarikian and Andrea Serrani*

Abstract— **We consider the systematic design of tracking controllers for Euler-Lagrange systems which are affected by unmeasurable harmonic disturbances. The problem addressed in the paper departs from the classic setup of the regulator problem and its solution based on the internal model principle in two aspects. First, the presence of an exogenous disturbance affecting the output channel as well as the input channel of the plant is taken into account. Second, we aim at designing a nominal tracking controller using standard techniques, independently of the device that is used to provide asymptotic rejection of the disturbance. This latter is then placed outside the stabilizing loop, in such a way that stability is preserved, and asymptotic cancellation of the disturbance is guaranteed under mild conditions. The device in question is referred to as an external model of the exogenous system, to emphasize the departure from the classic internal model-based design. The external model is endowed with an adaptation mechanism that allows to deal with uncertainties on the frequencies of the exogenous signals as well. To validate our approach, we provide experimental results for a 2-DOF helicopter model.**

I. INTRODUCTION

The problem of letting the output of a system track a given reference trajectory, while at the same time reject unwanted disturbances occupies a central role in control theory. In a large number of control applications involving the control of rotating mechanisms or electro-mechanical systems, the disturbances to be rejected take the form of harmonic or periodic signals (see, among others, [1], [3], [5], [9]). In general, the harmonic disturbance may not be assumed to be known or measurable directly. It is therefore important to consider explicitly the situation in which the magnitude, phase, and even the frequency of the single harmonic components of the disturbance are unknown, and only suitable bounds for these quantities are available to the control system designer. In the case in which the harmonic disturbance are matched with the control input of the plant, and this latter is described by a linear model, there are two widely used, and quite effective, methodologies to deal with asymptotic disturbance rejection. The first technique, commonly known as "adaptive feedforward control" [2], [14], bases its effectiveness on a feedforward control action that aims at cancelling the disturbance directly at its insertion point in a stabilized loop. Since this is generally accomplished by means of a recursive gradient optimization, the applicability of this method is limited to specific classes of systems that satisfy positive realness-like conditions. A more general technique is based on the so-called "internal model principle", which states that the controller must embed a proxy of the mechanism that generates the disturbance in order to be able to offset it asymptotically. A general solution to the problem of designing internal-model based controllers for linear systems has been known since the seminal works of Davison [4] and Francis and Wonham [7], [6], and the pioneering work of Isidori and Byrnes [12] for the case of nonlinear systems. In particular, the application of internal-model based control to nonlinear systems has been the subject of intense research efforts throughout the last ten years. Despite major theoretical advancement and many successful applications (see the references in [13]), there are still several limitations encountered in applying internal-model based techniques to the control of nonlinear systems. First, it is generally not possible to design a model-based controller in a systematic way (using well established techniques such as feedback linearization or passivity) and then account for the presence of the internal model of the disturbance in a second stage of the design. Secondly, internal-model based controllers are most effective in rejecting disturbances occurring at the plant input, but are of limited applicability if the disturbance acts at the level of the plant sensor. A problem of this sort arises in applications where unwanted mechanical vibrations or electromagnetic disturbances may be transmitted to the feedback loop via the physical interaction between the sensors and the environment. A systematic way of designing controllers to let the output of a nonlinear system track a reference trajectory, while asymptotically rejecting harmonic disturbances acting at the input and the sensor channels is not yet available.

In this paper, we take a step in this direction, providing a solution that is applicable to the significant class of nonlinear systems whose dynamics is described by Euler-Lagrange equations. The path we take is to replace the standard internal model with a device that is placed outside the stabilizing feedback loop and referred to as an *external model* of the disturbance, borrowing the terminology from [18]. The task of the external model is merely that of providing a converging estimate of the disturbances that is in turn used to offset the error. The challenge is to prevent the external model from interfering with the stability of the loop. The external model can be endowed with an adaptation mechanism (similar to the one adopted in [15] and [16]) to deal with uncertainties on the frequency of the exogenous disturbances. The proposed methodology is applied to a 2-DOF helicopter model, and validated by experimental results.

The authors are with the Department of Electrical and Computer Engineering,The Ohio State University, Columbus, OH.
*Corresponding author. Email: serrani@ece.osu.edu

0-7803-9098-9/05/$25.00 ©2005 AACC

II. Problem formulation

The model of fully-actuated Euler-Lagrange systems we consider in this paper is given by

$$M(q)\ddot{q} + C(q,\dot{q})\dot{q} + g(q) = \tau \tag{1}$$

where $q \in \mathbb{R}^n$ is the vector of Lagrangian coordinates, $\tau \in \mathbb{R}^n$ is the vector of control torques, $M(q) \in \mathbb{R}^{n\times n}$ is the inertia matrix, $C(q,\dot{q}) \in \mathbb{R}^{n\times n}$ is the matrix of centripetal and Coriolis forces and $g(q) \in \mathbb{R}^n$ is the vector of gravitational torque. The model satisfies the following standard properties:

Property 2.1: The inertia matrix $M(q)$ is positive definite, and satisfies $m_1 I \leq M(q) \leq m_2 I$, for some $m_1, m_2 > 0$ and all $q \in \mathbb{R}^n$.

Property 2.2: The matrix $C(q,\dot{q})$ satisfies $C(q, v+w) = C(q,v) + C(q,w)$, for all q, v, and w in $\mathbb{R}^n$. Also, there exists $c > 0$ such that $\|C(q,w)\| \leq c\|w\|$ for all $q, w \in \mathbb{R}^n$.

Letting $x_1 = q$ and $x_2 = \dot{q}$, we write equation (1) in state-space form

$$\begin{array}{rcl} \dot{x}_1 &=& x_2 \\ M(x_1)\dot{x}_2 &=& -C(x_1,x_2)x_2 - g(x_1) + \tau\,. \end{array} \tag{2}$$

Since the main objective is to track a given reference trajectory $y_r(t)$, we first design a controller that achieves this goal in absence of external disturbances. Defining $e_1 = x_1 - y_r$ and $e_2 = x_2 - \dot{y}_r$, we select the control $\tau = \tau_{\text{fl}} + \tau_{\text{st}}$, where τ_{fl} is a feedback linearization control given by

$$\tau_{\text{fl}} = C(x_1,x_2)x_2 + g(x_1)\,, \tag{3}$$

and τ_{st} is a linear tracking controller given by

$$\tau_{\text{st}} = M(x_1)[\ddot{y}_r - K_1 e_1 - K_2 e_2]\,, \tag{4}$$

where K_1 and K_2 are positive definite gain matrices. We assume that the signals $(y_r, \dot{y}_r, \ddot{y}_r)$ are continuous and bounded. We consider the situation in which the plant input τ is corrupted by an additive harmonic disturbance d_{i}, and the velocity sensors are corrupted by another additive harmonic disturbance d_{s}, such that the available input to the controller becomes $\tau_a = \tau + d_{\text{i}}$, and the available velocity measurements become $x_{2,a} = x_2 + d_{\text{s}}$. Each element of $d_{\text{i}} \in \mathbb{R}^n$ and $d_{\text{s}} \in \mathbb{R}^n$ is given by a finite sum of sinusoidal signals of unknown amplitude, frequency and phase, that is, $d_{\text{i},i}(t) = \sum_{k=1}^{p_i} a_{ik}\sin(\sigma_{ik}t + \phi_{ik})$, and $d_{\text{s},i}(t) = \sum_{k=1}^{p_i} b_{ik}\sin(\sigma_{ik}t + \psi_{ik})$ for $i = 1,..,n$. For the sake of simplicity, we assume that each component of d_{i} and d_{s} contains harmonics of the same frequency but different amplitude and phase. For this reason, d_{i} and d_{s} can be represented as outputs of the state space systems

$$\begin{array}{rcl} \dot{w}_{\text{i}} &=& S w_{\text{i}} \\ d_{\text{i}} &=& \Gamma w_{\text{i}}\,, \end{array} \tag{5}$$

$$\begin{array}{rcl} \dot{w}_{\text{s}} &=& S w_{\text{s}} \\ d_{\text{s}} &=& \Gamma w_{\text{s}} \end{array} \tag{6}$$

with states $w_{\text{i}}, w_{\text{s}} \in \mathbb{R}^{2p}$, where $p = \sum_{i=1}^n p_i$, and matrices $S \in \mathbb{R}^{2p\times 2p}$, $\Gamma \in \mathbb{R}^{n\times 2p}$. Note that there is no loss of generality in assuming that the pair (S,Γ) is observable. The matrix S depends on a vector $\sigma \in \mathbb{R}^p$ of unknown parameters playing the role of the unknown *distinct* frequencies of the harmonic components of the disturbances, while the initial conditions $w_{\text{i}}(0)$ and $w_{\text{s}}(0)$ contain all the information about the unknown amplitudes a_{ik}, b_{ik} and phases ϕ_{ik}, ψ_{ik}. We make the following mild assumptions on the exosystem model:

Assumption 2.1: The vector σ ranges on a given compact set $\Sigma \subset \mathbb{R}^p$.

Assumption 2.2: The initial conditions of (5) and (6) range over a compact and invariant set $W \subset \mathbb{R}^{2p}$.

For reasons that will become clear later, we look for a more convenient realization of (5) and (6). Following [15], let $F \in \mathbb{R}^{2p\times 2p}$ and $G \in \mathbb{R}^{2p\times 1}$ be chosen arbitrarily in such a way that the pair (F,G) is controllable, and F is Hurwitz. The unique nonsingular solution $M_\sigma \in \mathbb{R}^{2p\times 2p}$ of the Sylvester equation $M_\sigma S - F M_\sigma = G\Gamma$ defines a parameter-dependent change of coordinates $\bar{w}_{\text{i}} = M_\sigma w_{\text{i}}$ and $\bar{w}_{\text{s}} = M_\sigma w_{\text{s}}$, which yields the canonical parametrization of the exosystem for the plant input disturbance

$$\begin{array}{rcl} \dot{\bar{w}}_{\text{i}} &=& (F + G\Psi_\sigma)\bar{w}_{\text{i}} \\ d_{\text{i}} &=& \Psi_\sigma \bar{w}_{\text{i}}\,, \end{array} \tag{7}$$

and for the velocity sensor disturbance

$$\begin{array}{rcl} \dot{\bar{w}}_{\text{s}} &=& (F + G\Psi_\sigma)\bar{w}_{\text{s}} \\ d_{\text{s}} &=& \Psi_\sigma \bar{w}_{\text{s}}\,, \end{array} \tag{8}$$

where $\Psi_\sigma = \Gamma M_\sigma^{-1}$. The canonical realization of the exosystem will be especially useful when designing an adaptive law for estimating the unknown disturbance frequencies, since all the relevant information is embedded in the matrix Ψ_σ.

III. Disturbance at the plant input

We first consider a simpler case in which the plant input is corrupted by the disturbance d_{i}, under the assumption that the vector σ is known, and that the velocity measurements are not corrupted, i.e., $x_{2,a} = x_2$. Applying the modified control

$$\tau = \tau_{\text{fl}} + \tau_{\text{st}} + \tau_d\,, \tag{9}$$

where τ_{fl} and τ_{st} are given by (3) and (4) respectively, and τ_d is an additional term to be defined, we write (2) as

$$\begin{array}{rcl} \dot{x}_1 &=& x_2 \\ M(x_1)\dot{x}_2 &=& \tau_{\text{st}} + \tau_d + \Psi_\sigma \bar{w}_{\text{i}}\,. \end{array} \tag{10}$$

Then, to reject d_{i} we employ an estimate generated by the following external model of (7)

$$\begin{array}{rcl} \dot{\xi}_{\text{i}} &=& (F + G\Psi_\sigma)\xi_{\text{i}} + N_{\text{i}}(x_1, x_2, \zeta_{\text{i}}) \\ \dot{\zeta}_{\text{i}} &=& -\mu(\zeta_{\text{i}} - M(x_1)x_2) + \dot{M}(x_1)x_2 \\ && +\tau_{\text{st}} + \tau_d + \Psi_\sigma \xi_{\text{i}} \\ \hat{d}_{\text{i}} &=& \Psi_\sigma \xi_{\text{i}} \end{array} \tag{11}$$

where $\xi_{\text{i}} \in \mathbb{R}^{2p}$ is the state of a copy of (7), $\zeta_{\text{i}} \in \mathbb{R}^n$ is the state of an observer for $M(x_1)x_2$, $\mu > 0$ is a design parameter and $N_{\rm i}(x_1, x_2, \zeta_{\rm i})$ is an interconnection term to be determined. Defining $\tilde{\zeta}_{\rm i} = \zeta_{\rm i} - M(x_1)x_2$ and changing coordinates as $\chi_{\rm i} = \xi_{\rm i} - \bar{w}_{\rm i} - G\tilde{\zeta}_{\rm i}$, we obtain

$$\dot{\chi}_{\rm i} = F\chi_{\rm i} + FG\tilde{\zeta}_{\rm i} + \mu G\tilde{\zeta}_{\rm i} + N_{\rm i}(x_1, x_2, \zeta_{\rm i}),$$

which suggests the choice

$$N_{\rm i}(x_1, x_2, \zeta_{\rm i}) = -(FG + \mu G)\tilde{\zeta}_{\rm i}.$$

In the new coordinates, system (11) reads as

$$\begin{array}{rcl} \dot{\chi}_{\rm i} & = & F\chi_{\rm i} \\ \dot{\tilde{\zeta}}_{\rm i} & = & -(\mu I - \Psi_\sigma G)\tilde{\zeta}_{\rm i} + \Psi_\sigma \chi_{\rm i} \\ \hat{d}_{\rm i} & = & \Psi_\sigma \xi_{\rm i}. \end{array} \tag{12}$$

Since F is Hurwitz and Ψ_σ is bounded as a function of σ, selecting μ large enough renders the origin of (12) globally exponentially stable. Finally, we select the control τ_d as $\tau_d = -\hat{d}_{\rm i}$, and write system (10) in the error coordinates (e_1, e_2) as

$$\begin{array}{rcl} \dot{e}_1 & = & e_2 \\ \dot{e}_2 & = & -K_1 e_1 - K_2 e_2 - M^{-1}(x_1)[\Psi_\sigma \chi_{\rm i} + \Psi_\sigma G\tilde{\zeta}_{\rm i}]. \end{array}$$

Since the matrix $M^{-1}(x_1)$ is bounded, and $(\chi_{\rm i}(t), \tilde{\zeta}_{\rm i}(t))$ converges to zero exponentially, the tracking error $e(t) = \mathrm{col}(e_1(t), e_2(t))$vanishes exponentially as $t \to \infty$.

IV. Disturbance at the output sensor

The next step is to deal with the presence of disturbances in the available measurements of x_2, and with uncertainty on the value of σ. The idea is to replace the input x_2 and the term Ψ_σ in (11) with suitable converging estimates $\hat{x}_2$ and $\hat{\Psi}$, to be provided by an adaptive version of an external model of (8). The suggested form of the sensor external model is similar to (11), and given by

$$\begin{array}{rcl} \dot{\xi}_{\rm s} & = & (F + G\hat{\Psi})\xi_{\rm s} + N_{\rm s}(x_1, \zeta_{\rm s}) \\ \dot{\zeta}_{\rm s} & = & -\lambda(\zeta_{\rm s} - x_1) + x_{2,a} - \hat{\Psi}\xi_{\rm s} \\ \dot{\hat{\Psi}} & = & \phi(x_1, \zeta_{\rm s}, \xi_{\rm s}) \\ \hat{d}_{\rm s} & = & \hat{\Psi}\xi_{\rm s}, \end{array} \tag{13}$$

where $\xi_{\rm s} \in \mathbb{R}^{2p}$, $\zeta_{\rm s} \in \mathbb{R}^n$ is the state of a linear observer for x_1, and $\lambda > 0$ is a design parameter. Following [16], since the actual value of σ is not known, the term Ψ_σ has been replaced by an estimate $\hat{\Psi} \in \mathbb{R}^{n \times 2p}$ governed by an update law $\phi(x_1, \zeta_{\rm s}, \xi_{\rm s})$ to be determined. The output $\hat{d}_{\rm s}$ of (13) is an estimate of $d_{\rm s}$, and thus the required estimate of x_2 is readily obtained as $\hat{x}_2 = x_{2,a} - \hat{d}_{\rm s}$. To determine the update law, we first reparameterize $\hat{\Psi}$ as $\hat{\theta} = \mathrm{col}(\hat{\Psi}_1^T, \hat{\Psi}_2^T, \ldots, \hat{\Psi}_n^T) \in \mathbb{R}^{2p}$, and look for an adaptive law of the form $\dot{\hat{\theta}} = \phi(x_1, \zeta_{\rm s}, \xi_{\rm s})$. Defining $\tilde{\Psi} = \hat{\Psi} - \Psi_\sigma$, we let $\tilde{\theta}$ be the reparameterization of $\tilde{\Psi}$, and the matrix $\psi(\xi_{\rm s})$ be the regressor for $\tilde{\theta}$ such that $\tilde{\Psi}\xi_{\rm s} = \psi(\xi_{\rm s})\tilde{\theta}$. Finally, changing coordinates with $\chi_{\rm s} = \xi_{\rm s} - \bar{w}_{\rm s} - G\tilde{\zeta}_{\rm s}$ where $\tilde{\zeta}_{\rm s} = x_1 - \zeta_{\rm s}$, and choosing $N_{\rm s}(x_1, \zeta_{\rm s}) = -(FG + \lambda G)\tilde{\zeta}_{\rm s}$, system (13) in the new coordinates reads as

$$\begin{array}{rcl} \dot{\chi}_{\rm s} & = & F\chi_{\rm s} \\ \dot{\tilde{\zeta}}_{\rm s} & = & -(\lambda I - \Psi_\sigma G)\tilde{\zeta}_{\rm s} + \Psi_\sigma \chi_{\rm s} + \psi(\xi_{\rm s})\tilde{\theta} \\ \dot{\hat{\theta}} & = & \phi(x_1, \zeta_{\rm s}, \xi_{\rm s}). \end{array} \tag{14}$$

The Lyapunov function candidate

$$V(\chi_{\rm s}, \tilde{\zeta}_{\rm s}, \tilde{\theta}) = \chi_{\rm s}^T P \chi_{\rm s} + \frac{1}{2}\tilde{\zeta}_{\rm s}^T \tilde{\zeta}_{\rm s} + \frac{1}{2\gamma}\tilde{\theta}^T \tilde{\theta}, \tag{15}$$

where $P = P^T > 0$ solves the matrix Lyapunov equation $F^T P + PF = -I$ and $\gamma > 0$ is a design parameter, yields the choice for the update law as

$$\dot{\hat{\theta}} = -\gamma \psi^T(\xi_{\rm s})\tilde{\zeta}_{\rm s}.$$

Next, defining the vector $z = \mathrm{col}(\chi_{\rm s}, \tilde{\zeta}_{\rm s})$ and the matrices

$$A(\lambda) = \begin{pmatrix} F & 0 \\ \Psi_\sigma & -\lambda I + \Psi_\sigma G \end{pmatrix}, \quad B = \begin{pmatrix} 0 \\ I_{n \times n} \end{pmatrix},$$

$$C^T = \begin{pmatrix} 0 & I_{n \times n} \end{pmatrix},$$

we write the system (14) as

$$\begin{array}{rcl} \dot{z} & = & A(\lambda)z + B\psi(\xi_{\rm s})\tilde{\theta} \\ \dot{\tilde{\theta}} & = & -\gamma \psi^T(\xi_{\rm s})Cz, \end{array} \tag{16}$$

and claim the following:

Proposition 4.1: There exists $\lambda^\star > 0$ such that for any $\lambda \geq \lambda^\star$ the triplet $\{A(\lambda), B, C\}$ is strictly positive real for all $\sigma \in \Sigma$, with storage function given by (15). □

The proof is straightforward, and it is therefore omitted. Standard arguments [11] show that $(z(t), \tilde{\theta}(t))$ is bounded, and $z(t)$ vanishes asymptotically. Moreover, if the signal $\xi_{\rm s}(t)$ is persistently exciting (PE), then $\tilde{\theta}$ vanishes as well, which means that the frequency estimates converge to their true values. Since in the sequel an estimate of both $d_{\rm s}$ and Ψ_σ will be employed in (11), the PE condition cannot be easily relaxed. However, it is easy to prove the following:

Proposition 4.2: Assume that the initial condition $\bar{w}_{\rm s}(0)$ of the exosystem (8) excites all modes of $(F + G\Psi_\sigma)$. Then, $\xi_{\rm s}(t)$ in (16) is PE. □

The result follows from [11, Lemma 4.8.3], as z is easily shown to be in $\mathcal{L}_2$ (see also [16]). Therefore, if the sensor disturbance contains all the modes of the exosystem, the external model (13) provides converging estimates $\hat{d}_{\rm s}$ and $\hat{\Psi}$ of $d_{\rm s}$ and Ψ_σ, respectively.

V. Input and sensor disturbance combined

In this section, we turn our attention to the case in which the controller given by (3), (4), and (11) operates under the presence of the input disturbance $d_{\rm i}$, the disturbance $d_{\rm s}$ in the measurements of x_2, and uncertainty on Ψ_σ. We assume that the PE condition formulated in Proposition 4.2 is verified. It is worth noticing that connecting an adaptive external model of the form (13) to the plant (2) in closed loop with the dynamic controller (9)-(11), one

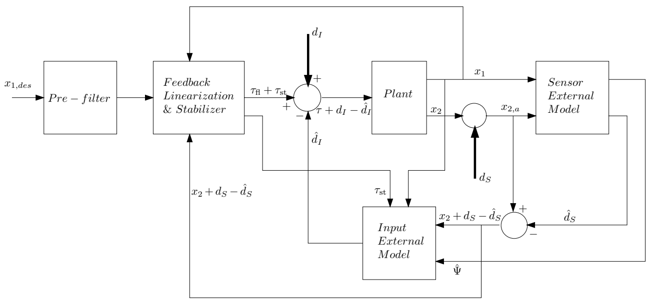

Fig. 1. Interconnection of two external models

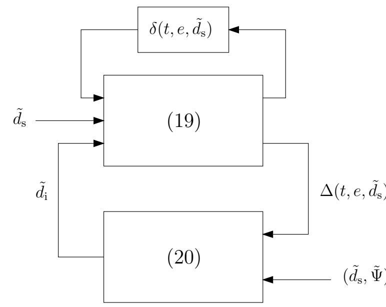

Fig. 2. Loop interconnection of systems (18) and (19)

obtains converging estimates of Ψ_σ and d_s regardless of the presence of the control laws, as long as the trajectories of the closed loop system are defined for all $t \geq 0$. Therefore, a natural strategy would be that of replacing x_2 and Ψ_σ in the controller with the estimates provided by (13), in such a way that boundedness of all trajectories is guaranteed. Figure 1 shows the resulting block diagram of the interconnection between the two external models and the main controller. The certainty-equivalence versions of the feedback linearization and the tracking controllers (3) and (4) are given by

$$\begin{aligned}\tau_{fl} &= g(x_1) + C(x_1,\hat{x}_2)\hat{x}_2 \\ \tau_{st} &= M(x_1)[\ddot{y}_R - K_1 e_1 - K_2(\hat{x}_2 - \dot{y}_r)]\,,\end{aligned}$$

where $\hat{x}_2 = x_{a,2} - \hat{d}_s$, being $\hat{d}_s$ the output of (13). The control τ_d is again given by $\tau_d = -\hat{d}_i$, where $\hat{d}_i$ is generated this time by the certainty-equivalence version of (11)

$$\begin{aligned}\dot{\xi}_i &= (F + G\hat{\Psi})\xi_i + N_i(x_1,\hat{x}_2,\zeta_i) \\ \dot{\zeta}_i &= -\mu(\zeta_i - M(x_1)\hat{x}_2) + \dot{\hat{M}}(x_1)\hat{x}_2 \\ &\qquad + \tau_{st} + \tau_d + \hat{\Psi}\xi_i \\ \hat{d}_i &= \mathrm{sat}_\ell(\hat{\Psi}\xi_i)\,,\end{aligned} \tag{17}$$

where $\mathrm{sat}_\ell(\cdot)$ is a vector-valued saturation function whose level $\ell > 1$ is chosen large enough to guarantee that

$$\mathrm{sat}_{\ell-1}(\Psi_\sigma \bar{w}_i) = \Psi_\sigma \bar{w}_i$$

for all $\sigma \in \Sigma$ and all $\bar{w}_i \in W$. The presence of the saturation is needed to ensure boundedness of the output of the external model (17), which feeds the certainty-equivalence controller. Finally, the interconnection term $N_i(\cdot)$ is chosen as

$$N_i(x_1,\hat{x}_2,\zeta_i) = -(FG + \mu G)(\zeta_i - M(x_1)\hat{x}_2)\,.$$

Let $\tilde{d}_i = d_i - \hat{d}_i$ and $\tilde{d}_s = d_s - \hat{d}_s$ denote the estimation error for the input and sensor disturbance respectively, and note that $\hat{x}_2 = x_2 + \tilde{d}_s$. After easy manipulations, the closed-loop system can be written as the interconnection of the controlled plant in error coordinates

$$\begin{aligned}\dot{e}_1 &= e_2 \\ \dot{e}_2 &= -K_1 e_1 - K_2 e_2 + M^{-1}(e_1 + y_r)\tilde{d}_i \\ &\qquad + \delta(t,e,\tilde{d}_s)\end{aligned} \tag{18}$$

where

$$\delta(t,e,\tilde{d}_s) = M^{-1}(x_1)[C(x_1,\hat{x}_2)\hat{x}_2 - C(x_1,x_2)x_2]\,,$$

and the certainty-equivalence external model in the $(\chi_i,\tilde{\zeta}_i)$-coordinates (see Section III) driven by the exosystem (7), that is, the system

$$\begin{aligned}\dot{\bar{w}}_i &= (F + G\Psi_\sigma)\bar{w}_i \\ \dot{\chi}_i &= F\chi_i + \Delta_1(t,e,\tilde{d}_s) \\ \dot{\tilde{\zeta}}_i &= -(\mu I - \Psi_\sigma G)\tilde{\zeta}_i + \Psi_\sigma\chi_i + \tilde{\Psi}\xi_i + \Delta_2(t,e,\tilde{d}_s) \\ \tilde{d}_i &= \Psi_\sigma\bar{w}_i - \mathrm{sat}_\ell\left(\Psi_\sigma\bar{w}_i + \Psi_\sigma[\chi_i + G\tilde{\zeta}_i] + \tilde{\Psi}\xi_i\right)\,,\end{aligned} \tag{19}$$

where

$$\Delta_1(t,e,\tilde{d}_s) = (FG + \mu G)M(x_1)\tilde{d}_s - G\Delta_2(t,e,\tilde{d}_s)$$

$$\begin{aligned}\Delta_2(t,e,\tilde{d}_s) = \;& \mu M(x_1)\tilde{d}_s + \dot{\hat{M}}(x_1)\hat{x}_2 - \dot{M}(x_1)x_2 \\ & -M(x_1)\delta(t,e,\tilde{d}_s)\,.\end{aligned}$$

In the above interconnection, depicted in Fig. 2, the term $\delta(\cdot)$ plays the role of a perturbation of the error system dynamics, while the terms $\Delta(\cdot) = \mathrm{col}(\Delta_1,\Delta_2)$ and $\tilde{d}_i$ represents the coupling between the two systems. The time-varying nature of $\delta(\cdot)$ and $\Delta(\cdot)$ is due to their dependence on the reference signal y_r and its first derivative. In this setup, the signals $\tilde{d}_s$ and $\tilde{\Psi}$ are regarded as exogenous inputs generated by the autonomous system (14). The main result is summarized in the following:

Proposition 5.1: Assume that the condition in Proposition 4.2 holds. Then, for any initial condition $\bar{w}_i(0) \in W$ and any initial condition $(e(0),\chi_i(0),\tilde{\zeta}_i(0)) \in \mathbb{R}^{2n+2p+n}$, the corresponding trajectory of the closed-loop system is defined for all $t \geq 0$ and satisfies

$$\lim_{t\to\infty}(e(t),\chi_i(t),\tilde{\zeta}_i(t)) = 0\,.$$

Proof: First, we show that the error dynamics (18) has the bounded-input/bouded-state property (BIBS) with respect to $\tilde{d}_i$, and the converging-input/bounded-state property (CIBS) with respect to $\tilde{d}_s$ (see [17] for more details on the terminology). The first property follows trivially by the fact that the unperturbed system is globally exponentially stable. As far as the second property is concerned, note that since the reference trajectory is bounded together with its first derivative, by virtue of Properties 2.1 and 2.2 the perturbation $\delta(\cdot)$ satisfies

$$\|\delta(t, e, \tilde{d}_{\mathrm{s}})\| \leq (a_0 + a_1\|e\|)\|\tilde{d}_{\mathrm{s}}\| \tag{20}$$

for some positive constants a_0, a_1. This, together with the global exponential stability of the unperturbed dynamics, implies the CIBS property with respect to $\tilde{d}_{\mathrm{s}}$. Moreover, as the norm of $\tilde{d}_{\mathrm{s}}$ can be bounded from above by a number which depends only on the initial conditions of (16), the inequality (20) also shows that the system (18) is globally Lipschitz, uniformly in t, and thus the trajectory $e(t)$ is defined for all $t \geq 0$. This result and Proposition 4.2 ensure that indeed $\tilde{d}_{\mathrm{s}}(t)$ and $\tilde{\Psi}(t)$ vanish asymptotically. Since the presence of the saturation function implies that $\tilde{d}_{\mathrm{i}}$ is a bounded signal, it follows that the trajectory $e(t)$ of (18) is bounded. Now, consider the system (19), and note that $\Delta(t, e, 0) = 0$ for all t and all $e \in \mathbb{R}^{2n}$. Since we have already shown that the solution $e(t)$ of (18) is bounded, it follows that the interconnection $\Delta(\cdot)$ satisfies $\lim_{t\to\infty} \Delta(t, e(t), \tilde{d}_{\mathrm{s}}(t)) = 0$. Since the unforced dynamics is globally exponentially stable, this implies convergence of the trajectory $(\chi_{\mathrm{i}}(t), \tilde{\zeta}_{\mathrm{i}}(t))$ to the origin. As a consequence, $\tilde{d}_{\mathrm{i}}(t)$ vanishes asymptotically, and so does $e(t)$. ■

VI. EXPERIMENTAL RESULTS

In this section, we provide experimental results to show the effectiveness of the proposed methodology when applied to real-time control systems. The experiments have been performed using the Quanser® 2-DOF helicopter [10] experimental setup. The helicopter model is equipped with two DC motors actuating two propellers, directed in such a way that the front (main) propeller produces a pitch motion, and the tail propeller a yaw motion. The inputs of the plant are the voltages $u = (u_{\mathrm{main}}, u_{\mathrm{tail}})^T$ applied to the two motors, while the pitch angle q_1 and the yaw angle q_2 are measured by means of optical encoders. The pitch and yaw rates are obtained using high-pass filtering of the respective encoder readings. Letting $q = (q_1, q_2)^T$, a mathematical model in the form (1) of the 2-DOF helicopter has been derived neglecting the propeller dynamics. A static relationship of the form $\tau = B(q)u$, with $B(q) \in \mathbb{R}^{2\times 2}$ nonsingular for all q, has been assumed to model the relationship between voltages applied to the motors and torques produced by the propellers. The system matrices in (1) are given by

$$M(q) = \begin{pmatrix} \vartheta_1 & 0 \\ 0 & \vartheta_2 + \vartheta_3 \cos^2 q_1 \end{pmatrix},$$

$$C(q, \dot{q}) = \begin{pmatrix} 0 & \vartheta_4 \sin(2q_1)\dot{q}_2 \\ -\vartheta_4 \sin(2q_1)\dot{q}_2 & -\vartheta_4 \sin(2q_1)\dot{q}_1 \end{pmatrix},$$

$$B(q) = \begin{pmatrix} -\vartheta_7 & \vartheta_8 \\ -\vartheta_9 \cos q_1 & -\vartheta_{10} \cos q_1 \end{pmatrix},$$

while the gravitational term reads as

$$g(q) = \begin{pmatrix} -\vartheta_5 \cos q_1 + \vartheta_6 \sin q_1 \\ 0 \end{pmatrix}.$$

The model parameters have been obtained experimentally by means of the energy-based system identification method of [8]. Letting $x_1 = q$ and $x_2 = \dot{q}$, where $x_1, x_2 \in \mathbb{R}^2$, the baseline tracking control becomes

$$u = B^{-1}(x_1)[C(x_1, x_2)x_2 + g(x_1) + \tau_{\mathrm{st}}] \tag{21}$$

where τ_{st} is the tracking control

$$\tau_{\mathrm{st}} = M(x_1)[\ddot{y}_r - K_1 e_1 - K_2 e_2 - K_3 \int e_1(s)\mathrm{d}s] \tag{22}$$

where the integral term has been added to remove steady state errors due to parameter mismatch (most noticeably, the effect of the residual gravitational forces). The controller gains, given by $K_1 = \mathrm{diag}(16.52, 20.93)$, $K_2 = \mathrm{diag}(11.53, 11.91)$ and $K_3 = \mathrm{diag}(3.16, 10)$, have been obtained by means of standard LQR design. The reference signal and its derivatives $(y_r, \dot{y}_r, \ddot{y}_r)$ have been obtained filtering a stepwise input command y_R with a second order linear reference model with damping ratio $\zeta = 1$ and natural frequency $\omega_n = 0.1$ rad/s. After the performance of the baseline controller have been successfully evaluated as the upcoming figures will show, in preparation for the disturbance rejection experiment, the controller has been augmented with the input and sensor external models (11) and (13). Then sinusoidal disturbances have been added to the plant input and to the measurement of the yaw and pitch rates. The disturbance signals have been initially selected as

$$d_{\mathrm{i}}(t) = \mathrm{col}(0.5\sin(t + \pi/6),\ 0.5\sin(2t))$$

and

$$d_{\mathrm{s}}(t) = \mathrm{col}(\sin(t),\ \sin(2t - \pi/6)),$$

respectively. The matrix F has been chosen in such a way that $\mathrm{spec}(F) = \{-1, -1, -2, -2\}$, while the scalar gains have been selected as $\lambda = 12$, $\mu = 6$, and $\gamma = 100$. The initial condition of the frequency estimates vector $\hat{\Psi} \in \mathbb{R}^4$ has been chosen at the origin, $\hat{\Psi}(0) = 0$. In this way, no a priori knowledge on the disturbance frequencies is assumed.

In the first experiment, the external models are disabled at the beginning of the experiments, and no disturbance is initially applied. Then, at time $t = 40$ s both disturbances are applied, while the external models become active at $t = 65$ s, switched off at time $t = 120$ s, and engaged again at time $t = 150$ s. The results in Figure 3, portraying the time history of the pitch and yaw angles and depicting the tracking error, show the effectiveness of the baseline controller and the external model-based disturbance rejection mechanism. The tracking error vanishes whenever the external model is enabled. Remarkably, the controller achieves asymptotic regulation of the tracking error in spite not only of uncertainties on the external disturbance, but

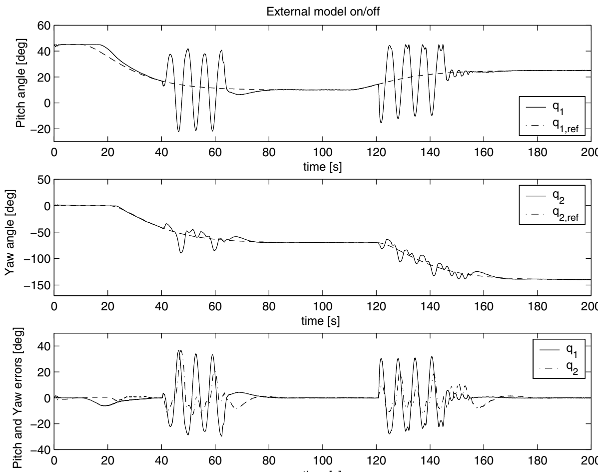

Fig. 3. Pitch and yaw tracking and tracking error. External models and disturbance active

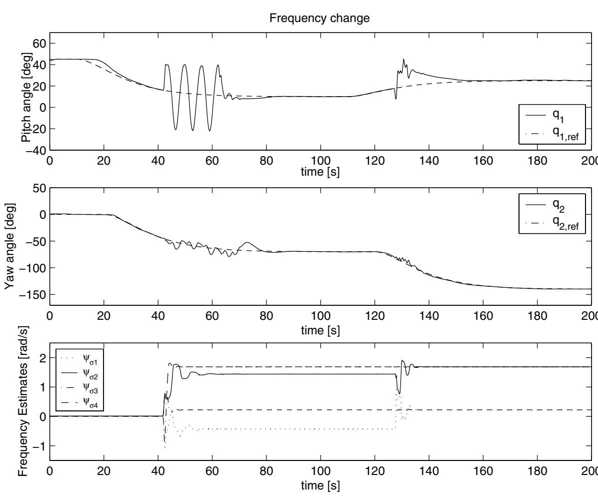

Fig. 4. Pitch and yaw tracking and frequency estimates $\hat{\Psi}(t)$. Change in the pitch disturbance frequency

also in spite of the unavoidable uncertainties affecting the plant model. To test the robustness and the self-tuning capability of the external model, experiments have been conducted in which the frequency, amplitude and phase of the harmonic disturbances have been changed during closed-loop operations. Figure 4 shows the response of the pitch and yaw angles for the same experimental setup as before, but this time, instead of switching the external model off at time $t = 130\,\mathrm{s}$, the frequency of the pitch disturbance (in both the input and the sensor channels) is changed from 1 to $2\,\mathrm{rad/s}$. It can be noticed that the change of frequency produces a vanishing transient behavior, and tracking is fully recovered. The third plot in the same figure shows the convergence of the corresponding frequency estimates $\hat{\Psi}(t)$.

VII. Conclusions

In this paper, we have proposed an external model-based approach to the problem of rejecting unknown harmonic disturbances while tracking arbitrary reference trajectories for Euler-Lagrange systems. An important feature of the methodology consists in the fact that the design of the device that aims at canceling the disturbance is performed independently of that of the stabilizing (tracking) unit, in such a way that well-established techniques can be employed for the latter. The external model provides simultaneous rejection of harmonic disturbances occurring at both the input and the sensor channels, and accounts for uncertainties in the frequencies of the harmonic components of the exogenous signals. Experimental results on a 2-DOF helicopter model have been provided to validate the theoretical findings.

References

[1] K.B. Ariyur and M. Krstic. Feedback attenuation and adaptive cancellation of blade vortex interaction on a helicopter blade element. *IEEE Transactions on Control Systems Technology*, 7(5):596–605, 1999.

[2] M. Bodson and S.C. Douglas. Adaptive algorithms for the rejection of sinusoidal disturbances with unknown frequency. *Automatica*, 33(12):2213–21, 1997.

[3] M. Bodson, A. Sacks, and P. Khosla. Experimental results of adaptive periodic disturbance cancellation in a high performance magnetic disk drive. *Transactions of the ASME. Journal of Dynamic Systems, Measurement and Control*, 118(3):416–24, 1996.

[4] E. J. Davison. The robust control of a servomechanism problem for linear time-invariant multivariable systems. *IEEE Transactions on Automatic Control*, AC-21:25–34, Feb. 1976.

[5] C.C. de Wit and L. Praly. Adaptive eccentricity compensation. *IEEE Transactions on Control Systems Technology*, 8(5):757–66, 2000.

[6] B. A. Francis. The linear multivariable regulator problem. *SIAM Journal of Control and Optimization*, 14:486–505, 1977.

[7] B. A. Francis and W. M. Wonham. The internal model principle of control theory. *Automatica*, 12:457–465, 1976.

[8] M. Gautier and W. Khalil. On the identification of the inertial parameters of robots. *Proceedings of the 27th IEEE Conference on Decision and Control*, pages 2264–2269, 1988.

[9] G. Hillerstrom. Adaptive suppression of vibrations - a repetitive control approach. *IEEE Transactions on Control Systems Technology*, 4(1):72–78, 1996.

[10] Quanser Inc. World Wide Web, `http://www.quanser.com`.

[11] P. Ioannou and J. Sun. *Robust adaptive control*. Prentice Hall, Upper saddle river, NJ, 1996.

[12] A. Isidori and C. I. Byrnes. Output regulation of nonlinear systems. *IEEE Transactions on Automatic Control*, AC-35:131–140, 1990.

[13] A. Isidori, L. Marconi, and A. Serrani. *Robust Autonomous Guidance: An Internal Model-Based Approach*. Advances in Industrial Control. Springer Verlag, London, UK, 2003.

[14] W. Messner and M. Bodson. Design of adaptive feedforward algorithms using internal model equivalence. *International Journal of Adaptive Control and Signal Processing*, 9(2):199–212, 1995.

[15] V.O. Nikiforov. Adaptive non-linear tracking with complete compensation of unknown disturbance. *European Journal of Control*, 4:132–139, 1998.

[16] A. Serrani, A. Isidori, and L. Marconi. Semiglobal nonlinear output regulation with adaptive internal model. *IEEE Transactions on Automatic Control*, AC-46(8):1178–1194, 2001.

[17] E.D. Sontag. Remarks on stabilization and input-to-state stability. In *Proceedings of the IEEE Conf. on Decision and Control*, pages 1376–1378, Tampa, FL, 1989.

[18] W.-C. Yang and M. Tomizuka. Disturbance rejection through an external model for nonminimum phase systems. *Transactions of the ASME. Journal of Dynamic Systems, Measurement and Control*, 116:39–44, 1994.